The Notion of Twenty-Two Shrutis

Frequency Ratios in Hindustani Classical Music

Dinesh S Thakur

This is an expository article with the intent of briefly explaining the possible logic behind a 22-*shruti* model for the system of musical notes in Hindustani (i.e., North Indian) classical music. It will also be explained how this model is consistent with the *Chatus-saaranaa* (meaning 'four translations') tuning procedure described in the famous manuscript *Natya Shastra* by Bharat Muni, dated somewhere between second century BC to third century AD [1]*.

Dinesh Thakur is currently Professor at the University of Rochester. Number theory is his principal subject of interest. He had the good fortune of growing up in a musical environment under the influence of his uncle P L Deshpande.

1. Introduction

The Sanskrit word, *shruti*, has been used in various ways, even in the context of North Indian classical music, also described as *Raaga Sangeet*. For our purposes, shrutis are musical notes used in Indian *raagas*. (A raaga is a melodic mode or frame. Its description is much more than just a scale, and includes not only allowed lines of ascent, descent, signature phrases, but also much more refined information on how the notes are approached, their relative emphasis, etc. Musicians are supposed to develop and create music, by exploring moods and emotions within the language/grammar of a particular raaga.) Long-forgotten understanding and awareness of this shruti system is evident from ancient texts such as the aforementioned *Natya Shastra*, which mentions that there are 22 shrutis [1]. However, the frequency tables obtained by many scholars [2, pp.129−130 and 3, p.24] in the past are inconsistent with one another. We explain an interpretation which seems robust. Finally, good experimentation has to decide among these models. The final table of the

* We use the quotations from [1],[2]; the latter mentions the date of [1] as the 3rd century latest.

Keywords
22 Shrutis, frequency ratios, Chatus-saaranaa Samvaad, meend, Dorian scale, consonance principle.

Indian classical music and most other music, especially the concept of raagas and *taals* rhythmic modes or metres, is based on the concept of *samvaad*, meaning dialog, accord or intercommunication between the parts of the whole.

shruti frequency ratios that we obtain is not new. The same table is given in other texts with a little different or no explanations [4, 5, 6, 3, 7].

Indian classical music (and most other music), especially the concept of raagas and *taals* (rhythmic modes or metres), is based on the concept of *samvaad*, meaning dialog, accord or intercommunication between the parts of the whole. We explain this logical shrutis model for Hindustani raag-sangeet based on some basic principles of consonance and certain simplifying assumptions. The possible discrepancy between the exact scientific model and the creative musical approach must not be overlooked. Music is not a rigid, fixed science; as listeners, our ears are culture-specific. The perception of what is musical and euphonious might have even evolved over time. Aspects that complicate the matter – alluding strictly to musical appreciation – include psychology, perception in ear, interpretation by brain, and conditioning.

Thus, this description of shrutis just provides a scientific framework and model conceived by the ancients in the face of conflicting requirements. It ultimately must be tested by connoisseurs to assess the musical validity of this approach.

I end this short introduction with a confession, a warning and a request. I am neither a trained performing musician, nor a historian of the development of music, nor a scientist who has performed relevant experiments. I am only reasonably curious about these issues. So be warned that the explanations below are not thorough in any of these respects. The reader should refer to the literature on the subject for more scholarly details. I will be very glad to receive any corrections, comments, and references.

2. Divisions of an Octave in 7, 12, 22 Notes

2.1 *Rough Positional Description*

Musical notes can be identified by their frequencies, and the ratio of relative frequencies determines the relative positioning perceived.

For example, notes that are an octave apart have a 2/1 frequency ratio and are identified by the same letter. The next important consonance (*samvaad*) corresponds to the next low complexity ratio 3/2, determining the *Shadja-Pancham* (first-fifth) dialog (*bhaav, samvaad*).

The 7 so-called *shuddha* (pure) *surs* (musical notes) in an octave are named:

Shadja=Saa=S, Rishabh=Re=R, Gandhaar=Ga=G, Madhyam=Ma=M, Pancham=Pa=P, Dhaivat=Dha=D, Nishaad=Ni=N.

(They roughly correspond to the western major scale or Do Re Mi Fa So La Ti or the indian scale named *bilaawal*. Saa frequency can be anything and is not a fixed frequency such as 'A' or 'C' in western notation).

