A Manual on Experiments in Physics
(To be done with the IASc Kit)

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Preface

Physics education in India today, both at the graduate and post-graduate levels, lacks good training in experimental physics. The curriculum is biased more towards theory than experiment. Even the few experiments that are done in the laboratory use outdated equipment and techniques.

In the year 2000, the Indian Academy of Sciences, Bengaluru, entrusted to one of us (RS), the responsibility of developing good, low-cost experiments in physics, and of conducting Refresher Courses to train teachers in doing these experiments. Dr Priolkar of Goa University was a part of this effort right from the start. Dr Ramesh of the National Aerospace Laboratories, Bengaluru, joined this effort in 2010. The authors of this manual have developed more than fifty experiments covering different aspects of physics. Some of these experiments are at the BSc, and some at the MSc or post-MSc levels. The experiments have all been designed to verify a physical law, or measure a physical property. All the electronic circuits are analog circuits. Our claim is that the circuits work for the purpose for which they have been designed, namely, to carry out reproducible measurements to an accuracy of a few per cent.

In 2010, the Indian Academy of Sciences, Bengaluru, licensed Messrs Ajay Sensors and Instruments in Bengaluru to produce the equipment and circuits developed by us, and sell them at prices fixed by the Academy.

Ninety-five Refresher Courses of two weeks, duration, using 25 of these experiments, have been conducted all over India till the end of January 2018. Eight Refresher Courses in more advanced experiments have been conducted in Bengaluru from 2013. About 2500 teachers and students have been trained to do these experiments. More than 250 standard kits to do some basic experiments have been sold so far. More than 150 institutions, including some IITs, IISERs, Central and State Universities and autonomous institutions are using some of these experiments in their curriculum. Since 2007, this effort has been supported by the Indian Academy of Sciences, Bengaluru, the Indian National Academy of Sciences, New Delhi and the National Academy of Sciences of India, Allahabad.

The Science Education Panel of the Academies suggested that the manual for the experiments may be published as a book. The manual has been revised completely.

We believe that this manual, along with the equipment developed, will be useful for improving the laboratory practice in colleges. It is our hope that the manual will also stimulate teachers to develop new experiments.

R Srinivasan K R Priolkar T G Ramesh

Bengaluru
1 July 2018.
Contents

(In the following, the letter A, within brackets, at the end of the title, signifies experiments that can be done in B.Sc classes. Experiments marked B can be done in M.Sc. Experiments marked C require advanced infrastructure, and are being done in the IASc laboratory in Jalahalli, Bengaluru)

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Chapter 1

Importance of Experiment in Physics
1.1 Importance of Experiments in Physics

1. The Path of Scientific Discovery

Science rests on the twin pillars of experiment and theory. An experiment is more than mere collection of data. One should have a credible hypothesis to explain the mass of data. For example, Tycho Brahe, the Danish astronomer, had accumulated a large volume of data on the position of planets in the sky over many years of careful observation. Kepler analyzed this complex maze of data, and deduced three simple laws of planetary motion. Newton then introduced the hypothesis of attraction between two masses, inversely proportional to the square of the distance between the two, and showed that all three laws of motion followed as natural consequences of this hypothesis. This is the path taken by any scientific discovery.

2. Primacy of Experiment

In the above process of development, experiment plays a dominant role. As Henri Poincaré, a theoretical physicist and philosopher of science of the nineteenth century, stated

"Experiment is the sole source of truth. It alone can teach us something new. It alone can give us certainty."

A theory stands if its predictions are verified by experiments. Take the following example. By the end of the nineteenth century, many experiments were performed on the photoelectric effect. The results of these experiments could not be explained by the wave theory of light. Max Planck showed that the spectrum of black-body radiation could only be explained if one assumes that the energy of a harmonic oscillator is quantized in units of $hv$, where $h$ is the Planck’s constant and $v$ is the frequency of the oscillator. Einstein made a giant leap. In 1905 he proposed that light of frequency $v$ propagates as packets of energy $hv$ each packet having a momentum $hv/c$, where $c$ is the velocity of light. From this he obtained his famous photoelectric equation. Many people were skeptical of this hypothesis, and Millikan was one such skeptic. Millikan was a great experimental physicist. He tried to test Einstein’s photoelectric equation by carefully designed experiments. In 1914 these experiments clearly demonstrated the truth of Einstein’s photoelectric equation, and yielded the most accurate value of $h$ at that time. Still, one had to wait till 1923, when Compton Effect was discovered, and the concept of the photon as a packet of energy and momentum was established beyond doubt. The photon concept now found universal acceptance.

In today’s world, theory often precedes experiment by many years. As an example, consider the Higgs boson, first postulated in 1964. The Higgs field was thought to confer mass on some fundamental particles. The confirmation of the existence of Higgs boson had to wait nearly five decades. It was finally confirmed in experiments with the Large Hadron Collider in CERN in 2012. The long gap between the theoretical hypothesis and its experimental verification, in this case, is due to the enormous complexity of the experiment. The experiment could be performed only after tremendous advances took place in accelerator and detector technology.

In view of the importance of experiments, it is clear that adequate care and time should be devoted to experimental techniques in physics education.

3. Types of Experiments

Experiments in physics education are of three types, namely: Demonstration experiments; Virtual experiments on a computer; Real experiments in a laboratory.

Demonstration experiments are very useful in introducing concepts to students. They should be used in school and in early years of college. A demonstration experiment should be so designed as to make a great visual impact on the audience. It should be followed with a detailed explanation of the physical principle that the experiment is designed to demonstrate. Without such an explanation, a demonstration will degenerate to magic. It will titillate the senses, but will not educate the mind.

Many demonstration experiments at the level of school and early years of college are simple to make, and do not need much money or effort.
Importance of Experiment in Physics

The demonstration experiment is qualitative. One does not carry out any precise measurement to verify quantitatively a physical law. For more serious experimentation, we have to design experiments which will give us numerical data.

Here, it is pertinent to recall an observation of Lord Kelvin.

“In physical science a first essential step in the direction of learning any subject is to find principles of numerical reckoning and methods for practicably measuring some quality connected with it. I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meagre and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science, whatever the matter may be.”

Quantitative measurements are essential for the advancement of science.

Virtual experiments are useful in illustrating the laws of physics quantitatively. These experiments need a computer with the necessary software to make the calculations following a physical law, and a display to show the results visually. Virtual experiments are very useful in teaching physics to a large class. However, they cannot be a substitute for real experiments, with real equipment and real materials.

This is what Martin Rees, a noted cosmologist and astrophysicist, has to say about virtual experiments.

“Crucial to science education is hands-on involvement: showing, not just telling; real experiments and field trips and not just ‘virtual reality’.”

A student must work with his/her hands to develop skills in techniques of measurement, understanding the limitations of the instruments he/she uses. This can be had in practical laboratory sessions. Since experiments and theory are equally important, the curriculum must devote adequate time for experiments and theory.

4. Present State of Education in Experimental Physics in India

The utility of practical education in physics is not measured in terms of the fraction of the time in the curriculum devoted to laboratory class, or in terms of the number of experiments done in the practical class. It is the quality of experiments that determines the utility of the laboratory classes. From this point of view, the current state of affairs in most of the universities and colleges in the country is disappointing.

A good laboratory experiment must be easy to do, must give reproducible and fairly accurate values of the quantities measured, and must validate a physical law, or illustrate a physical principle. Instructions for the experiment must be detailed, so that the student understands what he/she is measuring, how he/she is measuring, and why the experiment is being done. Unfortunately, the experiments in most of the colleges are done with ancient equipment and outdated techniques, and seldom give reliable answers. There is a strong feeling among the university teachers that a student will be overloaded if the instructions exceed a few sentences. So the instructions are very brief and do not inform the student regarding the theory behind the experiment. The student is asked to take some readings, and use a formula to calculate the result. The student is not informed how the formula was obtained, and what assumptions went into deriving the formula. Often teachers, who are assigned the duty of supervising the laboratory classes, are not present. If the student has a problem, he/she has no one to seek advice on how to overcome the problem. Often, there is no connection between what is taught in the class and what the student does in the laboratory. The student does not understand that theory and experiment go together.

There has been a rapid, and revolutionary, development in all branches of physics in the last hundred years. Eager to introduce the new developments, the curriculum has been chopped and altered, at the expense of teaching experimental techniques.

A student, who passes out with a high grade in BSc or MSc Physics course, is unable to answer simple questions, and find solutions to simple problems. One is reminded of the experience Feynman had with students in Brazil, when he spent ten months, around 1950, in that country teaching physics. His comments on education in Brazil at that time is included in the book *Surely you’re joking, Mr. Feynman!*
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pages 211 to 219. These comments are valid to the system of physics Education in our country today. He stated,

"After a lot of investigation, I finally figured out that the students had memorized everything, but they didn’t know what anything meant. When they heard ‘light that is reflected from a medium with an index’, they didn’t know that it meant a material such as water…”.

The conclusion is inescapable that our students after graduation are very deficient in practical skills. This is the view expressed by many top scientists and educators in the country. The students lack the ability to apply their knowledge to solve simple practical problems.

Unless we make a drastic revision in the curriculum and give experiment its due, we will be producing sub-standard physics graduates.

5. Suggestions for using the Book

This manual is written mainly to help Indian colleges and universities to improve laboratory education in physics. More than fifty experiments, covering all branches of physics, have been developed by the authors. The necessary analog circuits have been built and the required mechanical components have been designed. Most of the circuits and mechanical components are being manufactured by Ajay Sensors and Instruments, in Bengaluru, since 2010, under license from the Indian Academy of Sciences, Bengaluru. Many kits have been sold, and many institutions are using these experiments. The kit is well within the means of an ordinary college. The experiments work well. Some experiments can be done together within one laboratory period, thus saving time.

The experiments are collected under various heads, such as experiments in mechanics, experiments in heat, etc. Each experiment carries a letter A or B or C, within brackets, after the title. The experiments with the label A are suitable for BSc (Physics). The experiments with the label B are suitable for MSc (Physics). Some experiments, with the label C, need costly infrastructure like a cryo-cooler or spectrograph. Such infrastructure is available in the Indian Academy of Sciences Laboratory in Bengaluru. These experiments are done in advanced courses conducted by the authors in Bengaluru. Some institutions may have such infrastructure, and the experiments marked C can be done in such institutions.

For each experiment, a short introduction to the theory of the experiment is given. This is followed by a detailed description of the procedure to do the experiment. A sample set of readings is given, and the method of analyzing the readings is described in detail. It must be emphasized that the readings given are sample readings. The student should not expect to get identical readings when he/she does the experiment. A set of questions is appended in some sections.

All the experiments, except the experiments in optics, are done with equipment designed by us and manufactured by Ajay Sensors and Instruments. Some of the electronic circuits are used in many experiments. A description of the controls on such common electronic equipment is given in Chapter 2. The description of other electronic equipment or circuits, which are used only for experiments described in one chapter, is given in that chapter.

An introduction to a topic, relevant to the experiments in a given chapter, is included at the beginning of the chapter.

A brief description of errors and their analysis, common to all chapters, is given in the following section in Chapter 1.

It was felt that the addition of some brief information on topics of experimental interest, like sensors, cryogenic and vacuum techniques, and thin film deposition techniques will make the book more useful. A section on tensor properties of crystals, and how symmetry reduces the number of independent constants, is also included. These sections are collected as an Appendix in Chapter 18.

Apart from references at the end of some sections, a list of books is collected at the end of this book.

In an attempt to make each section self-contained, some amount of repetition is unavoidable. But this is kept at a minimum.

It requires commitment on the part of the teachers to revamp the experimental curriculum. The teachers must also be interested in developing new experiments. We hope that at least some of the teachers will be enthused by this book to embark on such useful activity.
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1.2 Introduction to Error Analysis

1. Introduction

In this section an introduction to error analysis is given. Error is also known as uncertainty. In any experiment, we use different types of instruments to make measurements. These instruments have their limitations, and any measurement made with an instrument is subject to errors. During the measurement, one tries to keep certain environmental parameters constant. However, there could be small variations in the environmental parameters, which may lead to an error, or uncertainty, in the measurement. Measurements are made on samples, the composition of which may not be exactly defined. Results will then vary from sample to sample. To see how accurate the results of a given measurement are, it is necessary to estimate the magnitude of such errors.

One must bear in mind the difference between precision and accuracy. This is discussed in §18.1 in the Appendix.

2. Types of Errors

One can distinguish between the following types of errors:

Reading Error of an Instrument

A meter scale is graduated in millimeters. If one wants to measure the length of a rod with this scale, one places one end of the rod to coincide with the zero reading on the meter scale, and then sees what the reading is at the other end of the rod. One may find the other end of the rod to lie between 50.0 cm and 50.1 cm. One cannot read the length to better than 1 mm with this scale. Therefore, one can say that the reading error of the scale is 1 mm.

The meter scale is an analog instrument. One may subjectively imagine the 1 mm interval on the scale to be subdivided into two equal intervals, and subjectively estimate that the end of the rod lies between the marked division 50.0 cm on the scale, and the imaginary division 50.05 cm. In this case one can say that the reading error is 0.05 cm. Thus, the reading error of an analog instrument is subjective, and varies from person to person. The maximum reading error is the interval between two successive divisions on the scale. But a person can estimate subjectively a reading error less than this.

Suppose one measures a current on a digital multimeter. Say the reading fluctuates between 2.44 and 2.45 A, where the last unit on the panel meter will be fluctuating between 4 and 5. If the current is between 2.445 and 2.455 A, the meter will indicate 2.45 A, while, if the current is between 2.435 and 2.445 A, the meter will indicate 2.44 A. The rounding off error is therefore half of the unit in the last digit. If one wants to measure the current to a higher precision, one should use a meter with more digits in the display.

Calibration Error

In an experiment one uses a variety of sensors and diverse instruments for measurement. These sensors and instruments are generally calibrated against standards. However, there could be errors in calibration. A meter scale, divided into millimeters, may not be accurately 1 m in length. Even assuming the graduations are uniform, a length measured with such a scale will be either systematically more, or less, than the actual length. Similarly, a thermometer may have a calibration error, so that all temperatures measured with that thermometer will always differ from the actual value in the same sense and by the same percentage. The detection of systematic error in an instrument is difficult. One will have to measure certain standard quantities with such instruments to detect systematic errors. Small calibration errors will not change the verification of a physical principle or law. Suppose one wants to verify that the heat conducted along a rod is proportional to the temperature gradient along its length. If the thermometer has a calibration error the temperature gradient will be in error. But all values of temperature gradient, for different amounts of heat transported, will suffer the same percentage of error in the same sense. So, a plot of the quantity of heat conducted against the erroneous temperature gradients, measured with this thermometer, will still lead to a linear relation. Only the value of thermal conductivity will differ by a
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small percentage from the actual value. Thus, small calibration errors are not very important for student experiments.

Random Errors

Random errors arise due to factors beyond our control. Environmental parameters may undergo sudden random changes during the course of the measurement. In electrical measurements, the starting or stopping of a motor near the place of the experiment may affect the reading transitorily due to electromagnetic interference. Random errors are always present in any experiment, and lead to deviations in the value of the measured quantity from one run to the other. We shall consider an example below, which includes considerations of random error.

3. Analysis of Data – an Illustration

All experimental measurements are subject to different types of error. To illustrate these different types of errors, consider the following simple experiment to measure the acceleration due to gravity. Suppose we drop a ball from a height $h$ and measure the time $t$ it takes to reach the ground. From this measurement we can calculate $g$ from the formula

$$h = \frac{1}{2} gt^2.$$  \hspace{1cm} (1)

The experimental arrangement is shown in Figure 1.

There is a stand with a horizontal rod and a clip. A ball is fixed to the clip. With a spring latch, we can open the jaws of the clip to release the ball. A pointer is attached to the rod, to read the height of the rod from the bottom of the stand, on a scale fixed to the stand.

We adjust the height of the center of the ball above the ground to be 1000 mm. This implies several things. We must know the diameter of the ball. We must position the ball every time within the clamps at the same position. We must know the distance between the marker and the center of the ball when it is clamped.

All these factors we may know correct to one mm. Then, when we say that the ball is clamped so that its center is at a height 1000 mm, we mean that the height of the center of the ball from the ground is between 999.5 mm to 1000.5 mm. This uncertainty in the position of the ball is called the reading uncertainty of our scale. In analog measurements, one may use one’s judgment to say that the marker is between 999.75 mm to 1000.25 mm. In this case the reading uncertainty is only $\pm 0.25$ mm.

We measure the time on a digital timer. The last digit on the timer is in units of 0.001 s. A time interval between 0.5005 s and 0.5015 s will be indicated as 0.501 s on the clock, while a time interval between 0.5005 s and 0.4995 s will be indicated as 0.500 s. The reading error here is half the unit in the last digit.

We drop the ball from a height of 1000 mm and record the time it takes for the ball to reach the ground. Our sense impressions convey to us when we have released the ball, and then we start the clock. When the ball has reached the ground, we hear the thud and stop the clock. These sense impressions are subjective, and are subject to random variations. We repeat the observations twenty times and record the time $t$ of fall. We get the data recorded in Table 1.
We see that the measured time interval varies randomly between 0.447 s and 0.458 s. This difference is much larger than the reading error of the clock. This variation arises due to random errors. These errors are caused by the variations in sensory impressions. There may also be errors caused by air currents, etc.

We plot a histogram in which we count the number of times a given value of \( t \) occurs in Table 1. This is shown in Figure 2 as the discontinuous black curve joining various points.

![Figure 2. Histogram showing the number of times a given value occurs.](image)

From these observations we can calculate the mean value \( \langle t \rangle \) of time of fall. This is defined by

\[
\langle t \rangle = \frac{1}{N} \sum_{i=1}^{N} t_i.
\]

Here the sum is over the \( N \) observations (in this example \( N = 20 \)). This mean value of \( \langle t \rangle \) comes out to be 0.4522 s.

Then we calculate the sum of squares of the deviations of the measured values from the average value.

\[
\text{Sum of deviations squared} = \sum_{i=1}^{N} (t_i - \langle t \rangle)^2.
\]

This is always positive, though the deviations may have either sign. From the sum of deviations, we can calculate the variance defined as

\[
\text{Variance} = \frac{\text{Sum of deviations squared}}{(N - 1)}.
\]
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This comes out to be \(6.34 \times 10^{-8}\) s\(^2\). The standard deviation \(\sigma\) is defined as

\[
\text{Standard deviation} \quad \sigma = (\text{Variance})^{1/2}.
\]  

(5)

This comes out to be \(0.0025\) s.

In Figure 2 the continuous curve (red) follows the equation

\[
N = N_0 \exp \left(-\left(\frac{t - \langle t \rangle}{\sigma}\right)^2\right).
\]  

(6)

In drawing the above curve we take \(\langle t \rangle\) to be 0.4522, the standard deviation \(\sigma\) to be 0.0025 and \(N_0\) to be 4.5 (obtained by integrating equation (6) between \(-\infty\) to \(+\infty\), and equating it to the total number of measurements). This curve is called the normal distribution or the Gaussian distribution. We see that this curve roughly represents the histogram of the twenty trials. If we take a very large number of readings, then the histogram will approximate more and more closely to the normal distribution.

From the normal distribution it follows that the probability for the measured value of \(t\) to lie between \(\langle t \rangle - \sigma\) and \(\langle t \rangle + \sigma\) is approximately 70%.

We then express the result of our measurement of the time of fall as

\[ t = \langle t \rangle \pm \sigma. \]

From our measurements

\[ t = 0.4522 \pm 0.0025 \text{ s} \]  

(7)

or

\[ t = 0.452 \pm 0.003 \text{ s}, \]  

(8)

since the reading error of the clock is 0.0005 s.

4. Propagation of Errors

Let us now calculate the value of \(g\), taking \(h\) to be 1 m and \(t\) to be 0.452 s, from the equation

\[ g = \frac{2h}{t^2}. \]  

(9)

The value of \(g\) comes out to be 9.789 m/s\(^2\). The error in \(h\) and the error in \(t\) will contribute to an error in the calculated value of \(g\). Let us now calculate the maximum possible error in \(g\), due to errors in \(h\) and \(t\). Taking the logarithm of both sides of equation (9) and differentiating we get

\[
\frac{\Delta g}{g} = \frac{\Delta h}{h} + 2 \frac{\Delta t}{t}.
\]  

(10)

Here \(\Delta g/g\) is the fractional error in \(g\) due to the fractional error \(\Delta h/h\) and the fractional error \(\Delta t/t\). The fractional errors in \(h\) and \(t\) can be positive or negative. So the maximum possible error in \(g\) is obtained from the equation

\[
\frac{\Delta g}{g} = \left|\frac{\Delta h}{h}\right| + 2 \left|\frac{\Delta t}{t}\right|.
\]  

(11)

The reading error \(\Delta h\) is 0.5 mm and \(h\) is 1 m. The total error in \(t\), \(\Delta t\), is 0.003 s and \(t = 0.452\) s. Substituting in equation (11) we get

\[
\frac{\Delta g}{g} = \frac{0.5}{1000} + 2 \left(\frac{0.003}{0.452}\right)
\]

\[ = 0.0005 + 0.0133 = 0.0138 \approx 0.01. \]
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The percentage error in $g$ is obtained by multiplying the fractional error by 100. So the percentage error in $g$ is 1%. The major contribution to this error comes from the error in the time of fall. We then say

$$g = 9.8 \pm 0.1 \text{ m/s}^2.$$  

The result must be quoted up to the first significant digit in error.

This example shows how errors propagate. Any quantity which occurs to a power $m$ in an equation will contribute $m$ times to the fractional error. If $m$ is large then we should take special care to measure that quantity more accurately than the other quantities.

This gives a brief introduction to error analysis. The value of taking many readings to get an accurate value of standard deviation cannot be over-emphasized.

5. **Questions**

1. A mercury-in-glass thermometer is graduated in 0.2° interval. What is the reading error of the thermometer?

2. The last digit of a digital ammeter is 1 mA. What is the reading error of the digital ammeter?

3. One can measure the viscosity $\eta$ of water by capillary flow. Here the water flows due to a pressure difference $h\rho g$ of a water column, where $h$ is the height of the water column, $\rho$ is the density of water and $g$ is the acceleration due to gravity. The volume $V$ of water flowing through the capillary in one second is

$$V = \frac{\pi (h\rho g) r^4}{8\eta l},$$

where $l$ is the length of the capillary, and $r$ its radius.

Describe how you will measure $h$, $l$, $r$ and $V$ to arrive at a value of viscosity? Which quantity needs to be measured more accurately?

In a capillary we cannot be sure that the cross section is perfectly circular along its length. We have to measure the average cross section along its length and get an average value of $r$. How will you do this?
Chapter 2

Some Commonly Used Electronic Equipment
2.1 Regulated DC Power Supply

1. Introduction

A regulated DC power supply is a source of DC electrical power, the voltage of which can be controlled between zero and a maximum value. Such a power supply is required for heating a furnace, and maintaining it at a given high temperature. This will enable experiments to be carried out on the temperature variation of some physical properties of a material, such as the electrical resistance.

The regulated power supply provided with the kit can give a maximum current of 30 V and 2 A.

![Regulated DC power supply](image)

**Figure 1.** Regulated DC power supply.

There are two knobs on the left for adjusting voltage. One is marked Coarse and the other Fine. Similarly there are two knobs on the right for adjusting current. If we want to set the voltage, we use the first two knobs, and the voltage will be indicated on the panel meter, marked V. If we want to set the current, we use the other two knobs, and the current is indicated on the panel meter, marked A. The load is connected to the banana terminals at the right bottom of the front panel.

