Thermalization in integrable models and conformal field theories

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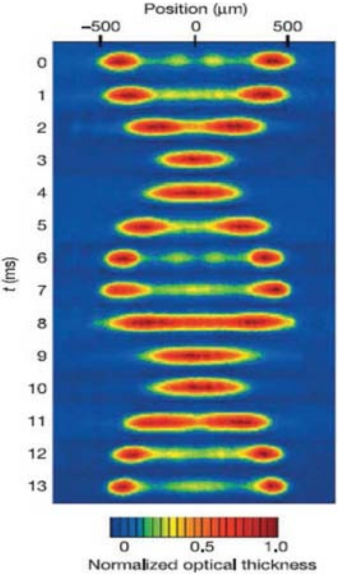
TIFR, Mumbai

Mid-year meeting, IASc, 23 June, Bengaluru

Based on: GM, R. Sinha, N. Sorokhaibam (1405.6695, 1501.04580), GM, T. Morita (1302.0859), P. Caupta, GM, R. Sinha (1306.4974), ongoing work with S. Paranjape, R. Sinha, N. Sorokhaibam and T. Ugajin

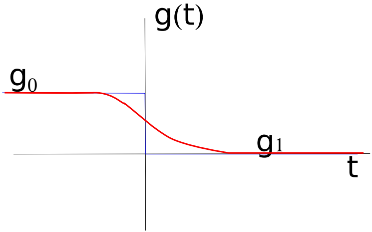
To thermalize or not to thermalize?



Quantum quench

Consider a quantum system in its ground state. Turn on a time-dependent coupling $g(t)$ for some time up to $t = t_1$.

$$\text{e.g.} \quad H(t) = -J \sum_{i=1}^{L} [\sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z]$$



The post-quench dynamics is described by a final Hamiltonian H and an 'initial state' $|\psi_1\rangle$, which depends on $g(t)$.

Late time dynamics: thermalization

Post-quench:

$$|\psi(t)\rangle = \exp[-iH(t - t_1)]|\psi_1\rangle$$

Does the system reach a steady state at 'late times'?

Does the final state 'forget' most features of the initial state? In particular, <u>is the state 'thermal'</u>?

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Of course, we cannot have pure state $\rightarrow$ mixed state.

A more accurate statement of thermalization is ...

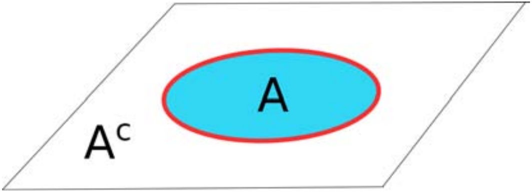
Definition of thermalization

$$\langle\psi_1|O_1(x_1,t)...O_n(x_n,t)|\psi_1\rangle \xrightarrow{t>t_{eqm}} (O_1(x_1)...O_n(x_n)\rho_{eqm})$$

Equivalent statement in terms of density matrix of subsystem A

$$\rho_A(t) \xrightarrow{t>t_{eqm}} \rho_{A,eqm}$$

where $\rho_A(t) = \mathrm{Tr}_{A^c}|\psi(t)\rangle\langle\psi(t)|, \ \rho_{A,eqm} = \mathrm{Tr}_{A^c}\rho_{eqm}$



<u>This formalizes the concept of the rest of the system as a ‘bath’.</u>

Quantum Ergodic Hypothesis

QEH: An equilibrium state ρ_{eqm} exists, and it is given by the microcanonical ensemble

$$\rho_{eqm} = \rho_{micro}$$

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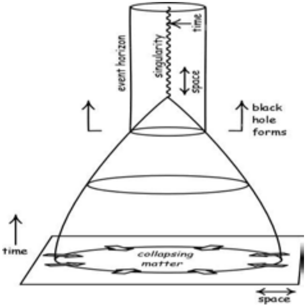
Is the QEH true?