The frequencies of the upper half octave $PDNS$ are obtained from the lower half octave $SRGM$ frequencies by multiplication with 3/2. The intervals SR, RG, MP, PD, DN are roughly (exactly in equal tempered instruments) in the same ratio and thus, are called whole tone intervals; whereas GM and NS frequency ratios are roughly (exactly in equal tempered instruments) half powers of the ratios above, and thus are called semitone intervals.

Of these seven notes, the first and fifth (Saa, Pa) are considered immovable (*achal*). When we include the *komal* (meaning soft and corresponding roughly to flat versions) R_1, G_1, D_1, N_1 and *teevra* (meaning harsh or sharp) M_2 versions of the rest of the 5 surs (which we now denote by R_2, G_2, M_1, D_2, N_2 to distinguish), we get $2 + 2*5 = 12$ surs, all roughly (exactly in equal tempered

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instruments) a semitone apart. (In equal temperament, k-th sur – starting with Saa corresponding to $k = 0$ – has the ratio $2^{k/12}$ with Saa.) Each of the ten surs X_i (except achal Saa and Pa) gets two versions X_{i1}, X_{i2} with the ratio 81/80 between them, as described below thus giving 2 + 2*10 = 22 shrutis. The ratios between the successive frequencies (if we order them in increasing order) are no longer even approximately the same. The ratios between the two versions of the same (komal or shuddha, say) sur are smaller than those between the different surs.

What we write as $R_1, R_2, R_{11}, R_{12}, R_{21}, R_{22}$ is written as $r, R, r1, r2, R1, R2$ respectively in another common notation (which we will not use), by using capital letters for shuddha and small letters for komal versions.

In practice, in a given raaga, when we want to say R_{21}, we just say R_2 by abuse of notation, and use 12 sur notation to describe a raaga. This is tolerated, because usually a raaga is supposed to use only one of the two shrutis of a given sur. There are exceptions, such as the Lagan-Gandhaar [8], where more accurate notation is really needed. But since the shrutis are prescribed in the context of the *chalan* (meaning ascending–descending pattern rules) of the raaga, more notation can be avoided. After all, most Indian music is not even written, but learned from the *guru* (teacher) in oral tradition!

2.2 *Frequency Ratios*

Let us provide tables followed by the explanation and logic behind the frequency determination.

Relative frequency ratios for the seven shuddha notes (Bilawal scale (*thaat*))

S	R	G	M	P	D	N
1	9/8	5/4	4/3	3/2	27/16	15/8

If we change the scale by calling R the new S (so that the old G becomes the new R with ratio $(5/4)(8/9) = 10/9$

and so on), we get the following seven notes (*Kaafi* scale *thaat*). This roughly corresponds to the Dorian scale in Western music, and there might be a more specific name for this tuning.

S	R	G_1	M	P	D	N_1
1	10/9	32/27	4/3	3/2	5/3	16/9

Note that the ratio between the two shrutis of R in the two tables is $(9/8)(9/10) = 81/80$, which is very close to 1.

We get the following 22 shrutis with the same ratio of 81/80 between the two versions of the 10 surs (other than the 2 achal surs S and P).

Table of frequency ratios of 22 shrutis

S	R_{11}	R_{12}	R_{21}	R_{22}	G_{11}	G_{12}	G_{21}	G_{22}
1	$\frac{256}{243}$	$\frac{16}{15}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{32}{27}$	$\frac{6}{5}$	$\frac{5}{4}$	$\frac{81}{64}$

M_{11}	M_{12}	M_{21}	M_{22}
$\frac{4}{3}$	$\frac{27}{20}$	$\frac{45}{32}$	$\frac{729}{512}$

P	D_{11}	D_{12}	D_{21}	D_{22}	N_{11}	N_{12}	N_{21}	N_{22}
$\frac{3}{2}$	$\frac{128}{81}$	$\frac{8}{5}$	$\frac{5}{3}$	$\frac{27}{16}$	$\frac{16}{9}$	$\frac{9}{5}$	$\frac{15}{8}$	$\frac{243}{128}$

The above table will be referred to in this article a number of times. Note that the upper half octave ($P - N_{22}$) ratios are again 3/2 times the lower half ($S - G_{22}$) ratios.