This power supply is used in the following experiments:

(i) Calibration of a silicon diode and a copper–constantan thermocouple as temperature sensors § 5.1
(ii) Neutral temperature of an iron–copper thermocouple § 4.2
(iii) Thermal diffusivity of brass § 4.6
(iv) Temperature coefficient of resistance (TCR) of copper § 5.1
(v) Verification of Curie–Weiss law for the electrical susceptibility of a ferroelectric material in the paraelectric state § 7.5
(vi) B–H curve in a hard ferromagnetic material and in a soft ferrite § 9.2
(vii) Chua circuit for nonlinear dynamics § 14.2
(viii) Energy band gap of silicon using a silicon diode § 15.1
(ix) Integral method for evaluating the Seebeck coefficient of constantan as a function of temperature § 15.2
(x) Martensite to austenite phase transition in shape memory alloy nitinol § 16.2
(xi) Paraelectric to ferroelectric transition in modified barium titanate § 16.3
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(xii) Dependence of ferromagnetic (FM) to paramagnetic (PM) transition temperature on strontium concentration in lanthanum strontium mangenate (LSMO), and FM to PM transition in a Ni-Zn ferrite bead using AC susceptibility setup § 16.5
(xiii) Metal-insulator transition in a pellet of Sr-doped lanthanum manganite § 16.6
(xiv) Superconducting transition in yttrium barium copper oxide (YBCO) § 16.7
(xv) Variable-range hopping in a composite § 17.2

2.2 Temperature Indicator and Controller

![Temperature Indicator Image]

**Figure 1.** Front panel of the temperature indicator box.

1. **Front Panel**

   There is a panel meter which indicates the temperature in degrees Celsius. There are four pads marked **Menu, >, <, and Enter**. This temperature indicator is normally calibrated for use with a platinum resistance thermometer Pt 100. Connection diagrams are given on the top cover of the indicator. At the back are the connectors. Connectors marked ‘Line’ are to be connected to the mains. The power supply to the furnace is connected through the connectors marked ‘Relay’ if the temperature has to be controlled. The relay is normally closed and will open when the set temperature is reached.

   At the rear of the indicator are the connectors. The connectors marked Line are to be connected to the mains voltage. The connectors to a thermometer and a relay are provided. The connection diagrams are given on top of the indicator. The relay is normally closed.

2. **Mode of Use**

   For studying the temperature variation of a physical property of a material, it is necessary to put the sample in a furnace and heat the sample, taking readings at different temperatures. The furnace is heated with the regulated power supply described in the previous section. A temperature indicator is used to read the temperature of the furnace.

   The temperature sensor is placed within the furnace, and monitors its temperature. If we want the temperature to be controlled, the regulated power supply is connected to the furnace through the relay in the temperature controller box as shown in Figure 2.
Some Commonly Used Electronic Equipment

![Schematic diagram of a temperature controlled furnace. A: Regulated DC power supply; B: Temperature controller; C: Temperature sensor; D: Furnace.](image)

**Figure 2.** Schematic diagram of a temperature controlled furnace. A: Regulated DC power supply; B: Temperature controller; C: Temperature sensor; D: Furnace.

A power chord is connected to two terminals, designated Line, at the rear bottom of the temperature controller, to give power to the temperature controller.

The red (positive) terminal of the regulated power supply is connected to one of the banana plugs on the furnace. The other banana plug on the furnace is connected to one of the pair of designated screw terminals (relay) at the back of the temperature controller. The other terminal of the (relay) pair is connected to the black terminal on the regulated power supply. There is a relay in the temperature controller box. This relay is normally closed. Then the power supply is connected to the furnace. When the power supply is switched on, a current will flow through the heating coil of the furnace and the furnace gets hot.

By pressing the pads at the bottom, one may set the temperature to be reached on the panel meter of the temperature indicator. When the furnace is heated, the temperature goes up. The reading on the panel meter increases. When the reading on the panel meter crosses the set temperature reading, the relay in the temperature controller box opens, cutting off the current to the furnace. Because of its thermal inertia the furnace continues to heat to a temperature beyond the set value and then starts to cool. When the temperature of the furnace goes below the set value, the relay in the controller box closes, restoring the current to the furnace. The furnace starts to get hot. Thus the temperature of the furnace oscillates about the set value. The voltage across the furnace is adjusted, so that the oscillation about the set temperature is about ±0.5 °C.

Sometimes one does not wish to control the temperature, but one would like to heat the furnace slowly at about 1 °C/minute up to, let us say, 150 °C. Then the controller can be set at a temperature higher than 150 °C, and the voltage on the regulated power supply adjusted to give the desired heating rate. *One may also connect the furnace directly to the power supply bypassing the relay.*

### 2.3 Constant Current Source

![Constant current source.](image)

**Figure 1.** Constant current source.

1. **Introduction**

A constant current source (CCS) is a versatile instrument used in many experiments. It produces a constant DC current, independent of the load, over a range of load values. As long as the potential difference across the load does not exceed a given value, the current remains reasonably constant.
2. Description of the Source

The front panel of the commercial constant current source is shown in Figure 1. On the bottom left of the panel is the switch marked Mains. When the switch is down, as shown in the figure, a red light will glow, indicating that the instrument is on. Power is now given to the current source.

The current source operates in two ranges:
- Low range: 100 microamps to 20 milliamps.
- High range: 20 milliamps to 0.3 amps in continuous operation; up to 0.6 amps in intermittent operation.

If the current is set at more than 0.3 amps for a long time in the high current range, the IC voltage regulator chips may become very hot, and the current may start decreasing with time. So, one should use the instrument, in this range above 0.3 amps, only for a short time of a few minutes.

There are two range switches to the right of the On light. When these switches are put in the position marked Low, the instrument operates in the low current range. When the switches are set in the position High, the instrument operates in the high current range.

There are three knobs, one in the middle right, and two at the top of the panel. The knobs can be turned ten turns. The knob, marked Low, is used to set the current in the low range. The current is at its minimum value when the knob is in the extreme left position. The current increases when the knob is turned to the right. Near the extreme right position, the current will change rapidly as the knob is turned. The knob must be turned very slowly as the extreme right position is reached.

The two knobs, on the top right, control the high current. Before switching on the instrument, make sure that both the knobs are in the extreme left position. The two knobs can be used to adjust the current to a precise value. For example, to set a current of 0.300 A, first turn the left knob till you reach a current below 0.300 A (say, 0.290 A). Then turn the right knob till you reach the desired current. Do not turn the knobs fast at the right end of the pots. The current will increase very rapidly.

On the top left of the front panel is a meter which reads the set current in milliamperes. In the low current range, the current will be indicated to the first decimal place. For a more accurate reading of the current in the low current range, one should connect a DMM (digital multimeter) in the 20 milliamp range in series with the load, or measure the voltage across a known resistance in series with the load.

The two banana plugs, marked O/P on the bottom right of the panel, are to be connected to the load.

3. Load Regulation of Current

(a) Low Current Range

![Figure 2(a)](image-url)  
*Figure 2(a).* Current vs. Load in the low current range.
Some Commonly Used Electronic Equipment

We see from Figure 2(a) that the current remains constant as long as the load does not exceed a given value. The maximum load, for a given current, is inversely proportional to the set value of the current. Figure 2(b) shows that, for a given current, the voltage across the load increases in proportion to the load, keeping the current constant. When the voltage reaches about 20 V, the voltage remains constant. The current then decreases as the load is increased.

For a given current $I$, the maximum load $R_{\text{max}}$ up to which the current will remain constant, is given by

$$R_{\text{max}} = \frac{20}{I}.$$  \hspace{1cm} (1)

(b) High Current Range

Figure 3(a). Variation of current against load in the high current range.
Some Commonly Used Electronic Equipment

Figure 3(b). Variation of voltage against load for a current of 100 mA.

The behavior is similar to that in the low current range. The current regulation is however not as good as in the low current range. The current decreases slowly (by about 5%) till the maximum load is reached. Then it drops precipitously as seen in Figure 3(a). However, in many experiments, the load does not change much, and so the current remains quite constant within this change in load, in the high current range also.

The constant current source is used in the experiments noted below.

(i) Calibration of secondary thermometers § 4.1
(ii) Stefan’s constant § 4.3
(iii) Thermal and electrical conductivity of copper § 4.4
(iv) Thermal conductivity of a poor conductor § 4.5
(v) TCR (temperature coefficient of resistance) of copper § 5.1
(vi) Techniques for measuring resistivity of thin films and pellets § 5.2
(vii) DC Hall effect in semiconductors § 15.4
(viii) Superconducting transition in YBCO § 16.7
(ix) Percolation threshold for resistance in composites § 17.1
(x) Variable-range hopping in a composite § 17.2

2.4 DC Differential Amplifier

Figure 1. DC differential amplifier (also called DC microvoltmeter).
Some Commonly Used Electronic Equipment

1. Introduction

The DC differential amplifier is used to measure small DC signals, of a few microvolts. Thermocouples, measuring temperature differences of a few degrees, provide such signals.

2. Description of the Instrument

The mains switch is at the bottom left corner of the front panel. When it is switched on, a red light will come on. Four different sources of emf can be connected to four pairs of banana terminals, marked I₁, I₂, I₃ and I₄ on the back side of the instrument. A selector switch, at the bottom right of the front panel, will select the channel to be measured. A panel meter directly indicates the voltage in millivolts.

3. Mode of Use of the Instrument

First, short the input terminals I₄. Put the selector switch in the channel 4 position. Switch on the mains. The panel meter may not indicate zero. It may indicate a small positive or negative voltage. Press the touch switch marked >, on the bottom of the front panel, to make this reading zero. Then the amplifier is ready to measure the voltage connected to any of the channels.

4. Experiments in which the Instrument is Used

This instrument is used in the experiments described below:

(i) Calibration of secondary thermometers § 4.1
(ii) Neutral temperature of iron–copper thermocouple § 4.2
(iii) Stefan’s constant § 4.3
(iv) Thermal conductivity and electrical conductivity of copper § 4.4
(v) Thermal conductivity of a poor conductor § 4.5
(vi) Thermal diffusivity of brass § 4.6
(vii) Techniques for measurement of resistivity of thin films and pellets § 5.2
(viii) Differential method for measuring the absolute Seebeck coefficients of n-and p-type bismuth telluride § 15.3
(ix) DC Hall effect in semiconductors § 15.4
(x) Martensite to austenite phase transition in a shape memory alloy § 16.2
(xi) Superconducting transition in YBCO § 16.7

2.5 Signal Generator

Figure 1. Signal generator.
1. **Introduction**

The signal generator produces square, triangular and sinusoidal waves, from 1 Hz to 1 MHz. There are four knobs in a row at the top. The knob marked frequency is used to tune to the frequency required. The range of Frequency can be reduced by a factor of ten, by pressing the button marked \( \rightarrow \), and can be increased by a factor of ten by pressing the button marked \( \leftarrow \). The knob marked Amplitude, at the extreme right top, sets the peak-to-peak amplitude output of the signal generator. This peak-to-peak amplitude can be changed up to a maximum of 20 V. If \( V_{pp} \) is the peak-to-peak amplitude, the RMS amplitude output is given by \( V_{RMS} = V_{pp}/(2\sqrt{2}) \). There are two buttons marked 20 dB and 40 dB at the bottom right corner. If the 20 dB button is pressed, the output amplitude is reduced by a factor of 10. If the 40 dB button is pressed, the output amplitude is reduced by a factor of 100. If both buttons are pressed the output amplitude is reduced by a factor of 1000. The peak-to-peak amplitude and the frequency of the output are indicated on a panel meter, and can be read by pressing the button F/A. The output of the signal generator is taken from the BNC connector at the right corner of the front panel. The waveform is chosen by pressing the button marked Waveform. An indication about the chosen waveform appears on the panel meter. If a pulsed waveform is chosen, the pulse width can be set by the knob marked Pulse Width. There is another knob marked DC Offset. For all the experiments in this manual we keep the offset at zero.

In all our experiments we use the sinusoidal waveform **without** a DC offset.

The signal generator will provide a current of a few milliamperes, and so can be used directly to excite a circuit requiring such low currents. It cannot deliver large power. If large power is required, it should be used along with a power amplifier.

2. **Experiments in which the Signal Generator is Used**

The signal generator is used in the following experiments:

(i) Rigidity modulus by torsion vibrations of a wire § 3.2
(ii) Young’s modulus by flexural vibrations of a bar § 3.3
(iii) All AC experiments in Chapter 6
(iv) Verification of Curie–Weiss law for a ceramic capacitor § 7.5
(v) Determination of \( e \) by shot noise § 8.3
(vi) Measurement of thermal relaxation time of a serial light bulb § 10.1
(vii) Calibration of a lock-in amplifier § 11.2
(viii) Mutual inductance with lock-in amplifier § 11.3
(ix) Measurement of low resistance using a lock-in amplifier § 11.4
(x) Elastic and piezoelectric properties of PZT § 15.7
(xi) AC susceptibility setup § 16.5

2.6 **Power Amplifier**

![Figure 1. Power amplifier.](image)
Some Commonly Used Electronic Equipment

1. Introduction

The power amplifier is used, along with the signal generator, for driving a loud speaker, or a coil of wire to produce oscillating magnetic fields.

2. Description of the Instrument

On the left bottom of the front panel is the 'Mains' switch. When it is pressed down the instrument is on. The signal generator output is connected to the RCA socket marked In on the front panel. The load (a loudspeaker or coil) is connected to the RCA socket marked Output on the front panel.

The power amplifier works in the audio range of frequencies, i.e., from 30 Hz to 30 kHz. If a loudspeaker is connected to the power amplifier, it will emit a sound at the pitch of the input from the signal generator. The loudness of the sound increases as the amplitude of the input is varied, by turning the amplitude knob on the signal generator.

The power amplifier will deliver a few watts of power (about 10 watts) to a matched load between 4 and 8 ohms.

3. Experiments in which the Instrument is Used

The instrument is used in the following experiments:

(i) Rigidity modulus by torsion vibrations of a wire § 3.2
(ii) Young’s modulus by flexural vibrations of a bar § 3.3

2.7 Furnace

1. Introduction

A furnace is used for measurements on samples at different temperatures, higher than room temperature. A photograph of the box furnace is shown in Figure 1.
A nichrome heater is wound on a ceramic tube of about 3 cm diameter. The heater has a resistance of about 15 ohms. It can carry a current of 2 A. The heater leads are brought to a socket on the powder-coated rectangular steel box. Fibre glass thermal insulation is provided inside the box, to thermally insulate the wall of the box from the heater. A stand is fixed to the box, with a horizontal Perspex plate. A circular hole is made in the Perspex plate. For different experiments, different inserts are made, with leads coming from the sample and thermometer to circular terminal blocks. The terminal block will rest on the Perspex plate, while the sample and thermometer will hang inside the furnace near its centre.

The furnace is heated by connecting it to a DC power supply described in Section 2.1. The top and bottom ends of the ceramic tube will have to be closed with a wad of cotton to prevent convection currents. With a DC voltage of about 8 V, the temperature of the sample in the furnace will increase at a rate of 1 to 1.5 degrees Celsius per minute. The DC voltage will have to be increased slowly as the temperature increases, to maintain this rate of heating. When the temperature reaches 100 °C, the voltage applied to the furnace will be about 12 V. Most of the experiments are done up to 150 °C. But the furnace will go up to 400 °C.

This furnace is used in the following experiments:

1. Calibration of a silicon diode and a copper–constantan thermocouple as temperature sensors § 4.1
2. TCR of copper § 5.1
3. Verification of the Curie–Weiss law for a ceramic capacitor § 7.5
4. Energy band gap of silicon using a silicon diode § 15.1
5. Integral method for measuring thermo-emf § 15.2
6. Percolation threshold in composites § 17.1

(For the experiment on percolation we use a similar furnace built with a porcelain tube of 5 cm diameter).
Chapter 3

Experiments in Mechanics
3.1 Introduction to Elastic Properties of Solids

1. Strain

Consider a wire of length \( l \) clamped at one end. If a load is added at the other end, the length of the wire increases to \( l' \). The change in the length of the wire is

\[
\Delta l = l' - l.
\]

The fractional change in the length of the wire is

\[
e = \frac{\Delta l}{l},
\]

where \( e \) is called the longitudinal strain of the wire.

Consider a rectangular slab of thickness \( d \) (Figure 1).

![Figure 1. Rectangular parallelepiped that is sheared.](image)

ABCD is the top face and EFGH is the bottom face. The thickness of the slab is \( d \). The bottom face is clamped, and a force \( F \) is applied on the face ABCD, parallel to its length AB. There will be an equal and opposite force on the clamped face EFGH. The face ABCD will move to the position \( A'B'C'D' \) relative to EFGH, due to this pair of forces. The rectangular parallelepiped becomes deformed. The face ABCD is displaced by \( \Delta x \) (from AB to \( A'B' \)) in the direction of the force. The angle \( \phi \) between AE and \( A'E \) is called the angle of shear. This angle is given by

\[
\tan \phi = \frac{\Delta x}{d},
\]

\( \tan \phi \approx \phi \), if \( \Delta x \) is very small compared to \( d \). The angle \( \phi \) is called the shear strain.

Note that the longitudinal strain \( e \), and the shear strain \( \phi \), are dimensionless numbers.

Consider a cube of volume \( V \) as shown in Figure 2.

![Figure 2. A cube to which equal and opposite forces are applied normal to all six sides.](image)

The original volume of the cube is \( V \). The cube is subjected to a hydrostatic pressure \( p \). Then an equal and opposite force will be applied normal to all the six faces of the cube. The volume of the cube will decrease to \( V' \). The change in volume of the cube is

\[
\Delta V = V' - V.
\]
The volume strain $\kappa = \Delta V/V$ will be negative, since $V'$ will be less than $V$. $\kappa$ is also a dimensionless number.

2. Stress

A load $F$ is applied to a wire to produce a strain $\varepsilon$. Here, the load $F$ is parallel to the length of the wire. The ratio $F/A$ is called the longitudinal stress $T_{\text{long}}$, where $A$ is the area of cross section of the wire.

In Figure 1, the force $F$ is applied to the face ABCD, the normal to which is along EA, i.e., the thickness $d$ of the rectangular parallelepiped. The ratio of $F$ to the area of the face ABCD is called the shear stress $T_{\text{shear}}$.

Note that the longitudinal stress is produced by a force applied perpendicular to the area of cross section, while the shear stress is produced when the force is parallel to the area ABCD.

In Figure 2, on each face, a force $F$ is applied, which is opposite in direction to the outward normal to the face. So, each face has a stress equal to $-F/($Area of the face$)$. This means that each side of the cube experiences a longitudinal stress $T_{\text{long}}$, which is negative, and the value of $T_{\text{long}}$ on each pair of opposite faces of the cube is identical. We then say that the cube is subjected to a pressure $p = -T_{\text{long}}$. Such a pressure can be realized by placing the cube in a bath of oil, and compressing the oil. So, this pressure is called hydrostatic pressure. Under the action of a hydrostatic pressure, the volume strain $\kappa$ will be negative.

Stress has the dimension Newton/m$^2$.

3. Elastic Constants

A material is said to be isotropic if its properties are the same in all directions. The normal materials that one comes across, are isotropic materials. A wire of copper will contain grains of copper oriented at all angles. Each grain of copper may not be isotropic. But the properties of the wire will be properties of the grain, averaged over all orientations. So, the material of the wire behaves as if it is isotropic.

If the longitudinal strain $\varepsilon$ is small, it will be proportional to the longitudinal stress $T_{\text{long}}$. Similarly, if the shear strain $\phi$ is small, it will be proportional to the shear stress $T_{\text{shear}}$. If the volume strain $\kappa$ is small, it will be proportional to the hydrostatic stress. This is Hooke’s law.

When the strain is small, stress is proportional to strain.

The ratio of stress to strain is called an elastic modulus. One can define three elastic moduli for isotropic materials.

\[
\text{Young’s modulus } Y = \frac{T_{\text{long}}}{\varepsilon} = \frac{\text{longitudinal stress}}{\text{longitudinal strain}}.
\]

\[
\text{Rigidity modulus } n = \frac{T_{\text{shear}}}{\phi} = \frac{\text{shear stress}}{\text{shear strain}}.
\]

\[
\text{Bulk modulus } K = \frac{-p}{\kappa} = \frac{\text{hydrostatic pressure}}{\text{volume strain}}.
\]

In addition to these three moduli, there is one more elastic property called the Poisson’s ratio $\sigma$. When a longitudinal stress is applied to a wire, its length $l$ increases, and its radius $r$ decreases. The longitudinal strain $\varepsilon$ is positive. The ratio $\sigma = -(\Delta r/r)/(\Delta l/l)$ is called the Poisson’s ratio. It is a number between 0 and 0.5.

The dimensions of the three elastic moduli are the same as the dimension of stress. Elastic moduli are usually expressed in $10^9$ Newton/m$^2$ or gigapascals (GPa).

For an isotropic material, the four elastic constants are not independent. Only two are independent. We may choose $n$ and $\sigma$ as the independent constants. Then we may express the Young’s modulus $Y$, and the bulk modulus $K$, in terms of $n$ and $\sigma$.

\[
Y = 2n (1 + \sigma).
\]

\[
K = \left(\frac{2}{3}\right) n \left[ \frac{(1 + \sigma)}{(1 - 2\sigma)} \right] = \frac{(1/3) Y}{(1 - 2\sigma)}.
\]

The Young’s modulus cannot be less than twice the rigidity modulus, because the Poisson’s ratio cannot be negative.
If \( K \) becomes negative, the volume of the material will increase under a hydrostatic pressure. This is not possible. \( \text{So, } 2\sigma \text{ cannot be greater than 1 or } \sigma \text{ cannot be greater than 0.5.} \)

Table 1 gives the Young’s modulus \( Y \) and the rigidity modulus \( n \) of some common materials in GPa.

<table>
<thead>
<tr>
<th>Material</th>
<th>( Y ) (GPa)</th>
<th>( n ) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>69</td>
<td>25</td>
</tr>
<tr>
<td>Brass</td>
<td>102–125</td>
<td>40</td>
</tr>
<tr>
<td>Copper</td>
<td>117</td>
<td>45</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>193–204</td>
<td>77</td>
</tr>
</tbody>
</table>

Experimentally, one can determine the Young’s modulus of a wire by measuring the increase in length when a load is applied. If the material is in the form of a bar, one can determine its Young’s modulus by applying a load at the center of a bar, which is supported on two knife edges, and measuring the depression at the center of the bar, as a function of load. In these experiments, the strain and stress remain constant, independent of time. So, they are called static methods.

One can determine the Young’s modulus of the material of the bar by finding the natural frequency of vibration of the bar, clamped at one end and free at the other end. This is a dynamic method, since the stress and strain vary with time as the bar vibrates. Frequency can generally be measured very accurately. So, this method will yield very good results even for bars which are made of a material like steel, which has a large Young’s modulus.

To measure rigidity modulus, one can take a rod of the material clamped at one end, and apply a torque to the other end. This will produce a shear strain which can be measured. This is again a static method.

One can also measure the natural frequency of a wire by exciting its torsion oscillations. This is a dynamic method. We will describe a tabletop experiment to measure the rigidity modulus of a wire using such oscillations.

4. Questions

1. There is a material in which the electrical resistance, measured in two different directions, is different. Is this material isotropic or anisotropic?

2. A rod is compressed by applying equal and opposite forces normal to its cross section at the two ends. What is the sign of the longitudinal stress?

3. A student measures the Young’s modulus and rigidity modulus of a material, as \( Y = 100 \) GPa and \( n = 60 \) GPa. Is the measurement correct? Justify your answer.

4. In an experiment, one measures the Young’s modulus as 120 GPa, and the rigidity modulus as 30 GPa. Is this measurement correct? Justify your answer.

5. Calculate the bulk modulus and Poisson’s ratio for the materials given in Table 1.

3.2 Rigidity Modulus of Brass by Torsion Vibrations of a Wire

1. Introduction

An experiment to measure the frequency of torsion oscillations of a wire will be described in this section. This experiment is usually done with a long wire hanging from the roof of the laboratory. Here we will use a tabletop model to measure the rigidity modulus of the material of the wire.

2. Couple per Unit Twist of the Wire

The rigidity modulus can be determined by a dynamic method by measuring the frequency of oscillations of a pendulum of the wire, when torsion oscillation is excited. The torsion pendulum is a long wire AB of
the material with a disc C at one end (Figure 1). The pendulum is clamped at the end A. If the disc is twisted
and let go, the pendulum executes torsion oscillations.

