Quantum Zeno effect: a watched pot never boils!

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The Quantum Zeno Effect

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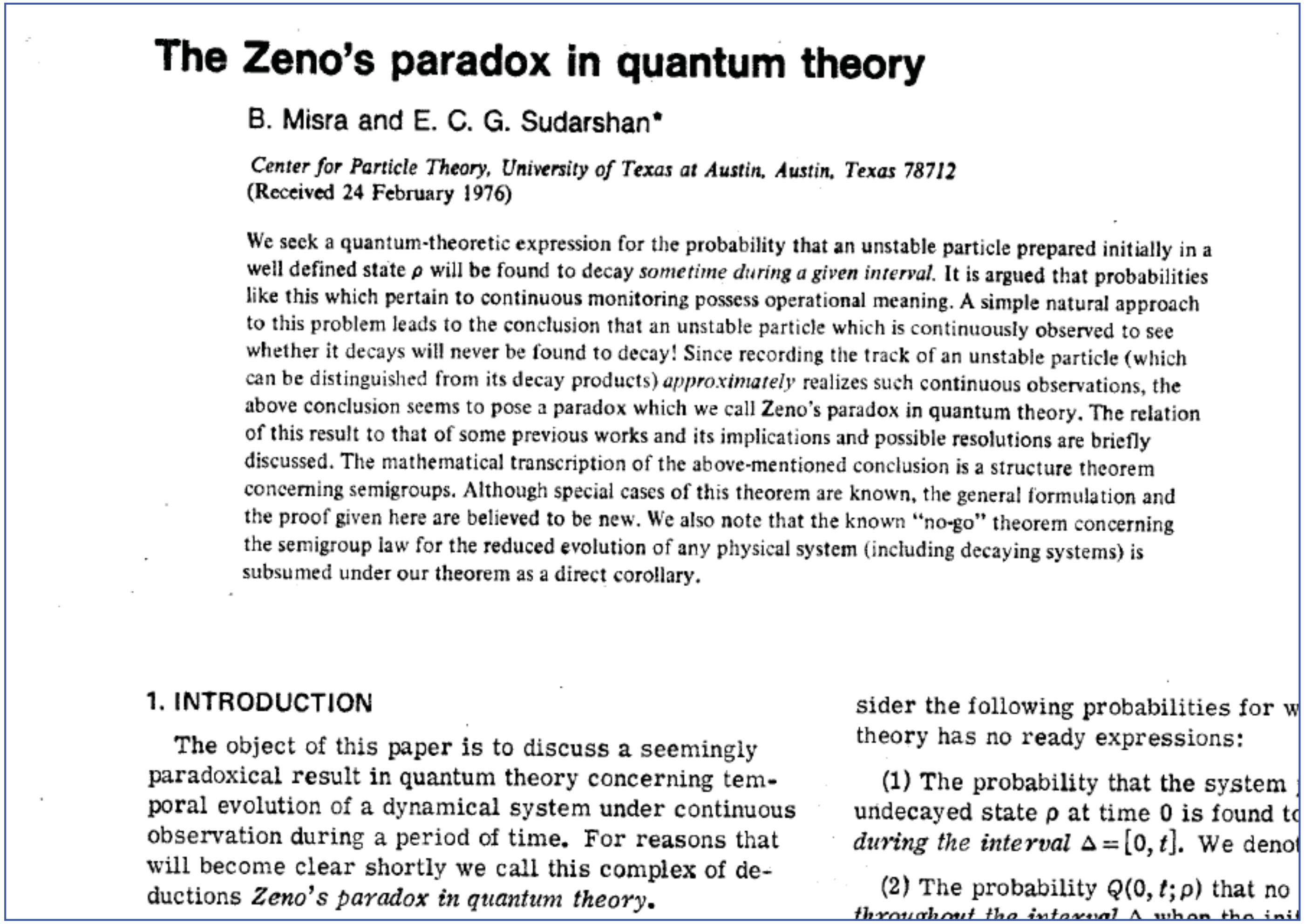
Life and Work of Prof. ECG Sudarshan
Annual Meeting - Indian Academy of Sciences -University of Hyderabad - **9-11- 2019**

"The Zeno's Paradox in quantum theory" B. Misra and E. C. G. Sudarshan
Journal of Mathematical Physics **18**, 756 (1977)



The Zeno's paradox in quantum theory

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(Received 24 February 1976)

We seek a quantum-theoretic expression for the probability that an unstable particle prepared initially in a well defined state ρ will be found to decay *sometime during a given interval.* It is argued that probabilities like this which pertain to continuous monitoring possess operational meaning. A simple natural approach to this problem leads to the conclusion that an unstable particle which is continuously observed to see whether it decays will never be found to decay! Since recording the track of an unstable particle (which can be distinguished from its decay products) *approximately* realizes such continuous observations, the above conclusion seems to pose a paradox which we call Zeno's paradox in quantum theory. The relation of this result to that of some previous works and its implications and possible resolutions are briefly discussed. The mathematical transcription of the above-mentioned conclusion is a structure theorem concerning semigroups. Although special cases of this theorem are known, the general formulation and the proof given here are believed to be new. We also note that the known "no-go" theorem concerning the semigroup law for the reduced evolution of any physical system (including decaying systems) is subsumed under our theorem as a direct corollary.