The ratios between the successive shrutis are:

(i) 81/80 for the two versions X_{i2} and X_{i1} of the same

Since $S \leftrightarrow 1$, $P \leftrightarrow 3/2$ are very powerful samvaads, going down 3/2 from upper S corresponding to 2, we get (shuddha i.e., komal) $M \leftrightarrow 4/3$. By changing P to S, the new P would be R in the upper octave, thus, R corresponds to the ratio (3/2)(3/2)(1/2) = 9/8. If we continue this procedure, it will never end as $(3/2)^k \neq 2^m$, for positive integers k, m.

sur X_i, (ii) 25/24 between the upper shruti X_{12} of the komal version X_1 of the (non-achal) sur X and the lower shruti X_{21} of the shuddha version X_2 of the sur X, and (iii) 256/243 between the highest shruti X_{22} (X, if it is achal) of sur X and the lowest shruti Y_{11} (Y if it is achal) of the next sur Y.

Note that $(81/80)^{10}(256/243)^7(25/24)^5 = 2$.

2.3 *Explanation based on Samvaad/Consonance Principles*

Since $S \leftrightarrow 1$, $P \leftrightarrow 3/2$ are very powerful samvaads, going down 3/2 from upper S corresponding to 2, we get (shuddha i.e., komal) $M \leftrightarrow 4/3$. By changing P to S, the new P would be R in the upper octave, thus, R corresponds to the ratio $(3/2)(3/2)(1/2) = 9/8$. If we continue this procedure, it will never end as $(3/2)^k \neq 2^m$, for positive integers k, m. But, $(3/2)^{12}$ is very close to 2^7. In fact, going up and down (k positive and negative) and choosing among these fractions those with the lowest possible complexities (i.e., with the lowest of the maximum of the numerator and denominator size), namely, using $-5 \leq k \leq 6$, we get twelve shrutis, one for each sur. Then the ratio 81/80 multiplication/division gives 10 more corresponding to the non-achal surs. Whether we go up or down (i.e., multiply or divide) is determined immediately by keeping the equal-tempered note in between.

Let us make this more explicit:
Going up by (3/2)'s successively and adjusting for octaves, we get $S = 1$, $P = 3/2$, $R_{22} = 9/8$, $D_{22} = 27/16$, $G_{22} = 81/64$, $N_{22} = 243/128$, $M_{22} = 729/512$.
On the other hand, going down by 3/2s and adjusting for octaves, we get $M_{11} = 4/3$, $N_{11} = 16/9$, $G_{11} = 32/27$, $D_{11} = 128/81$, $R_{11} = 256/243$.

Had we continued once more, we would have got 1024/729 very close to M_{22} (the ratio being $(3/2)^{12}/2^7$ very close to one) and thus, the more complicated ratio 1024/729 is discarded as a version for M_{22}.

Another way to remember whether to go up or down by 81/80 is as follows. By going up 3/2 sequence, we get 'upper' shrutis, so we go down by 81/80 to get the 'lower' versions, whereas we do the opposite in the other five shrutis obtained by going down by (3/2)'s.

The whole 81/80 business thus comes up, since rather than considering $G_{22} = 81/64$ obtained above as the Gandhaar, most musical traditions prefer the simpler, lower complexity ratio 5/4 as a natural or *swayambhu* (meaning naturally born) Gandhaar for greatest samvaad (with Saa). Note that the best samvaads are obtained by ratios $1/1, 2/1, 3/2, 4/3, 5/4$ i.e. with low complexity. This is explained (see below) by the harmonic consonance principle and the *tanpura* (aka tambora, the most common drone instrument used in Indian clasical music, consisting of 4 strings, two tuned to S, one to lower octave S and one to lower octave P and these are continuously plucked in a certain way, progression and tempo) harmonics distributions as well as the combinatorial tones of the tanpura.

The *tanpura* (aka tambora) is the most common drone instrument used in Indian classical music, consisting of 4 strings, two tuned to *S*, one to lower octave *S* and one to lower octave *P* and these are continuously plucked in a certain way, progression and tempo.

This change to 5/4 is responsible for two versions 9/8 and 10/9 of R which leads to 81/80 as described above.