Thermalization in gravity: gravitational collapse



No hair theorem: different forms and descriptions of matter, collapse into a black hole characterized by only the total mass (and angular momentum and charge) of the collapsing matter.

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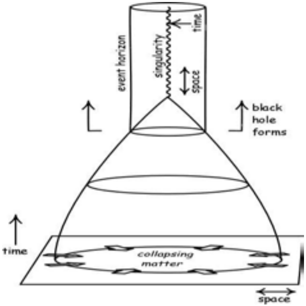

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Correct way to understand this is in terms of the reduced density matrix: $|\rho_A(t)\rangle \rightarrow \rho_{A;M,J,Q}$

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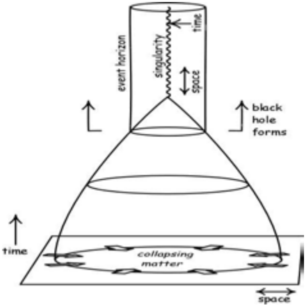

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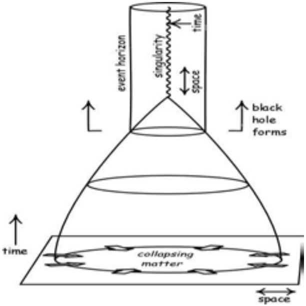

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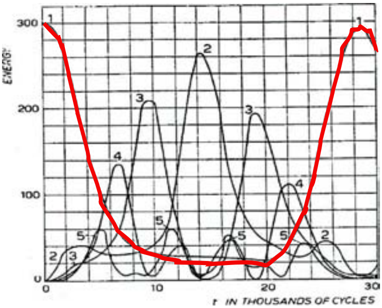
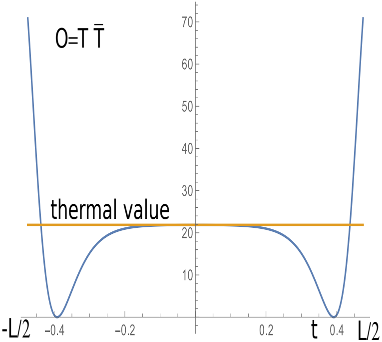
AdS/CFT: gravitational collapse= thermalization in field theory.

Non-thermalization



Weakly anhamonically coupled chain of oscillators show 'revival'

Fermi, Pasta, Ulam 1953



1+1 dimensional critical system with spatial boundaries; shows periodicity $\Delta T = L/2$.

Mandal, Sinha, Ugajin 2015; see also Cardy 2014, Kuns, Marolf 2014

AdS/CFT: For gravitational duals of non-ergodic systems, see Balasubramanian, Buchel, Green, Lehner, Leibling 2014

Integrable systems: recent surprises

Expect: integrability $\Rightarrow$ non-ergodicity.

But, examples of 2D integrable models have been discovered in the last 8 years, where QEH holds.

Transverse field Ising (Calabrese et al 2005)

$H = -J \sum_{i=1}^{L} \left[\sigma_i^x \sigma_{i+1}^x + h(t)\sigma_i^z\right]$

Hard core boson chain (Rigol et al 2007)

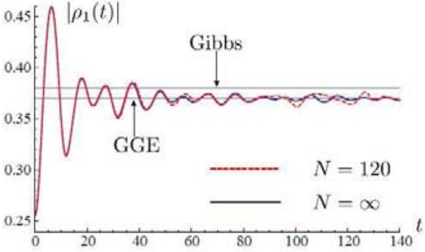
$H = -J \sum_{i=1}^{L(t)} b_i^\dagger b_{i+1} + \mathrm{h.c.}$

Massive Scalar(Sotiriadis, Cardy 2010)

$S = \int d^2x \left[(\partial\phi)^2 - m^2(t)\phi^2\right]$

Matrix QM model (Morita, GM 2013)

$S = \int dt \left[\mathrm{Tr}(U^\dagger \partial_t U + a(t)(U + U^\dagger)\right]$



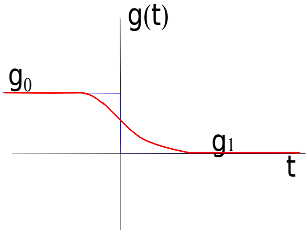
(a) $\rho_1(t)$ and GGE vs. Gibbs ensemble

Morita, GM 2013

Critical Quench

RG flow to a critical point leads to universality classes of systems.