1. INTRODUCTION

The object of this paper is to discuss a seemingly paradoxical result in quantum theory concerning temporal evolution of a dynamical system under continuous observation during a period of time. For reasons that will become clear shortly we call this complex of deductions *Zeno's paradox in quantum theory.*

sider the following probabilities for
theory has no ready expressions:

(1) The probability that the system
undecayed state ρ at time 0 is found
during the interval $\Delta=[0,t]$. We

(2) The probability $Q(0,t;\rho)$ that no

Zeno of Elea was a pre-Socratic Greek philosopher of southern Italy and a member of the Eleatic School founded by Parmenides. He is best known for his paradoxes, which Bertrand Russell has described as "immeasurably subtle and profound”

Four paradoxes of motion

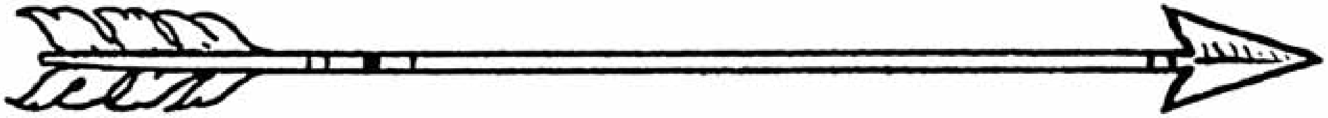
Achilles and the tortoise

The arrow

The dichotomy

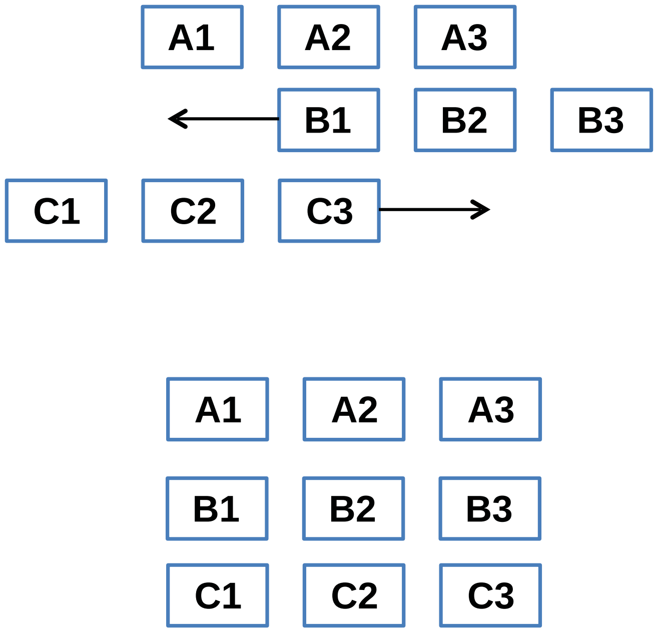
The sophisticated stadium

Achilles and the tortoise



The Arrow

The dichotomy



The sophisticated stadium

Zeno’s paradoxes remained unresolved for over 2,500 years - satisfactory resolutions came after the development of the concepts of real numbers, calculus of infinitesimals (Newton, Leibniz), functions, limits continuity, infinite series convergence, etc.

Attempts to understand Zeno’s paradoxes by philosophers helped shape modern concepts and definitions. Our ideas of space, time, motion, infinite, infinitesimal, line, point, derivative and measure would not be the same without Zeno’s input - in 2000 years, his arguments contributed to the foundations of modern mathematics.

1977 The return of Zeno

- in a scenario involving the time evolution of a quantum system which is subject to “observations” over a period of time

Misra & Sudarshan introduced the name “Zeno’s paradox” for the effect studied in their Journal of Mathematical Physics paper (**1977**) - their groundbreaking result activated over three decades of theoretical and experimental explorations into the subject.

Quantum Zeno Effect (QZE)

common term used to describe similar situations in various quantum systems

Misra and Sudarshan were the first to call the effect by the name “Zeno”, but closely related work was done earlier in **1957** by L A Khalfin working in the USSR

J. Von Neumann “Mathematical Foundations of Quantum Mechanics” (**1955**)

“The Zeno’s Paradox in quantum theory”
B. Misra and E. C. G. Sudarshan
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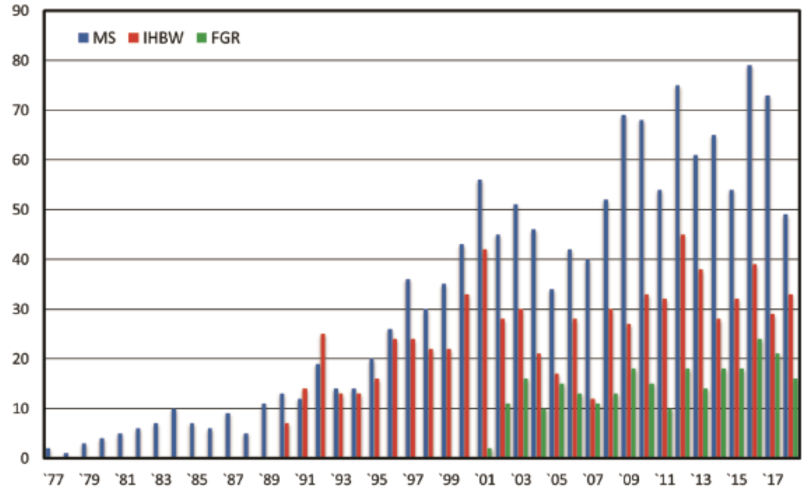


Figure 1. Graph of the number of citations per year to Misra and Sudarshan[2] (MS) in blue, Itano *et al.*[6] (IHBW) in red and Fischer *et al.*[8] (FGR) in green, from 1977 through late 2018.