2.4 *Derivation Based on Bharat's Chatus-saaranaa Tuning Description*

Bharat's *Chatus-saaranaa* (meaning four transpositions or four changes of scales) experiment [1, 9] starts with two veenaas (plucked string instrument somewhat similar to the sitar) tuned to the natural scale (which was the 7 sur Kaafi scale given above, as mentioned in various commentaries). One veena is fixed (achal) throughout, whereas the second one is transposed four times to establish all the shrutis. Bharat describes what happens in detail, but in words rather than in numbers. We quote these descriptions and show that we get the table above.

The first transposition is to lower the P of the veenaa so that the ratio between the achal veenaa (Kaafi) $R = 10/9$ and the new P is $SM = 4/3$, so that new $P = 40/27 = (10/9)(4/3)$, establishing the inter-shruti ratio of the first transposition $y_1 = 81/80 = (27/40)(3/2)$.

The second transposition reduces the $N_1 = 16/9$ (which has now become $(16/9)(80/81)$) to achal veenaa $D = 5/3$, hence the new ratio is $y_2 = (16/9)(80/81)(3/5) = 256/243$.

The third transposition reduces the D (which is now at $(5/3)(80/81)(243/256) = 25/16$) to achal veenaa $P = 3/2$ giving the ratio $y_3 = (25/16)(2/3) = 25/24$.

The final fourth transposition reduces the S (which is now at $2(80/81)(243/256)(24/25) = 9/5$) to achal veenaa $N_1 = 16/9$ giving again the ratio $y_4 = y_1 = (9/5)(9/16) = 81/80$.

Agreeing with Bharat's description, we do get (i) $MP = G_1M = N_1S = y_1y_2y_3y_4$ after 4 transpositions, (ii) $RG_1 = DN_1 = y_1y_2$ after 2 transpositions, and (iii) $SR = PD = y_1y_2y_3$ after 3 transpositions.

(The same table can be derived using these equations and less precise information on the Kaafi scale than we have used here.)

2.5 *Strengths and Weaknesses*

No scale will have all basic samvaads and transposition properties, because products of powers of 3s or 5s cannot be powers of 2s.

We have seen that *no scale will have all basic samvaads and transposition properties*, because products of powers of 3s or 5s can not be powers of 2s. So all fixed, finite scales represent some compromises. This system of shrutis achieves this by dumping the complexities at the *teevra/vikrut* (meaning harsh/distorted) Madhyam M_2, halfway between the two parts of the octave (or near the achal surs by samvaad). The Madhyam shrutis (except for M_{11}) do not have corresponding shrutis in the scale giving 3/2 samvaads of type SP, or downwards 3/4 samvaad of SM type (or even 20/27 samvaad of SM type, except for M_{12} itself), while other shrutis do.

(In [3, pp.73–76, 86], in several places it is wrongly stated that these samvaads between teevra Madhyam shrutis exist.) We leave it to the reader to figure out which other (SG, SM, etc.) samvaads are present in the table. These compromises represent a weakness of this system of specific 22 shrutis. Since Indian classical music does not use different surs simultaneously, and often uses *meends*, which are smooth glidings from one note to another by touching intermediate notes in the raaga, rather than straight jumps between distant surs, even the premise of the model has some weakness.

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2.6 *Approximations and Comparisons*

Most people are more comfortable with approximation, percentages, and addition rather than exact fractions representing multiplication ratios. So we now convert the frequency ratio f into $1200 * \log_2 f$, so that the octave ratio 2 becomes 1200, and each semitone interval becomes 100 (called hundred cents, in standard terminology) approximately (exactly in equal-tempered version), facilitating the comparison. Then we round to the nearest integer. Here is the approximate table of shrutis in cents that we obtain.

S	R_{11}	R_{12}	R_{21}	R_{22}	G_{11}	G_{12}	G_{21}	G_{22}
0	90	112	182	204	294	316	386	408
M_{11}	M_{12}	M_{21}	M_{22}	P	D_{11}	D_{12}	D_{21}	D_{22}
498	520	590	612	702	792	814	884	906
N_{11}	N_{12}	N_{21}	N_{22}					
996	1018	1088	1110					

Note that the inter-shruti differences in cents are approximately $22, 70, 90$ (better approximations would be $21.5, 70.7, 90.2$) and $1200 = 22 * 10 + 90 * 7 + 70 \times 5$. Minimum deviation from equal tempered is 2 cents (for

P, M_{11}, which explains why these basic chords do not sound too bad on equal-tempered instruments, but the difference is still noticeable because of the low denominators) and maximum 20 cents (for M_{12}).

