Thermalization in integrable models and conformal field theories

Gautam Mandal
TIFR, Mumbai

Mid-year meeting, IASc, 23 June, Bengaluru

Based on: GM, R. Sinha, N. Sorokhaibam (1405.6695, 1501.04580), GM, T. Morita (1302.0859), P. Caupta, GM, R. Sinha (1306.4974), ongoing work with S. Paranjape, R. Sinha, N. Sorokhaibam and T. Ugajin
To thermalize or not to thermalize?
Quantum quench

Consider a quantum system in its ground state. Turn on a time-dependent coupling $g(t)$ for some time up to $t = t_1$.

\[
H(t) = -J \sum_{i=1}^{L} \left[ \sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z \right]
\]

e.g. $H(t) = J \sum_{i=1}^{L} \left[ \sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z \right]$

The post-quench dynamics is described by a final Hamiltonian $H$ and an ‘initial state’ $|\psi_1\rangle$, which depends on $g(t)$.
Late time dynamics: thermalization

Post-quench:

$$|\psi(t)\rangle = \exp[-iH(t - t_1)]|\psi_1\rangle$$

Does the system reach a steady state at ‘late times’?

Does the final state ‘forget’ most features of the initial state? In particular, is the state ‘thermal’?
Late time dynamics: thermalization

Post-quench:

\[ |\psi(t)\rangle = \exp[-iH(t - t_1)]|\psi_1\rangle \]

Does the system reach a steady state at ‘late times’?

Does the final state ‘forget’ most features of the initial state? In particular, is the state ‘thermal’?

Of course, we cannot have pure state \(\rightarrow\) mixed state.

A more accurate statement of thermalization is ...
Definition of thermalization

\[ \langle \psi_1 | O_1(x_1, t) ... O_n(x_n, t) | \psi_1 \rangle \xrightarrow{t \to t_{eqm}} (O_1(x_1) ... O_n(x_n) \rho_{eqm}) \]

Equivalent statement in terms of density matrix of subsystem \( A \)

\[ \rho_A(t) \xrightarrow{t \to t_{eqm}} \rho_{A,eqm} \]

where

\[ \rho_A(t) = \text{Tr}_{A^c} |\psi(t)\rangle\langle \psi(t)|, \quad \rho_{A,eqm} = \text{Tr}_{A^c} \rho_{eqm} \]

This formalizes the concept of the rest of the system as a ‘bath’.
Quantum Ergodic Hypothesis

QEH: An equilibrium state $\rho_{eqm}$ exists, and it is given by the microcanonical ensemble

$$\rho_{eqm} = \rho_{\text{micro}}$$

where the microcanonical ensemble is defined by the energy of the pure state $|\psi_1\rangle$. 
Quantum Ergodic Hypothesis

QEH: An equilibrium state $\rho_{eqm}$ exists, and it is given by the microcanonical ensemble

$$\rho_{eqm} = \rho_{micro}$$

where the microcanonical ensemble is defined by the energy of the pure state $|\psi_1\rangle$.

- Besides the energy, all other details of the quench are forgotten at late times.
Quantum Ergodic Hypothesis

QEH: An equilibrium state $\rho_{eqm}$ exists, and it is given by the microcanonical ensemble

$$\rho_{eqm} = \rho_{micro}$$

where the microcanonical ensemble is defined by the energy of the pure state $|\psi_1\rangle$.

- Besides the energy, all other details of the quench are forgotten at late times.

Is the QEH true?
No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.
No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.

\[
\rho_{\text{pure}} = |\psi_1\rangle\langle\psi_1| \xrightarrow{\text{?}} \rho_{M,J,Q} \quad \text{information loss}
\]

Correct way to understand this is in terms of the reduced density matrix:

\[
|\rho_A(t)\rangle \rightarrow \rho_{A;M,J,Q}
\]
Thermalization in gravity: gravitational collapse

No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.

\[ \rho_{\text{pure}} = |\psi_1\rangle \langle \psi_1| \rightarrow \rho_{M,J,Q} \quad \text{information loss} \]

Correct way to understand this is in terms of the reduced density matrix: \[ |\rho_A(t)\rangle \rightarrow \rho_{A;M,J,Q} \]

Quantum ergodic hypothesis holds for gravitational collapse.
No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.

\[ \rho_{\text{pure}} = |\psi_1\rangle\langle\psi_1| \xrightarrow{?} \rho_{M,J,Q} \]

Correct way to understand this is in terms of the reduced density matrix:

\[ |\rho_A(t)\rangle \rightarrow \rho_{A;M,J,Q} \]

Quantum ergodic hypothesis holds for gravitational collapse.