“The quantum Zeno paradox, 42 years on” Wayne M. Itano, Current Science, Vol. 116, No. 2, 25 January 2019

The quantum Zeno effect is the name used to refer to the inhibition of transitions between quantum states due to frequent measurements

Measurements in quantum mechanics

Quantum Measurement

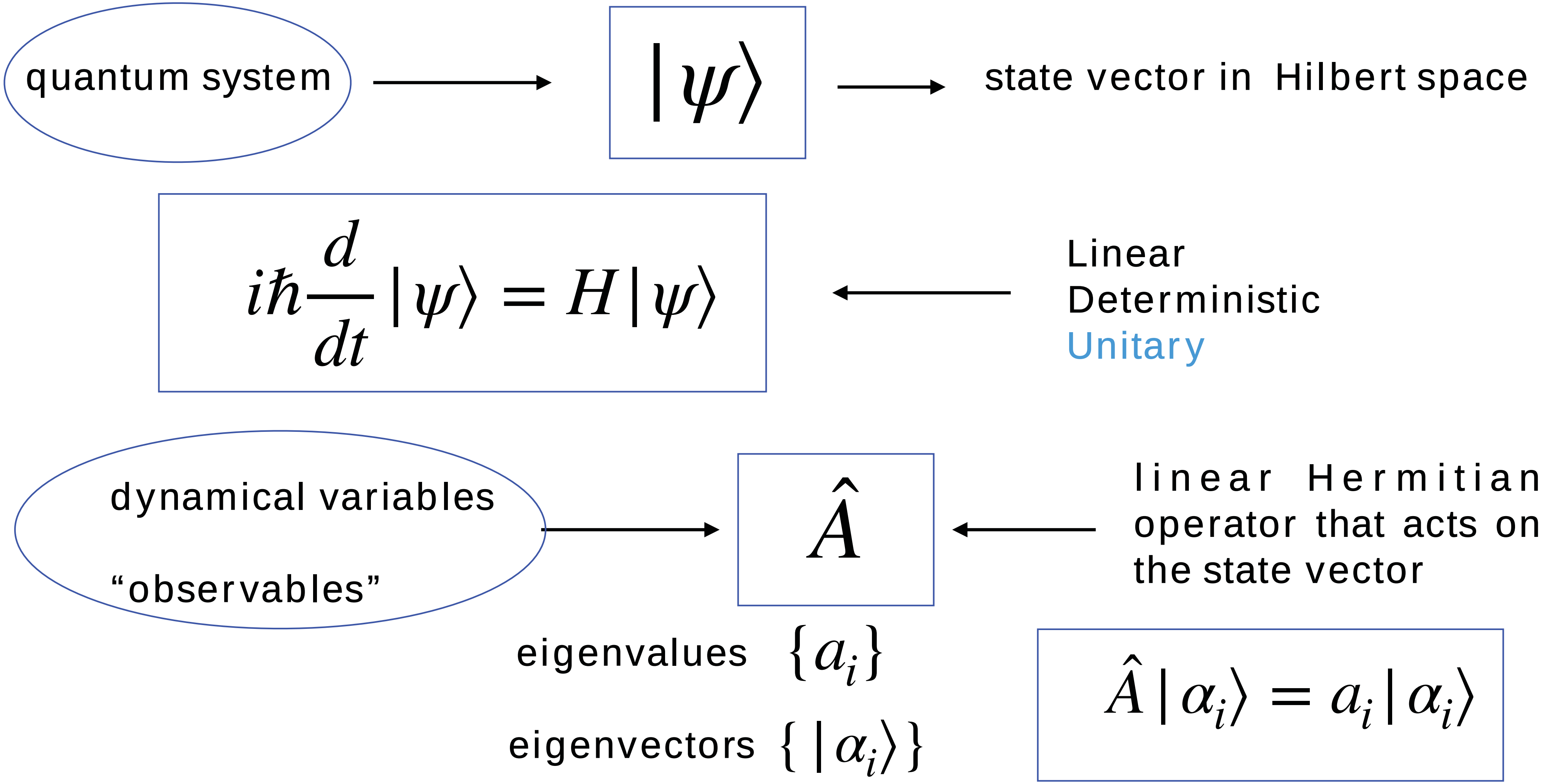
Quantum Mechanics - currently accepted as the most elegant and satisfying description of phenomena at the atomic scale - the fundamental theory of nature. Stunningly powerful but counterintuitive - compels us to reshape our ideas of reality and notions of cause, effect and measurement

An unobserved particle does not possess physical properties that exist independent of observation

The act of measurement on a system disturbs the quantum state of the system

This standard interpretation of quantum measurement is attributed to von Neumann (1932)

Quantum Measurement



quantum system → $|\psi\rangle$ → state vector in Hilbert space

$$i\hbar\frac{d}{dt}|\psi\rangle = H|\psi\rangle$$

← Linear
Deterministic
Unitary

dynamical variables

“observables” → $\hat{A}$ ← linear Hermitian operator that acts on the state vector

eigenvalues $\{a_i\}$

eigenvectors $\{|\alpha_i\rangle\}$

$$\hat{A}|\alpha_i\rangle = a_i|\alpha_i\rangle$$

Quantum Measurement

a measurement of $\hat{A}$ on the state $|\psi\rangle$

$$|\psi\rangle = \sum c_i |\alpha_i\rangle$$

upon a measurement of $\hat{A}$ *only* *one* of the eigenvalues, a_i , will be obtained, with probability $|c_i|^2$

The measurement of the observable $\hat{A}$ is followed by a collapse of the state vector $|\psi\rangle$ to the eigenstate $|\alpha_i\rangle$

$$|\psi\rangle = \sum c_i |\alpha_i\rangle \xrightarrow{\text{collapse}} |\alpha_i\rangle$$