We remark that the ratio $3^{12}/2^{19}$ mentioned above translates to about 23.5 cents. So there is indeed room for other possible compromises. It is quite possible that scholars who have come up with different tables have made modifications giving more importance to different requirements.

3. Theory vs. Practice, Questions and Experiments

3.1 *Use of Shrutis in Various Raagas*

Probably the way shrutis are used currently in raagas does not really follow the model frequencies that closely, though most musicians seem to accept the qualitative upper/lower version description. Many great musicians have mentioned that some shrutis of some raagas are extra special (see below for specific references) implying that different versions may be involved.

In a given raaga, just as we use only a few of the 12 notes, we use only some of the 22 shrutis. Usually, only one out of the two versions of a sur (but there are exceptions such as the raaga Lagan-Gandhaar) is used, depending on what samvaads and effects one wants to achieve to create the mood of the raaga. Many well-known musical texts (e.g., [9]) prescribe what samvaads are used in a particular raaga. This, together with the model described above, suggests which shrutis are used in that raaga. But again this may not be automatic, as the samvaads are not chords played simultaneously, and sliding used in practice does not need to follow exact consonance principles used above. In [3, pp.87–94], this issue is studied and shrutis are suggested for various common raagas. Also, in [9] and many other books, prescriptions are sometimes given whether we use lower or upper versions of each shruti in a given raaga.

We now discuss three examples of this (see also [3]). The famous raagas Bhoop and Deskar (whose variants are found in various other musical cultures) are both based on shuddh surs $SRGPD$, but the respective shrutis are $SR_{21}G_{21}PD_{21}$ and $SR_{22}G_{22}PD_{22}$, respectively. The corresponding basic samvaadi pairs in Bhoop are SP, RD with 3/2 ratio, GD with $4/3 = M_{11}$ and RP with $27/20 = M_{21}$ ratios, and SG is Shadja-swayambhu Gandhaar pair, whereas, in Deskar, we have SP, RD with 3/2 ratio, and RP and GD with 4/3.

Another famous raaga Todi is described by

$$SR_1G_1M_2D_1N_2$$

(with P also used in certain special ways in some versions) and it is also said that the Rishabh shruti is lower [3, p.73]. So, in our model, it would mean R_{11} shruti for R_1, and that would suggest choosing D_{11} shruti at 3/2 ratio, which would then suggest choosing G_{11} shruti at 3/4 of this. Using such repeated multiplication by 3/2, (with adjustment for octaves) and discarding the surs not in the raaga, as a prescription, [3, p.73] gives M_{21}, N_{21} as the right shrutis for M and N, but says wrongly that RM and MN are Shadja–Madhyam samvaads. (See 2.5). Even the higher version R_{12}, which is mentioned for the raaga Gujari Todi in [3], (a higher version is used also in some famous Todi recordings) does not have this samvaad as explained in 2.5.

Whether the suggested rough choices (upper/lower) of shrutis as well as the specific frequency ratios agree with the current usage in a given raaga, should be determined by experiments with co-operation from the masters.

3.2 Comparison with Western Music

In contrast to Western music, Indian music uses a fixed

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tonic Saa, does not usually change/transpose the key within the music and uses samvaadi/sangati/harmonical notes in succession rather than simultaneously as in chords ('melody rather than harmony'). So the severe *requirements of the transposition independence as well as of the presence of all the chordal intervals are absent in Indian music.*

Note that these twenty-two shrutis frequency ratios (and many other variants) have also been found in Western music literature (see table [3, p.61] for the corresponding names). They were probably discovered later, but independently in Western music and were viewed as possible alternatives, rather than a 'complete' system as in the model here.

The ratio 81/80 is known as a syntonic comma, whereas $3^{12}/2^{19}$ is known as a Pythagorean comma. The half octaves $SRGM$ and $PDNS$ are called tetrachords.

Refer to the references given in the bibliography below for vast literature on scales/temperament in Western music.

3.3 *Perceived Frequency/Pitch, Consonance, Tanpura*

Unlike the tuning fork sound which corresponds to a `pure' sine wave of one frequency, when we perceive a musical sound of frequency *f*, it really corresponds to superposition of pure waves of different frequencies

Perception of pitch is not a straightforward phenomenon because of the complexity of the ear-brain system. But we give here a simplistic explanation, which hopefully is substantially correct.