Figure 1. Torsion pendulum.

Consider a solid cylinder of radius \( R \) and length \( l \) as shown in Figure 2(a). The cylinder is clamped at
the top. Consider an inner concentric section of radius \( r \) and same length. This is on the right in Figure 2(b).
Consider a line \( A'B' \) of length \( l \) on this cylinder. \( OA' \) is its radius \( r_r < R \). When the free end is twisted
through an angle \( \theta \) this line \( A'B' \) moves to \( A''B'' \), and \( A'B' \) makes an angle \( B''O''B'' \) with the original line. Let this angle be \( \phi \). This is shown in Figure 2(b).

Figure 2. A cylinder of length \( l \) and radius \( r \) is clamped at the top.

(a) CD is a line on the surface parallel to the length of the cylinder. O is the center of the circular top and
OC is the radius \( R \).

(b) An inner concentric section of radius \( r \). The top face of the cylinder is held fixed and the bottom face
is twisted through an angle \( \theta \). The original point \( B' \) goes to \( B'' \). Angle \( B'O'B'' \) is \( \theta \). Angle \( B'A'B'' \)
is \( \phi \).

From the figure it is evident that the displacement \( B'B'' \) is equal to \( r \theta \), where \( \theta \) is the angle of twist
\( B'O'B'' \). \( B'B'' \) is also equal to \( l \phi \). So

\[
r \theta = l \phi. \tag{1}
\]

The angle \( \phi = r \theta / l \) is the shear strain. The shear force applied to the bottom face is perpendicular to
the radius. We take a small element of area \((rd\xi) dr\) between \( r \) and \( r + dr \), and polar angles \( \xi \) and \( \xi + d\xi \). The
shearing force on this area is

\[
dF = n \left( (rd\xi) dr \right) \phi = n \left( \frac{r^2}{l} \right) \theta \ dr \ d\xi, \tag{2}
\]

where \( n \) is the rigidity modulus of the material. This force is tangential to the circle. The torque due to this
force about the axis of the cylinder is

\[
d\tau = dF \ r = n \left( \frac{r^3}{l} \right) \theta \ dr \ d\xi. \tag{3}
\]
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To get the total torque on the cylinder of radius $R$, we integrate over $d\varepsilon$ from 0 to $2\pi$, and over $dr$ from 0 to $R$. This gives

$$\tau = \left( \frac{\pi n R^4}{2I} \right) \theta. \quad (4)$$

Here, $\tau / \theta$ is $C$, the couple per unit twist of the wire.

3. Experimental Setup

A brass wire 1.5 mm diameter, and 50 cm in length, is clamped at its two ends in a frame. There are two moveable clamps, which can be moved symmetrically on either side of the center of the wire, so that the length of the wire between the two clamps can be varied. The brass wire at its center carries a cylindrical disc of brass, which is 0.5 cm in thickness and 3.5 cm in diameter. It is brazed to the brass wire. Two magnets are attached to this disc on two diametrically opposite prongs. Two coils wound around iron cores are placed in front of the magnets. The pole pieces of the magnets facing the coils are suitably arranged so that when one coil pulls one magnet, the other coil pulls the other magnet in the opposite direction. When the coils are excited by an AC current, the wire will be subject to a torque, oscillating at the same frequency as the frequency of the AC current. This excites forced torsion oscillations of the wire. The arrangement is shown in Figure 3.

4. Procedure

Fix the movable clamps on either side of the center of the wire, so that each clamp is at a distance of 20 cm from the center of the brass disc. Connect a signal generator to a power amplifier. The output of the power amplifier is connected to the wires from the coils on the experimental setup. When the signal generator is turned on, and the signal amplitude is kept around two volts, the power amplifier sends a current through the two magnet coils in series. The coils will exert a force on the magnets fixed to prongs that are attached to the central brass disc. This will exert a torque on the brass wire, and will make it execute forced torsion oscillations at the frequency of the signal generator output. The central disc has a moment of inertia $I = \frac{mR^2}{2}$ about the axis of the wire. Here $m$ is the mass of the disc and $R'$ is its radius ($3.5 / 2 = 1.75$ cm). The mass of the disc is $\pi R'^2 \rho$, where $t$ is the thickness of the disc (0.5 cm), and $\rho$ is the density of brass (8500 kg/m$^3$). Each of the two magnets has a mass $m'$ and the distance between the magnets is $d'$. The total moment of inertia $I$ about the axis of the wire is

$$I = \frac{mR'^2}{2} + 2m' \left( \frac{d'^2}{4} \right). \quad (5)$$
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The torque per unit twist of the brass wire is \((\pi n R^4/2l) \times 2\), where \(n\) is the rigidity modulus of the wire, \(R\) its radius, and \(l\) is the distance of the movable clamp from the center of the brass disc. The factor 2 arises because each half of the wire, from the center to a fixed clamp, produces a torque given by equation (4). The equation of motion of the wire is

\[ I \frac{d^2}{dt^2} \theta + \left( \frac{\pi n R^4}{l} \right) \theta = \tau \exp(i\omega t). \] (6)

Here, \(\theta\) is the twist of the wire at the disc, \(\tau\) is the torque exerted by the coils on the wire, and \(\omega = 2\pi f\), where \(f\) is the frequency of the AC excitation. When the frequency of the excitation coincides with the natural frequency \(f_n\) of torsion oscillations of the wire, one gets the maximum amplitude of oscillation. The natural frequency is the solution to the equation

\[ I \frac{d^2}{dt^2} \theta + \left( \frac{\pi n R^4}{l} \right) \theta = 0. \] (7)

Writing \(\theta = \theta_0 \exp(i2\pi f_n t)\), we get

\[ f_n = \left( \frac{1}{2\pi} \right) \left( \frac{\pi n R^4}{(l/2)} \right)^{1/2}. \] (8)

The natural frequency \(f_n\) will vary as \((1/l)^{1/2}\), when the distance between the center of the brass disc and the movable clamps is changed.

Starting with a frequency of about 80 Hz, we lower the frequency of the signal generator. Resonance is indicated by the maximum amplitude of vibration of the disc. At resonance, the frequency of the signal generator is \(f_n\).

The length is changed from 20 cm to 5 cm, in steps of 2.5 cm, and the natural frequency is measured by observing resonance. The diameter of the wire should be measured with a screw gauge.

A sample set of data is shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Rigidity modulus of a brass wire.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of brass</td>
</tr>
<tr>
<td>Diameter of brass wire</td>
</tr>
<tr>
<td>Moment of inertia of disc</td>
</tr>
<tr>
<td>Mass of each magnet</td>
</tr>
<tr>
<td>Distance between magnets</td>
</tr>
<tr>
<td>MI of magnets about axis</td>
</tr>
<tr>
<td>Total moment of inertia</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(l) (cm)</th>
<th>(f) (Hz)</th>
<th>(1/l) (m⁻¹)</th>
<th>(f^2) (s⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>18.8</td>
<td>5</td>
<td>353.44</td>
</tr>
<tr>
<td>17.5</td>
<td>20.4</td>
<td>5.7143</td>
<td>416.16</td>
</tr>
<tr>
<td>15</td>
<td>22.5</td>
<td>6.6667</td>
<td>506.25</td>
</tr>
<tr>
<td>12.5</td>
<td>24.6</td>
<td>8</td>
<td>605.16</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>10</td>
<td>729</td>
</tr>
<tr>
<td>7.5</td>
<td>32.6</td>
<td>13.333</td>
<td>1062.76</td>
</tr>
<tr>
<td>5</td>
<td>41.7</td>
<td>20</td>
<td>1738.89</td>
</tr>
</tbody>
</table>
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A plot of $f^2$ vs $1/l$ is shown in Figure 4.

\[ f^2 \text{ vs. } 1/l \]

\[ \text{Figure 4. } f^2 \text{ vs. } 1/l. \]

From the plot, the slope of $f^2$ against $1/l$ is 91.4 m/s$^2$. The rigidity modulus in calculated from equation (8) to be $n = (4\pi I/R^4) \times \text{slope} = 40.5$ GPa.

The elastic moduli of brass have the following values (from the internet):

- Young’s modulus $Y$: 100 to 125 GPa.
- Rigidity modulus $n$: 37 to 45 GPa.
- Bulk modulus $K$: 120 to 140 GPa.

Note: Brass is an alloy of zinc and copper in different proportions. If the composition of the sample used in the Young’s modulus experiment is different from the composition of the sample in the rigidity modulus experiment, the Poisson’s ratio, calculated from these values may come out to be greater than 0.5. Poisson’s ratio for a give material cannot be greater than 0.5. If this happens, it is likely that the compositions of the bar and the wire are different.

5. Questions

1. The natural frequency of a torsion pendulum, of length $l$ and moment of inertia $I$ is 20 Hz. The length is decreased to $l/2$, and the moment of inertia is increased to $2I$. What is the new frequency of the torsion pendulum?

2. The rigidity modulus of material A is twice the rigidity modulus of material B. Consider wires of materials A and B of equal length and radius. If the torque for twisting wire A through an angle $\theta$ is $\tau$, what is the torque required to twist the material B through the same angle?

3. Can you suggest a simple method to measure the amplitude of the angular twist of the torsion pendulum.

4. Suppose an impulse $\tau dt$ is given to the torsion pendulum. Here, $dt$ is a small time interval compared to the period $T$ of the pendulum. Calculate the throw $\theta$ of the pendulum, by equating the kinetic energy of the pendulum acquired due to the torque, to its potential energy at the end of the throw. Does this relate to the theory of the ballistic galvanometer?

3.3. Young’s Modulus by Flexural Vibrations of a Bar

1. Introduction

Consider a bar of length $l$, width $b$, and thickness $d$, clamped at one end. If the thickness is small, the bar can be bent easily in the direction of its thickness (downwards in Figure 1). There is a line N1N4 (see Figure 1) the length of which is not changed when the bar is bent.
The line N1N4 (shown in solid red) before bending, becomes N1N4' (shown in dashed red) after bending; but it does not change its length. This is called a neutral line. All such lines in the plane N1N2N3N4 have the property that their length remains unchanged on bending. This plane is called the neutral plane. Any filament along the length of the bar, above the neutral plane, suffers an extension; and any filament along the length, below the neutral plane, suffers a compression.

The tensile stress on an element will increase as the height of the element above the neutral plane increases. Similarly, as one moves from the neutral plane towards the bottom of the cross section, the compressive stress will increase. Any cross section of the beam, perpendicular to its length, is subjected to a bending moment. This is given by \( YAk^2/R \). Here \( Ak^2 \) is the geometric moment of inertia of the cross section; \( R \) is the radius of curvature of the neutral line, at the point \( x \) of the cross section. \( Y \) is the Young's modulus of the material of the bar. For a rectangular bar of width \( b \) and thickness \( d \), this geometric moment of inertia is \( bd^3/12 \). For a bar of circular cross section of radius \( a \), it is \( \pi a^4/4 \).

When this bar is set in flexural vibrations, the equation of motion is

\[
\rho A \frac{\partial^2 y}{\partial t^2} = -YAk^2 \frac{\partial^4 y}{\partial x^4}
\]

Here, \( y \) is the displacement at a point \( x \) along the length of the bar, \( \rho \) is the density and \( A \) is the area of cross section of the bar.

Assuming harmonic vibration with an angular frequency \( \omega = 2\pi \nu \) (\( \nu \) is the frequency in Hertz), the equation becomes

\[
YAk^2 \frac{d^4 y}{dx^4} - \rho A \omega^2 y = 0.
\]

For a rectangular bar, \( Ak^2 = bd^3/12 \) and \( A = bd \), and the equation becomes

\[
\frac{d^4 y}{dx^4} - a^4 y = 0,
\]

where

\[
a^4 = \frac{12 \rho \omega^2}{(Yd^2)}.
\]

The general solution of this equation is

\[
y(x) = A \cosh (ax) + B \sinh (ax) + C \cos (ax) + D \sin (ax).
\]

Note that there are four constants \( A, B, C \) and \( D \), which have to be determined from the boundary conditions.

The boundary conditions are:

At the clamped end, \( x = 0 \) (i) \( y = 0 \) and (ii) \( \frac{dy}{dx} = 0 \).

At the free end, \( x = l \) (iii) \( \frac{d^3 y}{dx^3} = 0 \) and (iv) \( \frac{d^3 y}{dx^3} = 0 \).

Applying the first two boundary conditions we get

\[
A + C = 0
\]

and

\[
B + D = 0.
\]
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Applying the boundary conditions (iii) and (iv) at the free end \( x = l \), we get

\[ A \cosh al + \cos al + B \sinh al + \sin al = 0 \]  
(8)

and

\[ A \sinh al - \sin al + B \cosh al + \cos al = 0. \]  
(9)

There will be non-trivial solutions only if

\[ (\cosh al + \cos al)^2 - (\sinh^2 al - \sin^2 al) = 0 \]  
(10)

or

\[ 2(1 + \cosh al \cos al) = 0 \]  
(11)

or

\[ -\cos al = \frac{1}{\cosh al}. \]  
(12)

The first two solutions of equation (12) are

\[ \alpha_1 l = 1.875 \quad \text{or} \quad \omega_1 = \left( \frac{1.875}{l} \right)^2 \left( \frac{Yd^2}{12\rho} \right)^{1/2} \]  
(13)

and

\[ \alpha_2 l = 4.694 \quad \text{or} \quad \omega_2 = \left( \frac{4.694}{l} \right)^2 \left( \frac{Yd^2}{12\rho} \right)^{1/2}. \]  
(14)

The lowest natural flexural vibration frequency of the bar is \( \nu_1 \), and the next higher frequency is \( \nu_2 \). For a given bar, clamped at one end, the two frequencies are in the ratio

\[ \frac{\nu_2}{\nu_1} = \left( \frac{4.694}{1.875} \right)^2 = 6.26. \]  
(15)

The lowest frequency is proportional to the thickness of the bar, and inversely proportional to the square of its length.

If one determines the frequency \( \nu_1 \), one can obtain the Young’s modulus of the material of the bar.

2. Aim of the Experiment

To verify that the fundamental frequency of vibration of a brass bar, clamped at one end, is inversely proportional to the square of its length and to measure the Young’s modulus of the bar.

3. Apparatus Required

Signal generator, power amplifier, Young’s modulus setup, and multimeter to measure frequency to two decimal places.

4. Experimental Setup

A brass bar B, one foot in length is clamped between two metal plates F, so that its breadth is vertical. At the free end of the bar, a small magnet \( m \) is fixed with superglue. The length of the bar from the clamped to the free end can be changed by pushing the bar into the clamping plates F. The vibrations of the bar are excited by a coil C, with a core consisting of steel wires insulated from one another with shellac varnish. If a solid iron core is used, it will get hot due to eddy currents. The coil can be moved on a horizontal rail R so that, as the length of the bar is changed, the coil can always be positioned to excite the bar at its free end. The coil is connected to the output of a power amplifier. The input of the power amplifier is connected to a signal generator. Since the power amplifier delivers sufficient power only if the load is between 4 to 8 ohms, a nichrome coil is connected in series with the copper excitation coil to make the total resistance approximately four ohms. The output of the power amplifier is connected to this load. The panel meter on the signal generator will indicate the frequency only to a precision of 0.5 Hz. So a multimeter, which can measure the frequency to two decimal places, is used.
5. Procedure

The bar is clamped so that the length of the bar from the free end to the edge of the clamp F is 20 cm. The excitation coil C is positioned opposite the free end of the bar, and the distance between the coil and the bar is adjusted to be approximately 1 cm. The power amplifier output is connected to the wires attached to the coil on the Young’s modulus setup. The input of the power amplifier is connected to the signal generator output. The signal generator is put in the frequency range 10 to 100 Hz. The amplitude and frequency knobs of the signal generator are turned to the extreme left. The power amplifier and the signal generator are then switched on. The amplitude knob on the signal generator is slowly turned to the right, till the output peak-to-peak amplitude is about 3 V. The frequency of the signal generator is set at 50 Hz. Touch the nichrome wire on the Young’s modulus setup. If it is hot, then power is passing to the magnet coil C.

Slowly reduce the frequency. At one point, the bar starts vibrating, and as the frequency is slowly decreased the amplitude reaches a maximum value. Then, the amplitude starts decreasing with a further decrease in frequency. Note the frequency value, \( f_1 \), on the DMM (digital multimeter), when the amplitude reaches the maximum value. This is the fundamental frequency for the length \( l \) of the bar.

Then, turn the frequency knob to a frequency nearly seven times the fundamental frequency. Reduce the distance between the excitation coil and the free end of the bar. Otherwise the amplitude of vibration will be too small to detect. Now adjust the frequency till the free end of the bar vibrates with maximum amplitude. If the apparatus is on a wooden table, one can hear a loud humming noise at this second frequency of resonance. Otherwise, one can feel the vibrations if one places one’s fingers lightly on the clamp. This is the overtone frequency \( f_2 \).

Now, change the length of the bar to 18 cm by loosening the screws on the clamp, and pushing the bar in. Move the coil on the rail, so that the coil comes opposite the free end of the bar. Again, measure the fundamental and overtone frequencies, by adjusting the frequency knob of the signal generator, and measuring the frequency at which the amplitude of vibration becomes maximum. Repeat the measurements by decreasing the length of the bar by 2 cm each time, till a length of 8 cm is reached.

Remove the bar from the clamp and measure its thickness \( d \) with a screw gauge.

(Do not assume the thickness to be the value given in the sample data below. Different bars may have different thicknesses.)

A specimen set of readings are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Young’s modulus of brass.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density of brass</td>
</tr>
<tr>
<td>Thickness of brass bar, ( d )</td>
</tr>
<tr>
<td>length ( l ) (cm)</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>18</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>14</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>Average</td>
</tr>
</tbody>
</table>
Experiments in Mechanics

A plot of $l^2$ against $1/f_1$ is shown in Figure 3.

The slope $\beta$ of the graph comes out to be 0.52 m$^2$/s.

Then, the Young’s modulus is calculated from the relation

$$Y = \left( \frac{4\pi^2}{1.875^4} \right) \left( \frac{12\rho}{d^2} \right) (\beta)^2.$$ (16)

The Young’s modulus comes out to be $1.09 \times 10^{11}$ Pa (1 Pa = 1 Newton/m$^2$) = 109 gigapascals. For brass, the Young’s modulus varies from 100 to 120 GPa, depending on its composition.

Next we calculate the values of $f_2/f_1$. We find the average value to be 6.66 which is in reasonable agreement with the theoretical value of 6.26.

6. Questions

1. For a sonometer wire, the frequency is inversely proportional to the length of the wire, while for the flexural vibrations of the bar, the frequency is inversely proportional to the square of the length. Why is it so?

2. We want a platform to be mounted on pillars so that sensitive instruments on the platform are not affected by vibrations due to a large motor in the same hall rotating at 500 rpm. Should we choose the dimensions of the pillar such that the natural vibration frequency of the pillar is very different from the rotational frequency of the motor? Justify your answer.

3. A reed is a bar of small thickness, width and length. Suppose there are two reeds of the same material, clamped at one end. The reeds have identical thickness and width. What should be the ratio of their lengths, if their fundamental frequencies are to differ by an octave?

3.4 Viscosity of Oil by Stokes Fall

1. Introduction

When two layers of a fluid, separated by a distance $dz$, move with different velocities $v_x$ and $v_x + dv_x$ along the x-direction, there is a velocity gradient $\partial v_x/\partial z$ in the z-direction. A shear stress is developed along the x-direction in the xy-plane which opposes this velocity gradient. This is shown in Figure 1.
Experiments in Mechanics

Figure 1. Two layers of liquid flowing with different velocities.

This shear stress $\sigma_{xz}$ is given by

$$\sigma_{xz} = -\eta \frac{\partial v_x}{\partial z}. \quad (1)$$

The quantity $\eta$ is a property of the liquid and is called the dynamic viscosity of the liquid. Shear stress and velocity have the dimensions of Newton/m$^2$ and m/s respectively. So, the dynamic viscosity has the dimension

$$\frac{\text{Newton/m}^2}{\text{(m/s)/m}} = \frac{\text{newton second}}{\text{m}^2} = \text{pascal second}.$$ 

The unit of viscosity is called a poise P, which is 0.1 Pa s. This is also equal to 1 g/cm s. The dynamic viscosity of liquids is usually given in centipoise (cP) which is 0.01 poise = 1 millipascal second.

We can also define a quantity called kinematic viscosity $\mu$ of a liquid by

$$\mu = \frac{\eta}{\rho}, \quad (2)$$

where $\rho$ is the density of the liquid. The dimension of $\mu$ is m$^2$/s. The unit is called stokes and

$$1 \text{ stokes} = \frac{1 \text{ cm}^2}{\text{s}} = 10^{-4} \text{m}^2/\text{s}.$$ 

The kinematic viscosity of liquids is usually given in centistokes (cS) which is 0.01 stokes.

2. Newtonian and Non-Newtonian Fluids

There are two types of fluids – Newtonian and non-Newtonian. In the Newtonian fluid, the shear stress $\sigma_{xz}$ varies linearly with the rate of shear strain $\partial v_x/\partial z$, i.e. $d(\partial v_x/\partial z)/dt$, for all values of the rate. This is shown in Figure 2.

Figure 2. Variation of shear stress with rate of shear strain in Newtonian and non-Newtonian fluids.

For a non-Newtonian fluid, the shear stress varies nonlinearly with rate of shear strain. There are two types of variation. In the first type, the slope of the graph (i.e., the dynamic viscosity) is more for low rates of shear strain, and decreases as the rate increases. In the second type, the slope is less for low rates of shear strain, and increases as the rate increases. A non-Newtonian type 1 fluid is also called shear
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thinning fluid. Example of a shear thinning fluid is blood. A non-Newtonian fluid of type 2 is called a shear thickening fluid. An example is corn-starch dissolved in water. Liquids like water, coconut oil, and castor oil are examples of Newtonian fluids. Hereafter, our discussion will only refer to Newtonian fluids.

3. Temperature Dependence of Viscosity of a Liquid

The viscosity of a liquid is strongly temperature dependent. As the temperature increases, the viscosity decreases. The density of the liquid also decreases as the temperature increases, but much more slowly. Figure 3 shows the temperature dependence of the kinematic viscosity and density of SAE 30 engine oil.

![Figure 3](from www.viscopedia.com/viscosity tables)

The values of viscosity of a few liquids are collected in Table 1.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Dynamic Viscosity $\eta$ (centipoise)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acetone</td>
<td>0.306</td>
</tr>
<tr>
<td>Benzene</td>
<td>0.604</td>
</tr>
<tr>
<td>Castor oil</td>
<td>985</td>
</tr>
<tr>
<td>Mercury</td>
<td>1.526</td>
</tr>
<tr>
<td>Motor oil SAE 30</td>
<td>175</td>
</tr>
<tr>
<td>Water</td>
<td>0.894</td>
</tr>
</tbody>
</table>

4. Determination of Viscosity by Stokes Fall Method

There are many methods for measuring the dynamic viscosity of a liquid:

(a) Capillary flow method: In this method, one measures the volume $Q$ of the liquid flowing through a capillary, of length $l$ and radius $r$ in one second, due to a pressure gradient.

(b) Rotating viscometer method: In this method, the liquid is contained between two coaxial cylinders. The outer one is rotated with an angular velocity $\omega$. The torque $\tau$ on the inner cylinder is measured.
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(c) Stokes fall method: In this method, a spherical ball of a material with a density greater than that of the liquid, is dropped in a long column of the liquid, and the terminal velocity of the ball is measured.

Of these methods, the Stokes fall method is the simplest and will be described here. When a spherical ball of radius \( r \) and density \( \rho \) is dropped in a liquid of density \( \rho' \), the net downward force on the ball is

\[
F = \left( \frac{4\pi}{3} \right) r^3 (\rho - \rho') g. \tag{3}
\]

The viscous force opposing this is

\[
F_{\text{viscous}} = 6\pi \eta rv. \tag{4}
\]

In the above, \( g \) is the acceleration due to gravity, and \( v \) is the downward velocity of the ball.

