ECG and the Heralding of nonclassical optics 1963

- ECG's passing interest in optics dates back to his days at MCC.
- Manuel A. Thangaraj – portrait at imsc
- Classical theory of optical coherence was developed by Emil Wolf.
  for free fields:
  $\langle E^-(\vec{x}_1, t_1) \cdots E^-(x_n, t_n)\, E^+(x_1', t_1') \cdots E^+(x_m', t_m') \rangle$
- Analytic signal – Denis Gabor
- Mandel's theory connects the photo-electron counting statitis to such coherence functions
- Hanbury-Brown & Twiss effect (1956) is explained as intensity-intensity correlation $(m = n = 2)$

- ECG was always proud of his role in bringing Mandel to Rochester.

1963

Optics is joined by two high energy physicists.

- Glauber, "Photon Correlations"
  PRL – submitted 27 Dec 62
  published 1 Feb. 63

- Glauber, "The Quantum Theory of Optical Coherence"
  Phys. Rev. ~~sub. 15 June 63 pub~~
  sub 11 Feb 63 pub. 15 Jun 63

- ECG: "Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams"
  PRL. sub. 1 Mar 63 pub. 1 Apr. 63

- Glauber, "Coherent and Incoherent States of the radiation field"
  Phys. Rev. sub. 29 Apr. 63 pb. 15 Sep 63

- Coherent states are eigenstates of the positive frequency part $E^+(z,t)$ of the field operator (annihilation)
- Coherent state

$$|z\rangle = \exp\left(-\tfrac{1}{2}|z|^2\right)\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle$$

$$\hat{a}|z\rangle = z|z\rangle$$

- If a state ~~is~~ ρ is diagonal in the coherent state basis:

$$\hat{\rho} = \int \phi(z)\,|z\rangle\langle z|\,d^2z$$

then evaluation of the expectation value of a normally ordered operator

$$\hat{F} = \sum_{n,m} f_{nm}\,(\hat{a}^\dagger)^m\hat{a}^n$$

is simple:

$$\langle\hat{F}\rangle = tr(\rho\hat{F}) = \int \phi(z)\; z^{*m} z^n f_{nm}\, d^2z$$

= phase space average of the corresponding classical function

$$F = \sum_{n,m} z^{*m} z^n f_{m,n}$$

52, 337 (1962).

[6]L. Mandel, J. Opt. Soc. Am. 52, 1337, 1408 (1962).

[7]O. S. Heavens, Suppl. Appl. Opt. 1, 4 (1962).

[8]W. Bothe, Z. Physik 41, 345 (1927).

[9]E. M. Purcell, Nature 178, 1449 (1956).

[10]L. Mandel, Proc. Phys. Soc. (London) 72, 1037 (1958); 74, 233 (1959).

[11]E. Wolf, Proc. Phys. Soc. (London) 76, 424 (1960).

[12]C. T. J. Alkemande, Physica 25, 1145 (1959).

[13]E. Wolf, Proceedings of the Third Symposium on Quantum Electronics, Paris, February, 1963 (to be published).

[14]L. Mandel, Proceedings of the Third Symposium on Quantum Electronics, Paris, February, 1963 (to be published).

[15]R. J. Glauber, Proceedings of the Third Symposium on Quantum Electronics, Paris, February, 1963 (to be published).

[16]L. Mandel, J. Opt. Soc. Am. 51, 797 (1961).

[17]Since this was written, E. C. G. Sudarshan [Phys. Rev. Letters 10, 277 (1963)] has demonstrated that the semiclassical and the quantum mechanical descriptions are completely equivalent to each other as long as no nonlinear effects are considered.

EQUIVALENCE OF SEMICLASSICAL AND QUANTUM MECHANICAL DESCRIPTIONS OF STATISTICAL LIGHT BEAMS

E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York
(Received 1 March 1963)

With the advent of the laser, attention has been focused on the problem of the complete description of the electromagnetic field associated with arbitrary light beams. The classical theory of optical coherence[1] works almost exclusively with two-point correlations; and this theory is adequate for the description of the classical optical phenomena of interference and diffraction in general. More sophisticated experiments on intensity interferometry and photoelectric counting statistics necessitated special higher order correlations. Most of this work[2] was done using a classical or a semiclassical formulation of the problem. On the other hand, statistical states of a quantized (electromagnetic) field have been considered recently,[3] and a quantum mechanical definition of coherence functions of arbitrary order presented. It is the aim of this note to elaborate on this definition and to demonstrate its complete equivalence to the classical description as long as no nonlinear effects are considered.