Unlike the tuning fork sound which corresponds to a 'pure' sine wave of one frequency, when we perceive a musical sound of frequency f, it really corresponds to superposition of pure waves of different frequencies kf (called the k-th harmonic or the k-th partial of f), as k varies through the positive integers. The amplitudes of the components (i.e., the strengths of the harmonics present) are different for different k and can be zero or non-zero. Sometimes it can be zero even for

$k = 1$. These superpositions provide texture and rich tonal quality to the musical sound. For higher k's, the amplitudes eventually become negligible and also the corresponding frequencies are out of listening range, so only relatively smaller ks dominate.

This explains the consonance of two notes with a small complexity ratio: If the frequency ratio of two notes is a fraction with a small denominator, then many of the partials of the two notes coincide, giving nice resonance.

Depending on the strength of the k-th partial present, sometimes we perceive a note corresponding to $k/2^n$. For example, while strong second and third/sixth partials just reinforce S and P respectively, the strong presence of the 5th partial in the tanpura sound leads to the perception of the swayambhu shuddha Gandhaar $G_{21} = 5/4$. Somewhat weaker 9th, 15th partials (or 3rd and 5th of P string) give the perception of $R_{22} = 9/8$, $N_{21} = 15/8$ respectively [10].

Sometimes, one perceives 'combinatorial tones' $m \pm n$, $2n - m$, etc., in presence of two notes m, n due to the non-linearity (quadratic, cubic...) of the ear membrane mechanism. Mathematics and science behind this is based on school trigonometric identities such as

$$(\sin(mt) + \sin(nt))^2 = \frac{\cos((m-n)t) - \cos((m+n)t)}{2} + 1 - \frac{\cos(2nt) + \cos(2mt)}{2}.$$

If m and n are close, the identity

$$\sin(mt) + \sin(nt) = 2\sin\left(\frac{t(m+n)}{2}\right)\cos\left(\frac{t(m-n)}{2}\right)$$

shows that we hear the average frequency, but with the amplitude modulated by a slow cosine wave of the frequency $(m-n)/2$. This gives us a sensation of $m - n$ beats (useful for tuning!).

Talks with master performers suggest that they do not consciously try to get these exact samvaads using lower and upper versions, but rather rely on tradition and intuition to use notes in a range to create the right feeling and mood.

So the tanpura Gandhaar can also arise as $5/4 = 1/2 + 3/4 = 2 - 3/4$ by lower S, P and S combinations [11].

Note we may perceive bad tones such as $7/4 = 1+3/4$ in presence of strong 'wrong' harmonics. Is such harmonic distribution related to perception of '*surel awaaz*' or a beautiful voice?

3.4 *Open Questions and Suggested Experiments*

There are many natural questions which arise. For example, are all twenty-two shrutis used by the great experts, or do they even use more? Is there a consistency of the frequency ratios within or between raagas, or is there only rough positional (upper vs. lower version, say) consistency? In raagas, where some samvaadi pair is described, but actual theoretical samvaad ratios are absent in the table above, are the shrutis modified accordingly in actual practice? Are the high complexity shrutis used exactly or by their low denominator approximations (obtained by truncating continued fraction expansions say)? How often and accurately are the high complexity shrutis G_{22}, M_{12} (rather than the low complexity 'natural' G_{21}, M_{11}) or M_{22} used in practice?

While the table we mentioned is quite uniform, that may not be the right criterion. Some high complexity ratios in the table above, as well as preference for some SG, SM samvaads that are absent in it, may be the reason why some scholars have given other variations on the table. Talks with master performers suggest that they do not consciously try to get these exact samvaads using lower and upper versions, but rather rely on tradition and intuition to use notes in a range to create the right feeling and mood. They often mention (see below) that a particular shruti of a particular raaga is extra special.

The shruti frequencies should be tested in experiments with available high quality recordings of great masters such as Kumar Gandharva, Mallikarjun Mansur, Kishori

Amonkar and Bhimsen Joshi. Here are some concrete suggestions for experiments.