As the ball falls due to the force \( F \), it gets accelerated till the velocity reaches a value at which \( F = F_{\text{viscous}} \). Thereafter, the ball travels with a constant velocity \( v_{\text{terminal}} \) given by

\[
v_{\text{terminal}} = \left( \frac{2}{9} \right) r^2 \frac{\rho - \rho'}{\eta} g. \tag{5}
\]

The terminal velocity is measured by the time taken by the ball to cover a given distance \( s \). Then the dynamic viscosity of the liquid is evaluated from equation (5). This is the principle of the method.

Usually, the experiment is done with a jar more than 1 m in height, containing an oil of high viscosity, and steel balls of small radius are dropped. Steel has a density of 8050 kg/m\(^3\), while oil has a density of 870 kg/m\(^3\). So, unless the ball has a very small radius, the gravitational force will be very large, and the ball will have to travel a large distance in oil before it reaches terminal velocity.

In the experiment described here, we use water drops instead of steel balls. Water has a density of 1000 kg/m\(^3\) and so \((\rho - \rho')\) is only about 130 kg/m\(^3\) compared to 7180 kg/m\(^3\) for steel balls. Due to surface tension, water drops in oil will be spherical. We can get water drops with diameters of the order of 2 to 4 mm easily. Because the radius is large, the small gravitational force makes the drops reach terminal velocity in a shorter length of the oil column than with steel balls.

The mass of a water drop of radius \( r \) is

\[
m = \left( \frac{4\pi}{3} \right) r^3 \rho. \tag{6}
\]

When this ball is dropped in a liquid of viscosity \( \eta \) the equation of motion is

\[
\frac{mdv}{dt} + 6\pi \eta rv = \left( \frac{4\pi r^3}{3} \right) (\rho - \rho') g. \tag{7}
\]

If the velocity at \( t = 0 \) is zero, the velocity will vary with time as

\[
v(t) = v_{\text{terminal}} \left( 1 - e^{-\frac{t}{\tau}} \right). \tag{8}
\]

Here \( v_{\text{terminal}} \) is given by equation (5) and

\[
\tau = \frac{m}{(6\pi \eta r)}. \tag{9}
\]

When \( t = 5\tau \), the velocity would have reached a value of 0.993\( v_{\text{terminal}} \). If we take a water drop of 2 mm radius, the mass \( m \) is 0.0335 grams. Taking the viscosity of oil as 100 cP (1 g/cm s), the relaxation time \( \tau \) comes out to be

\[
\tau = \frac{0.0335}{(6\pi \times 0.2)} = 8.89 \times 10^{-3} \text{ s} \tag{10}
\]

and

\[
v_{\text{terminal}} = \left( \frac{2}{9} \right) (0.2)^2 (1 - 0.870) 980 = \frac{1.13 \text{ cm}}{\text{s}} \tag{11}
\]
In a time $5\tau$, the drop will travel a distance less than

$$s = v_{\text{Terminal}} \times 5\tau = 1.13 \times 5 \times 8.89 \times 10^{-3} = 0.05 \text{ cm.} \quad (12)$$

If we start measuring the velocity of the drop after it has travelled a distance of 5 cm through the oil, we may be sure that it has reached the terminal velocity.

The jar needs to be only about 50 cm tall, and the terminal velocity can be measured by timing the fall through the middle $1/3$ of the height of the jar. It is easy to get drops of water of the same size, falling in regular succession. So the time of fall can be measured for six drops, one after the other, and the average taken. To measure the radius of the drop, we collect twenty drops of water in a watch glass, and weigh it.

After several such experiments, water collects at the bottom of the tube, and the oil column floats on the water. The water can be tapped off periodically.

5. Experimental Arrangement

Figure 4 shows the experimental arrangement.

A cylindrical glass jar, of diameter 5 cm and height 50 cm, is fixed upright in a stand. At the bottom end, it has a stopcock to drain the water collected in the jar during the experiment. This jar is filled to a height of 40 cm with motor oil 10 W-30, which is used in scooters as a lubricant. A water dispenser rests on the top of the jar. This has a nozzle of a small diameter. When filled with water to a certain height, drops of water form at the end of the nozzle, and fall into the oil at regular intervals. The number of water drops falling into the oil in one minute will depend on the height to which water is filled in the dispenser. But the size of the drops is independent of this height. The height of water is adjusted so that two to three drops fall into the oil per minute. Two marks A and B are made at a distance of 15 cm in the middle $1/3$ of the jar. The drops of water are spherical in the oil, and one can track the falling drops as they are visible to the eye. The drops travel a distance of about 5 to 10 cm in the oil before reaching the mark A, and attain terminal velocity by the time they reach mark A. Using a stop watch, one can measure the time taken by the drop to travel from A to B.

This measurement can be repeated on six drops, and the average time taken by a drop to travel from A to B can be calculated.

The water dispenser is taken out, put on another stand, and twenty drops of water are collected on a weighed watch glass. The watch glass is weighed again. The difference in weight gives the mass of 20 drops of water, from which the mass $m$ of each drop is calculated. The radius of the drop can then be found from the equation
Experiments in Mechanics

\[ r = \left( \frac{3}{4\pi \rho} \right)^{1/3} m \]  

The experiment can be repeated with water dispensers having three or four different sizes of the nozzle.

A sample set of data is given in Table 2.

<table>
<thead>
<tr>
<th>Run No.</th>
<th>Time (s)</th>
<th>Mass of 20 drops of water</th>
<th>Mass of 1 drop of water</th>
<th>Radius of drop</th>
<th>Dynamic viscosity of oil</th>
<th>Kinematic viscosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20.98</td>
<td>0.51 g</td>
<td>0.026 g</td>
<td>0.18 cm</td>
<td>126 cP</td>
<td>141.8 cS</td>
</tr>
<tr>
<td>2</td>
<td>21.04</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20.63</td>
<td></td>
<td></td>
<td>0.18 cm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20.77</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>21.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>20.83</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Column 2 gives the time for a drop to fall a distance of 15 cm. The average time of fall is calculated. The terminal velocity is determined by dividing 15 cm by this average time of fall. From the mass of 20 drops of water, the average mass of each drop is found. The radius of the drop is calculated, taking the density of water as 1 g/cc. The dynamic viscosity is calculated from the radius of the drop and the terminal velocity, using equation (5). The kinematic viscosity is obtained by dividing the dynamic viscosity by the density of the oil.

Two other measurements with different drop sizes gave the dynamic viscosity as 130.5 and 120.6 cP and the kinematic viscosity as 146.6 and 135.5 cS. So the average value of dynamic viscosity is 126±5 cP and the average value of kinematic viscosity is 141±6 cS.

From the tables of viscosity for SAE 30 oil given in www.viscopedia, the dynamic viscosity at 30 °C is 128.4 cP and the kinematic viscosity is 146.7 cS. 10W-30 oil is a mixture of oils. But the number 30 refers to the SAE viscosity grade 30. The temperature dependence of the viscosity for 10W 30 oil may not be exactly the same as the temperature dependence shown for SAE 30 oil in Figure 1. The agreement of our value with the Viscopedia data for SAE 30 oil is within the error limits calculated from the three measurements.

This is a very simple method for measuring the viscosity of motor oil at room temperature.

6. Questions

1. If the experiment is performed in a place where the temperature is 15 °C, will the time of fall increase or decrease?

2. If a plastic ball of density 0.85 g/cc is used, can the experiment be performed with 10W-30 oil?
Chapter 4

Experiments in Heat
4.1 Calibration of a Si Diode and a Copper–Constantan Thermocouple as Temperature Sensors

1. Introduction

One uses a thermometer for temperature measurement in the laboratory. Thermometry is based on the variation of some property of the thermometer with temperature. The platinum resistance thermometer is based on the variation of the electrical resistance of high purity platinum wire with temperature. The platinum resistance thermometer is convenient, because (1) it has an appreciable temperature coefficient of resistance, and (2) the variation of resistance with temperature is very nearly linear over a wide range of temperature. For example, consider a platinum resistor with a nominal resistance of 100 ohms at 0°C. Such a resistor is called Pt 100. The resistance of this Pt 100 thermometer increases by 0.39 ohms for every degree Centigrade rise in temperature.

Two other convenient thermometers are (1) the thermocouple, and (2) the diode thermometer. In a thermocouple, there are two junctions between two dissimilar materials like copper and constantan. When the junctions are maintained at two different temperatures $T_1$ and $T_2$, a thermo-emf appears across the thermocouple leads. This emf is not a linear function of temperature difference, over a wide range of temperature difference. The value of the thermo-emf vs. $T_1$, when $T_2$ is held constant, is a curve which can be fitted to a polynomial. The slope of this curve, at any temperature, gives the thermo-power $S$ of the thermocouple at that temperature. For example, near room temperature, a copper–constantan thermocouple has a thermo-power of about 40 $\mu$V/°C. Because the thermo-emf is small, one has to amplify the output with a DC differential amplifier. The advantage of the thermocouple is its small thermal capacity. A thermocouple will track rapidly changing temperatures because its thermal capacity is small.

Another thermometer is a diode. One can send a small current (about 1 to 2 mA) through a diode in the forward direction, and measure the voltage drop across the diode. If the current through the diode is kept constant, this forward voltage decreases, as the temperature increases, nearly linearly over a limited temperature region. This decrease is measurable with a digital voltmeter. The diode thermometer can be used for controlling the temperature. The forward voltage of a diode changes with the age of the diode. For precision thermometry, a diode is seldom used.

In this experiment, we study the temperature variation of the thermo-emf of a copper–constantan thermocouple, and the temperature variation of the forward voltage of a silicon diode at a constant current. The temperature is measured with a Pt 100 resistance thermometer.

2. Apparatus Required

Regulated power supply, temperature indicator with a Pt100 thermometer, furnace, insert containing diode and copper–constantan thermocouple, constant current source, DC differential amplifier, and a multimeter, reading in DC 2V range.

3. Experimental Procedure

Connect the stabilized power supply to the furnace. The furnace has an insert. At the bottom end of the insert, there is a length of aluminium rod, of square cross section of 1 cm $\times$ 1 cm, and length about 5 cm. An axial hole of 3 mm diameter is bored in the aluminium rod. On one end, a Pt 100 sensor is inserted into the hole. One junction of a copper–constantan thermocouple is inserted in the hole from the other end. The other junction of the copper–constantan thermocouple is fixed to the terminal block, from which the insert hangs into the furnace.

On one of the lateral faces of the aluminium rod, the Si diode is fixed with Araldite. Care is taken to see that there is no shorting of any of these elements with the rod. The aluminium rod is suspended inside the furnace from a terminal block, on a stand fixed to the furnace.

The terminal block of the insert is shown in Figure 1.

On the terminal block of the insert to the furnace, we have a RCA socket (marked TC) connected to the thermocouple leads, a pair of banana terminals (marked Pt 100) connected to the Pt 100 sensor, and another pair of banana terminals (marked Si diode) connected to a silicon diode.
The Pt100 terminals on the terminal block are connected to the appropriate terminals at the back of the temperature controller, as indicated on the information sheet pasted on top of the temperature controller.

The output terminals of a constant current source are connected to the Si diode, the red terminal on the constant current source to the terminal marked + on the terminal block, and the black terminal of the CCS to the other terminal of the Si diode on the terminal block. Current will flow in the forward direction through the diode. The two switches on the current source must be in the low current mode. The constant current source is switched on, and the LC pot is set to give a current of 1 or 2 mA through the diode. If this current is not achieved, it means the current is not flowing in the forward direction, and the connections have to be checked. A DMM in the DC 2 V range is connected to the Si diode terminals, and the forward voltage across the Si diode is measured.

The RCA socket on the terminal block is connected to the terminals I1 of the DC differential amplifier. The panel meter of the DC differential amplifier indicates directly the voltage \( V \) of the thermocouple in millivolts. Switch on the differential amplifier.

Connect the stabilized DC power supply to the furnace. Switch on the stabilized power supply and adjust the voltage output of the power supply to about 7 to 8 V.

Put a wad of cotton to close lightly the top of the furnace tube. Otherwise there will be convection currents of air in the furnace, and the furnace will not reach the required temperature even when the applied voltage reaches 20 V.

The temperature of the furnace will increase slowly, at the rate of about 1 to 1.5 degrees per minute. As the temperature rises, one will have to increase, slowly, the applied voltage to the furnace to maintain this rate of heating. This applied voltage will not exceed about 15 V for the furnace to reach a temperature of 150 °C.

Measure the voltage across the Si diode at intervals of 10 °C, as indicated on the temperature indicator. Similarly, measure, at intervals of 10 °C, the output of the differential amplifier connected to the copper–constantan thermocouple. Go only up to 150 °C because the contacts to the Si diode and the thermocouple are soft soldered, and the solder will melt around 180 °C.

At the end of the experiment, do not forget to switch off the power supply and all the other equipment.

### Table 1

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>( V_{\text{for}} ) (volts)</th>
<th>( V_{\text{cal}} )</th>
<th>Error</th>
<th>Sum of</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.5</td>
<td>0.578</td>
<td>0.578</td>
<td>7.56E−08</td>
<td></td>
</tr>
<tr>
<td>50.2</td>
<td>0.551</td>
<td>0.551</td>
<td>2.26E−06</td>
<td></td>
</tr>
<tr>
<td>62.2</td>
<td>0.526</td>
<td>0.526</td>
<td>2.97E−06</td>
<td></td>
</tr>
<tr>
<td>77.3</td>
<td>0.494</td>
<td>0.494</td>
<td>3.46E−06</td>
<td></td>
</tr>
<tr>
<td>89.9</td>
<td>0.468</td>
<td>0.468</td>
<td>3.97E−06</td>
<td></td>
</tr>
<tr>
<td>105</td>
<td>0.436</td>
<td>0.436</td>
<td>4.69E−06</td>
<td></td>
</tr>
<tr>
<td>117.5</td>
<td>0.409</td>
<td>0.409</td>
<td>4.97E−06</td>
<td></td>
</tr>
<tr>
<td>135.2</td>
<td>0.372</td>
<td>0.372</td>
<td>4.98E−06</td>
<td></td>
</tr>
<tr>
<td>149.8</td>
<td>0.341</td>
<td>0.341</td>
<td>6.73E−06</td>
<td></td>
</tr>
<tr>
<td>Variance</td>
<td>8.41E−07</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Std Dev</td>
<td>9E−04V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A sample set of readings for Si diode is given in Table 1. The current through the diode is kept at 1 mA. In Table 1, the first column gives the temperature, in Celsius, as recorded on the temperature indicator. The second column gives the forward voltage across the silicon diode, as indicated on the DMM. The third and fourth columns are described in the next paragraph.

\[ V = 0.6574 - 0.00211 \times T \]

The quality of the fit is shown by calculating \( V \), using this equation, at the temperatures of measurement. The calculated values are given in the third column of Table 1. The fourth column gives the sum of \((V_{\text{meas}} - V_{\text{calc}})^2\) up to the row in which this value is entered. The value in the fourth column, at the last temperature measured, gives the sum of the square of the error for all measured values. The variance is obtained by dividing this value by \((N - 1)\), where \(N\) is the number of measurements taken. The standard deviation is the square root of the variance. The standard deviation of the measured value is within \(\pm 1\) mV. Since the diode voltage changes by 2.11 mV for every degree temperature change, this standard deviation will correspond to an error in temperature of \(1/(2.11) \approx 0.5^\circ C\). With this calibration graph, one can measure temperature with the Si diode with an accuracy of \(\pm 0.5^\circ C\).

NOTE: The readings shown above are sample readings. The forward voltage will vary with the current and the diode. However, for all diodes, the slope will lie between 1.8 mV/°C to 2.2 mV/°C.

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>Th-emf (mV)</th>
<th>Th-emf calc (mV)</th>
<th>Sum of Error^2</th>
<th>Thermo-power (mV/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.5</td>
<td>0.395</td>
<td>4.0</td>
<td>2.71E−05</td>
<td>0.04</td>
</tr>
<tr>
<td>50.2</td>
<td>0.905</td>
<td>0.92</td>
<td>2.44E−04</td>
<td>0.041</td>
</tr>
<tr>
<td>62.2</td>
<td>1.455</td>
<td>1.421</td>
<td>1.40E−03</td>
<td>0.042</td>
</tr>
<tr>
<td>77.3</td>
<td>2.05</td>
<td>2.066</td>
<td>1.66E−03</td>
<td>0.043</td>
</tr>
<tr>
<td>89.9</td>
<td>2.63</td>
<td>2.617</td>
<td>1.84E−03</td>
<td>0.044</td>
</tr>
<tr>
<td>105</td>
<td>3.285</td>
<td>3.291</td>
<td>1.87E−03</td>
<td>0.045</td>
</tr>
<tr>
<td>117.5</td>
<td>3.85</td>
<td>3.861</td>
<td>2.00E−03</td>
<td>0.046</td>
</tr>
<tr>
<td>135.2</td>
<td>4.68</td>
<td>4.688</td>
<td>2.06E−03</td>
<td>0.047</td>
</tr>
<tr>
<td>149.8</td>
<td>5.395</td>
<td>5.386</td>
<td>2.15E−03</td>
<td>0.048</td>
</tr>
</tbody>
</table>

\[ \text{Variance} = 2.68E−04 \]
\[ \text{Std Dev} = 1.6E−02 \text{ mV/K} \]
The thermo-emf of the copper–constantan thermocouple was measured on a DC differential amplifier with the same setup. The results are given in Table 2.

In Table 2, the first column gives the temperature, in Celsius, indicated by the temperature indicator. The next column gives the thermo-emf, measured in mV on the differential amplifier. Figure 3 shows a plot of thermo-emf $V$ against temperature $T$. Using a scale, one can see that the variation is not linear.

\[
v = -1.068 + 0.0378 T + 3.499 \times 10^{-5} T^2
\]

This variation is fitted with a second-degree polynomial

\[
V = -1.0680 + 0.0378 \times T + 3.499 \times 10^{-5} T^2.
\]

Here, $V$ is the thermo-emf in mV, and $T$ is the temperature in ¹C. The calculated value of the emf, using this formula, is given in column 3 of Table 2. The sum of the square of the error in the fit is given in column 4. The variance and standard deviation are also given in the table. The calculated thermo-power $dV/dt$ from equation (2) is given in the last column of the table.

From the last column, we see that the thermo-power is $\approx 40 \mu \text{V/}^º\text{C}$. The standard deviation is $16 \mu \text{V}$. So the calibration will be accurate to $16/40 = 0.4$ ¹C.

4. Questions

In this experiment, you have used three different temperature sensors: Platinum resistance thermometer with the temperature indicator, Si diode, and the thermocouple.

1. The response of the platinum thermometer is linear with temperature. Of the other two sensors, which one shows a linear response in the temperature range measured?

2. Can you use a Si diode thermometer to control the temperature?

3. Which of the two sensors, the Si diode or the thermocouple, will have a lower mass? Which thermometer will respond fast when the temperature changes rapidly?

4. Can you think of any other property that can be used to measure temperature?

5. Can you suggest a way of controlling the temperature of a bath using Si diode thermometer?
4.2 The Neutral Temperature of Iron–Copper Thermocouple

1. Introduction

In general, the thermo-emf of a thermocouple will increase monotonically with temperature, as seen in the case of the copper–constantan thermocouple in the previous section. But for iron–copper and iron–silver thermocouples, the behavior is significantly different. The thermo-emf goes through a maximum value at a certain temperature. At this temperature, the thermo-electric power $S$ becomes zero. This temperature is called the neutral temperature.

2. Experimental Procedure

Between two flat heaters, we have a junction of iron–copper, and a junction of chromel–alumel. The chromel–alumel thermo-emf is used to measure temperature. The two thermocouples are connected to Channels 1 and 2 of a differential amplifier. The heater is connected to a regulated power supply, and the voltage is periodically adjusted so that the thermo-emf of the chromel–alumel thermocouple goes up at the rate of about $200 \mu$V per minute. Readings are taken of the thermo-emf of the Fe–Cu thermocouple and the chromel–alumel thermocouple every two minutes, till the thermo-emf of the chromel–alumel thermocouple is about 16 mV. Then the heater and the differential amplifier are switched off.

We note the room temperature $T_0$ in Celsius. From the NBS Table given in the Appendix, we note the thermo-emf $V(T_0)$ of the chromel–alumel thermocouple at temperature $T_0$. In the NBS, Table, the reference junction is at ice point. If the thermo-emf of the chromel–alumel thermocouple is $V$ mV, we calculate $\Psi = (V + V(T_0))$. This voltage $\Psi$ will be the thermo-emf at temperature of measurement, when the reference junction is at ice point. From the NBS Tables, we note the temperature corresponding to the voltage $\Psi$.

Figure 1 shows how the thermo-emf of an iron–copper thermocouple varies with temperature.

![Figure 1. Thermo-emf of iron–copper thermocouple as a function of temperature.](image)

The thermo-emf $V_{Fe-Cu}$ is fitted to a second degree polynomial

$$V_{Fe-Cu} = A + B_1 T + B_2 T^2. \quad (1)$$

The fit is shown by the continuous curve in Figure 1.
The thermo-power $S_{\text{Fe-Cu}}$ is obtained by differentiating $V_{\text{Fe-Cu}}$ with respect to $T$ giving

$$S_{\text{Fe-Cu}} = B_1 + 2B_2 T.$$  \hfill (2)

Figure 2 shows the variation of $S_{\text{Fe-Cu}}$ with respect to temperature.

The Seebeck coefficient $S_{\text{Fe-Cu}}$ goes from positive to negative values. It becomes zero at around 260 °C. This is the neutral temperature of the iron–copper thermocouple.

$$S_{\text{Fe-Cu}} = S_{\text{Fe}} - S_{\text{Cu}}.$$  \hfill (3)

The Seebeck coefficient for copper is given by

$$S_{\text{Cu}} = CT + D, \quad C = 4.989 \times 10^{-3} \mu \text{V/}^\circ \text{C}^2, \quad D = 1.7 \mu \text{V/}^\circ \text{C}.$$  \hfill (4)

From equation (4), one can calculate $S_{\text{Cu}}$ at different temperatures, and then calculate $S_{\text{Fe}}$ from equation (3), as a function of temperature. A plot of the Seebeck coefficients $S_{\text{Fe}}$ and $S_{\text{Cu}}$ as functions of temperature is shown in Figure 3. We see that the two curves intersect at the neutral temperature.

This experiment is of historic significance. From thermodynamic arguments Thomson showed that the Seebeck coefficient for any material should be a constant, and independent of temperature, if the only thermoelectric effects are the Seebeck and Peltier effects. This will imply that for any thermocouple, the thermo-emf will vary linearly with temperature. This experiment showed clearly that this is not so. So Thomson postulated a third thermoelectric effect, namely the Thomson effect.

When a current $I$ passes through a small element of a wire with a temperature gradient, Thomson postulated that the heat generated per second is

$$Q = I^2 R + \mu I \frac{dT}{dx}.$$  \hfill (5)

Here $R$ is the resistance of the element, and $dT/dx$ is the temperature gradient. $\mu$ is a constant of the material, called the Thomson coefficient. The first term is the irreversible heat generated through the resistance. The second term is the reversible heat generated by the temperature gradient. It arises because the charge carriers have a specific heat, and heat is absorbed or liberated when the temperature of the charge carriers is raised or lowered. Thomson derived a relation between $\mu$ and the temperature variation of the Seebeck coefficient, namely
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\[ \mu = T \cdot \frac{dS}{dT}. \] (6)

One can determine the absolute Seebeck coefficient of a material by measuring the Thomson coefficient as a function of temperature.

Figure 4 shows the temperature dependence of the Seebeck coefficients for chromel and alumel, and the relative Seebeck coefficient \( S_{Cr-Al} \).