Measurement of an observable culminates with the state collapsing to one of the eigenstates

Projection postulate (von Neumann)

non-unitary

"The Zeno's Paradox in quantum theory" B. Misra and E. C. G. Sudarshan
Journal of Mathematical Physics **18**, 756 (1977)

The Quantum Zeno Effect (QZE) is a name name given to the phenomenon of the inhibition of (spontaneous or induced) transitions between quantum states by frequent measurements. In their study, Misra and Sudarshan looked at an unstable quantum system and concluded that if an unstable quantum system is kept under continuous observations, it does not decay!

time evolution of a quantum system

initial state of the quantum system ("undecayed") → $|\psi_0\rangle$

state of the quantum system after time t → $|\psi(t)\rangle$

$$|\psi(t)\rangle = U(t)|\psi_0\rangle \; ; U(t) = e^{-iHt}$$

"Survival Probability"

$$P(t) = |\langle\psi_0|U(t)|\psi_0\rangle|^2$$

$$P(t) = |\langle\psi_0|e^{-iHt}|\psi_0\rangle|^2 \approx 1 - t^2\Delta H^2 + \ldots\ldots$$

$$\Delta H = \sqrt{\langle\psi_0|H^2|\psi_0\rangle - \langle\psi_0|H|\psi_0\rangle^2}$$

$$P(t) \approx 1 - \enclose{circle}{t^2}\Delta H^2 + \ldots\ldots$$

At short times the survival probability deviates from the exponential law and is quadratic in time

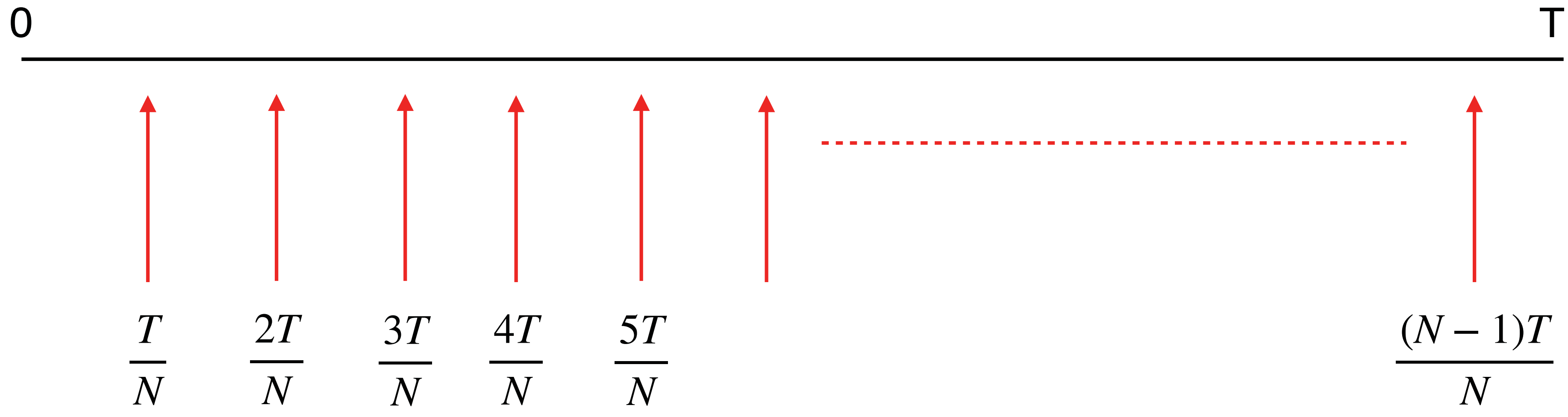
$$P(t) \approx 1 - \frac{t^2}{\tau_Z^2}\ ;\ \tau_Z = \frac{1}{\Delta H}$$

continuously “observing” the system
frequent quantum measurements

make N equally spaced instantaneous measurements over a time period [0, T]

$T = N\tau$

τ : time between two measurements



measurements ⟶ Projection postulate (von Neumann) what is the survival probability after N measurements at time intervals $\tau = T/N$ to check whether the system is still in its initial state $|\psi_0\rangle$?

After each measurement the system's state is “projected" back onto its initial state $|\psi_0\rangle$ and the time evolution starts anew with initial condition $|\psi_0\rangle$

The survival probability at the end of the interval T is

$$P^N(T) = P(\tau)^N = P\left(\frac{T}{N}\right)^N \approx \left(1 - \frac{T^2}{N^2\,\tau_Z^2}\right)^N \longrightarrow \lim_{N\to\infty} P^N(T) \to 1$$