We begin with an outline of the analytic function representation[4] of canonical creation and destruction operators. If a and $a^\dagger$ satisfy the relations

$$[a, a^\dagger] = 1,$$

every irreducible representation is equivalent to the Fock representation in terms of the states $\psi(n)$, satisfying

$$a^\dagger a \psi(n) = n\psi(n); \quad (\psi(m), \psi(n)) = \delta_{mn}.$$

The matrix elements of a and $a^\dagger$ in this representation are

$$(\psi(m), a\psi(n)) = \sqrt{n}\,\delta_{m,n-1},$$

$$(\psi(m), a^\dagger\psi(n)) = (n+1)^{1/2}\delta_{m,n+1}.$$

One could, however, introduce an overcomplete set of eigenstates of the destruction operator given by

$$|re^{i\theta}\rangle \equiv |z\rangle = \exp(-\tfrac{1}{2}|z|^2)\sum_{n=0}^{\infty}\frac{z^n}{(n!)^{1/2}}\psi(n), \qquad (1)$$

satisfying

$$a|z\rangle = z|z\rangle; \quad \langle z|a^\dagger = z^*\langle z|; \quad \langle z|z\rangle = 1,$$

for every complex number z. These states are all normalized but not orthogonal[5]; they are complete in the sense that they furnish a resolution of the identity

$$1 = (1/\pi)\iint r\,dr\,d\theta\,|re^{i\theta}\rangle\langle re^{i\theta}|.$$

More generally,

$$\int\frac{d\theta}{2\pi}|re^{i\theta}\rangle\langle re^{i\theta}| = e^{-r^2}\sum_{n=0}^{\infty}\frac{r^{2n}}{n!}\psi(n)\psi^\dagger(n). \qquad (2)$$

We can make use of the overcompleteness[6] of the states to represent every density matrix,

$$\rho = \sum_{n=0}^{\infty}\sum_{n'=0}^{\infty}\rho(n,n')\psi(n)\psi^\dagger(n'),$$

optical coherence[1] works almost exclusively with two-point correlations; and this theory is adequate for the description of the classical optical phenomena of interference and diffraction in general. More sophisticated experiments on intensity interferometry and photoelectric counting statistics necessitated special higher order correlations. Most of this work[2] was done using a classical or a semiclassical formulation of the problem. On the other hand, statistical states of a quantized (electromagnetic) field have been considered recently,[3] and a quantum mechanical definition of coherence functions of arbitrary order presented. It is the aim of this note to elaborate on this definition and to demonstrate its complete equivalence to the classical description as long as no nonlinear effects are considered.

We begin with an outline of the analytic function representation[4] of canonical creation and destruction operators. If a and $a^\dagger$ satisfy the relations

$$[a, a^\dagger] = 1,$$

every irreducible representation is equivalent to the Fock representation in terms of the states $\psi(n)$, satisfying

$$a^\dagger a \psi(n) = n\psi(n); \quad (\psi(m), \psi(n)) = \delta_{mn}.$$

The matrix elements of a and $a^\dagger$ in this represen-

EQUIVALENCE OF SEMICLASSICAL AND QUANTUM-MECHANICAL DESCRIPTIONS OF STATISTICAL LIGHT BEAMS

E. C. G. Sudarshan
Department of Physics and Astronomy, University of Rochester, Rochester, New York

The purpose of this brief presentation is to furnish a bridge between the generalized classical theory of coherence[1] of optical wavefields and coherence notions in quantum electrodynamics. Our present understanding of the properties of light beams and the interaction of light and matter are consistent with quantum electrodynamics and the wave particle dualism entailed. On the other hand, classical coherence theory has been adequate to handle interference and diffraction phenomena; and with the generalization to include higher order correlation functions, to handle intensity interferometry and photon counting statistics as well. Traditionally, a classical treatment of a quantum-mechanical system has been valid in the correspondence principle limit of multiple quantum excitations; and one is tempted to invoke the correspondence principle to "explain" the success of the (generalized) classical coherence description. This asymptotic connection is, however, misleading since, in most optical phenomena, the excitations of the various photon modes is by no means large.

In the sequel, we develop the connection between the classical coherence description and a quantum (field theoretic) description from an entirely different point of view. We show[2] that the classical and quantum mechanical *descriptions* are completely equivalent, in the sense that to every probability distribution function (or, more appropriately, the probability functional) there corresponds linearly a unique quantum-mechanical density matrix. Further, this correspondence is invertible; there is a classical probability distribution function corresponding to every density matrix. The correspondence is such that the expectation values of classical dynamical variables calculated using the classical probability distribution coincide with the expectation values of normal ordered (operator) dynamical variables calculated using the quantum-mechanical density matrix.