Suppose in course of a quench, $g(t)$ stops at a value g_1 for which the system is critical.



Does the ‘dynamics’ of various systems show some universality in this case?

Rest of the talk

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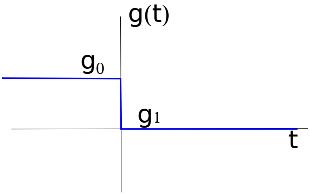
Critical quench: Calabrese-Cardy ansatz

For a sudden quench to a quantum critical coupling g_1

$$S = S_{CFT} + \delta g \int d^2x \theta(-t) O_\Delta(x,t), \quad \delta g \equiv g_0 - g_1$$



Calabrese and Cardy (2005) modelled the state at $t = 0$ as $|\psi_1\rangle = e^{-\kappa H}|Bd\rangle$

Here $\kappa \sim (\delta g)^{\frac{1}{\Delta - 2}}$ = [length]. $|Bd\rangle$ =conformal boundary state.

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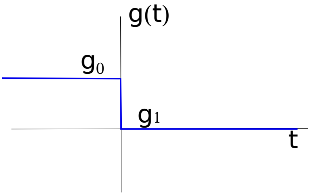

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Logic: At low energies $\kappa E \ll 1$, $\langle \psi_k | E\rangle = f(\kappa E)$= constant ($\Rightarrow$ conformality)

Generalized Calabrese-Cardy states

CC state fails to consider:
(a) Multiple scales: when the quench protocol $g(t)$ has more scales, e.g. $g(t) = g_0\, f(t/\delta t)$,
(b) Multiple charges: when there are other conserved charges, the simple QEH $\rho_{eqm} = \rho_{micro}(E)$ is inadequate.

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We propose that in a CFT with additional charges W_n we can solve both issues with a generalized Calabrese-Cardy (gCC) state

$$|\psi_1\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \ldots\right)} |Bd\rangle,$$

Require: W_n obtained from local currents which are primary or quasiprimary operators.

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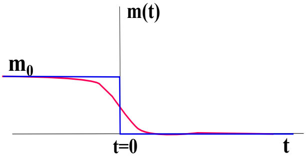
$$|\psi_1\rangle = e^{-\left(\kappa_2 H + \sum_n \kappa_n W_n + \dots\right)} |Bd\rangle,$$

This includes integrable conformal theories with ∞ number of conserved charges.

Example of scalar field theory

$$S = \int d^2x \left[(\partial\phi)^2 - m(t)^2 \phi^2 \right]$$



Time-dependent coupling induces a Bogoliubov transformation:

$a_{in}(k) \propto a_{out}(k) - \gamma(k) a^{\dagger}_{out}(k)$..

Hence

$|0, in\rangle = e^{\sum_k \gamma(k) a^{\dagger}_{out}(k) a^{\dagger}_{out}(-k)} |0, out\rangle$

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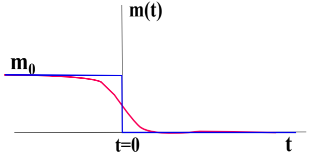

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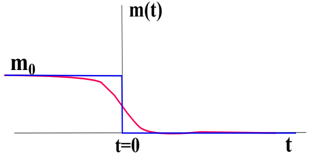

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Here $W_n \sim \sum_k k^{n-1} a^\dagger_{out}(k)a_{out}(k)$, $n = 2, 4, 6, \ldots$

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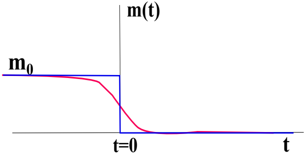

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- This example verifies the generalized quench model (gCC).

