Applied Gravity

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Successes of General Relativity

- Einstein's theory of general relativity has had a remarkably successful last hundred years
- It provides the basic framework for cosmology. The gravitational waves it predicts are likely to provide us with a new 21st century window to the universe (LIGO, LISA?). Black holes appear to have been observed. At a more mundane level GR effects are programmed into GPS's...
- Fair to say that GR provides a remarkably successful description of all observed gravitational phenomena.
- This talk. Things are even better. GR also provides a remarkably 'successful' description of several non gravitational phenomena.

Classical physics at large N

- How can a theory of gravity describe non gravitational phenomena? This happens through a strange route.
- Quantum systems admit two kinds of classical limits. The first of these is obtained by taking $\hbar \to 0$ (i.e. taking the coupling to zero). Reduce to a known classical system (classical physics with the same Lagrangian as the quantum theory).
- Second much less familiar clasical limit in theories whose variables are $N \times N$ matrices. Effectively classical in terms of 'trace' variables in an appropriate large N limit. Classical even away from weak coupling. Basic idea is the law of large numbers. Traces are averages whose fluctuations, appropriately measured, are $1/\sqrt{N^2}$ suppressed.
- In this second limit simple counting arguments assure us that traces become effectively classical (fluctuations are suppressed). However these arguments dont determine the classical system.

Large N limits are hard

- It is possible to solve very simple large N toy models (e.g. gauged large N matrix quantum mechanics with a single matrix) and work out the effective classical description.
- However even a slight increase in complexity (e.g. gauged matrix quantum mechanics with two matrices) makes the problem intractable. We know physics is classical at large N, but dont know what the effective classical description is.
- A problem of great interest - but one that so far seems completely out of reach of direct field theory analysis - is that of large N QCD.

Applied Gravity

- Many attempts to find classical description of large N QCD. No success. Consolation (20 years ago): perhaps no great loss. Description likely to be horribly complicated, nonlocal, probably impossible to deal with, even if we could find it.
- Maldacena, 1997: the large N limit of $U(N)$ $\mathcal{N} = 4$ Yang Mills theory (maximally supersymmetric cousin of QCD) at large coupling is far from a mess. Governed by the Einstein's equations, the most beautiful equations in physics.
- Have a question you are interested in knowing the answer to in this susy cousin of QCD, at large N and strong coupling? Can answer it using Einstein's equations of gravity. This remarkable AdS/CFT correspondence opens the door to the study of applied gravity.

Who cares about N=4 Yang Mills?

- Some of you are thinking: that sounds like a great application for experts on $\mathcal{N} = 4$ Yang Mills theory. But are there also some lessons for the rest of us who are unconcerned about that particular field theory?
- Answer yes. There are several phenomena that all quantum field theories are expected to exhibit. Some of these phenomena are have never been understood in full detail in *any* field theory. A detailed understanding of these processes in a particular strongly interacting QFT like $\mathcal{N} = 4$ Yang Mills could sharpen understanding of these proccesses in all quantum field theories.
- In the rest of this talk I will explain lessons of this broader variety that have been learnt.

Hydrodynamics: Variables

- On intuitive grounds we believe that every nontrivial interactng field theory at high energy densities is described, in a long wavelength limit, by the equations of hydrodynamics.
- The basic idea here is that at high enough energy densities stuff equilibriates locally very rapidly. However the parameters of this equilibrium (local temperature, chemical potential, velocity) vary from point to point, and form the variables of the long wavelength effective theory.

Hydrodynamics: Equations

- The equations of hydrodynamics are conservation of energy - momentum and other charges. These become equations for local thermodynamical fields once we have constitutive relations that express stress tensor and currents as functions of of local thermodynamical fields in an expansion in derivatives.
- We expect the stress tensor and charge currents to be expressible as functions of thermodynamical fields precisely because local lumps of the fluid are equilibriated.
- Several questions here. What is the form of allowed expressions in constitutive relations? What are the values of parameters? Great deal of bottom up analysis of the first condition starting with Landau and collaborators.

Hydrodynamics and Gravity (via AdS/CFT)

- We have just explained that all field theories - so in particular N=4 Yang Mills - are expected to admit a classical effective description in terms of hydrodynamics in appropriate conditions.
- However strongly coupled large N $\mathcal{N} = 4$ Yang Mills is also expected to admit a classical description in terms of Einstein's equations under all circumstance.
- These two expectations can both be true only if Einstein's equations (in an asymptotically AdS_5 space) effectively reduce, in the appropriate limits, to the equations of hydrodynamics in 4 dimensions, with particualar constitutive relations and particular values for dissipative parameters. Is this true?

Hydrodynamics from gravity

- Quite remarkably yes. We and others have shown that in an appropriate long wavelength limit, Einstein's equations in AdS_5 reduce to the equations of conformal hydrodynamics in $d = 4$. The proof of this claim is constructive. It yields the specific equations of hydrodynamics (i.e. constitutive relations) for this theory
- I do not have enough time to describe the details of this remarkable connection between the equations of gravity and hydrodynamics here. However I should atleast mention that black holes play a major role in the story. Gravitational configurations dual to fluid flows are black hole spacetimes with wavy and rippling event horizons.

- Once the connection has been established, one can sit back and look at the result. In particular we have a prediction for the constitutive relations for large N, strongly coupled $\mathcal{N} = 4$ Yang Mills fluid dynamics. Is there anything to learn from the answer?
- Yes. There is a major surprise. The form of constitutive relations obtained from gravity can be compared with the general parameterization of allowed constitutive relations in the text of Landau and Lifshitz. Our answers fall into the Landau Lifshitz framework when internal charges are set to zero. However we have disagreement if other conserved charges are nonzero. The gravitational fluid dynamics has a new term in the charge current proportional to vorticity.
- Redoing the computation of Landau and Lifshitz it was indeed found that they had overlooked a subtlety. Redoing their computation keeping this subtlety in mind not only allows such a term but forces it. The coefficient of this term determined by an anomaly and so is always computable.