AdS/CFT: gravitational collapse = thermalization in field theory.
Non-thermalization

Weakly anhamonically coupled chain of oscillators show ‘revival’

Fermi, Pasta, Ulam 1953

1+1 dimensional critical system with spatial boundaries; shows periodicity $\Delta T = L/2$.

Mandal, Sinha, Ugajin 2015; see also Cardy 2014, Kuns, Marolf 2014

**AdS/CFT**: For gravitational duals of non-ergodic systems, see Balasubramanian, Buchel, Green, Lehner, Leibling 2014
Integrable systems: recent surprises

Expect: integrability $\Rightarrow$ non-ergodicity.

But, examples of 2D integrable models have been discovered in the last 8 years, where QEH holds.

Transverse field Ising (Calabrese et al 2005)

$$H = -J \sum_{i=1}^{L} \left[ \sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z \right]$$

Hard core boson chain (Rigol et al 2007)

$$H = -J \sum_{i=1}^{L(t)} b_i^\dagger b_{i+1} + \text{h.c.}$$

Massive Scalar (Sotiriadis, Cardy 2010)

$$S = \int d^2x \left[ (\partial \phi)^2 - m^2(t)\phi^2 \right]$$

Matrix QM model (Morita, GM 2013)

$$S = \int dt \left[ \text{Tr}(U^\dagger \partial_t U + a(t)(U + U^\dagger)) \right]$$

Morita, GM 2013
Critical Quench

RG flow to a critical point leads to universality classes of systems.

Suppose in course of a quench, $g(t)$ stops at a value $g_1$ for which the system is critical.

Does the ‘dynamics’ of various systems show some universality in this case?
Rest of the talk

• Develop a general description of critical quench in 1+1 dimensions; extend to integrable models.
Rest of the talk

- Develop a general description of critical quench in 1+1 dimensions; extend to integrable models.
- Prove QEH for an arbitrary integrable conformal field theory.
Rest of the talk

- Develop a general description of critical quench in 1+1 dimensions; extend to integrable models.
- Prove QEH for an arbitrary integrable conformal field theory.
- Compute the relaxation rate of the density matrix to equilibrium, leading to new universality relations.
Rest of the talk

- Develop a general description of critical quench in 1+1 dimensions; extend to integrable models.
- Prove QEH for an arbitrary integrable conformal field theory.
- Compute the relaxation rate of the density matrix to equilibrium, leading to new universality relations.
- Holographic interpretation. QEH can be interpreted as gravitational collapse to black holes with infinite number of extra charges! CFT Relaxation rate= Quasinormal frequency of BH.
Rest of the talk

- Develop a general description of critical quench in 1+1 dimensions; extend to integrable models.
- Prove QEH for an arbitrary integrable conformal field theory.
- Compute the relaxation rate of the density matrix to equilibrium, leading to new universality relations.
- Holographic interpretation. QEH can be interpreted as gravitational collapse to black holes with infinite number of extra charges! CFT Relaxation rate= Quasinormal frequency of BH.
Critical quench: Calabrese-Cardy ansatz

For a sudden quench to a quantum critical coupling $g_1$

$$S = S_{CFT} + \delta g \int d^2 x \theta(-t) O_\Delta(x, t), \quad \delta g \equiv g_0 - g_1$$

Calabrese and Cardy (2005) modelled the state at $t = 0$ as $|\psi_1\rangle = e^{-\kappa H} |Bd\rangle$