Reduced density matrix

Consider a 2-site ‘lattice’ of quantum Ising spins. Prepare a wavefunction

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow,\downarrow\rangle \; + \; |\downarrow,\uparrow\rangle \right).$$

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The reduced density matrix of the 1st spin (=subsystem A) is given by

$$\langle s_1|\rho_A|s_1'\rangle = \sum_{s_2=\uparrow,\downarrow} \langle s_1, s_2|\psi\rangle\langle\psi|s_1', s_2\rangle = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix}$$

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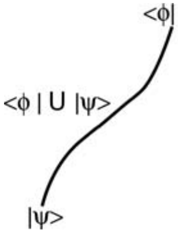
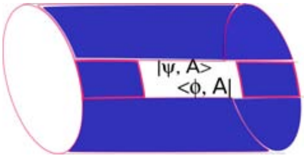
ρ_A can calculate all operators on site 1. Note $\rho_A^2 \neq \rho_A$.

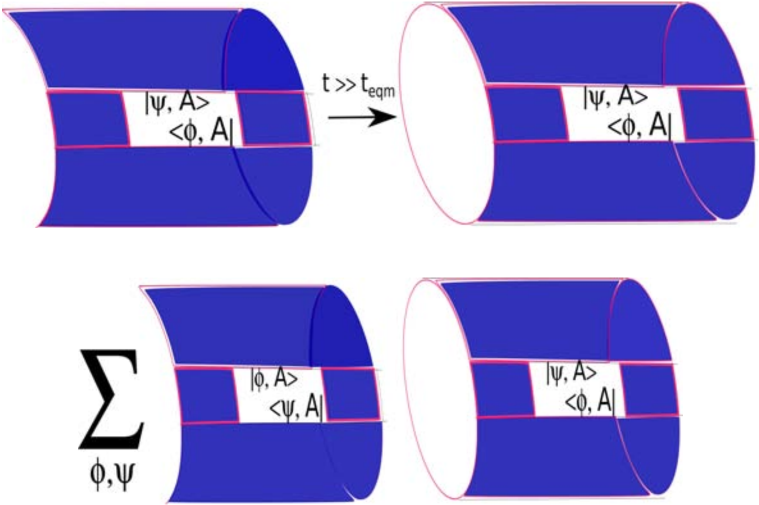
Reduced density matrix- contd.





Functional integral representation of the density matrix element $\langle\phi, A|\rho_A|\psi, A\rangle =$ $\sum_n \langle n, A^c; \phi, A|\rho|\psi, A; n, A^c\rangle$. In the picture $\rho \sim \exp[-\beta H]$.

Proof of Quantum ergodicity



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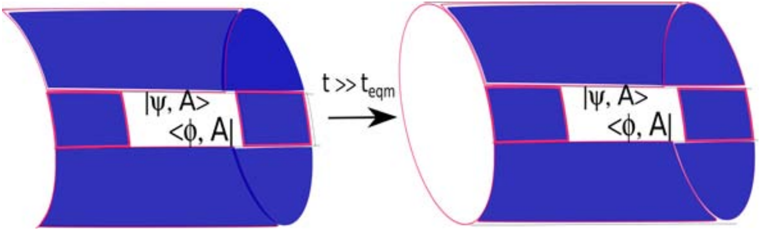

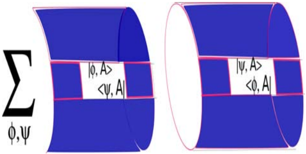

$$= \sum_{k_1,k_2} C_{k_1,k_2} \langle O_{k_1} \rangle_{strip} \langle O_{k_2} \rangle_{cylinder}.$$
$$= 1 - \alpha e^{-2\gamma t}$$

Thus,

$\rho_A(\kappa_2, \{\kappa_n\}; t) \xrightarrow{t \gg t_{eqm}} \rho_A(\beta, \{\mu_n\})$ where $\beta = 4\kappa_2, \mu_n = 4\kappa_n$.