We see that the Seebeck coefficient of chromel is positive, while the Seebeck coefficient of alumel is negative. The relative Seebeck coefficient, which is the difference between the Seebeck coefficients for the individual metals, is therefore large (about 40 \( \mu \text{V/K} \)), and nearly independent of temperature over a wide range. This is the reason that chromel–alumel is a standard thermocouple.
3. Questions

1. Why can the iron–copper thermocouple not be used for measuring temperature?

2. We make three junctions, copper–chromel, chromel–constantan, and constantan–copper. We place the first and the second junctions at the same temperature \( T_1 \) and the third junction at a temperature \( T_2 < T_1 \). Show that the emf measured between the two copper wires is

\[
V = \int_{T_2}^{T_1} S_{\text{Cu-constantan}} \, dT.
\]

The presence of the intermediate metal chromel is immaterial.

This is called the Law of Intermediate Metals.

3. The Seebeck coefficient of a superconductor is zero. Why can one not measure the absolute Seebeck coefficient of any other metal, like chromel, over a wide range of temperature, by forming a thermocouple of chromel with a superconductor?

4.3. Stefan’s Constant of Radiation

1. Introduction

A hot body emits radiation that is absorbed by another body. The amount of energy radiated depends on its surface area, the absolute temperature of the body, and a property depending on the nature of the surface of the body. This property is called emissivity, and is denoted by the symbol \( \varepsilon \). To understand emissivity we have to define a perfect black body.

A perfect black body absorbs all the radiation that falls on it, whatever be the frequency of radiation. This is an ideal. No body is a perfect black body.

The emissivity \( \varepsilon \) is the ratio of the energy radiated by unit area of the given surface, to that radiated by unit area of a perfectly black body, when both are at the same temperature. The total energy radiated per second \( Q \) by any body is given by

\[
Q = \varepsilon \sigma A T^4.
\]

Here \( A \) is the surface area of the body. \( \sigma \) is called the Stefan–Boltzmann constant. It is one of the important constants of physics.

\[
\sigma = \frac{\pi^2/60}{h^2c^2},
\]

where \( k \) is the Boltzmann constant, \( h \) is the Planck’s constant divided by \( 2\pi \), and \( c \) is the velocity of light in vacuum.

A body can lose heat to its surroundings by conduction, convection and radiation. To measure the Stefan’s constant, we must reduce the fraction of heat lost by conduction and convection to a few percent of the total heat lost, so that radiation dominates. This is achieved by the following steps: (a) blackening the surface of the body and that of the enclosure to make \( \varepsilon \) have a maximum value of nearly unity, (b) increasing the resistance to heat conduction by using very thin wires for the electrical measurements, and by using a nylon thread of poor conductivity to suspend the sample, and (c) using a closed enclosure to reduce convection losses due to currents of air.

2. Experimental Arrangement

Figure 1 shows a schematic of the setup for measuring Stefan’s constant. A is a lightweight hollow aluminium cylinder, about 3 cm long and 2 cm in outer diameter, with wall thickness of 0.5 mm. To heat the sample, we require a heater which can generate about 1 W power. A simple way of making the heater is the
A schematic of the experimental arrangement.

Figure 1. A schematic of the experimental arrangement.

A Teflon tube of outer diameter 10 mm and wall thickness 0.5 mm is cut to a length of 2.6 cm. A series of five 1 mm diameter holes are made around the circumference of the tube at the top and bottom. Five 100-ohm 1/4 W resistors are taken, and the leads of each resistor are inserted into the top and bottom holes in the Teflon tube. The five resistors are connected in parallel, and lie flat on their length on the surface of the Teflon tube. Leads for current are insulated by winding a thin Teflon tape around them, and are brought out from the top of the aluminium cylinder. The total resistance of the heater is 20 (i.e., 100/5) ohms, and it can generate up to 1 W of heat. Two or three layers of thin Teflon tape are wound on the outer surface of the resistors. The Teflon tube is placed within the aluminium cylinder. The interspace between the inner wall of the aluminium cylinder and the outer wall of the Teflon tube is filled with aluminium foil to provide a good thermal link (Figure 2).

Figure 2. Mounting of the heater in the Al cylinder.

A copper–constantan thermocouple is made by taking about 5 cm of insulated constantan wire (about SW 40), and two long insulated copper wires of the same gauge. After removing the insulation of the wires to a length of 3 to 5 mm at the ends, two copper–constantan junctions are made by twisting the bare wires. When the solder is just melting on the soldering iron, the two junctions are pushed into the molten solder and pulled out. A drop of superglue (a very quick and strong rapid-cure adhesive available in small tubes) is applied on the aluminium cylinder, near the middle, where it is desired to place one of the thermocouple junctions. In a few minutes the liquid dries over the surface to form a thin electrically insulating layer. A drop of superglue is applied on the thermocouple junction. One should be careful not to apply too much superglue. If the glue forms a thick layer between the junction and the surface of the aluminium cup, there will be an appreciable temperature drop in this layer of glue. The thermocouple will measure a temperature lower than the actual temperature of the aluminium surface. After a few minutes, the superglue sets, and forms a tight joint between the thermocouple and the aluminium cylinder. The aluminium cylinder can then be painted black on the outside with enamel paint. The paint must be allowed to dry. The cylinder is hung by a thin nylon thread from the top lid of a small stainless steel can E. The inside of the can is also painted black. On the top lid D, banana sockets to the current leads, and RCA socket to the thermocouple leads are fixed. The second junction of the thermocouple is fixed to the lid D of the stainless steel can E.

The copper–constantan thermocouple has a thermo-electric power of 40 microvolts per degree Kelvin near room temperature. For a 10 degree difference in temperature of the two junctions, the thermo-emf will be about 400 microvolts. This can be measured by a DC differential amplifier.


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The Stefan’s constant box is shown in Figure 3.

![Figure 3. Stefan’s constant box.](image)

3. Apparatus Required

Constant current source, DC differential amplifier, Stefan’s constant box.

4. Procedure

The connection diagram for the Stefan’s constant experiment is shown in Figure 4. A constant current source, CCS, is connected to the heater terminals of the Stefan’s constant box. The two switches on the constant current source are put in the high current mode. The input terminals I₁ of a DC differential amplifier, DDA, are connected to the RCA socket (TC) for thermocouple outlet on the Stefan’s constant box (SB).

![Figure 4. Connection diagram for Stefan’s constant experiment.](image)

The current through the heater is adjusted to be 200 mA by turning the two pots at the top right corner of the front panel of the CCS. This is the maximum current you should pass through the heater. Do not exceed this current. A voltage will appear on the DMM, which will increase in magnitude with time. Wait for one hour for the steady state to be reached. Note the value of the output DC voltage $V$ in millivolts.

Then, reduce the current in steps of twenty mA, till you reach 100 mA. For each value of current, wait for 45 minutes before taking a reading of $V$.

The total power dissipated in the resistor is

$Q = I^2 R.$

Here, $I$ is the current through the heater, and $R$ is the resistance of the heater. The resistance of the heater is marked on the Stefan’s constant box.

The temperature difference between the cylinder and the surrounding can is
Here, the thermo-power of the copper–constantan thermocouple is taken as 40 microvolts per degree Kelvin. The factor $10^3$ arises because we are dividing millivolts by microvolts. The room temperature $T_1$ is measured with a mercury thermometer. Knowing $T_1$ and $\Delta T = (T_1 - T_2)$ from measurement, $T_2$ is calculated. All temperatures should be expressed in Kelvin.

If all the heat is lost only by radiation, then

$$ Q = \varepsilon \sigma A \left( T_2^4 - T_1^4 \right). $$

Here, $\varepsilon$ is the emissivity of the black surface. For different types of black paints, the emissivity varies between 0.9 and 0.95, the latter being the value for enamel paints. We take 0.95 as the emissivity of the black surface, since we have used enamel paint.

The area $A$ of the aluminium cup radiating heat is

$$ A = 2\pi r(r + l). $$

Here, $r$ is the radius and $l$ the length of the cylinder. The values of $r$ and $l$ are given on the Stefan’s constant box.

A graph is plotted on a computer with $(T_1^4 - T_2^4)$ on the X axis, and $Q$ on the Y-axis. A straight line is fitted to the points, and the slope of the straight line is found. The Stefan’s constant is calculated from the slope, knowing the area $A$ of the aluminium cylinder.

A sample set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Stefan’s constant. Room temperature $T_1 = 299$ K.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of Al Cup</td>
</tr>
<tr>
<td>Height of Al Cup</td>
</tr>
<tr>
<td>Area of surface</td>
</tr>
<tr>
<td>Resistance of heater</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$I$</th>
<th>$Q$</th>
<th>$V$</th>
<th>$\Delta T$</th>
<th>$T_2$</th>
<th>$T_2^4 - T_1^4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(amps)</td>
<td>(watts)</td>
<td>(mV)</td>
<td>(K)</td>
<td>(K)</td>
<td>(K$^4$)</td>
</tr>
<tr>
<td>0.203</td>
<td>0.8283</td>
<td>1.780</td>
<td>44.5</td>
<td>343.5</td>
<td>5.93E+09</td>
</tr>
<tr>
<td>0.182</td>
<td>0.6658</td>
<td>1.546</td>
<td>38.66</td>
<td>337.7</td>
<td>5.01E+09</td>
</tr>
<tr>
<td>0.161</td>
<td>0.521</td>
<td>1.22</td>
<td>30.49</td>
<td>329.5</td>
<td>3.79E+09</td>
</tr>
<tr>
<td>0.141</td>
<td>0.3996</td>
<td>0.951</td>
<td>23.78</td>
<td>322.8</td>
<td>2.86E+09</td>
</tr>
<tr>
<td>0.121</td>
<td>0.2943</td>
<td>0.707</td>
<td>17.68</td>
<td>316.7</td>
<td>2.06E+09</td>
</tr>
<tr>
<td>0.101</td>
<td>0.205</td>
<td>0.498</td>
<td>12.44</td>
<td>311.4</td>
<td>1.42E+09</td>
</tr>
</tbody>
</table>

In this table, column 1 gives the current through the heater in amperes. The heater power $Q$ in watts is given in column 2. Column 3 gives $V$ in mV. From this, the temperature difference $\Delta T = T_2 - T_1$ is calculated by multiplying $V$ (in mV) by 25, and this value is given in column 4. Knowing $T_1$, the temperature $T_2$ of the surface of the aluminium block is calculated, and is given in column 5. Column 6 gives $(T_2^4 - T_1^4)$ in units of K$^4$. Figure 5 gives a plot of $Q$ against $(T_2^4 - T_1^4)$.

From the figure, the slope of the curve is found to be $1.382 \times 10^{-10}$ W/K$^4$. The Stefan’s constant is calculated from equation (1), with the area of surface given in Table 1, and taking the emissivity of black paint to be 0.95. The value of $\sigma$ comes out as $5.38 \times 10^{-8}$ W/m$^2$K$^4$, in reasonable agreement with the actual value $5.67 \times 10^{-8}$ W/m$^2$K$^4$.

The sources of systematic errors in this experiment are the following:

1. The emissivity of the black paint used may be lower than 0.95. This is the most important source.
2. Some heat may be lost by conduction and convection.

The sources of systematic errors in this experiment are the following:

1. The emissivity of the black paint used may be lower than 0.95. This is the most important source.
2. Some heat may be lost by conduction and convection.
If one would like to measure the emissivity of a polished surface, one can have a second setup with a thin copper cylinder, with the outer surface electroplated with nickel. The above measurements can be repeated. Knowing the value of $\sigma$, the emissivity of the electroplated nickel surface can be found.

5. Time Constant

In all steady-state heat experiments, time constant plays an important role. The temperature grows from its initial value, at room temperature, to its final value as

$$T_1 + \theta(t) = T_1 + \theta(\infty)\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]. \quad (7)$$

Here, $\theta(\infty)$ is $(T_2 - T_1)$, $T_2$ being the steady-state value of the temperature, and $\tau$ is the relaxation time. Theoretically, the steady-state temperature is attained after infinite time. But in practice, if one waits for a time $t$ longer than $5 \tau$, the temperature is within 1% of its steady-state value. It is necessary to understand the factors governing $\tau$ so that the experiment can be designed with a value of $\tau$ of the order of 10 minutes.

Then, the reading of the thermocouple for a given heater power can be taken after forty five minutes to one hour, and the experiment can be repeated for at least four different heat inputs within three to four hours.

At a time $t$, part of the heat input goes to raise the temperature of the block, and part is lost as radiation, conduction and convection. If the temperature rise is not too large, we may lump all the heat loss mechanisms into a factor $C \theta(t)$. This is the Newton’s law of cooling.

$$ms \left(\frac{d\theta(t)}{dt}\right) + C \theta(t) = Q. \quad (8)$$

Here, $ms$ is the total thermal capacity of the metal block including the thermal capacity of the heater and the thermometer.

The solution to this equation is

$$\theta(t) = \left\{\frac{Q}{C}\right\} \left[1 - \exp\left\{-\left(\frac{C}{ms}\right) t\right\}\right]. \quad (9)$$
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From this equation, we identify $\frac{Q}{C}$ to be $\theta(\infty)$, and $(ms/C)$ to be $\tau$. For reducing the relaxation time $\tau$, we should reduce the mass of the cup. That is why it is preferable to use a lightweight cup, made of aluminium or thin copper.

To calculate what mass we should choose, we assume that heat is lost only by radiation. Then, using equation (6), we have

$$C\theta = \varepsilon\sigma A \left( (T_1 + \theta)^4 - T_1^4 \right) \approx 4\varepsilon\sigma A T_1^3 \theta$$

(10)

when $\theta$ is small. $T_1$ is about 300 K and $\theta$ about 30 K. So,

$$C \approx 4\varepsilon\sigma A T_1^3$$

(11)

and

$$\tau \approx \frac{ms}{(4\varepsilon\sigma A T_1^3)}.$$ 

(12)

Taking $\sigma$ to be $5.7 \times 10^{-3}$ W/m²K, $\varepsilon$ to be 1, $T_1$ to be 300 K, and $A$ to be $2 \times 10^{-3}$ m², we get

$$C \approx 0.0125.$$

If $\tau$ is to be less than 600 s,

$$ms < C \times 600 = 7.51/{K}.$$

The specific heat of aluminium is 0.910 J/gK. This means that the mass of the aluminium cylinder should be less than about 8 gm. A cylinder with diameter of 2 cm, length $l$ of 3 cm, and wall thickness $t$ of 0.5 mm will have a volume $(2\pi rl + \pi r^2)t = 1.09$ cc. Density of aluminium is 2.7 g/cc. The mass of the cylinder in the experiment is about 2.9 g. To this, the weight of the Teflon cylinder and the five resistors must be added. This is the reason why you should wait for at least 45 minutes for steady state to be reached.

6. Questions

1. Why is the inside of the wall of a thermos flask silvered?

2. Liquid nitrogen is stored in an evacuated double-walled stainless steel container. The radius of the outer wall is 10 cm, and the length of the cylinder is 1 m. The outer wall is at room temperature (300 K). The container is filled with liquid nitrogen. Its boiling point is 77 K. Assuming the emissivity of stainless steel to be 0.1, calculate the heat reaching the liquid nitrogen from the outer wall. If the latent heat of vaporization is 160 kilojoules/liter of liquid nitrogen, calculate the boil-off rate of liquid nitrogen.

3. If we put 10 layers of aluminized Mylar foils between the outer wall and the inner wall of the double-walled vessel, will it reduce the heat radiation reaching the liquid nitrogen? If the emissivity of the foils is taken to be the same as the emissivity of stainless steel, by what factor will the heat reaching liquid nitrogen change?

4.4 Thermal and Electrical Conductivity of Copper to Determine its Lorentz Number

1. Introduction

In a pure metal, the conduction of heat is almost entirely due to electrons. Therefore, one would expect the thermal conductivity of a metal to be related to its electrical conductivity. There is a relation called the Wiedemann–Franz relation for metals. This relation states that

$$\frac{K}{\sigma} = LT.$$ 

(1)
Here, $K$ is the thermal conductivity, $\sigma$ is the electrical conductivity, $T$ is the absolute temperature and $L$ is called the Lorentz number. In the free electron theory of metals, this Lorentz number is a universal number, related to the charge on an electron and the Boltzmann’s constant.

$$L = \left( \frac{\pi^2}{3} \right) \left( \frac{k}{e} \right)^2. \quad (2)$$

The Wiedemann–Franz relation is found to be satisfied by many metals, like copper, silver and gold, especially at and above room temperature. A measurement of $L$, therefore, allows one to get an approximate value for the ratio of $k/e$.

2. Experimental Arrangement

This arrangement is shown in Figure 1.

![Figure 1](image.png)

Figure 1. Copper tube with heater, thermocouples 1 and 2, current leads CL1 and CL2, voltage leads VL1 and VL2, and heater leads HL.

A 3/8" OD copper tube, of length 15 cm, and of refrigerator tube quality, is used for this purpose. A heater is made of a 2 W resistor of about 22 ohms, which is placed inside the tube near its center. Leads are taken from the two ends of the resistor. A thin layer of Teflon tape is wound on the resistor to insulate the wires from shorting with the copper tube. The interstices between the Teflon tape and the copper tube are filled with aluminium foil. With such a heater, the maximum current that can be passed should not exceed 260 mA. The exact resistance of the heater is given on the thermal conductivity box.

On each side of the copper tube we fix differential copper–constantan thermocouples to measure the temperature gradient. A few turns of enamel-coated copper wire (SWG 36) is wound tight on the copper tube, at the point where one wants to put the thermocouple junction. The enamel-coated wire is twisted tightly around the tube to make a good thermal contact. After twisting the wire, enamel is scratched out over part of the twisted wire, and the junction of a copper–constantan thermocouple is soldered to it. This way the junction is electrically insulated from the copper tube, but the thermal contact between the junction and the tube is good. The two junctions of each thermocouple are fixed on each side of the copper tube in this fashion, as shown in Figure 1. The distance between the junctions of each thermocouple is $d_1$ (about 6 cm). The exact value of $d_1$ is given on the thermal conductivity box. Two copper current leads, CL1 and CL2, are soldered to the copper tube at its two ends. Between the two current leads, two more voltage leads, VL1 and VL2, are soldered to the copper tube. The distance between the voltage leads is $d_2$. The value of $d_2$ is given on the thermal conductivity box.

A layer of cotton is wound on this copper tube. On this, a layer of paper is wound. On the paper, an aluminium foil is wound. Again, a layer of paper and a layer of aluminium foil are wound. In this process, care is taken to bring the wires out of the wrappings. The cotton and the aluminium foil wrappings serve to reduce heat loss by convection and radiation.

The copper tube, with its wrappings, is laid on a loose bed of cotton in a box. It is covered loosely with cotton and the leads are soldered to terminals on the front panel of the box. The front view of the box is shown in Figure 2.
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Figure 2. Setup for thermal and electrical conductivity of copper.

The red and black banana terminals (marked H1 H2) at the bottom are the heater connections. The two RCA sockets on either side are the thermocouple connections for the two differential thermocouples ThC1 and ThC2. The four banana terminals at the top are for measuring the electrical conductivity. The two extreme banana terminals (marked CL1 and CL2) are connected to the current leads, and the two banana terminals in between (marked VL1 and VL2) are connected to the voltage leads.

3. Apparatus Required

Constant current source, thermal conductivity of copper box, DC differential amplifier.

4. Procedure for Measuring Thermal Conductivity

1. Check that the two switches in the constant current source are in the high position, and the coarse and fine potentiometers of the high current mode are rotated anticlockwise to their extreme positions. Connect the banana terminals on the constant current source to the banana terminals marked H1 H2 on the thermal conductivity box.

2. Connect the RCA socket ThC1 on the thermal conductivity box to input terminals I1 on the differential amplifier, and the RCA socket marked ThC2 on the thermal conductivity apparatus to the input terminals I2 of the differential amplifier.

3. Switch on the constant current source, put it in high current mode, and set the current at 260 mA. Switch on the differential amplifier. Wait for 45 minutes before taking a reading of the thermocouple voltages. Steady state is attained in this time.

4. Put the selector switch on the differential amplifier in position I1, and note the reading $V_1$ in mV for the thermocouple ThC1. Then put the selector switch on the front panel of the differential amplifier in the position I2, and measure $V_2$.

5. Reduce the current to 240 mA. Wait for 30 minutes before taking readings $V_1$ and $V_2$. Reduce the current in steps of 20 mA, each time wait for 30 minutes and take the readings of $V_1$ and $V_2$. Take the readings for four values of the heater current.

From these readings, the thermal conductivity is calculated as follows. The heat input, when a current $I$ is passed through the heater, is
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\[ Q = I^2 R \text{ watts.} \]  \hspace{1cm} (3)

The resistance \( R \) of the heater is marked on the thermal conductivity box.

A part \( Q' \) of this heat is conducted along the left half of the tube where ThC1 is located. The remaining part \( Q'' = Q - Q' \) is conducted along the right half of the tube where the thermocouple ThC2 is located. If the measured thermo-emf of ThC1 is \( V_1 \text{ mV} \), it will correspond to a temperature difference between the two junctions of ThC1 given by

\[ \Delta T_1 (\text{in } ^\circ \text{C}) = \left( \frac{V_1}{40} \right) \times 10^3 = 25V_1 \text{ (in mV)}. \]  \hspace{1cm} (4)

Here the thermo-power of the copper–constantan thermocouple is taken as 40 \( \mu \text{V}/^\circ \text{C} \) and the factor 10\(^3\) arises from the division of millivolt (in which the voltage \( V \) is expressed) by microvolt (in which the thermo-power is expressed).

In the steady state

\[ Q' = K \left( \frac{A}{d_1} \right) \Delta T_1. \]  \hspace{1cm} (5)

Here \( A \) is the area of cross section of the copper tube.

Similarly, from the measurement of \( V_2 \text{ mV} \) of ThC2, we calculate the temperature drop \( \Delta T_2 \) across the junctions of ThC2

\[ \Delta T_2 (\text{in } ^\circ \text{C}) = \left( \frac{V_2}{40} \right) \times 10^3 = 25V_2 \text{ (in mV)}. \]  \hspace{1cm} (6)

So,

\[ Q'' = K \left( \frac{A}{d_1} \right) \Delta T_2. \]  \hspace{1cm} (7)

It is to be noted that the distance between the two junctions of ThC1 and ThC2 is the same at the value \( d_1 \).

Adding the two equations (6) and (7), and assuming no loss of heat due to radiation and convection,

\[ Q = Q' + Q'' = K \left( \frac{A}{d_1} \right) \left( \Delta T_1 + \Delta T_2 \right) \]  \hspace{1cm} (8)

or

\[ K \left( \frac{A}{d_1} \right) = \frac{Q}{(\Delta T_1 + \Delta T_2)} = \frac{Q}{(\Delta T)}. \]  \hspace{1cm} (9)

For different heat inputs, we find \( \Delta T_1 \) and take the average value of \( [Q/\Delta T] \). From the outer diameter OD (9.5 mm) of the copper tube and the wall thickness \( t \) (0.75 mm) given on the top of the box, we calculate the inner diameter of the tube as,

\[ \text{ID} = \text{OD} - 2t. \]  \hspace{1cm} (10)

So the area of cross section \( A \) of the tube is

\[ A = \left( \frac{\pi}{4} \right) (\text{OD}^2 - \text{ID}^2) = 0.207 \text{ cm}^2. \]

Taking \( d_1 \) to be 6 cm, \( d_1/A = 6 \times 10^{-2} \text{ m} \times 0.207 \times 10^{-4} \text{ m}^2 = 2910/\text{m.} \)

(Note: In the calculation, you should use the value of \( d_1 \) marked on the box.)

A sample set of readings are given in Table 1.
Experiments in Heat

Table 1. Thermal conductivity of copper.