Here $\kappa \sim (\delta g)^{\frac{1}{\Delta-2}} = [\text{length}]$. $|Bd\rangle =$conformal boundary state.
Critical quench: Calabrese-Cardy ansatz

For a sudden quench to a quantum critical coupling $g_1$

\[ S = S_{CFT} + \delta g \int d^2x \theta(-t) O_\Delta(x, t), \quad \delta g \equiv g_0 - g_1 \]

Calabrese and Cardy (2005) modelled the state at $t = 0$ as $|\psi_1 \rangle = e^{-\kappa H} |Bd\rangle$

Here $\kappa \sim (\delta g)^{\frac{1}{\Delta-2}} = [\text{length}]$. $|Bd\rangle = \text{conformal}$ boundary state.

Logic: At low energies $\kappa E \ll 1$, $\langle \psi_k | E \rangle = f(\kappa E) = \text{constant} (\Rightarrow \text{conformality})$
Generalized Calabrese-Cardy states

CC state fails to consider:
(a) Multiple scales: when the quench protocol \( g(t) \) has more scales, e.g. \( g(t) = g_0 \ f(t/\delta t) \),
(b) Multiple charges: when there are other conserved charges, the simple QEH \( \rho_{eqm} = \rho_{micro}(E) \) is inadequate.
Generalized Calabrese-Cardy states

CC state fails to consider:
(a) Multiple scales: when the quench protocol $g(t)$ has more scales, e.g. $g(t) = g_0 \cdot f(t/\delta t)$,
(b) Multiple charges: when there are other conserved charges, the simple QEH $\rho_{eqm} = \rho_{micro}(E)$ is inadequate.

We propose that in a CFT with additional charges $W_n$ we can solve both issues with a generalized Calabrese-Cardy (gCC) state

$$|\psi_1\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \ldots\right)} |Bd\rangle,$$

Require: $W_n$ obtained from local currents which are primary or quasiprimary operators.
Generalized Calabrese-Cardy states

CC state fails to consider:
(a) Multiple scales: when the quench protocol $g(t)$ has more scales, e.g. $g(t) = g_0 \, f(t/\delta t)$,
(b) Multiple charges: when there are other conserved charges, the simple QEH $\rho_{eqm} = \rho_{micro}(E)$ is inadequate.

We propose that in a CFT with additional charges $W_n$ we can solve both issues with a generalized Calabrese-Cardy (gCC) state

$$|\psi_1\rangle = e^{-(\kappa_2 H + \sum_n \kappa_n W_n + \ldots)} |Bd\rangle,$$

This includes integrable conformal theories with $\infty$ number of conserved charges.
Example of scalar field theory

\[ S = \int d^2x \left[ (\partial \phi)^2 - m(t)^2 \phi^2 \right] \]

Time-dependent coupling induces a Bogoliubov transformation:
\[ a_{in}(k) \propto a_{out}(k) - \gamma(k) a_{out}^\dagger(-k) \]
Hence
\[ |0, in\rangle = e^{\sum_k \gamma(k) a_{out}^\dagger(k) a_{out}^\dagger(-k)} |0, out\rangle \]
Example of scalar field theory

\[ S = \int d^2x \left[ (\partial \phi)^2 - m(t)^2 \phi^2 \right] \]

Time-dependent coupling induces a Bogoliubov transformation:

\[ a_{in}(k) \propto a_{out}(k) - \gamma(k) a_{out}^\dagger(k). \]

Hence

\[ |0, in\rangle = e^{\sum_k \gamma(k) a_{out}^\dagger(k) a_{out}^\dagger(-k)} |0, out\rangle \]

For a simple form \( m^2(t) = m_0^2 \left( 1 - \tanh(t/\delta t) \right)/2 \), explicit value of \( \gamma(k) \): Birrell, Davies 1994

\[ |\psi_1\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \ldots\right)} |\text{Dirichlet}\rangle, \]

where \( \kappa_2, \kappa_4 \) depend on \( m_0, \delta t \).
Example of scalar field theory

\[ S = \int d^2 x \left[ (\partial \phi)^2 - m(t)^2 \phi^2 \right] \]