Presented at the Symposium on Optical Masers,
Polytechnic Institute of Brooklyn, April 16, 17, 18, 19, 1963.

- ECG proved that EVERY state ρ of an oscillator is diagonal in the coherent state basis:

$$\hat{\rho} = \int \phi(z) \, |z\rangle\langle z| \, d^2 z$$

- Proved by presenting the inversion formula – $\phi(z)$ in terms of $\hat{\rho}$.

Hermiticity of $\hat{\rho} \Longleftrightarrow$ reality of $\phi(z)$

$tr\,\hat{\rho} = 1 \Longleftrightarrow \int d^2 z \, \phi(z) = 1$

But $\hat{\rho} \geq 0$ does not transcribe to pointwise nonnegativity of $\phi(z)$. It can take negative value in some small regions.

$\phi(z)$ can be very singular

– like $(2n)^{th}$ derivative of Dirac delta function

– like Fourier transform of $\phi(z)$ can be a growing gaussian.

- $\phi(z)$ always occur as kernel in an integral.

ECG's diagonal weight function $\phi(z)$ is the first time quasi-probability entered optics.

If $\phi(z)$ ~~[illegible]~~ of the state of a radiation field is a true probability, the ~~corr.~~ state can be described exactly by statistical ensemble of classical fields.
otherwise, it is beyond the scope of classical statistical optics.
It is a nonclassical state!

Nonclassical states are hard to come by!

They are quite fragile!

- A laser beam is typically in a coherent state.
- It is the most classical state we have.

- Nonclassical light is a rare commodity, and high value resource.
- Glauber was never able to put his finger on one of them.
- For a coherent state

$$|z\rangle = \exp\left(-\tfrac{1}{2}|z|^2\right)\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{n!}}|n\rangle$$

$$\langle n\rangle = |z|^2 = \langle z|a^{\dagger}a|z\rangle$$

$$\langle n^2\rangle - \langle n\rangle^2 = \langle n\rangle$$

$$p(n) = \exp\left(-|z|^2\right)\frac{\left(|z|^2\right)^n}{n!}$$

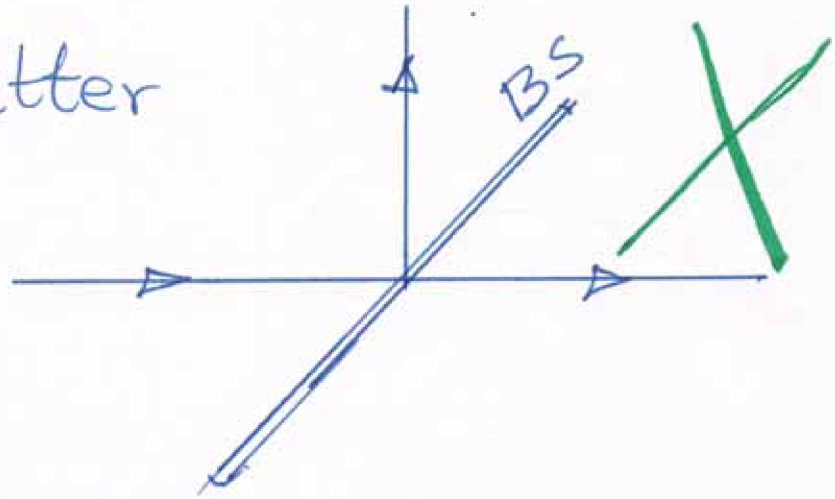
(1) $\langle n^2\rangle - \langle n\rangle < \langle n\rangle \Rightarrow$ antibunching or negative H-B-T $\Rightarrow$ nonclassicality.

(2) For Vacuum, fluctuation in any quadrature is $\frac{1}{2}$. True for $|z\rangle$ also.