Hydrodynamics: lessons

- Gravity has taught us important lessons about the structure of the equations of charged relativistic hydrodynamics at first order in the derivative expansion. There are new terms in the hydrodynamical equations.
- Very surprising because hydrodynamics is an old and mature subject. Unexpected that there is something new to learn about the structure of the fundamental equations. Potential applications to neutron stars and the RHIC and LHC experiment.

Entagnglement associated with regions of space

- We will now turn our attention away from hydrodynamics to 'entanglement'. Let me explain what I mean.
- Associated with any quantum system we have a Hilbert Space. The Hibert space of every local quantum field theory is the tensor product of an infinite number of sub Hilbert spaces, one associated with each point in space (I'm ignoring some subtleties here).
- In order to understand the nature of states (or phases) of quantum field theories, one often wants to know how 'entangled' the Hilbert space associated with a region R_1 is with the complement of the Hi.lbert space associated with the compliment of R_1. If the full state is a tensor product $|\psi\rangle = |\psi_1\rangle|\psi_2\rangle$ we say the state is unentagled.

Entagnglement entropy

- A useful quantitative measure of entanglement is obtained as follows. Given any state $|\psi>$ in the QFT, we construct the corresponding density matrix $\rho = |\psi><\psi|$. ρ can be thought of as a matrix. We then define ρ^{red} by tracing over the part of the Hilbert Space associated with the compliment of R_1.
- We then define the entanglement entropy of the region R_1 (for the state $|\psi>$) as $S = \mathrm{Tr}\rho^{red} \ln \rho^{red}$.
- Entanglement entropy is an interesting quantity that carries a lot of information about the quantum structure of a state. Unfortunately this quantity is very difficult to directly compute even in free field theories- and usually impossible to compute in interacting theories (single intervals in 2 d CFTs are an exception)

Entagngment Entropy from gravitational geometry

- Fascinatingly, it turns out that we can use gravity to understand entanglement entropy in $\mathcal{N} = 4$ Yang Mills in exquisite detail.
- This is how it goes. Any particular state of $\mathcal{N} = 4$ Yang Mills is associated with an asmyptotically AdS_5 gravitational geometry (solution to Einstein's equations). The boundary of AdS_5 is conformal to R^4, and may be thought of as the space on which the dual field theory lives.
- Consider a region R_1 in the dual field theory and so on the boundary of AdS_5. In order to find the entanglement entropy of the region R_1 we are supposed to extremize the area of codimension one surfaces that venture out into the bulk but end on the boundary of R_1. The entanglement entropy is given by $S = A/(4G)$.

Properties of EE from gravity

- This dictionary element establishes a completely precise and entirely remarkable connection between entanglement and geometry.
- Known but difficult to prove inequalities, E.g. $S_A + S_B \geq S_{AUB} + S_{A\mathrm{int}B}$ reduce to almost obvious statements about area extremizaion problems (modulo a subtlety).
- Entanglement entropy is so easy to now compute in strongly coupled $\mathcal{N} = 4$ Yang Mills that it is being computed in detail in all sorts of interesting states in this theory. E.g. how does the entanglement entropy of a region behave as a function of time during the process of thermalization (dual to the proces of black hole creation in the bulk). This study is leading to a detailed intuitive understanding of the behaviour and evolution of entanglement entropies that is hard to imagine obtaining by any other method.

Scattering Amplitudes

- Scattering amplitudes are fundamental observables in quatnum field theories, and are of great interest to high energy phenomenologists, because they are directly measurable in particle accelerators.
- $\mathcal{N} = 4$ Yang Mills is a theory of massless gluons. Natural question: can one use gravity to compute the S matrix of these gluons at strong coupling.
- Answer yes. The problem of computing scattering amplitudes at strong coupling may be reformulated as a soap bubble or minimal area problem of a 2d (string) surface. One needs to compute the minimal area of a 'soap bubble' that ends on a polygon at the boundary of AdS_5 space. The polygon has only null edges, the length of each being the momentum of one of the scattering problems
- Thus AdS/CFT converts the comutation of a scattering amplitude into a well posed minimal area problem in AdS_5 geometry. The corresponding soap bubble problem turns

- The study of this problem has led to explicit results for several scattering amplitudes at strong coupling and the discovery of a new invariance of these scattering amplitudes called dual conformal symmetry in $\mathcal{N} = 4$ Yang mills. Results from the fact that *AdS* space is selfdual under 'T duality'.
- While dual conformal symmetry is believed to be a special property of $\mathcal{N} = 4$ Yang Mills scattering amplitudes that dont generalize to, e.g. QCD amplitudes, the much improved understanding of scattering amplitudes this theory seems likely to soon impact the study of scattering amplitudes in related theories like pure QCD.

Discussion

- A remarkable feature of our physics based description of the real world has always been the unity and economy of ideas. A few underlying universal frameworks describe a huge variety of phenomena
- In this talk I have attempted to show you that this unity runs even deeper than had been originally anticipated. The study of string theory has revealed deep connections between areas of physics that seem superficially completely unrelated.
- This development has given rise to an entirely unexpected use for Einstein's equations: as a tool for exploring and understanding non gravitational physics. Many concrete results about non gravitational physics have already been obtained in this way.
- Its natural to wonder if this is the tip of the iceberg. Are more such unsuspected relations waiting to be discovered between already well developed frameworks of theoretical