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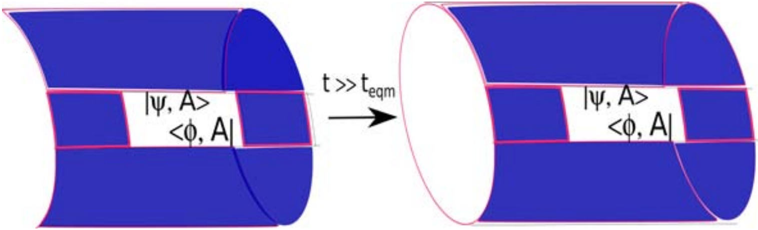

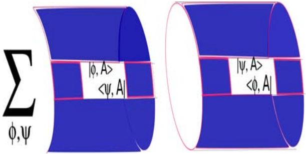

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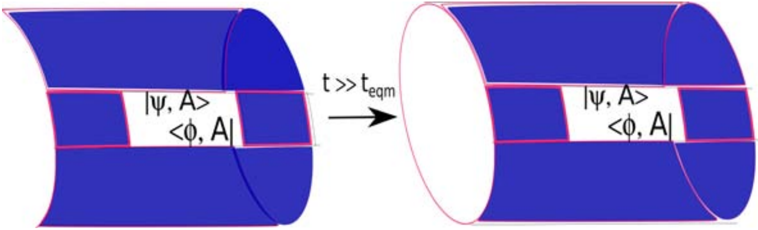

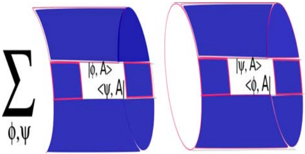

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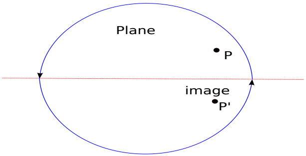
Relaxation rates $\Leftarrow$ critical exponents



$$\langle\kappa|O_\Delta(t)|\kappa\rangle \sim \mathrm{Tr}(\rho_\beta O_\Delta) + Ce^{-\gamma t}, \quad \gamma = 2\pi\Delta/\beta. \quad \beta = 4\kappa.$$

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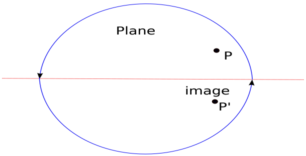

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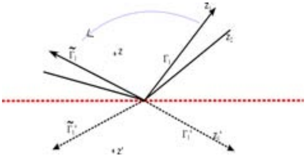
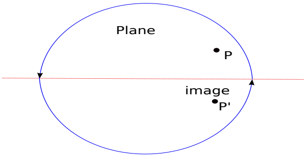
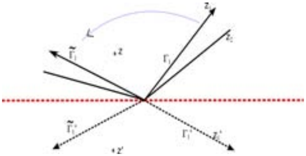


$$\langle\kappa_n|O_\Delta(t)|\kappa_n\rangle \sim \mathrm{Tr}(\rho_{\beta,\mu_n} O_\Delta) + Ce^{-\gamma t}, \quad \text{where}$$

Relaxation rates $\Leftarrow$ critical exponents



$$\langle \kappa | O_\Delta(t) | \kappa \rangle \sim \mathrm{Tr}(\rho_\beta O_\Delta) + C e^{-\gamma t}, \quad \gamma = 2\pi\Delta/\beta. \quad \beta = 4\kappa.$$



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$$\gamma_i = \frac{2\pi}{\beta}\left[\Delta_i + \sum_n \tilde{\mu}_n Q_{n,i} + O(\tilde{\mu}^2)\right], \ \tilde{\mu}_n \equiv \frac{\mu_n}{\beta^{n-1}}$$

New Universality relations

Cardy-Calabrese state: ratios $\gamma_i/\gamma_j = \Delta_i/\Delta_j$ are universal; independent of quench protocol.