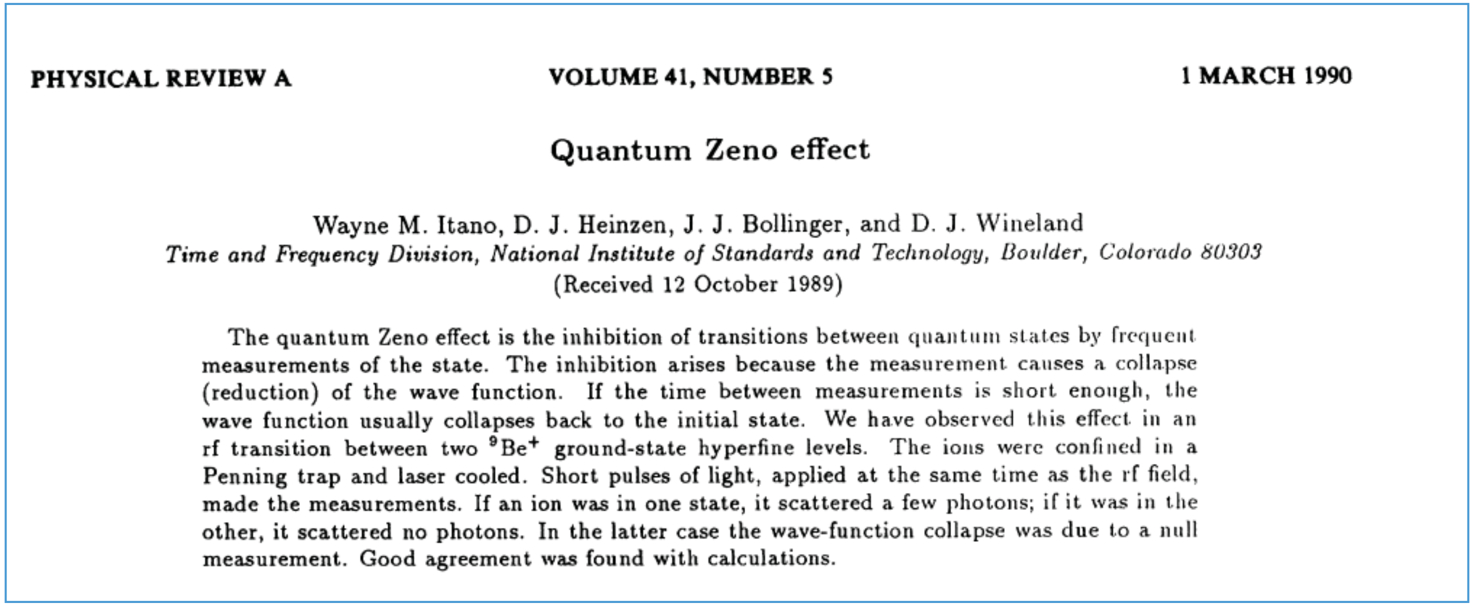
continuous measurements arrests the time evolution, “freezing” the system in its initial state - the quantum Zeno effect

1. quadratic short time behaviour of the survival probability
2. continuous quantum measurements as von Neumann projections (wave function collapse)

Can we see the QZE in in an experiment?

This effect (inhibition of spontaneous decay) could be very difficult to access experimentally – the interval during which the probability grows quadratically is very short compared to the time required to make a measurement

The experiment of Itano, et al at NIST Colorado (1990)
Proposal by Cook (1988)



PHYSICAL REVIEW A **VOLUME 41, NUMBER 5** **1 MARCH 1990**

Quantum Zeno effect

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(Received 12 October 1989)

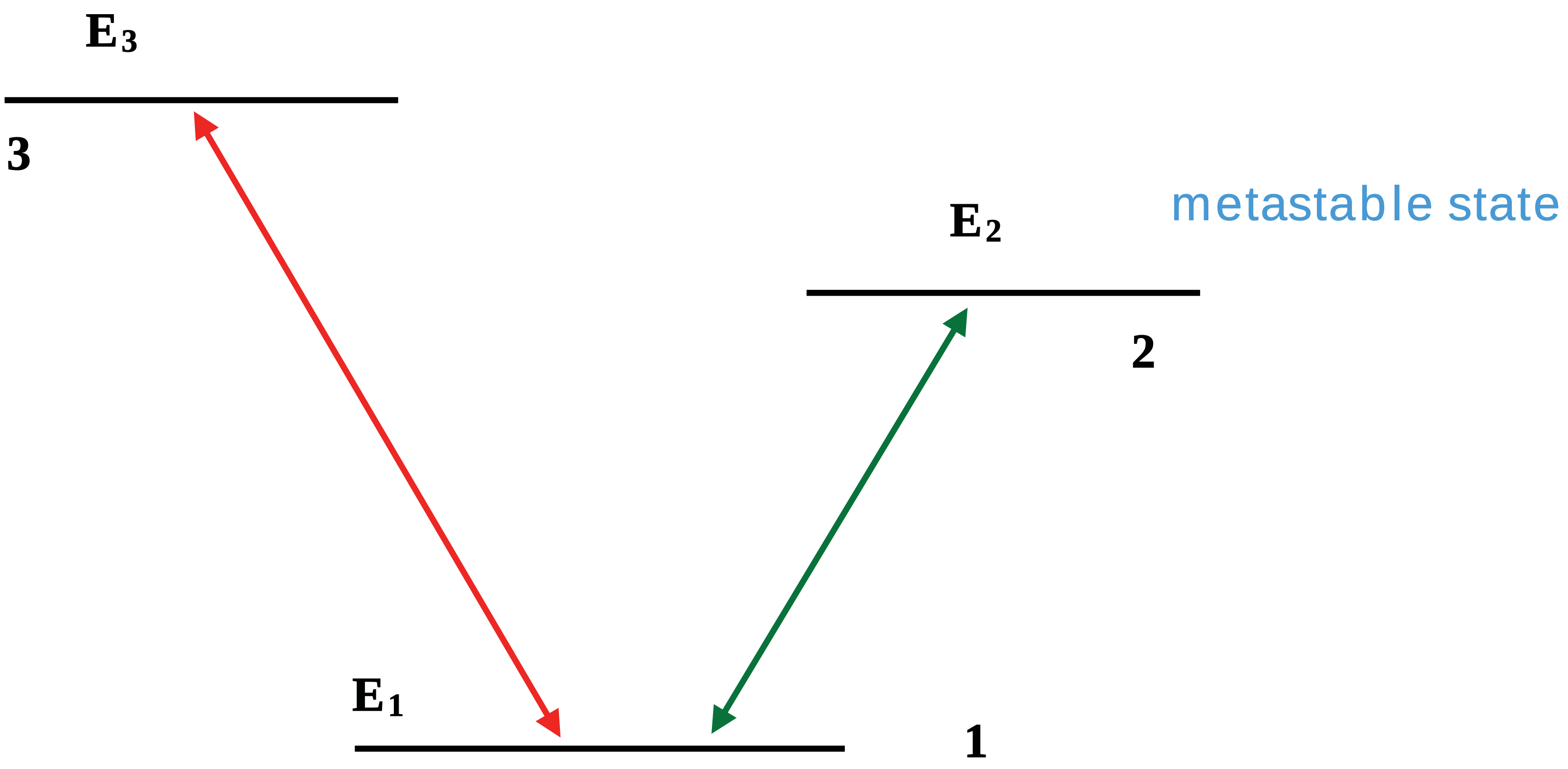
The quantum Zeno effect is the inhibition of transitions between quantum states by frequent measurements of the state. The inhibition arises because the measurement causes a collapse (reduction) of the wave function. If the time between measurements is short enough, the wave function usually collapses back to the initial state. We have observed this effect in an rf transition between two $^9\mathrm{Be}^+$ ground-state hyperfine levels. The ions were confined in a Penning trap and laser cooled. Short pulses of light, applied at the same time as the rf field, made the measurements. If an ion was in one state, it scattered a few photons; if it was in the other, it scattered no photons. In the latter case the wave-function collapse was due to a null measurement. Good agreement was found with calculations.