Time-dependent coupling induces a Bogoliubov transformation:

\[ a_{in}(k) \propto a_{out}(k) - \gamma(k) a_{out}^\dagger(k). \]

Hence

\[ |0, in\rangle = e^{\sum_k \gamma(k) a_{out}^\dagger(k) a_{out}^\dagger(-k)} |0, out\rangle \]

For a simple form \( m^2(t) = m_0^2 \left( 1 - \tanh(t/\delta t) \right) / 2 \), explicit value of \( \gamma(k) \): Birrell, Davies 1994

\[ |\psi_1\rangle = e^{-\left( \kappa_2 H + \sum_n \kappa_n W_n + \cdots \right)} |\text{Dirichlet}\rangle, \]

where \( \kappa_2, \kappa_4 \) depend on \( m_0, \delta t \).

Here \( W_n \sim \sum_k k^{n-1} a_{out}^\dagger(k) a_{out}(k), n = 2, 4, 6, \ldots \)
Example of scalar field theory

\[ S = \int d^2 x \left[ (\partial \phi)^2 - m(t)^2 \phi^2 \right] \]

Time-dependent coupling induces a Bogoliubov transformation:

\[ a_{in}(k) \propto a_{out}(k) - \gamma(k)a_{out}^\dagger(-k) \]

Hence

\[ |0, in\rangle = e^{\sum_k \gamma(k)a_{out}^\dagger(k)a_{out}^\dagger(-k)} |0, out\rangle \]

For a simple form \( m^2(t) = m_0^2(1 - \tanh(t/\delta t))/2 \), explicit value of \( \gamma(k) \): Birrell, Davies 1994

\[ |\psi_1\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \ldots\right)} |\text{Dirichlet}\rangle, \]

where \( \kappa_2, \kappa_4 \) depend on \( m_0, \delta t \).

- This example verifies the generalized quench model (gCC).
Consider a 2-site ‘lattice’ of quantum Ising spins. Prepare a wavefunction

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle)$$.
Consider a 2-site ‘lattice’ of quantum Ising spins. Prepare a wavefunction

\[ |\psi\rangle = \frac{1}{\sqrt{2}} \left( |\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle \right). \]

The reduced density matrix of the 1st spin (=subsystem A) is given by

\[ \langle s_1 | \rho_A | s'_1 \rangle = \sum_{s_2=\uparrow,\downarrow} \langle s_1, s_2 | \psi \rangle \langle \psi | s'_1, s_2 \rangle = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}. \]
Consider a 2-site ‘lattice’ of quantum Ising spins. Prepare a wavefunction

\[ |\psi\rangle = \frac{1}{\sqrt{2}} ( |\uparrow, \downarrow\rangle + |\downarrow, \uparrow\rangle ) . \]

The reduced density matrix of the 1st spin (=subsystem A) is given by

\[
\langle s_1 | \rho_A | s'_1 \rangle = \sum_{s_2=\uparrow, \downarrow} \langle s_1, s_2 | \psi \rangle \langle \psi | s'_1, s_2 \rangle = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} .
\]

\( \rho_A \) can calculate all operators on site 1. Note \( \rho_A^2 \neq \rho_A \).
Reduced density matrix - contd.

Functional integral representation of the density matrix element

\[ \langle \phi, A | \rho_A | \psi, A \rangle = \sum_n \langle n, A^c ; \phi, A | \rho | \psi, A ; n, A^c \rangle. \]

In the picture \( \rho \sim \exp[-\beta H] \).
Proof of Quantum ergodicity
Proof of Quantum ergodicity

\[
\sum_{\phi, \psi} \langle \phi, A \rangle \langle A, \phi \rangle = \sum_{k_1, k_2} C_{k_1, k_2} \langle O_{k_1} \rangle_{\text{strip}} \langle O_{k_2} \rangle_{\text{cylinder}}.
\]