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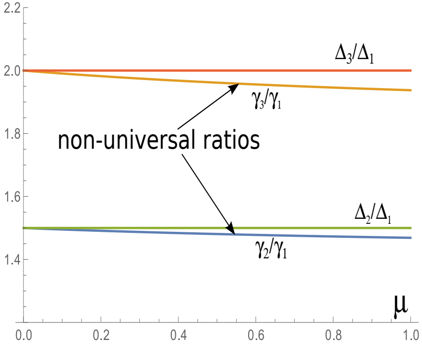
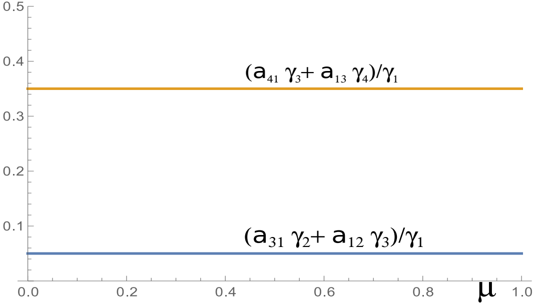
Generalized CC state (multi-scale quench): these ratios $\neq$ universal. E.g. in the presence of one extra charge, we have $\gamma_i = \frac{2\pi}{\beta}\left[\Delta_i + \tilde{\mu} Q_i\right]$.

The ratios γ_i/γ_j now depend on μ:

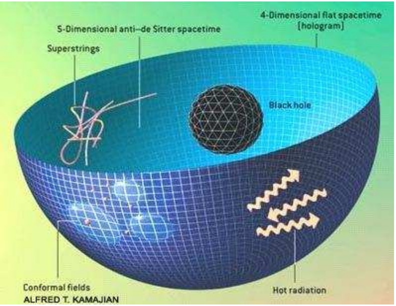


New Universality relations- contd.

μ-dependence can be eliminated by considering new ratios: $(a_{31}\gamma_2 + a_{12}\gamma_3)/\gamma_1$, $(a_{41}\gamma_3 + a_{13}\gamma_4)/\gamma_1$, with $a_{ij} = \Delta_{[i}Q_{j]}$, are independent of μ, and depend only on the spectrum of the final CFT.



AdS/CFT Dictionary (holography)



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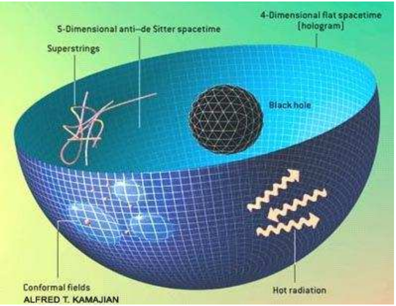

Thermalization in the boundary field theory= gravitational collapse into a black hole in the bulk!

Thermalization and holography

CFT	Gravity
Vacuum	Anti de Sitter space
Thermal state (β)	AdS-Schwarzschild ($M(\beta)$)
Thermal state with μ	Charged BH
Quantum Quench	Gravitational collapse
Ergodicity	No hair theorem
Thermal decay	Quasinormal mode
Thermalization/relaxation rate	Quasinormal frequency (QNF)
Integrable CFT (2D)	**?**
GGE (generalized Gibbs ensemble)	**?**
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QNF has been calculated for bulk scalar Cabo-Bizet, Gava, Giraldo-Rivera, Narain 2014 for a HS BH with a single chemical potential μ_3

Match made!

Imaginary part of QNF

$$\mathrm{Im}\,\omega = \frac{2\pi}{\beta}\left(1 + \lambda + \frac{\tilde{\mu}_3}{3}(1+\lambda)(2+\lambda)\right)$$

Thermalization rate of the dual CFT operator is

$$\gamma_{CFT} = \frac{2\pi}{\beta}\left[\Delta + \sum_n \tilde{\mu}_n Q_n\right]?$$

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