The quantum Zeno effect was observed in induced transitions between quantum states

↓

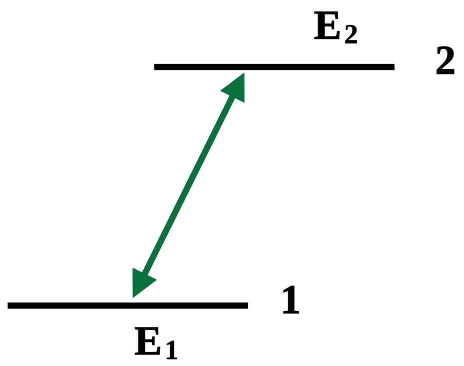
transitions between the ground state hyperfine levels of cooled and trapped Beryllium Ions

The experiment of Itano, et al at NIST Colorado (1990)
Proposal by Cook (1988)



The experiment of Itano, et al at NIST Colorado (1990)
Proposal by Cook (1988)

a two-level quantum system in the presence of a resonant driving field
Rabi Oscillations between levels 1 and 2



Rabi frequency →

$$\Omega = \frac{E_2 - E_1}{\hbar}$$

probability for the system to be in level 1 or level 2 →

$$P_1(t) = \cos^2\left(\frac{\Omega t}{2}\right)$$

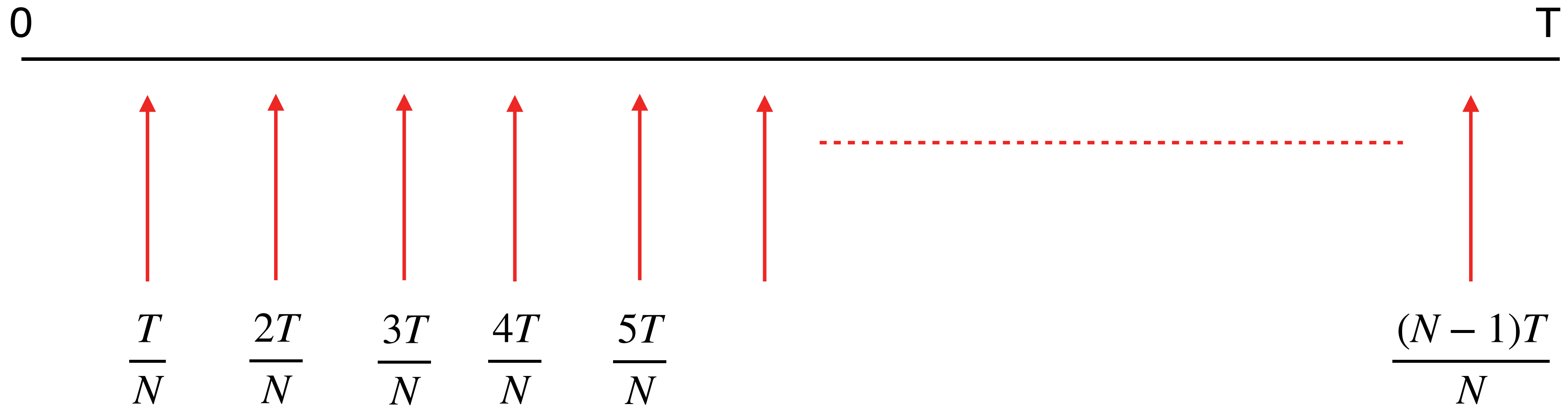
$$P_2(t) = \sin^2\left(\frac{\Omega t}{2}\right)$$