Thus,

\[
\rho_A(\kappa_2, \{\kappa_n\}; t) \xrightarrow{t \gg t_{\text{eqm}}} \rho_A(\beta, \{\mu_n\}) \quad \text{where} \quad \beta = 4\kappa_2, \mu_n = 4\kappa_n.
\]
Proof of Quantum ergodicity

$$\sum_{k_1, k_2} C_{k_1, k_2} \langle O_{k_1} \rangle_{\text{strip}} \langle O_{k_2} \rangle_{\text{cylinder}} = 1 - \alpha e^{-2\gamma t}$$

Thus,

$$\rho_A(\kappa_2, \{\kappa_n\}; t) \xrightarrow{t \gg t_{\text{eqm}}} \rho_A(\beta, \{\mu_n\}) \text{ where } \beta = 4\kappa_2, \mu_n = 4\kappa_n.$$  

QEH is proved. $\rho_{\text{eqm}} = \rho_{\text{GGE}}$. Integrable 2D CFT’s thermalize.
Proof of Quantum ergodicity

\[ \sum_{k_1, k_2} C_{k_1, k_2} \langle O_{k_1} \rangle_{\text{strip}} \langle O_{k_2} \rangle_{\text{cylinder}} = 1 - \alpha e^{-2\gamma t} \]

Thus,

\[ \rho_A(\kappa_2, \{\kappa_n\}; t) \xrightarrow{t \gg t_{\text{eqm}}} \rho_A(\beta, \{\mu_n\}) \]

where \( \beta = 4\kappa_2, \mu_n = 4\kappa_n \).

QEH is proved. \( \rho_{\text{eqm}} = \rho_{\text{GGE}} \). Integrable 2D CFT's thermalize.
Relaxation rates ↔ critical exponents

\[ \langle \kappa | O_\Delta(t) | \kappa \rangle \sim \text{Tr}(\rho_\beta O_\Delta) + Ce^{-\gamma t}, \quad \gamma = \frac{2\pi \Delta}{\beta}. \quad \beta = 4\kappa. \]
Relaxation rates $\leftrightarrow$ critical exponents

\[
\langle \kappa | O_{\Delta}(t) | \kappa \rangle \sim \text{Tr}(\rho_\beta O_{\Delta}) + Ce^{-\gamma t}, \quad \gamma = \frac{2\pi \Delta}{\beta}, \quad \beta = 4\kappa.
\]

\[
\langle \kappa_n | O_{\Delta}(t) | \kappa_n \rangle \sim \text{Tr}(\rho_\beta,\mu_n O_{\Delta}) + Ce^{-\gamma t}, \quad \text{where}
\]
Relaxation rates $\leftrightarrow$ critical exponents

$$\langle \kappa | O_\Delta (t) | \kappa \rangle \sim \text{Tr}(\rho_\beta O_\Delta) + Ce^{-\gamma t}, \quad \gamma = 2\pi \Delta / \beta, \quad \beta = 4\kappa.$$ 

$$\langle \kappa_n | O_\Delta (t) | \kappa_n \rangle \sim \text{Tr}(\rho_\beta, \mu_n O_\Delta) + Ce^{-\gamma t}, \quad \text{where}$$

$$\gamma_i = \frac{2\pi}{\beta} \left[ \Delta_i + \sum_n \tilde{\mu}_n Q_{n,i} + O(\tilde{\mu}^2) \right], \quad \tilde{\mu}_n \equiv \frac{\mu_n}{\beta^{n-1}}$$
New Universality relations

Cardy-Calabrese state: ratios $\gamma_i/\gamma_j = \Delta_i/\Delta_j$ are universal; independent of quench protocol.
Cardy-Calabrese state: ratios $\gamma_i / \gamma_j = \Delta_i / \Delta_j$ are universal; independent of quench protocol.

Generalized CC state (multi-scale quench): these ratios $\neq$ universal. E.g. in the presence of one extra charge, we have $\gamma_i = \frac{2\pi}{\beta} [\Delta_i + \tilde{\mu} Q_i]$.