(i) Kumar Gandharva's Lagan-Gandhaar raaga record should be used to see whether the two komal Gandhaar shrutis that he mentions [8, p.181] are the same as in the table above or different.

(ii) In raaga Ahimohini [8, p.188] komal Gandhaar and komal Dhaivat are higher versions in a 'special manner', possibly implying not the usual two versions.

(iii) In raaga Madhasuraja [8, p.160], komal Rishabh and teevra Madhyam are mentioned to be higher versions. (This particular use of teevra Madhyam with support of komal Madhyam is not typical!)

My colleagues at Tata Institute of Fundamental Research tell me that because of the *meend* (common practice in North Indian classical music of smooth sliding/gliding from one note to another) and the oscillating notes (rather the practice of very quickly moving/touching neighboring notes e.g., *aandolit* (oscillating) Nishaad of the raaga Bhimpalas, komal Gandhaar of the raaga Darbaari etc), it is very hard to determine the exact numerical frequencies from the recordings. Current technology is capable of determining only a reasonable range, so statistical analysis might be needed [12].

Of course, such experiments should be conducted by well-trained musicians, who can provide their opinions on the use of different shrutis in various raagas using the shruti-harmonium. (The shruti-harmonium or melodium is actually made by Oke in the conventional version, and earlier by Arnold and Bell in a digital version, both based on the table above [3, 4]. The shruti-harmoniums were also made many years ago based on tables by Clement, Deval, Acharekar).

Modern, digital technology enables us to manufacture new instruments with an easily changeable basic frequency

In Nirguni Bhajans, it is the slight dissonance with shadja that creates the desired musical effect.

and relative frequencies scale. With the co-operation of top musicians, it would be easier to determine, using such an instrument, which shrutis and relative frequencies are used in which ragaas rather than trying to determine them from recordings.

My recent interest in the subject was initiated by discussions with Satyasheel Deshpande, who actually tried out the sugestions in [3] on the Oke's shruti-harmonium and compared them with his vast knowledge of Indian raagas and great expertise in tuning. He feels that the top musicians in raaga-sangeet do use other shrutis (i.e., relative frequencies not in the table above) depending on the musical statements they are trying to establish. He explains that while performing, most often the relation is with tanpura shadja, pancham and natural shrutis occuring there, rather than SM or SP consonance principle, so shrutis can be different. Also, according to him, in Nirguni Bhajans, it is the slight dissonance with shadja that creates the desired musical effect.

A wise and famous composition: '*sur sangat raaga vidyaa,... jo kantha kar dikhaye, wako jano guni*' roughly meaning that the knowledge of raagas is related to the mastery of surs and shrutis, (but) we will (only) acknowledge those who can sing, perform and illustrate (and not those who can only discuss and theorize!).

Finally, music is an art. We end this article by quoting a wise and famous composition: '*sur sangat raaga vidyaa,... jo kantha kar dikhaye, wako jano guni*' roughly meaning that the knowledge of raagas is related to the mastery of surs and shrutis, (but) we will (only) acknowledge those who can sing, perform and illustrate (and not those who can only discuss and theorize!).

Acknowledgments

The first draft of this article was written and circulated in summer 2009 at the suggestion of my friends Manjul Bhargava, Noam Elkies and Satyasheel Deshpande. Even after pointing out that it is very old knowledge, they suggested that it would be useful to have such an exposition. I thank them for their interest, encouragement, and various comments, which helped improve the exposition. My interest in this subject started in 1974,

when my uncle P L Deshpande, knowing my interest in music and mathematics, gave me a copy of *Sangeet and Vidnyan* talking about the relative frequency of the swayambhu (natural) Gandhaar [11]. In college, interpolating between various texts and reading about Bharat's Chatus-saaranaa experiment, I came up with the shruti table described above, but soon found out that this table was known in literature (among various other contradictory tables published and discussed). I have also discussed these issues with S S Bhave and P Pandya at Tata Institue of Fundamental Research, and worked with their software trying to use the recordings of Kumar Gandharva's tanpura as well as Lagan-Gandhaar. They did various frequency and harmonic spread experiments with actual music and software.

This article is dedicated to the memory of Sudhir Phadke.

Suggested Reading

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Address for Correspondence
Dinesh Thakur
Department of Mathematics
University of Rochester
Rochester, New York 14627
USA
Email:
dinesh.thakur@rochester.edu