continuously “observing” the system
frequent quantum measurements

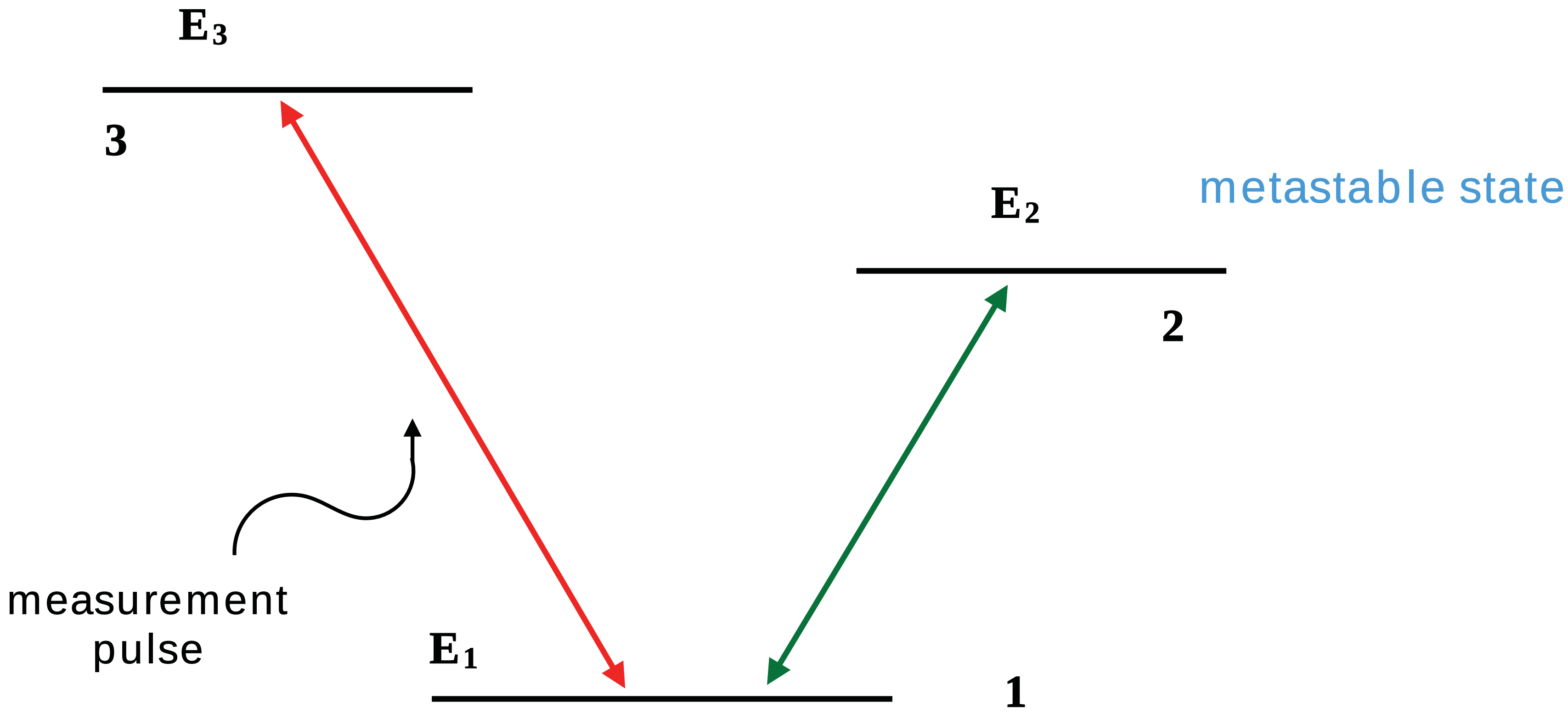
make N equally spaced measurements over a time period [0, T]

$$T = \frac{\pi}{\Omega}$$

π pulse



How were these measurements done?



make N equally spaced measurements over a time period [0, T], the time duration of a π pulse

At the end of N “measurements”

$$P_2(t) = \frac{1}{2}\left[1 - \cos^N\left(\frac{\pi}{N}\right)\right]$$

In the limit of large N
(continuous measurements)

$$P_2(t) = \lim_{N\to\infty} \frac{1}{2}\left[1 - \cos^N\left(\frac{\pi}{N}\right)\right] \to 0$$