<table>
<thead>
<tr>
<th>$d_1/A$ (m$^{-1}$)</th>
<th>$R$ (ohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2910</td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$I$ (amps)</th>
<th>$Q$ (W)</th>
<th>$V_1$ (mV)</th>
<th>$\Delta T_1$ (K)</th>
<th>$V_2$ (mV)</th>
<th>$\Delta T_2$ (K)</th>
<th>$\Delta T$ (K)</th>
<th>$Q/\Delta T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.26</td>
<td>1.487</td>
<td>0.234</td>
<td>5.85</td>
<td>0.303</td>
<td>7.58</td>
<td>13.43</td>
<td>0.111</td>
</tr>
<tr>
<td>0.24</td>
<td>1.267</td>
<td>0.198</td>
<td>4.96</td>
<td>0.26</td>
<td>6.5</td>
<td>11.46</td>
<td>0.111</td>
</tr>
<tr>
<td>0.22</td>
<td>1.065</td>
<td>0.161</td>
<td>4.03</td>
<td>0.224</td>
<td>5.59</td>
<td>9.62</td>
<td>0.111</td>
</tr>
<tr>
<td>0.2</td>
<td>0.88</td>
<td>0.142</td>
<td>3.54</td>
<td>0.184</td>
<td>4.6</td>
<td>8.14</td>
<td>0.108</td>
</tr>
</tbody>
</table>

Average 0.110

The value of thermal conductivity is

$$K = \left( \frac{Q}{\Delta T} \right) \times \left( \frac{d_1}{A} \right) = 320 \text{ W/m.K.}$$

For very pure copper, the thermal conductivity is 401 W/m.K. However, copper tubes used for refrigeration are called deoxidized, high phosphorus containing, copper – DHP copper. It has a copper (+Ag) content of 99.85% with phosphorous between 0.013 to 0.04%. From the Plumber’s Handbook, 8th Edition, page 7, published by Australian copper industries, the thermal conductivity of this type of copper is between 305 and 355 W/m.K. From the Internet, I found that Minerex AG, a company supplying tubes of DHP, has given the thermal conductivity as 340 W/m.K.

The thermal conductivity of copper has been measured with eight different thermal conductivity boxes. All of them gave a value between 310 and 330 W/m.K. So, we may take the correct value of thermal conductivity of copper used in the conductivity box as 320 ± 10 W/m.K.

5. Procedure for Measuring the Electrical Conductivity

Connect the constant current source between CL1 and CL2. Connect the terminals I3 on the differential amplifier to VL1 and VL2 and set the selector switch on the differential amplifier to I3. Pass a current $I$ of about 300 mA. Note the output voltage of the differential amplifier in mV. *There is no need to wait.* Increase the current in steps of 50 mA up to 600 mA and find the potential difference across the voltage leads for each current. Draw a graph between the potential difference in microvolts and the current in amps. Fit a straight line. The slope of the line gives the resistance $R$ in micro-ohms.

Note: The value of $d_2$ is given on the box.

A sample set of readings are given in Table 2.

Table 2. Electrical conductivity of copper.

<table>
<thead>
<tr>
<th>$d_2 = 15$ cm, $d_2/A = 7250$ m$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I$ (A)</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>0.3</td>
</tr>
<tr>
<td>0.35</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.45</td>
</tr>
<tr>
<td>0.5</td>
</tr>
<tr>
<td>0.55</td>
</tr>
<tr>
<td>0.6</td>
</tr>
</tbody>
</table>

Figure 3 shows a plot of the potential difference in microvolts against the current in amps.
Experiments in Heat

Figure 3. Plot of potential difference in microvolts against current in amps.

The slope of the graph is 194 micro-ohms. The electrical conductivity $\sigma$ is calculated from

$$\sigma = \frac{(d_2/A)}{\text{slope}}.$$ 

Therefore, $\sigma$ is $3.82 \times 10^7$ Siemens. For pure copper, $\sigma$ is $6 \times 10^7$ Siemens. The Plumber’s Handbook gives the specific electrical resistance for annealed DHP copper as 0.0192 to 0.0238 micro-ohm-meter. This implies a value for the conductivity from $4.3 \times 10^7$ to $5.2 \times 10^7$ Siemens. We get a lower electrical conductivity probably because the tube is not annealed.

The Lorentz number is

$$\frac{K}{(\sigma T)} = \frac{319}{(3.82 \times 10^7 \times 300)} = 2.87 \times 10^{-8} \left(\frac{V}{K}\right)^2.$$ 

The Lorentz number from the free electron theory is $2.44 \times 10^{-8} \left(\frac{V}{K}\right)^2$.

Note: The values of the heater resistance $R$, and the distances $d_1$ and $d_2$ for each setup are noted on the box. These may be different for different boxes. The values of $R$, $d_1$ and $d_2$ used in the write-up is for the box used in the experiment.

6. Questions

1. Do you think one can use a similar setup for measuring thermal conductivity for a poor conductor of heat?

2. Estimate the value of $k/e$ from equation (2) and compare with the actual value.

4.5 Thermal Conductivity of a Poor Conductor

1. Introduction

One may use a steady-state method for measuring the thermal conductivity $K$ of any material. In this method, one applies a heat power $Q$ at one end of the specimen and measures the temperature difference $\Delta T$ at two points separated by a distance $d$. Then
Experiments in Heat

\[ Q = KA\frac{(\Delta T)}{d}. \]  

Here, \( A \) is the area of cross section of the material. To obtain a measurable temperature difference of a few degrees Centigrade, and at the same time keep the heat power to about 1 W, one must decrease the area of cross section \( A \) and increase the distance \( d \) for a good conductor like copper. That is why, in the experiment on thermal conductivity of copper, we used a tube (area of cross section is determined by the diameter and thickness of the tube; area is small), and measured the temperature difference between two points separated by a distance of a few centimeters. The surface area, over which heat is lost by radiation and convection, is large in this case. However, for a good conductor, most of the heat is conducted, and only a small part is lost by radiation. This loss can be reduced by lagging the tube, to reduce convection, and by surrounding the lagging by a layer of aluminium foil to reduce radiation.

For a poor conductor (\( K < 1 \text{ W/m.K} \)), a small amount of heat will produce a large temperature gradient. To make the material conduct a heat flux of about 1 W, we have to increase the area over which heat is conducted. At the same time, to keep the temperature difference to a few degrees Centigrade, we have to reduce the thickness. For poor conductors, therefore, one uses a plate method in which the plate has a surface area of a few square centimeters and a thickness of the order of 1 mm.

2. Description of the Setup

A copper cup of about 2.54 cm (1") diameter, is made into two halves as shown in Figure 1.

![Figure 1](image1.png)

**Figure 1.** Construction of the heater.

A heater \( H \) made of six 120-ohm \( \frac{1}{4} \) W resistors in parallel is insulated by a thin layer of Teflon tape and is placed in the cup, cut in two halves, between two pads of aluminium foil. The foil helps to conduct the heat away from the heater on both sides.

The setup is shown in Figure 2. Two circular discs \( B \) are cut from 1” diameter Perspex rod. The discs have equal thickness, between 1 to 2 mm. These discs are clamped between the heater on one side, and two circular copper plates \( A \) on the other side, as shown in the figure. The height of each half of the cylindrical cup \( C \) containing the heater is 5 mm. One junction of a copper–constantan thermocouple \( E \) is fixed with super glue to the bottom face of the upper copper plate. The other junction is fixed to the top cylindrical surface of the upper half of the copper heater cup. Another pair of junctions of a copper–constantan thermocouple is similarly fixed to the top face of the bottom copper plate, and near the edge of the cylindrical surface of the bottom half of the heater cup. The two plates of Perspex are smeared with a very thin layer of Vaseline, and clamped by 4 bolts and nuts on the rim of the copper plates.

![Figure 2](image2.png)

**Figure 2.** Setup for thermal conductivity of a poor conductor.  
A: Copper base plate each 5 cm dia, 5 mm thick  
B: Perspex discs 2.5 cm dia and about 0.2 cm thickness  
C: Two halves of a copper cup 2.5 cm dia, outer height 5 mm, inner height 3 mm  
H: Heater made of six 120 ohm \( \frac{1}{4} \) W resistors in parallel  
E: Cu–constantan thermocouples ThC1 and ThC2
The thin layer of Vaseline under pressure provides an improved and acceptable thermal contact. The lateral surface of the heater cup is wrapped with a layer of cotton, a layer of aluminium foil and a second layer of cotton. This is not shown in the figure. The heater leads are brought to a pair of banana terminals on the front panel of a box in which the assembly is kept. The terminals of the two thermocouples are likewise brought to two RCA sockets on the front panel of the box. The box is loosely filled with cotton to prevent convection.

Note that the differential thermocouple measures the sum of the temperature drop across the Perspex plate, and the temperature drop across the copper plates, from the surface of the Perspex to the point at which the thermocouple junctions are fixed on the copper plates. But because the thermal conductivity of copper is about a thousand times larger than the thermal conductivity of Perspex, the latter contribution to the measured temperature drop is small. One may take the measured thermo-emf as arising solely from the temperature drop across the Perspex plate.

3. Apparatus Required

Constant current source, thermal conductivity of a poor conductor box, DC differential amplifier.

4. Procedure

Connect a constant current source to the heater terminals and set it in the high current mode. Connect the two thermocouple outputs to I\textsubscript{1} and I\textsubscript{2} terminals of the DC differential amplifier.

Set a current of about 250 mA from the constant current source. Do not exceed 260 mA of current. For this current, wait for forty five minutes for steady state to be reached. Check that the band switch on the differential amplifier is in position I\textsubscript{1}, and measure the output V\textsubscript{1} in mV of thermocouple ThC1. Then change the band switch on the differential amplifier to I\textsubscript{2}, and measure V\textsubscript{2} in mV of thermocouple 2.

Then decrease the current in steps of 25 mA, till you reach a current of 125 mA. For each value of the current, wait for half an hour for steady-state to be reached. Measure the outputs V\textsubscript{1} of thermocouple 1 and V\textsubscript{2} of thermocouple 2 in mV, as before, for each current.

The diameter of the Perspex plate is 2.54 cm. The thickness of each Perspex plate is 0.170 cm. The resistance \( R \) of the heater is 20.5 ohms.

Note: Values of \( R \), diameter of the plate and its thickness are marked on each box. Use these values in your measurement.

From the measured thermo-emf \( V \) in mV, for each of the two differential thermocouples, the temperature drop across the corresponding Perspex plate is

\[
\Delta T (^{\circ}C) = \left( \frac{V}{40} \right) \times 1000 = 25 V \text{ (in mV).} \tag{2}
\]

The thermo-power of copper–constantan thermocouple is 40 \( \mu V/^{\circ}C \), and a factor of 1000 arises from dividing mV by \( \mu V \). From the voltage measurements of the two thermocouples, one can find the temperature drop \( \Delta T_1 \) across plate 1, and the temperature drop \( \Delta T_2 \) across plate 2. Both the plates have the same area \( A \) and thickness \( d \). So,

\[
Q = \left( \frac{KA}{d} \right) (\Delta T_1 + \Delta T_2). \tag{3}
\]

A specimen set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>Table 1. Thermal conductivity of a poor conductor.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of the Perspex plate</td>
</tr>
<tr>
<td>Thickness ( d ) of the Perspex plate</td>
</tr>
<tr>
<td>( d/A ) for the Perspex plate</td>
</tr>
<tr>
<td>Resistance ( R ) of the heater</td>
</tr>
</tbody>
</table>
Experiments in Heat

<table>
<thead>
<tr>
<th>$I$ (amps)</th>
<th>$Q$ (watts)</th>
<th>$V_1$ (mV)</th>
<th>$\Delta T_1$ (°C)</th>
<th>$V_2$ (mV)</th>
<th>$\Delta T_2$ (°C)</th>
<th>$\Delta T = (\Delta T_1 + \Delta T_2)$ (°C)</th>
<th>$Q/\Delta T$ (W/K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125</td>
<td>0.32</td>
<td>0.086</td>
<td>2.15</td>
<td>0.086</td>
<td>2.16</td>
<td>4.31</td>
<td>0.074</td>
</tr>
<tr>
<td>0.15</td>
<td>0.461</td>
<td>0.124</td>
<td>3.09</td>
<td>0.134</td>
<td>3.35</td>
<td>6.44</td>
<td>0.072</td>
</tr>
<tr>
<td>0.175</td>
<td>0.628</td>
<td>0.176</td>
<td>4.11</td>
<td>0.172</td>
<td>4.29</td>
<td>8.7</td>
<td>0.072</td>
</tr>
<tr>
<td>0.2</td>
<td>0.82</td>
<td>0.222</td>
<td>5.55</td>
<td>0.232</td>
<td>5.81</td>
<td>11.36</td>
<td>0.072</td>
</tr>
<tr>
<td>0.225</td>
<td>1.038</td>
<td>0.28</td>
<td>7.01</td>
<td>0.288</td>
<td>7.2</td>
<td>14.21</td>
<td>0.073</td>
</tr>
<tr>
<td>0.25</td>
<td>1.281</td>
<td>0.352</td>
<td>8.8</td>
<td>0.354</td>
<td>8.85</td>
<td>17.65</td>
<td>0.073</td>
</tr>
</tbody>
</table>

$K = 3.434 \times 0.073 = 0.25$ W/m.K

Average 0.073

For different values of the current, the quantity $Q/\Delta T$ must remain constant. The last column gives the value of $Q/\Delta T$ and this is found to be constant within ±0.001. Find the average value of $Q/\Delta T$. Then the thermal conductivity is

$$K = \left( \frac{d}{A} \right) \left( \frac{Q}{\Delta T} \right).$$

(4)

The value of thermal conductivity comes out to be 0.25 W/m.K. This is about the value of $K$ for Perspex from the internet.

Note:

The dimensions of the Perspex disc are chosen so that one can pass the same currents through the heater as in the case of thermal conductivity of copper setup (hereafter called setup A). So one can connect the constant current source to the heater on setup A in series with the heater on the setup B (which is for the thermal conductivity of poor conductor). The waiting time for both setups, to reach steady state, is about the same. After this waiting time, connect the thermocouples ThC1 and ThC2 on setup A to I1 and I2 of the differential amplifier and measure the emfs. Remove the connections to I1 and I2 of setup A, connect the thermocouples of setup B to I1 and I2, and measure the thermo-emfs of setup B. Then change the current through the heater, wait for 30 minutes and repeat the procedure. This way the two experiments can be done simultaneously with a large saving in time.

### 4.6 Thermal Diffusivity of Brass

1. Introduction

The thermal diffusivity of a material is defined by the ratio $K/\rho c$, where $K$ is the thermal conductivity, $\rho$ is the density and $c$ is the specific heat per unit mass of a material. If heat is generated at a point in the material, the speed with which it diffuses out from the point is determined by this ratio. The dimension of this ratio is (joule/s.m.K)/[(kg/m$^3$) (joule/kg K)] = m$^2$/s. It indicates the spread, in m$^2$ per second, of the locally applied heat. For a poor thermal conductor like glass, $K = 1$ W/m.K, $\rho = 2500$ kg/m$^3$ and $c = 1$ kJ/kg K, and thus the thermal diffusivity is $4 \times 10^{-7}$ m$^2$/s. For copper, the diffusivity has a much higher value. Heat spreads much faster in copper than in glass.

In steady-state measurements of heat transport, only the conductivity plays a role. The diffusivity has no role to play. But diffusivity determines how the temperature varies with space and time in the medium, if the heat supplied varies as a function of time.

For such a case, the equation satisfied by the temperature $T$ for a one-dimensional problem is obtained as follows.

![Figure 1](image-url)  

**Figure 1.** Heat diffusing along a rod along the X-axis.
Experiments in Heat

Let the cross section of the rod be \( A \). Heat is propagated along the length of the rod, which is taken to be along the X-direction (Figure 1).

Consider two cross sections of the rod at B and C separated by an infinitesimal distance \( dx \). The temperature at the cross section at B is \( T \). The amount of heat crossing the section at B in the direction of the X-axis is 

\[-KA \left( \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} dx \right) \]

The temperature gradient is that at the point B. The heat passing out through the section at C is 

\[-KA \left( \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} dx \right) \] \( dx \). The temperature gradient at C is different and is expanded in a Taylor series about B. We retain the term up to the first power in \( dx \), since \( dx \) is infinitesimal. So, the net amount of heat flowing out of the element of length \( dx \) in one second is given by

\[-KA \left\{ \frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2} dx \right\} = -KA \frac{\partial^2 T}{\partial x^2} dx. \tag{1} \]

The mass of material between the two sections is \( \rho A \ dx \). If the temperature varies with time, then the amount of heat used up in one second in heating this element is

\[ \rho A \ dx \ e^{\frac{\partial T}{\partial t}}. \tag{2} \]

In addition, there may be heat loss from the surface to the surroundings. Using the simple Newton’s law of cooling, the heat lost from the surface area of the element between the sections B and C is

\[ \varepsilon P \ dx \ (T - T_0), \tag{3} \]

where \( T_0 \) is the temperature of the surroundings, \( P \) is the perimeter of the cross section of the rod, and \( \varepsilon \) is a constant in the Newton’s law of cooling.

The total heat used up by the element in one second is

\[ \rho A \ dx \ e^{\frac{\partial T}{\partial t}} - KA \frac{\partial^2 T}{\partial x^2} dx + \varepsilon P \ dx \ (T - T_0). \tag{4} \]

An amount of heat \( \lambda A \ dx \) is produced for unit time in the element BC, where \( \lambda \) is the heat generated per unit volume per unit time in the material. The law of conservation of energy tells us that the heat supplied must be equal to the heat used up. So

\[ \lambda A \ dx = \rho A \ dx \ e^{\frac{\partial T}{\partial t}} - KA \frac{\partial^2 T}{\partial x^2} dx + \varepsilon P \ dx \ (T - T_0) \]

or

\[ -KA \frac{\partial^2 T}{\partial x^2} + \rho \varepsilon \frac{\partial T}{\partial t} + \varepsilon \left( \frac{P}{A} \right) (T - T_0) = \lambda. \tag{5} \]

This is the partial differential equation satisfied by the temperature \( T(x, t) \).

Let us consider a case where we put a heater at one of the ends (the end at \( x = 0 \)) of the rod and there is no distributed heat source along the rod. Then, \( \lambda = 0 \) all along the rod. The equation satisfied by the temperature \( T \) is

\[ K \frac{\partial^2 T}{\partial x^2} - \rho \varepsilon \frac{\partial T}{\partial t} - \varepsilon \left( \frac{P}{A} \right) (T - T_0) = 0 \quad (0 < x < L) \tag{6} \]

or

\[ \frac{\partial^2 T}{\partial x^2} - \left( \frac{\rho \varepsilon}{K} \right) \frac{\partial T}{\partial t} - \varepsilon \left( \frac{P}{AK} \right) (T - T_0) = 0. \tag{7} \]

Let us assume that over the end face, the heat flowing per unit time, \( Q \), varies as \( \exp(iot) \), i.e.,

\[ Q = Q_0 \exp(iot). \tag{8} \]

Then, the temperature at every point along the rod will also vary as

\[ T(x, t) = \theta(x) \exp(iot) + T_0. \tag{9} \]

Equation (8) then becomes
Experiments in Heat

\[ \frac{d^2 \theta}{dx^2} - i \left( \frac{\rho c \omega}{K} \right) \theta - \varepsilon \left( \frac{P}{AK} \right) \theta = 0. \]  

(11)

Writing \( \varepsilon (P/AK) = a \) and \( (\omega pc/K) = b \), equation (11) can be written as

\[ \frac{d^2 \theta}{dx^2} - (a + ib) \theta = 0. \]  

(12)

This equation has to be solved subject to certain boundary conditions. One condition at \( x = 0 \) will be determined by

\[-KA \left( \frac{\partial T}{\partial x} \right)_{x=0} = Q. \]  

(13)

The other boundary condition at \( x = l \) can be imposed by keeping the end of the rod at \( l \) at a fixed temperature \( T_0 \).

Writing

\[ a + ib = \eta^2 \]  

(14)

and putting \( u = \eta x \), equation (12) becomes

\[ \frac{d^2 \theta}{du^2} - \theta = 0. \]  

(15)

The solution of this equation is

\[ \theta (x) = E \exp (\eta x) + F \exp (-\eta x). \]  

(16)

Therefore, the temperature \( T (x, t) \) varies as, from equation (10),

\[ T(x, t) = \left[ E \exp (\eta x) + F \exp (-\eta x) \right] \exp (i \omega t) + T_0. \]  

(17)

Writing \( \eta \) as \( \alpha + i \beta \), and using equation (14), we get

\[ \eta = \alpha + i \beta = (a + ib)^{1/2} = (a^2 + b^2)^{1/4} \exp \left( \frac{i \phi}{2} \right), \]  

(18)

where

\[ \tan \phi = \frac{b}{a}. \]  

(19)

So,

\[ T(x, t) = \left[ E \exp (\alpha x) \exp (i \beta x) + F \exp (-\alpha x) \exp (-i \beta x) \right] \exp (i \omega t) + T_0. \]  

(20)

where

\[ \alpha = (a^2 + b^2)^{1/4} \cos \left( \frac{\phi}{2} \right), \]  

(21)

\[ \beta = (a^2 + b^2)^{1/4} \sin \left( \frac{\phi}{2} \right). \]  

(22)

Note that \( \alpha \) and \( \beta \) have the dimension of inverse of length. If \( \alpha l > 5 \), we may take \( T = T_0 \) at \( l \) and put \( E = 0 \). We assume this to be true and put \( E = 0 \).

Then, the temperature distribution is given by

\[ T(x, t) = F \exp (- (\alpha + i \beta) x) \exp (i \omega t) + T_0. \]  

(23)

The temperature gradient at \( x = 0 \) is then given by

\[ \frac{dT}{dx} = -(\alpha + i \beta) F \exp (i \omega t). \]  

(24)
Experiments in Heat

The boundary condition at \( x = 0 \) is

\[-KA \left( \frac{dT}{dx} \right) = Q_0 \exp(i\omega t).\]  \hspace{1cm} (25)

This gives

\[F = \left[ \frac{Q_0}{(KA)} \right] \left[ \frac{1}{(a^2 + b^2)^{1/4}} \right] \exp\left( \frac{-i\phi}{2} \right).\]  \hspace{1cm} (26)

So, the temperature distribution is given by

\[T(x, t) = \left[ \frac{Q_0}{(KA)} \right] \left[ \frac{1}{(a^2 + b^2)^{1/4}} \right] \exp(-\alpha x) \exp i\left( \omega t - \beta x - \frac{\phi}{2} \right).\]  \hspace{1cm} (27)

The temperature is not in phase with the heating. The amplitude of the temperature oscillation is a function of \( x \) and varies as

\[\text{Amp}(T(x)) = \left[ \frac{Q_0}{(KA)} \right] \left[ \frac{1}{(a^2 + b^2)^{1/4}} \right] \exp(-\alpha x).\]  \hspace{1cm} (28)

The ratio of the amplitudes at \( x_1 \) and \( x_2 \) is

\[
\frac{\text{Amp}(x_2)}{\text{Amp}(x_1)} = \exp[-\alpha(x_2 - x_1)].
\]  \hspace{1cm} (29)

The difference in phase between the temperature oscillations at \( x_1 \) and \( x_2 \) is

\[\zeta(x_1) - \zeta(x_2) = \beta(x_1 - x_2).\]  \hspace{1cm} (30)

One can find \( \alpha \) and \( \beta \), and hence the thermal diffusivity of the material, by measuring the ratio of amplitudes at two points separated by a known distance and by measuring the phase difference of the temperature wave between two points separated by a known distance.

However, producing sinusoidal heating at low frequencies is difficult. We can produce a periodic heating at the end \( x = 0 \) by supplying a known current to a heater in a periodic fashion as shown in Figure 2 and described below.

\[\text{Figure 2. Periodic heating at } x = 0.\]

The current is switched on for times \( nt \leq t < (n + f) \tau \), where \( n \) is integral and \( 0 < f < 1 \). The current is switched off for times \( (n + f) \tau < t < (n + 1) \tau \). Here, \( \tau \) is the period of current variation. So, the heat input \( H \) varies as

\[H = I^2R \quad \text{for } nt \leq t < (n + f) \tau \]

and

\[= 0 \quad \text{for } (n + f) \tau < t < (n + 1) \tau,\]

where \( R \) is the resistance of the heater.
Experiments in Heat

Such a periodic function $H(t)$ can be expanded in a Fourier series

$$H(t) = H_0 + \sum_{n=1}^{\infty} \left[ H_n \exp(\text{i} \omega_0 t) + H_{-n} \exp(-\text{i} \omega_0 t) \right],$$

(31)

where $H_n = H^*_{-n}$, $\omega_0 = \frac{2\pi}{\tau}$.