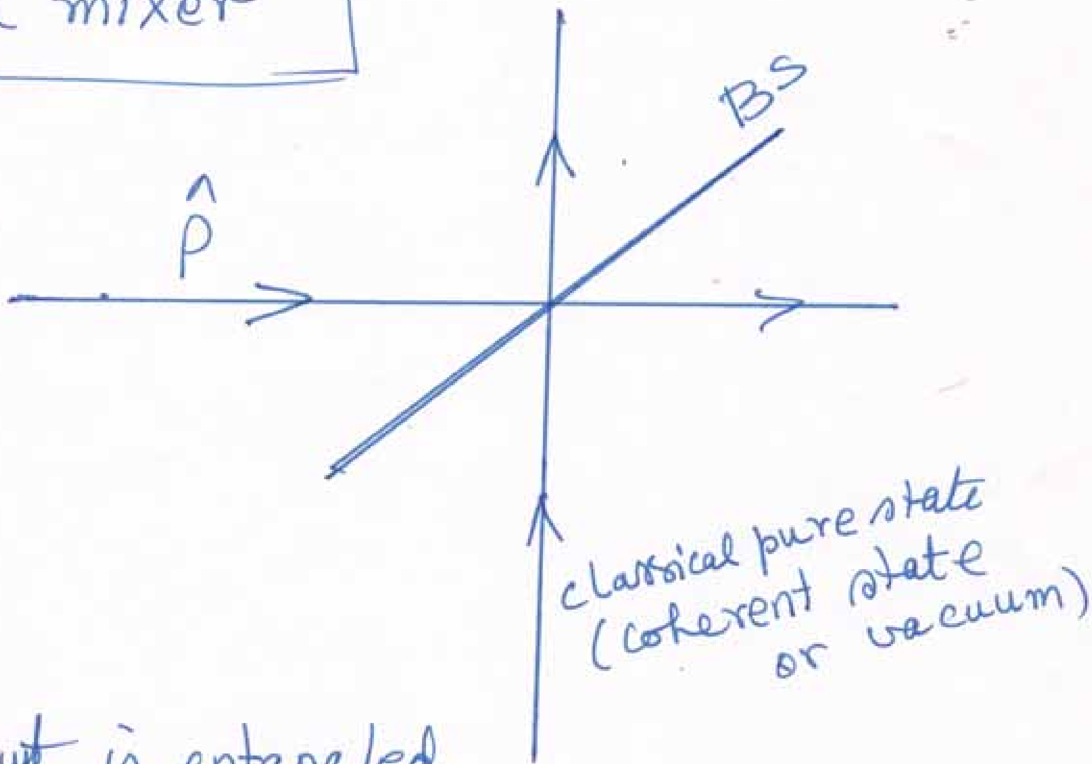
If it is $< \frac{1}{2}$ for some quadrature = squeezing $\Rightarrow$ nonclassicality.

Nonclassicality & Entanglement

Beam splitter



Not a splitter, but a mixer



output is entangled, iff $\hat{\rho}$ is nonclassical.

Measure of output entanglement
$\Rightarrow$ Measure of input nonclassicality?

where n is the photon number in the field. In the Hanbury Brown & Twiss phenomenon, the induced photons are few, and this was assumed to account for the factor of two; one photon may induce another one. In a laser, many photons contribute and one may predict a giant effect. On the other hand, there lingered an impression that quantum noise only supplied ripples on the field amplitudes of the classical fields. Thus random function theory would account for the observed effects. As an example we quote [8]: "In the conditions under which light fluctuations are usually measured by photoelectric detectors, the semi-classical treatment applies as readily to light of non-thermal origin as to thermal light, and to non-stationary as well as to stationary fields." The subsequent experimental progress rapidly proved the short-comings of the semi-classical approach. The correct theory was published by Roy Glauber in 1963 [9], and this has been the basis for all subsequent theoretical considerations.

Quantum theory of optical interference experiments
Glauber's 1963 contribution

In Ref. [9], Glauber presents the basic features of his quantum theory of optical coherence. The formal features were expanded on in two long articles [10] in the same year. This material was to form the basis for the development of Quantum Optics up to the present time.

In the 1963 publication, Glauber made the following points:

In interference experiments, the phase of the light is important, and then the state is best represented in terms of *coherent states,* and defining a distribution function on these, Glauber introduced the concept of a quasidistribution into Quantum Optics. These are quantum descriptions of the state, which have straightforward relations to classical phase space distributions. Glauber shows that in certain cases they can be given by a diagonal representation in the coherent states. They do, however, display clearly non-classical features; thus, for example, for some simple quantum states they do not satisfy the positivity of a probability distribution. If the distribution is positive, we can give the state a classical interpretation. Glauber shows that the thermal light sources correspond to a Gaussian distribution, thus justifying, in this case, the use of fluctuation theory. The case of an ideal laser source shows no correlations of the Hanbury Brown & Twiss type.

Given unto Glauber What is
not his!