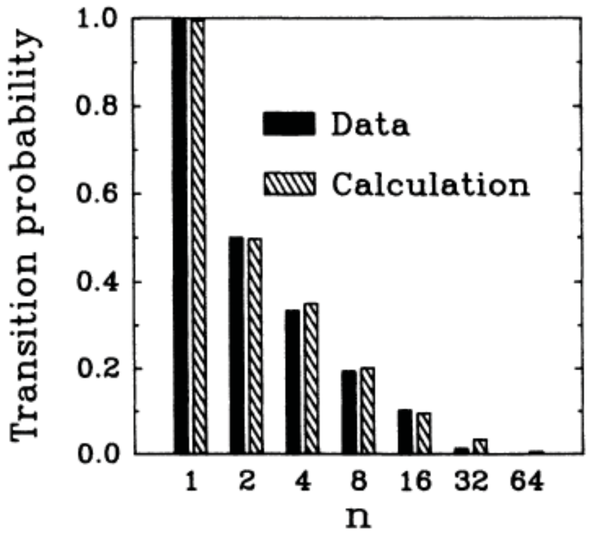
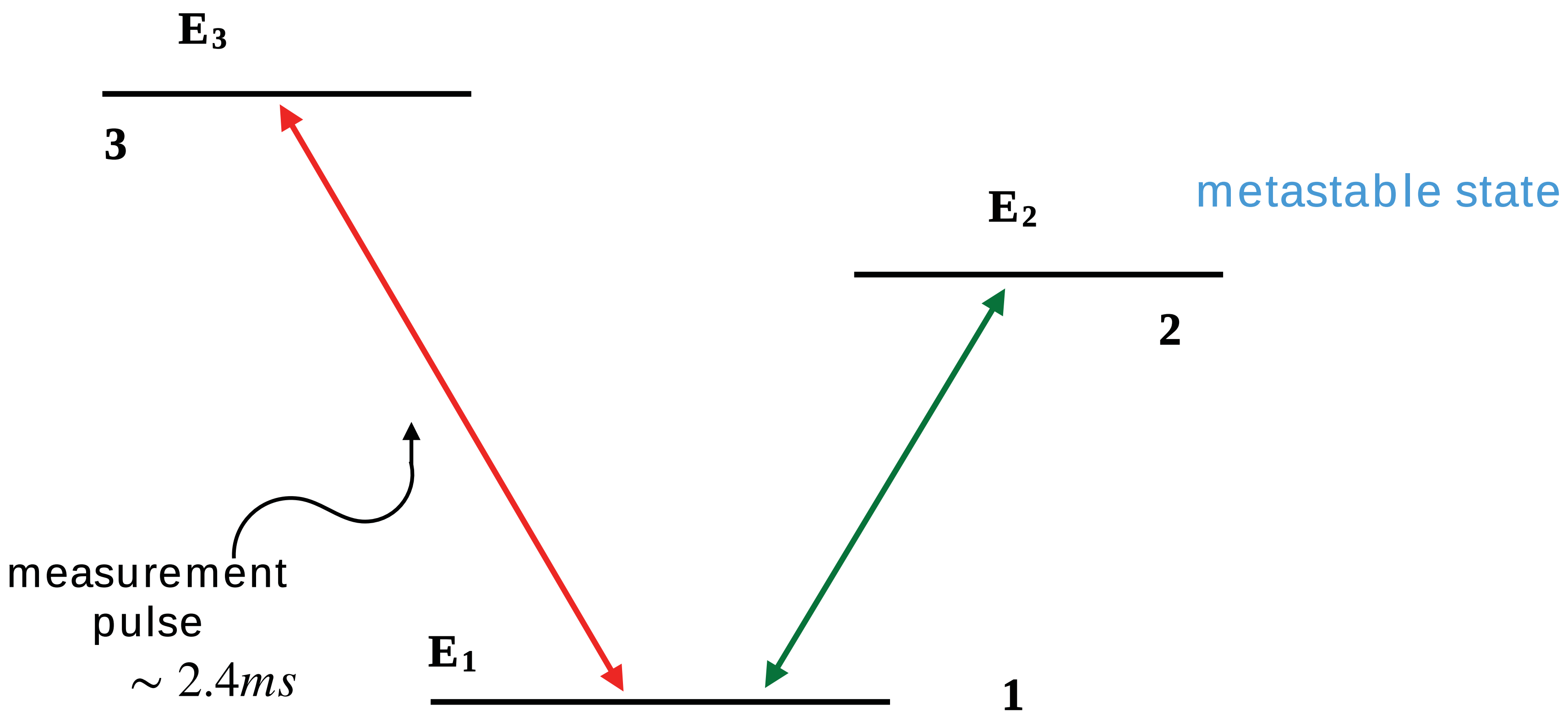


FIG. 3. Graph of the experimental and calculated 1 → 2 transition probabilities as a function of the number of measurement pulses n. The decrease of the transition probabilities with increasing n demonstrates the quantum Zeno effect.

The Quantum Zeno Effect

Continuous measurements
inhibit the transition from 1 to 2

The key to the success of the NIST experiment (the first direct demonstration of the QZE) is that in the quantum system chosen, the transition from one state to the other was slow enough to make a large number of “measurements”



The result of Kurizi and Kofman
“Acceleration of quantum decay processes by frequent observations”
[Nature, 2000] “The anti-Zeno effect”

whereas the inhibitory quantum Zeno effect may be feasible in a limited class of systems, the opposite effect—accelerated decay—appears to be much more ubiquitous. The Zeno and anti-Zeno effects had not yet been observed in any spontaneously decaying system.

“Observation of the Quantum Zeno and Anti-Zeno Effects in an Unstable System” - M. C. Fischer, B. Guti´errez-Medina, and M. G. Raizen, Phys. Rev. Lett. 87, 040402 (2001).

Sodium atoms trapped in a light wave - these atoms can escape only by quantum mechanical tunnelling. Left to its own devices, the system would decay slowly, with sodium atoms tunnelling across the barrier every now and then. When the system was observed every millionth of a second, the tunnelling rate slowed significantly. When they took measurements every five millionths of a second, tunnelling speeded up - Zeno- and Anti-Zeno-Effects

Sudarshan’s students Modi and Shaji theoretically modeled and reproduced all the results of this Quantum Zeno experiment (2004)

QZE - implications and potential applications

Misra and Sudarshan’s 1977 paper captured the imagination of the physics community, activating decades of theoretical and experimental explorations into the subject. Its implications range from the philosophical to the practical.

The quantum Zeno effect provides a fascinating insight into the foundational aspects of quantum mechanics, in particular, the quantum theory of measurement and its varied interpretations

Today, Misra and Sudarshan’s groundbreaking result has found relevance in experiments that control quantum dynamics in quantum information processing applications and could play a role in quantum computing

QZE could be a possible tool in the fight against decoherence, which has been the most crippling challenge of when it comes to storing a quantum state.

While in the classical world , Achilles overtakes the tortoise and all is well with the world, in the mysterious land of the quantum , watched pots stop boiling (or boil faster, maybe!) and the ghost of Zeno continues to make its presence felt in unimaginably interesting ways.