The coefficients $H_0$ and $H_n$ are obtained from the equations

$$H_0 = \left( \frac{1}{\tau} \right) \int_0^\tau H \, dt = Hf,$$

(32)

$$H_n = \left( \frac{1}{\tau} \right) \int_0^\tau H \exp(-\text{i} \omega_0 t) \, dt$$

(33)

$$= \left( \frac{H}{\tau} \right) \left( \frac{1}{\text{i} \omega_0} \right) \left[ 1 - \exp(-\text{i} 2\pi f \tau) \right].$$

The corresponding temperature distribution is given by

$$T(x,t) = T_0 + \left( \frac{Hf}{KA} \right) (l-x) + \sum_{n=1}^{\infty} \left[ F_n \exp(\text{i} \omega_0 t) + F_{-n} \exp(-\text{i} \omega_0 t) \right],$$

(34)

with

$$F_n = F^*_{-n},$$

and

$$F_n = \left[ \frac{H_n}{(KA)} \right] \left[ \frac{1}{(a_n^2 + b_n^2)^{1/4}} \right] \exp\left(-\sqrt{n}(\alpha + \text{i} \beta) x \right).$$

(35)

The constants $a_n$ and $b_n$ are proportional to $n \omega_0$. So, for a frequency $n \omega_0$, the constants $a_n$ and $b_n$ will be proportional to $n$ times $a$ and $b$ for the fundamental frequency $\omega_0$. This is the reason for the presence of $\sqrt{n}$ in the exponential term in the formula for $F_n$ in equation (35), and the constants $a$, $b$, $\alpha$, and $\beta$ in that equation are the constants for the frequency $\omega_0$.

Thus, the temperature at any point $x$ will display some periodic variation as shown in Figure 3.

![Figure 3. Temperature variation with time at a point x along the length of the rod.](image)

We measure the temperature vs. time at two points. We Fourier transform the graphs to get the Fourier component at the frequency $\omega_0$. This is done as follows. The complex amplitude $B_1 = F_1 \exp(-\alpha \text{i} \beta x)$ of the Fourier component varying as $\exp(\text{i} \omega_0 t)$ is found from the integral

$$F_1 \exp(-\alpha \text{i} \beta x) = \left( \frac{1}{\tau} \right) \int_0^\tau \theta(x_1,t) \exp(\text{i} \omega_0 t) \, dt.$$

(36)
Experiments in Heat

The real part, \( \text{RP} \), of the integral on the right is

\[
\text{RP of } I(x_1) = \left( \frac{1}{\tau} \right) \int_0^\tau \theta(x_1, t) \cos(\omega_0 t) \, dt
\]

and the imaginary part, \( \text{IP} \), is

\[
\text{IP of } I(x_1) = \left( \frac{1}{\tau} \right) \int_0^\tau \theta(x_1, t) \sin(\omega_0 t) \, dt.
\]

We divide the time interval from 0 to \( \tau \) into 20 equal parts, 0, 1, 2, ..., \( m \), \( m+1 \), ..., 20. Take the temperature \( \theta(x_1, m\tau/20) \) and multiply it by \( \cos(2\pi m/20) \) and \( \sin(2\pi m/20) \). We do this for each value of \( m \). The real part of the integral is then given by

\[
\text{RP of } I(x_1) = \left( \frac{1}{\tau} \right) \left\{ \sum_{m=0}^{m=20} \theta\left(x_1, \frac{m\tau}{20}\right) \cos\left(\frac{2\pi m}{20}\right) - \frac{1}{2} \left[ \theta(x_1, 0) + \theta(x_1, \tau) \right] \right\}
\]

and the imaginary part of the integral is

\[
\text{IP of } I(x_1) = \left( \frac{1}{\tau} \right) \left\{ \sum_{m=0}^{m=20} \theta\left(x_1, \frac{m\tau}{20}\right) \sin\left(\frac{2\pi m}{20}\right) \right\}.
\]

This is the Simpson’s rule for integration.

Once the \( \text{RP} \) and \( \text{IP} \) of \( I(x_1) \) are found, the amplitude of the temperature varying at frequency \( \omega_0 \) at \( x_1 \) is

\[
\text{Amp of } \theta(x_1, \omega_0) = \left[ (\text{RP of } I(x_1))^2 + (\text{IP of } I(x_1))^2 \right]^{1/2},
\]

and the phase \( \xi(x_1) \) is

\[
\text{Phase of } \theta(x_1, \omega_0) = \xi(x_1) = \tan^{-1}\left[ \frac{\text{IP of } I(x_1)}{\text{RP of } I(x_1)} \right].
\]

We repeat this calculation for the temperature at the point \( x_2 \) to get the amplitude and phase of \( \theta(x_2, \omega_0) \). From the ratio of the amplitude of \( \theta(x_1, \omega_0) \) to the amplitude \( \theta(x_2, \omega_0) \) using equation (29), one obtains \( \alpha \).

\[
\alpha = \ln \left( \frac{\text{Amp } \theta(x_1, \omega_0)}{\text{Amp } \theta(x_2, \omega_0)} \right) \left/ \frac{\text{Amp } \theta(x_1, \omega_0)}{\text{Amp } \theta(x_2, \omega_0)} \right. \right),
\]

Here \( \ln \) is logarithm to base \( e \).

One can also obtain \( \beta \) from the measurement of the phase difference using equation (30).

\[
\beta = \left[ \xi(x_2) - \xi(x_1) \right] \left/ (x_2 - x_1) \right. \right).
\]

From this, knowing \( r \), one obtains the diffusivity \( K/\rho c \).

\[
\alpha \beta = (a^2 + b^2)^{1/2} \cos\left(\frac{\phi}{2}\right) \sin\left(\frac{\phi}{2}\right).
\]

\[
= \left( \frac{1}{2} \right) (a^2 + b^2)^{1/2} \sin \phi = \frac{\rho c \omega_0}{2K}.
\]

So the diffusivity

\[
D = \frac{K}{\rho c} = \frac{\omega_0}{2\alpha \beta}.
\]

The diffusivity can be calculated from a measurement of \( \alpha \) and \( \beta \).
2. Apparatus Required

A DC power supply, thermal diffusivity box, DC differential amplifier.

3. Experimental Setup

The schematic of the experimental setup is shown in Figure 4. This experiment is designed to be performed on a brass rod. There is a brass rod (1) of 30 cm length and diameter about 5 mm (3/16”). A small heater (3) of about 10 ohms is wound on the centre of the brass rod. Two copper–constantan thermocouples (4) and (5) are attached to the brass rod with superglue. The distance between the junctions is 3 cm. Symmetrically to the right of the heater two more junctions (4’) and (5’) are fixed with superglue to the brass rod. The cold ends of the two thermocouples (4) and (5) are embedded in a copper block 2 so that they are at the same constant temperature. Similarly, the cold ends of the thermocouples (4’) and (5’) are embedded in another copper block 2’. 2 and 2’ are copper blocks in which the cold junctions of the thermocouple are embedded.

Figure 4. Thermal diffusivity box.

The distance between \(x_2\) and \(x_1\) is 3 cm. The leads of the four thermocouples are brought to the terminals marked TC1, TC2, TC3 and TC4. The heater leads are marked HTR. The brass rod (1) is enclosed inside an outer plastic tube (6) and the interstice between the brass rod and plastic tube is filled loosely with cotton wool to prevent convection currents. All the terminals come out through the right lid of the tube (6).

In the experiment, we connect the banana plug heater terminals to a DC voltage source and set the output voltage to about 5 V. To vary the current periodically, we remove or insert one of the banana plugs connected to the heater terminal periodically. We do not switch off and switch on the power supply. The power supply is kept on all the time.

The thermocouple terminals TC1 and TC2 are connected respectively to channel 1 and channel 2 of the DC differential amplifier. Using the band switch on the amplifier, we can measure the voltage in channel 1 or channel 2 in millivolts.

The thermocouples TC3 and TC4 are spare thermocouples. If the connection to the thermocouple 1 or 2 is broken, we will use TC3 and TC4.

We start counting the time from the instant the heater is switched on. We keep the current on for 300 seconds and the current off for 300 seconds. So, the period \(\tau\) of heating will be 600 sec.

For the first two cycles (i.e., for 1200 seconds from the start of the heating) no thermocouple readings are taken. In this time, the heat will spread to the ends of the rod and the temperatures at the thermocouple junctions will start varying periodically with time.

When the third cycle starts at 1200 seconds, we start reading the thermocouple voltage of TC1 every 30 seconds starting from 1215 seconds. One must not forget to switch off the heater at 1500 s and switch it on again at 1800 s. Thermocouple reading must be taken from 1215 s after switching on the heater, to 1815 seconds. This will cover one period.

After taking the reading at 1815 s, the selector switch is turned to I2. This will now read TC2. No readings are taken till the heater is switched on again at 2400 s. Then, readings of TC2 voltage are taken every 30 seconds from 2415 sec to 3015 seconds. One must not forget to switch off the heater current at 2700 seconds and switch it on again at 3000 seconds.
Experiments in Heat

Figure 5 illustrates schematically, the sequence in which readings are taken.

Figure 5. Schematic showing how readings are to be taken.

In the figure, numbers denote time in seconds. ON and OFF refer to switching heater current on and off.

A sample set of readings is shown in Table 1.

Table 1. Power supply voltage = 5 V. Period of heating $\tau = 600$ s. Distance between thermocouple junctions = 3 cm.

<table>
<thead>
<tr>
<th>time (s)</th>
<th>TC1 ($\mu$V)</th>
<th>TC1cos</th>
<th>TC1sin</th>
<th>time (s)</th>
<th>TC2 ($\mu$V)</th>
<th>TC2cos</th>
<th>TC2sin</th>
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<td>81.2</td>
<td>2415</td>
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<tr>
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<td>3015</td>
<td>606</td>
<td>598.5</td>
<td>94.9</td>
</tr>
</tbody>
</table>

The first column gives the time (measured from the time the heater is switched on before readings are taken). The second column gives the readings of TC1 in $\mu$V from 1215 s to 1815 s. In the third column are given values of (TC1 cos ($2\pi t/\tau$)) where time $t$ is from the first column and $\tau$ is 600 s, the period of heating. Similarly, the fourth column is (TC1 sin ($2\pi t/\tau$)). The row marked sum gives the sum of all rows in column 3 (or column 4) minus half the sum of the readings in the first and last rows. The column value next to $I_1 \cos$ is the $[(\text{sum in column 3}) \times 30/600]$, where 30 seconds is the interval at which temperatures are measured.

The diffusivity is $0.23 \text{ cm}^2/\text{s}$.
Similarly, the value next to \( I_1 \sin \) is the \(((\text{sum in column 4}) \times 30)/600\). \( \text{Amp}_1 \) is \( (I_1 \cos)^2 + (I_1 \sin)^2)^{1/2} \). Since both \( I_1 \cos \) and \( I_1 \sin \) are negative, the angle is in the third quadrant. So, Phase 1 is \( \text{atan}(I_1 \sin/I_1 \cos) + \pi \).

A similar analysis of the readings of TC2 gives Amp 2 and Phase 2. The value of \( \alpha \) is obtained from equation (43) and the value of \( \beta \) from equation (44). The value of diffusivity is calculated from the values of \( \alpha \) and \( \beta \) using equations (45) and (46). The diffusivity of brass has a value about 0.30 cm\(^2\)/s, and will vary with the composition of brass.

If the heater current is not switched off and on exactly with a period of 600 s and if the readings are not taken at exact intervals of 30 s, phase measurement will have a large error.

The aim of the experiment is (i) to illustrate how Fourier analysis is done and (ii) to measure thermal diffusivity. Fourier analysis occurs whenever a function varies periodically either in space or in time or both. For example, in a crystal, the electron charge density varies periodically in space. X-ray diffraction spots occur from different Fourier components of this charge density. Therefore, it is necessary to understand how Fourier analysis is done.

Measurement of thermal diffusivity is very important for materials of poor thermal conductivity. Here, the specimen will be thin and the frequency of periodic heating will be high. A whole branch of research called photo-acoustic spectroscopy uses such periodic heating methods.

Dr. Ramesh has made a square wave generator of period 600 seconds which controls a DC power supply to switch the current on and off. With this, the subjective error in switching on and off is eliminated. However, this square wave generator is not commercially available.

4. Questions

1. Consider an AC current which is perfectly sinusoidal with a frequency \( f \). If this signal is Fourier analyzed, at what frequencies will the Fourier components be non-zero?

2. White light is a mixture of colours. When it is passed through a prism, the colours are separated. Does the prism Fourier analyze the white light?

3. Will heat diffuse faster in a poor conductor than in copper?

4. In photo-acoustic spectroscopy, one can measure diffusivity as a function of the period \( T \) of heating. Indicate what uses can photo-acoustic spectroscopy be put to.
Chapter 5

Experiments to Measure Resistance
5.1 Temperature Coefficient of Resistance of Copper

1. Introduction

The electrical conductivity of a metal and a semiconductor is given by the expression

\[ \sigma = ne\mu. \]  

(1)

Here, \( n \) is the number of charge carriers per unit volume of the material, \( e \) is the magnitude of the charge, and \( \mu \) is a quantity called the mobility of the charge carrier. The mobility is the drift velocity acquired by the charge carrier per unit applied electric field. The mobility is determined by the rate at which the charge carriers are scattered by impurities, defects in the material, and by vibrations of the atoms. At high temperature and in pure materials, the dominant scattering process is the collision of the charge carrier with the vibrating atoms. As the temperature increases, the atoms vibrate more vigorously, and so the scattering probability is more. The mobility decreases as the temperature increases.

In a metal, the valence band is completely filled, while the conduction band is partially filled with electrons. A completely filled band will not contribute to electrical conduction. The electrons in the conduction band of a metal are responsible for electrical conduction in metals. The number of electrons per unit volume in a metal (of the order of \( 10^{27} \text{m}^{-3} \)) is not dependent on temperature. So, the temperature variation of conductivity \( \sigma \) arises only through the variation of mobility with temperature. Since the mobility decreases as the temperature increases, the conductivity of a metal comes down as it is heated. Conductivity \( \sigma \) is the reciprocal of specific resistance \( s \). So, as the temperature increases, the resistance of a metal increases. This is characteristic of metallic behavior.

If the resistance of a length of metallic wire is \( R(T_0) \) at some temperature \( T_0 \), and it changes to \( R(T) \) at a higher temperature \( T \), then the mean temperature coefficient of resistance is defined by

\[ \alpha = \left( \frac{1}{R(T_0)} \right) \frac{[R(T) - R(T_0)]}{[T - T_0]}. \]

(2)

This implies that in this temperature range we may write

\[ R(T) = R(T_0) \left[ 1 + \alpha (T - T_0) \right]. \]

(3)

2. Apparatus Required

Regulated DC power supply, temperature indicator, furnace, insert for measuring TCR of copper, constant current source and a DMM that can measure DC in millivolts.

3. Experimental Procedure

For this experiment, fine enamel coated copper wire (gauge 44) is used. It has a room temperature resistance of about 3.3 \( \Omega/\text{m} \). Approximately 7 to 8 meter long wire is wound on an insulating tube, and fitted into a hole in a rectangular aluminium rod 1 cm \( \times \) 1 cm, and 5 cm long. The rod also carries a Pt 100 temperature sensor, fitted into another hole in the aluminium block. The aluminium rod hangs inside the furnace from a terminal block. The terminal block is shown in Figure 1.

![Figure 1. Terminal block of the insert for TCR of copper.](image-url)
Experiments to Measure Resistance

The two ends of the copper wire are soldered to two current leads. The leads are brought to two small banana terminals, marked Coil, on the terminal block. The leads of the Pt100 are connected to the banana terminals marked Pt 100, on the terminal block.

Since the room temperature resistance of the copper coil is of the order of 25 ohms, ‘two terminal’ resistance measurements are sufficient. The current leads are also used for measuring the voltage drop across the copper wire when a current is passed through it. A current of 5 mA will give rise to a voltage of about 125 mV. Such a voltage can be easily read on a digital multi-meter. Furthermore, the temperature coefficient of resistance for copper is 0.0043/°C (choosing \( R(T_0) \) in equation (2) as the resistance value at 0 °C). This means that a rise in temperature of 10 °C would produce a change of about 10 mV in voltage. This change is measurable on a digital multimeter.

(NOTE: There are eight banana terminals on the terminal block. For TCR of copper we use only terminals 1 and 8 (Pt100 thermometer) and 2 and 3 (copper coil). If the numbering on the terminal block is not there, or if the numbering is done wrong, one can still determine which terminals are connected to the Pt resistor and which are connected to the copper coil, by measuring the resistance across adjacent terminals. The two adjacent terminals, giving a resistance of about 110 ohms, are connected to the Pt100 resistor. The two adjacent terminals, giving a resistance of about 25 ohms, are connected to the copper coil. The other terminals are for measuring the energy band gap of silicon. These terminals are not used in this experiment).

The aluminum block is suspended inside a furnace. The furnace is connected to a DC regulated power supply. The voltage of the regulated supply is adjusted so that the temperature at the center of the furnace increases slowly, at a rate of about 1 °C per minute. The required voltage for our furnace is about 7 to 8 V. The two open ends of the ceramic tube of the furnace should be lightly plugged with wads of cotton to prevent convection currents. The voltage is gradually increased, as the furnace temperature rises, to maintain approximately a uniform heating rate. At a temperature of 150 °C, this voltage will not exceed 15 volts.

A constant current source is connected to the current lead terminals (2 and 3) of the copper coil on the terminal block. The current source is put in the low current mode, and the current is set at 5 mA, as read on the panel meter of the constant current source. The same terminals of the copper coil are connected to a DMM in the DC 200 mV range. This will measure the voltage to an accuracy of 0.1 mV. The temperature of the furnace is read on the temperature indicator, connected to terminals 1 and 8 of the Pt100 thermometer on the terminal block. For every ten degree change in temperature, the digital multimeter reading is taken.

A sample set of readings is given in Table 1. Column 1 of the table indicates the temperature measured by the Pt100 thermometer. Column 2 gives the voltage in mV read on the DMM. Since the source supplies a constant current throughout the measurement, the voltage across the coil is proportional to its resistance. This measured voltage is plotted against temperature in Figure 2. A straight line fit to the curve is also shown.

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>( V_{\text{meas}} ) (mV)</th>
<th>( V_{\text{calc}} ) (mV)</th>
<th>Error²</th>
</tr>
</thead>
<tbody>
<tr>
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<td>156.2</td>
<td>156</td>
<td>4.00E–02</td>
</tr>
<tr>
<td>40.5</td>
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<td>162.9</td>
<td>2.50E–01</td>
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<tr>
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<td>60</td>
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<td>175.3</td>
<td>4.00E–02</td>
</tr>
<tr>
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<td>181.6</td>
<td>1.00E–02</td>
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<tr>
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<td>187.7</td>
<td>188</td>
<td>9.00E–02</td>
</tr>
<tr>
<td>90</td>
<td>194.9</td>
<td>194.3</td>
<td>3.60E–01</td>
</tr>
<tr>
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<td>2.50E–01</td>
</tr>
<tr>
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<td>207.1</td>
<td>1.60E–01</td>
</tr>
<tr>
<td>120</td>
<td>214</td>
<td>213.4</td>
<td>3.60E–01</td>
</tr>
<tr>
<td>130</td>
<td>219.1</td>
<td>219.8</td>
<td>4.90E–01</td>
</tr>
<tr>
<td>140</td>
<td>225.7</td>
<td>226.1</td>
<td>1.60E–01</td>
</tr>
<tr>
<td>150</td>
<td>232.3</td>
<td>232.5</td>
<td>4.00E–02</td>
</tr>
</tbody>
</table>

Variance 1.91E–01
Std Dev 4.37E–01
The slope of the graph is 0.636 mV/°C, and the intercept $V(0)$ (voltage across the coil at 0 °C for a current of 5 mA through the coil) is 137.1 mV from the fit. Since the resistance is proportional to the voltage, we may calculate the temperature coefficient of resistance, $\alpha$, from equation (1) by replacing $T_0$ by zero, $R(T_0)$ by $V(0)$ and $[(R(T) - R(T_0))/(T - T_0)]$ by the slope of the voltage vs. temperature graph.

$$\alpha = \frac{\text{slope}}{\text{intercept}} = \frac{0.636}{137.1} = 4.64 \times 10^{-3}/°C.$$ (4)

To see how well the straight line fits the observed values in Figure 2, we calculate the voltage from the equation

$$V_{\text{calc}}(T) = A + BT,$$ (5)

where $B$ is the slope and $A$ is the intercept) at the temperatures recorded in the first column of Table 1. These values are given in the third column headed $V_{\text{calc}}$ (mv). Column 4 shows the square of error difference between the measured and calculated values of the voltage at each temperature.

The variance is

$$\text{Variance} = \frac{\text{Sum of the error}^2}{(N - 1)},$$

where $N$ is the total number of measurements (13 in the present case). The variance is 0.191 (mV)$^2$. The standard deviation is the square root of the variance. This comes out to be 0.437 mV. The measured data deviate randomly from the linear fit by 0.44 mV. The measured value of $V(0)$ varies between 156.2 to 232.3 mV. So, the linear fit is good to within about 0.2%.

The value of $\alpha$ for pure copper is $4.3 \times 10^{-3}/°C$. The observed value is higher than this value beyond the source of error. So, there must be a reason for this difference. Commercial copper has some impurities. So the slightly higher value of $\alpha$ can be attributed to these impurities.

4. **Questions**

1. What is the unit of mobility?

2. Why does the resistance of a metal increase with temperature?

3. Can the change in resistance of a metal be used to measure temperature?

4. Two identical resistances, $R_1$ and $R_2$, of the same material, are connected in two arms of a Wheatstone bridge. In the other two arms, equal resistances from a resistance box are connected. The bridge is balanced at room temperature. The resistance $R_1$ is put in a furnace and heated. Will the balance
of the bridge be changed? If so, can you think of a temperature controller for the furnace using this phenomenon?

5.2 Techniques for Measuring Resistivity of Thin Films and Pellets

1. Introduction

Two techniques will be described below for the measurement of resistivity of thin films and pellets. The first is a collinear four probe resistivity technique, and the second is the Van der Pauw technique.

2. Collinear Four Probe Resistivity Technique

In the realm of electronic properties, the measurement of resistivity of a conducting sample in the form of a bulk material, a thin or a thick film, a homogeneous wire or a pellet is of primary importance. The electrical resistivity $\rho$ of a material is independent of its geometry, and characteristic of its electronic structure. It is worth noting that the resistivity is evaluated from a measurement of resistance, which is dependent on the geometry and dimensions of the sample.

The schematic of the collinear four probe resistivity technique is given in Figure 1.

![Figure 1](image1.png)

**Figure 1.** Collinear arrangement of current and voltage leads.

In this arrangement, the sample is contacted by four collinear probes of negligible size compared to any of the sample dimensions. The outer two probes carry a current $I$, while the inner two probes measure a potential difference $\Delta V$. From the measured potential difference and the known source current, the resistivity of the sample may be determined. We need to only determine, for various sample geometries, how the current flows and the resultant potential difference. We now consider the specific case of thin films.

Thin Film Geometry

Specifically, let us consider an infinite two-dimensional sheet, the thickness of which is small compared to the probe spacing, so that we may approximate the current density as uniform in the direction perpendicular to the film plane. Figure 2 shows the current distribution in an idealized two-dimensional geometry. The outer probes are separated by a distance $a$, while the inner probes are separated by a distance $b$.