The ratios $\gamma_i / \gamma_j$ now depend on $\mu$: 

![Graph showing non-universal ratios](image)
$\mu$-dependence can be eliminated by considering new ratios: 
$\left( a_{31} \gamma_2 + a_{12} \gamma_3 \right)/\gamma_1, \left( a_{41} \gamma_3 + a_{13} \gamma_4 \right)/\gamma_1$, with $a_{ij} = \Delta_{[i} Q_{j]}$, are independent of $\mu$, and depend only on the spectrum of the final CFT.
AdS/CFT Dictionary (holography)
AdS/CFT Dictionary (holography)

Thermalization in the boundary field theory = gravitational collapse into a black hole in the bulk!
## Thermalization and holography

<table>
<thead>
<tr>
<th>CFT</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>Anti de Sitter space</td>
</tr>
<tr>
<td>Thermal state ($\beta$)</td>
<td>AdS-Schwarzschild ($M(\beta)$)</td>
</tr>
<tr>
<td>Thermal state with $\mu$</td>
<td>Charged BH</td>
</tr>
<tr>
<td>Quantum Quench</td>
<td>Gravitational collapse</td>
</tr>
<tr>
<td>Ergodicity</td>
<td>No hair theorem</td>
</tr>
<tr>
<td>Thermal decay</td>
<td>Quasinormal mode</td>
</tr>
<tr>
<td>Thermalization/relaxation rate</td>
<td>Quasinormal frequency (QNF)</td>
</tr>
<tr>
<td>Integrable CFT (2D)</td>
<td>?</td>
</tr>
<tr>
<td>GGE (generalized Gibbs ensemble)</td>
<td>?</td>
</tr>
<tr>
<td>Quantum quench in integrable CFT</td>
<td>?</td>
</tr>
<tr>
<td>Thermal decay rate to GGE</td>
<td>?</td>
</tr>
</tbody>
</table>
## Thermalization and holography

<table>
<thead>
<tr>
<th>CFT</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>Anti de Sitter space</td>
</tr>
<tr>
<td>Thermal state ($\beta$)</td>
<td>AdS-Schwarzschild ($M(\beta)$)</td>
</tr>
<tr>
<td>Thermal state with $\mu$</td>
<td>Charged BH</td>
</tr>
<tr>
<td>Quantum Quench</td>
<td>Gravitational collapse</td>
</tr>
<tr>
<td>Ergodicity</td>
<td>No hair theorem</td>
</tr>
<tr>
<td>Thermal decay</td>
<td>Quasinormal mode</td>
</tr>
<tr>
<td>Thermalization/relaxation rate</td>
<td>Quasinormal frequency (QNF)</td>
</tr>
</tbody>
</table>

- Integrable CFT (2D)
- GGE (generalized Gibbs ensemble)
- Quantum quench in integrable CFT
- Thermal decay rate to GGE

- Higher spin gravity
- ?
- ?
## Thermalization and holography

<table>
<thead>
<tr>
<th><strong>CFT</strong></th>
<th><strong>Gravity</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>Anti de Sitter space</td>
</tr>
<tr>
<td>Thermal state ((\beta))</td>
<td>AdS-Schwarzschild ((M(\beta)))</td>
</tr>
<tr>
<td>Thermal state with (\mu)</td>
<td>Charged BH</td>
</tr>
<tr>
<td>Quantum Quench</td>
<td>Gravitational collapse</td>
</tr>
<tr>
<td>Ergodicity</td>
<td>No hair theorem</td>
</tr>
<tr>
<td>Thermal decay</td>
<td>Quasinormal mode</td>
</tr>
<tr>
<td>Thermalization/relaxation rate</td>
<td>Quasinormal frequency (QNF)</td>
</tr>
</tbody>
</table>

- Integrable CFT (2D)  
- GGE (generalized Gibbs ensemble)  
- Quantum quench in integrable CFT  
- Thermal decay rate to GGE  

- Higher spin gravity  
- Higher spin BH  
- ?  
- ?
## Thermalization and holography

<table>
<thead>
<tr>
<th>CFT</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>Anti de Sitter space</td>
</tr>
<tr>
<td>Thermal state ($\beta$)</td>
<td>AdS-Schwarzschild ($M(\beta)$)</td>
</tr>
<tr>
<td>Thermal state with $\mu$</td>
<td>Charged BH</td>
</tr>
<tr>
<td>Quantum Quench</td>
<td>Gravitational collapse</td>
</tr>
<tr>
<td>Ergodicity</td>
<td>No hair theorem</td>
</tr>
<tr>
<td>Thermal decay</td>
<td>Quasinormal mode</td>
</tr>
<tr>
<td>Thermalization/relaxation rate</td>
<td>Quasinormal frequency (QNF)</td>
</tr>
<tr>
<td>Integrable CFT (2D)</td>
<td>Higher spin gravity</td>
</tr>
<tr>
<td>GGE (generalized Gibbs ensemble)</td>
<td>Higher spin BH</td>
</tr>
<tr>
<td>Quantum quench in integrable CFT</td>
<td>Gravitational collapse to HS BH</td>
</tr>
<tr>
<td>Thermal decay rate to GGE</td>
<td>?</td>
</tr>
</tbody>
</table>

- Higher spin gravity
- Higher spin BH
- Gravitational collapse to HS BH
# Thermalization and holography

<table>
<thead>
<tr>
<th>CFT</th>
<th>Gravity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>Anti de Sitter space</td>
</tr>
<tr>
<td>Thermal state ($\beta$)</td>
<td>AdS-Schwarzschild ($M(\beta)$)</td>
</tr>
<tr>
<td>Thermal state with $\mu$</td>
<td>Charged BH</td>
</tr>
<tr>
<td>Quantum Quench</td>
<td>Gravitational collapse</td>
</tr>
<tr>
<td>Ergodicity</td>
<td>No hair theorem</td>
</tr>
<tr>
<td>Thermal decay</td>
<td>Quasinormal mode</td>
</tr>
<tr>
<td>Thermalization/relaxation rate</td>
<td>Quasinormal frequency (QNF)</td>
</tr>
<tr>
<td>Integrable CFT (2D)</td>
<td>Higher spin gravity</td>
</tr>
<tr>
<td>GGE (generalized Gibbs ensemble)</td>
<td>Higher spin BH</td>
</tr>
<tr>
<td>Quantum quench in integrable CFT</td>
<td>Gravitational collapse to HS BH</td>
</tr>
<tr>
<td>Thermal decay rate to GGE</td>
<td>QNF of HS BH</td>
</tr>
</tbody>
</table>
Introduction

Critical quench: model building

Quantum Ergodicity and integrability

Relaxation rates

Holography

Thermalization rate to GGE = QNF of HS BH

The holographic dual to 2D CFT = AdS gravity in 3D
The holographic dual to 2D CFT = AdS gravity in 3D

Higher spin gravity theories are a mini version of string theory with a single Regge trajectory. In 3D, this is characterized by an infinite dimensional symmetry and infinite number of conserved charges.
The holographic dual to 2D CFT = AdS gravity in 3D

Higher spin gravity theories are a mini version of string theory with a single Regge trajectory. In 3D, this is characterized by an infinite dimensional symmetry and infinite number of conserved charges.

These theories possess BH solutions. The BH’s carry an infinite number of conserved charges! (infinite number of ‘hairs’)
The holographic dual to integrable 2D CFT = Higher spin AdS gravity in 3D.

Higher spin gravity theories are a mini version of string theory with a single Regge trajectory. In 3D, this is characterized by an infinite dimensional symmetry and infinite number of conserved charges.

These theories possess BH solutions. The BH’s carry an infinite number of conserved charges! (infinite number of ‘hairs’).
The holographic dual to integrable 2D CFT = Higher spin AdS gravity in 3D

Higher spin gravity theories are a mini version of string theory with a single Regge trajectory. In 3D, this is characterized by an infinite dimensional symmetry and infinite number of conserved charges.

These theories possess BH solutions. The BH’s carry an infinite number of conserved charges! (infinite number of ‘hairs’)

QNF has been calculated for bulk scalar Cabo-Bizet, Gava, Giraldo-Rivera, Narain 2014 for a HS BH with a single chemical potential $\mu_3$.
Imaginary part of QNF

$$\text{Im } \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\tilde{\mu}_3}{3}(1 + \lambda)(2 + \lambda) \right)$$

Thermalization rate of the dual CFT operator is

$$\gamma_{CFT} = \frac{2\pi}{\beta} \left[ \Delta + \sum_{n} \tilde{\mu}_n Q_n \right]$$

Do these match?
Match made!

Imaginary part of QNF

$$\text{Im } \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\tilde{\mu}_3}{3} (1 + \lambda)(2 + \lambda) \right)$$

Thermalization rate of the dual CFT operator is

$$\gamma_{CFT} = \frac{2\pi}{\beta} \left[ \Delta + \sum_n \tilde{\mu}_n Q_n \right]$$

Do these match?

$$\Delta = 1 + \lambda, \text{ and } Q_3 = \frac{1}{3} (1 + \lambda)(2 + \lambda),$$  
Gaberdiel-Gopakumar 2010,

Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011
Imaginary part of QNF

\[
\text{Im } \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\tilde{\mu}_3}{3}(1 + \lambda)(2 + \lambda) \right)
\]

Thermalization rate of the dual CFT operator is

\[
\gamma_{\text{CFT}} = \frac{2\pi}{\beta} \left[ \Delta + \sum_n \tilde{\mu}_n Q_n \right]
\]

Do these match?

\[
\Delta = 1 + \lambda, \text{ and } Q_3 = \frac{1}{3}(1 + \lambda)(2 + \lambda), \quad \text{Gaberdiel-Gopakumar 2010, Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011}
\]

QNF = relaxation rate
Match made!

Imaginary part of QNF

\[ \text{Im} \, \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\tilde{\mu}_3}{3} (1 + \lambda)(2 + \lambda) \right) \]

Thermalization rate of the dual CFT operator is

\[ \gamma_{\text{CFT}} = \frac{2\pi}{\beta} \left[ \Delta + \sum_n \tilde{\mu}_n Q_n \right] \]

Do these match?

\[ \Delta = 1 + \lambda, \quad Q_3 = \frac{1}{3} (1 + \lambda)(2 + \lambda), \]

Gaberdiel-Gopakumar 2010,
Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011

QNF = relaxation rate

Integrable CFT’s thermalize. The thermalization is described by a new class of models (HS BH).
Match made!

Imaginary part of QNF

\[
\text{Im } \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\tilde{\mu}_3}{3}(1 + \lambda)(2 + \lambda) \right)
\]

Thermalization rate of the dual CFT operator is

\[
\gamma_{\text{CFT}} = \frac{2\pi}{\beta} \left[ \Delta + \sum_{n} \tilde{\mu}_n Q_n \right]
\]

Do these match?

\(\Delta = 1 + \lambda\), and \(Q_3 = \frac{1}{3}(1 + \lambda)(2 + \lambda)\), \text{Gaberdiel-Gopakumar 2010, Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011}\)

QNF = relaxation rate

**Integrable CFT’s thermalize.** The thermalization is described by a new class of models (HS BH).
Match made!

Imaginary part of QNF

\[ \text{Im } \omega = \frac{2\pi}{\beta} \left( 1 + \lambda + \frac{\bar{\mu}_3}{3} (1 + \lambda)(2 + \lambda) \right) \]

Thermalization rate of the dual CFT operator is

\[ \gamma_{\text{CFT}} = \frac{2\pi}{\beta} \left[ \Delta + \sum_n \bar{\mu}_n Q_n \right] \]

Do these match?
\[ \Delta = 1 + \lambda, \text{ and } Q_3 = \frac{1}{3} (1 + \lambda)(2 + \lambda), \]

Gaberdiel-Gopakumar 2010,

Gaberdiel-Hartman 2011, Ammon-Kraus-Gutperle 2011

QNF = relaxation rate

Integrable CFT’s thermalize. The thermalization is described by a new class of models (HS BH).