![Figure 2](image2.png)

**Figure 2.** Linear four-point-probe measurement on a thin film sample of thickness $d$. 
Experiments to Measure Resistance

In this case, the current is confined within the film’s thickness $d$. In the plane of the film, the current spreads evenly leading to circular equipotential surfaces. The crucial difference is that at a lateral distance $r$ from a current probe, the total current $I$ must pass through an area dictated by the circumference of a circle of radius $r$ and the film thickness $d$. Thus, the current density is $J(r) = I/2\pi rd$. By definition, $J(r) = \sigma E$, where $\sigma$ is the conductivity of the sample and $E$ is the electric field. Further $E = -\partial V/\partial r$. Thus, for thin film geometry, the potential $V$ at a distance $r$ is given by the expression

$$V(r) = \left(\frac{I\rho}{2\pi d}\right) \ln r.$$  

(1)

Here $\rho = 1/\sigma$ is the resistivity of the material, and $\ln$ is the logarithm to the base $e$. Using the above expression, it follows that the potential difference $\Delta V_+$ developed between the inner probes separated by distance $b$ due to current contact $I_+$ is given by the expression

$$\Delta V_+ = \left(\frac{I_+\rho}{2\pi d}\right) \ln \left(\frac{a+b}{a}\right).$$  

(2)

Similarly the potential difference $\Delta V_-$ between the two contacts due to the current $I_-$ is the same amount. So the total potential difference $\Delta V$ is

$$\Delta V = \Delta V_+ + \Delta V_- = \left(\frac{I\rho}{\pi d}\right) \ln \left(\frac{a+b}{a}\right).$$  

(3)

The bulk resistivity $\rho$ of the film is given by the expression

$$\rho = \frac{(\pi \Delta V d / I)}{\ln((a+b)/a)}.$$  

(4)

Here $I_+ = I_- = I$. For the special case of equally spaced collinear probes wherein $a = b$, the above expression gets simplified to

$$\rho = \frac{(\pi \Delta V d / I)}{\ln(2)}.$$  

(5)

$\pi/\ln2$ is close to 4.53 and 4.53($\Delta V/I$) is the measured value of sheet resistance $R_S$.

Thus

$$\rho = d R_S.$$  

(6)

The bulk resistivity of a thin film can thus be evaluated through an experimental measurement of the sheet resistance $R_S$, and a knowledge of the thickness $d$ of the film. The main assumptions for the validity of the above relation are that the thickness of the film should be much smaller than the distance between the probes and the other two dimensions, viz., length and breadth of the film, should be an order of magnitude larger than the distance between the probes.

However, for samples whose lateral dimensions are comparable to the spacing between the probes, the above relation gets modified with a multiplicative correction factor (instead of 4.53) which can be read from the plot given in Figure 3. In this figure, a rectangular film has a length $L$, width $w$ and thickness $d$. A circular film has a diameter $w$ and thickness $d$.

For a rectangular film, the correction factor can be read from curve 1 when $L/w = 1$ (a square film), and from curve 2 when $L/w \geq 3$ (rectangular film with an aspect ratio equal to or more than 3). For a circular film one uses the curve marked Circular.
Experiments to Measure Resistance

Experiment 1
Measurement of the resistivity of a thin film employing the ‘Collinear four probe technique’

A thin film of aluminium which is vacuum evaporated/sputtered, or a thin sheet of stainless steel can be studied using this technique. The thickness of the coated film can be estimated through the quartz thickness monitor. Current, of the order of 10 mA, is passed through the outer probes of the equally spaced collinear probe arrangement. The voltage developed across the inner two probes is measured through a high stability DC microvolt amplifier.

The bulk resistivity of aluminium is estimated through the measured value of $R_s$ and the thickness $d$ of the film.

3. Van der Pauw Resistance Measurement

Van der Pauw resistance measurement is the second most common resistance technique used in the semiconductor industry.

Assumptions:
1. Sample is homogenous.
2. Sample is isotropic.
3. Sample is two-dimensional (thickness small compared to breadth and width).
4. Sample boundary is sharply defined.

The Van der Pauw method exploits the reciprocity theorem of electromagnetism, which states that the current in a detector divided by the voltage at the source remains constant when source and detector are interchanged.

In the Van der Pauw method, electrical contacts are made on the perimeter of the sample to be studied. This means that the perimeter has to be sharply defined. The contacts cannot be too close to the edge of the sample nor too far from the edge of the sample. The basic measurement geometry is given in Figure 4.

Once the four contacts are established, two resistance measurements ($R_{ab\rightarrow cd}$ and $R_{bc\rightarrow da}$) are taken with the difference being that one each of adjacent current and voltage contacts are switched, as illustrated in the figure above. The sheet resistivity $\rho_s$, which is essentially the bulk resistivity per unit thickness, is given by the Van der Pauw formula, viz.,
Experiments to Measure Resistance

![Figure 4](image.png)

The contact arrangements for the two resistance measurements, $R_{ab\rightarrow cd}$ and $R_{bc\rightarrow da}$, used in the Van der Pauw resistivity measurement.

**Figure 4.** Van der Pauw contact geometry.

\[
\rho_s = \left( \frac{\pi}{\ln 2} \right) \left[ \frac{R_{ab\rightarrow cd} + R_{bc\rightarrow da}}{2} \right] f 
\]

(7)

where $f$ is a correction factor dependent on the resistance ratio of $R_{ab\rightarrow cd}$ and $R_{bc\rightarrow da}$. The correction factor is shown in Figure 5.

![Figure 5](image.png)

**Figure 5.** Correction factor $f$ for Van der Pauw geometry.

**Experiment 2**

**Measurement of resistivity of a stainless steel sheet employing van der Pauw geometry**

Commercially available stainless steel of nominal thickness 0.005 cm can be conveniently studied employing this technique. Samples of square, rectangle and irregular geometry were cut from this sheet, and electrical leads were spot welded for measurements as indicated in Figure 4. The entire information of the sample geometry is contained in the values of the resistance for the two configurations indicated in the figure. Some typical experimental data are given below which bring out clearly the power of the Van der Pauw technique.
Experiments to Measure Resistance

Vanader Paw Measurements on Stainless steel Sheet

Thickness : .005 cm.
Stainless steel sheet:

1. Rectangular sheet.
   \[ R_1 = 750 \text{ micro-ohm}, R_2 = 9920 \text{ micro-ohm} \]
   Resistivity = 78.9 micro-ohm cm.

2. Square sheet.
   \[ R_1 = 3500 \text{ micro-ohm}, R_2 = 3520 \text{ micro-ohm} \]
   Resistivity = 79.5 micro-ohm cm.

3. Irregular shape.
   \[ R_1 = 3690 \text{ micro-ohm}, R_2 = 3190 \text{ micro-ohm} \]
   Resistivity = 77.8 micro-ohm cm.

Adapted From

Quick & Dirty Review of Resistivity Measurement Techniques, by Michael J Burns

5.3 High Resistance by Leakage

1. Introduction

If an external resistance \( R \) is connected across a capacitor, with internal resistance \( R' \), the effective resistance through which the capacitor discharges is given by

\[
\frac{1}{R''} = \frac{1}{R} + \frac{1}{R'} .
\]  \( (1) \)

This experiment aims to measure a high resistance of the order of a few tens of mega-ohms or higher. When a capacitor, charged to a DC voltage \( V_0 \), is discharged through a high resistance \( R'' \), the voltage decreases exponentially with time as

\[
V(t) = V_0 \exp\left(\frac{-t}{\tau''}\right),
\]  \( (2) \)

where

\[
\frac{1}{\tau''} = \frac{1}{(CR'')}. \]  \( (3) \)

The capacitor is first charged to a voltage of about 2 V. The capacitor is then discharged through the resistance \( R'' \). When the high resistance \( R \) in the box is not connected in parallel with the capacitor, \( R'' = R' \), the internal resistance of the capacitor. When the resistance \( R \) is connected across the capacitor, the effective resistance \( R'' \) is given by equation (1).

In the first case, a plot of \( \ln(V) \) against time, in seconds, will give a slope \( \alpha' \) of \(-1/(CR)'\). In the second case, the slope \( \alpha'' \) of such a plot is given by

\[
\alpha'' = \frac{1}{(CR'')} = -\left[ \frac{1}{(CR)} + \frac{1}{(CR')} \right]. \]  \( (4) \)

The difference in slopes is

\[
\alpha' - \alpha'' = \frac{1}{(CR)}. \]  \( (5) \)
Experiments to Measure Resistance

This difference will be large if \( R' \gg R \). So, resistances with a value \( R \) which is a small fraction of \( R' \) can be measured by this technique. Since the tantalum capacitors are expected to have an internal resistance of a few hundred mega-ohms, one may measure resistances of the order of 100 mega-ohms or less by this apparatus.

2. Circuit for High Resistance by Leakage

Figure 1 shows the front panel of the box containing the circuit for high resistance by leakage, cum split power supply.

The front panel is divided into two areas. The area marked Split Power Supply contains three banana terminals marked +12 V, GRD and –12 V. One can use this to provide split power to a separate electronics circuit.

![Figure 1. Front panel of box containing the circuit for high resistance by leakage cum split power supply.](image-url)

The main area marked Measurement of High Resistance has all the controls on the front panel to measure a high resistance. From a regulated power supply inside the box, a potential divider provides about +2 V to charge a capacitor. The capacitor to be charged can be selected by a band switch. Three tantalum capacitors are provided, one of 10 \( \mu \)F capacitance, the second of 47 \( \mu \)F capacitance and the third of 100 \( \mu \)F capacitance. There are three toggle switches marked SW1, SW2 and SW3. When SW2 is put in the position marked CH, the capacitor selected by the band switch gets charged to approximately 2 V. When this toggle switch is put in the position marked DIS, the capacitor discharges through a high resistance. There is a high resistance of 130 megohms inside the box. By putting the switch SW3 in the position INT, the capacitor discharges through this resistance, in series with the switch SW1. The capacitor discharges through its internal resistance, when the switch SW1 is put in position O. When the switch is put in position S the capacitor discharges through the 130 megohm resistance, in parallel with its internal resistance. One may
Experiments to Measure Resistance

also connect a high resistance externally to the banana terminals marked EXT. If the switch SW3 is put in position marked EXT the capacitance discharges through the external resistance in series with the switch SW1.

When the capacitor is discharging, the voltage across the capacitor will decrease exponentially with time. The time constant is \( CR \) where \( C \) is the capacitance and \( R \) is the high resistance. This voltage cannot be measured with an ordinary multimeter, because the input impedance of the multimeter will be of the same order, or less than, the high resistance being measured, and will short the high resistance. There is a voltage follower circuit inside. The input impedance of the operational amplifier is higher than the resistance to be measured. By pressing the push switch marked PS the voltage across the capacitor will be applied to the input of the voltage follower. The output voltage of the voltage follower can be measured by connecting a DMM in the DC 2 volts range to the banana terminals marked DMM. The push switch, when pressed, connects the voltage across the capacitor to the input of the voltage follower for a short time, when the reading on the DMM is taken. For the rest of the time, the input of the voltage follower is isolated from the condenser terminals. This is an added precaution to reduce the effect of the input impedance altering the measured value of the high resistance.

Taking a measuring time of 3000 seconds as the upper limit, one may measure a resistance from 10 megohms to 50 megohms using 100 \( \mu F \) capacitor, a resistance of 50 megohms to 100 megohms with the 47 \( \mu F \) capacitor, and a resistance of 100 megohms to 500 megohms with the 10 \( \mu F \) capacitor. Of course, one can measure any high resistance with any of the capacitors, provided one takes readings for a time equal to or greater than the time constant.

3. Apparatus Required

High resistance by leakage circuit box and a DMM which can measure less than 2V DC to 3 decimal places.

4. Experimental Procedure

The circuit is switched on. For measuring the resistance connected internally, one puts the toggle switch 3 in the position marked INT. The selector switch for selecting the capacitor is put in the position 10 \( \mu F \). A DMM in the DC 2 volts range is connected to the banana terminals marked DMM. The toggle switch 2 is put in position marked CH. When the push switch is pressed, the DMM will indicate approximately 2 volts. This voltage is noted. The toggle switch 1 is put in position O. Then, the toggle switch 2 is put in position DIS. The condenser of 10 \( \mu F \) will now start discharging through its internal resistance \( R' \). The push switch is pressed every five minutes and the voltage reading on the DMM noted.

Then, the condenser is charged again by putting the toggle switch 1 in the position CH. The switch 2 is put in position S, and the switch 1 is put in position DIS. Then, the time versus voltage is recorded as before, but now at intervals of two minutes. The condenser will discharge faster, because now the resistance \( R \) is connected in parallel to the internal resistance \( R' \).

A plot of \( \ln (V) \) is made against time \( t \) in seconds, and a straight line is fitted to the points for each set of readings. If \( a' \) is the slope of the line when the switch 2 is in position O, and \( a'' \) is the slope when the switch 2 is in position S, then

\[
a' = \frac{-1}{\tau'} = \frac{-1}{(CR')}
\]

and

\[
a'' = \frac{-1}{\tau''} = \frac{-1}{(CR'')} = \frac{-1}{C} \times \left( \frac{1}{R} + \frac{1}{R'} \right).
\]

The value of the high resistance \( R \) is then found from

\[
R = \frac{1}{C (a' - a'')}. \tag{8}
\]

A sample set of data is given in Table 1. A plot of \( \ln (V) \) against time is shown in Figure 2.
Experiments to Measure Resistance

Table 1. Measurement of high resistance $C = 10 \, \mu F$.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>$V$ (volts)</th>
<th>$\ln (V)$</th>
<th>Time (s)</th>
<th>$V$ (volts)</th>
<th>$\ln (V)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.994</td>
<td>0.690</td>
<td>0</td>
<td>1.880</td>
<td>0.631</td>
</tr>
<tr>
<td>300</td>
<td>1.969</td>
<td>0.678</td>
<td>120</td>
<td>1.706</td>
<td>0.534</td>
</tr>
<tr>
<td>600</td>
<td>1.947</td>
<td>0.666</td>
<td>240</td>
<td>1.551</td>
<td>0.439</td>
</tr>
<tr>
<td>900</td>
<td>1.926</td>
<td>0.655</td>
<td>360</td>
<td>1.411</td>
<td>0.344</td>
</tr>
<tr>
<td>1200</td>
<td>1.906</td>
<td>0.645</td>
<td>480</td>
<td>1.285</td>
<td>0.251</td>
</tr>
<tr>
<td>1500</td>
<td>1.887</td>
<td>0.635</td>
<td>600</td>
<td>1.172</td>
<td>0.159</td>
</tr>
<tr>
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<td>1.866</td>
<td>0.624</td>
<td>720</td>
<td>1.071</td>
<td>0.069</td>
</tr>
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<td>2100</td>
<td>1.850</td>
<td>0.615</td>
<td>840</td>
<td>0.979</td>
<td>-0.021</td>
</tr>
<tr>
<td>2400</td>
<td>1.834</td>
<td>0.606</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 2. Plot of $\ln(V)$ as a function of time in seconds.

We see that the plots are straight lines with slopes $a' = -3.58 \times 10^{-5}/s$ and $a'' = -7.77 \times 10^{-4}/s$. The respective standard deviations in the slopes are $6.38 \times 10^{-7}/s$ and $4.26 \times 10^{-6}/s$. From this, and using

$$-\frac{1}{a'} = \varepsilon' = \frac{1}{CR'},$$

we get

$$R' = \frac{-1}{Ca'}.$$

This is the internal resistance of the 10 microfarad capacitance. Substituting for $a'$, we get $R' = 2865$ megohms.

The high resistance $R$ is

$$R = \frac{1}{[C(a' - a'')]}.$$

The value of $R$ comes out to be 135 megohms.

The error in $R$ is calculated from

Error in $R = Rx (\text{sum of the standard deviations})/(a' - a'')$

This comes out to be 0.9 megohms.

Similar measurements were made with $C = 47$ microfarad and $C = 100$ microfarad capacitors. The results are shown as plots in Figures 3 and 4.
From the values of $\alpha'$ and $\alpha''$ in the two figures, the values of $R'$ and $R$ are calculated. From the sum of the standard deviations the error in $R$ is calculated. These are collected in Table 2.

<table>
<thead>
<tr>
<th>Capacitance (μF)</th>
<th>$R'$ (megohms)</th>
<th>$R$ (megohms)</th>
<th>Error in $R$ (megohms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1401</td>
<td>133</td>
<td>1.8</td>
</tr>
<tr>
<td>47</td>
<td>102</td>
<td>158</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>2865</td>
<td>135</td>
<td>0.9</td>
</tr>
</tbody>
</table>

We see from this table that for both the 100 and 10 microfarad capacitors, the internal resistance is in the 1000 megohm range. So, they both have given closely agreeing values for the resistance $R$, which is in the 100 megohm range. On the other hand, the 47 microfarad capacitor has a lower internal resistance than the resistance $R$. The values of $\alpha'$ and $\alpha''$ are close to each other. The value of $R$ determined from the difference between $\alpha'$ and $\alpha''$ has a larger error.

In our experience, 10 μF tantalum capacitors available commercially, usually have a resistance in giga-ohms while 47 and 100 μF capacitors have lower resistances. So, to measure high resistance in the 100 megohm range, 10 μF capacitors are suitable. But, for measuring resistance in the 10 to 100 megohm range, 47 and 100 μF capacitors are quite okay.

An external high resistance can be measured by connecting it to the terminals marked EXT, and putting the switch 3 to position EXT.
Chapter 6

AC Experiments
6.1 Introduction to AC Circuits

1. Introduction

A current, varying periodically with time, is called an alternating current. When a current $I$ is passed through a coil of self-inductance $L$ it generates a magnetic field $B$. A magnetic flux $LI$ due to this current is linked with the coil. When the current changes with time, the flux linked with the coil also changes with time. It induces an emf, $-L(dI/dt)$, in the coil. If $I$ is a sinusoidal current with a frequency $f$,

$$I = I_0 \sin(2\pi ft),$$  \hspace{1cm} (1)

and the induced emf is

$$V_{\text{ind}} = -2\pi fL I_0 \cos(2\pi ft).$$  \hspace{1cm} (2)

The voltage applied across the coil should be equal and opposite to the induced voltage to maintain the oscillating current. The applied voltage is therefore

$$V_{\text{app}} = -V_{\text{ind}} = 2\pi fL I_0 \cos(2\pi ft).$$  \hspace{1cm} (3)

Figure 1 shows $I$ and $V_{\text{app}}$ as a function of $t/T$, where $T = 1/f$. Note that when the current goes through zero, the voltage is either a maximum or a minimum.

Since

$$\cos(\omega t) = \sin\left(\omega t + \frac{\pi}{2}\right),$$  \hspace{1cm} (4)

the phase (the argument of the trigonometric function) of the voltage is always more than the phase of the current by $\pi/2$. We say that the current lags behind the voltage by $\pi/2$. This is shown in Figure 1.

A coil has an inductance and a resistance. The voltage necessary to drive a current through a resistance is

$$V = IR = I_0 R \sin(\omega t).$$  \hspace{1cm} (5)

The voltage across a resistance is always in phase with the current.
AC Experiments

Since a coil has both resistance $R_L$ and inductance $L$, the voltage across the coil, when an alternating sinusoidal current passes through it, is the sum of two parts

$$V_{app} = I_0 [R_L \sin(\omega t) + \omega L \cos(\omega t)].$$

If we put $R_L = Z \cos \phi$, and $\omega L = Z \sin \phi$ in equation (6), we get

$$V_{app} = Z I_0 \sin(\omega t + \phi),$$

where

$$Z = (R_L^2 + \omega^2 L^2)^{1/2}$$

is called the impedance of the coil.

The applied voltage now leads the current by the phase angle given by

$$\tan \phi = \frac{\omega L}{R_L} \quad \text{or} \quad \phi = \tan^{-1} \left( \frac{\omega L}{R_L} \right).$$

($\tan^{-1}(\phi)$ is also called $\text{atan}(\phi)$).

The impedance of a coil increases as the frequency of the current is increased, and the phase angle increases from 0 and approaches $\pi/2$, as the frequency becomes very high.

When a capacitance $C$ is charged to a voltage $V$, a charge

$$q = CV$$

is stored on the positive capacitor plate, and an equal charge, but of opposite polarity, is stored on the negative capacitor plate.

If the voltage varies sinusoidally as

$$V = V_0 \sin(\omega t),$$

then the charge $q$ on the capacitor also varies sinusoidally as

$$q = CV_0 \sin(\omega t).$$

A time-varying charge corresponds to a current

$$I = \frac{dq}{dt} = C \omega V_0 \cos(\omega t).$$

If we write

$$I_0 = C \omega V_0 \quad \text{or} \quad V_0 = \frac{I_0}{\omega C},$$

then the current through the capacitor and the voltage across the capacitor may be written, in terms of $I_0$, as

$$I = I_0 \cos(\omega t)$$

and

$$V = \left( \frac{I_0}{\omega C} \right) \sin(\omega t) = V_0 \cos \left( \omega t - \frac{\pi}{2} \right).$$

The impedance of the capacitor is

$$Z = \frac{V_0}{I_0} = \frac{1}{\omega C}.$$ 

Note that, unlike with an inductance, the voltage across a capacitor lags behind the current through the capacitor by $\pi/2$.

We may represent the voltages across a series combination of resistance, inductance and/or capacitance, in relation to the current through the combination, in an Argand or phasor diagram. In this diagram, a vector along the X-axis represents a voltage in phase with the current. A voltage leading the current in phase by an angle $\phi$ is represented by a vector which makes an angle $\phi$ with the X-axis in the first quadrant. Similarly, a voltage lagging behind the current by a phase $\phi$ is represented by a vector in the fourth quadrant, making an angle $\phi$ with the X-axis.
AC Experiments

For example, when a current of amplitude $I$ and angular frequency $\omega$ passes through a coil that has a resistance $R = R_L$ and an inductance $L$, the Argand diagram in Figure 2 represents the voltage across the coil, in relation to the current through it.

![Figure 2](image)

Figure 2. Argand diagram for a current passing through a coil.

The vector AB represents the voltage drop across $R_L$ that is in phase with the current. The vector BC represents the voltage to overcome the induced emf due to the inductance $L$. The vector AC represents the vector sum of the above two voltages, and is the voltage applied to the coil. It is equal to $ZI$ and leads the current by angle $\phi$ in phase.

Suppose we have a resistance $R$ in series with the above coil, and an AC current $I$ at frequency $\omega$ flows through them. The Argand diagram for this case is shown in Figure 3.

![Figure 3](image)

Figure 3. Argand diagram for a resistance $R$ in series with a coil of resistance $R_L$ and inductance $L$.

The total RMS applied voltage $V_{\text{app}}$ (vector OC) is less than the sum of the RMS voltages $V_R$ (vector OA) across the resistance $R$, and $V_{\text{coil}}$ (vector AC) across the coil. This is because the voltage across the coil leads the current by an angle $\phi$. If we measure $V_{\text{app}}$, $V_R$ and $V_{\text{coil}}$, they should be related by

$$V_{\text{app}}^2 = V_R^2 + V_{\text{coil}}^2 + 2V_R V_{\text{coil}} \cos \phi.$$ 

(18)

The maximum and minimum values of $\cos(\phi)$ are +1 and −1 respectively. So, $V_{\text{app}}$ will have a value between $(V_R + V_L)$ and $(V_R - V_L)$.

From this, the phase angle $\phi$ can be obtained at any frequency $\omega$. The variation of $\phi$ with frequency can be studied.

Similarly, if we have a resistance $R$ in series with a capacitance $C$, we may construct the Argand diagram as shown in Figure 4.

![Figure 4](image)

Figure 4. Argand diagram for a resistance in series with a capacitor.

We can thus make an Argand diagram for any combination of resistances, inductances and capacitances.

2. RLC Box

The RLC box shown in Figure 5 is described below.
AC Experiments

A, B, C, and D are banana terminals on the box. Between A and B is connected a resistance R of 1 kΩ inside the box. Between B and C is connected a coil of inductance L of 100 mH. Between C and D is connected a capacitance C of 0.033 μF. One may choose other values depending on the availability.

If the signal generator is connected between A and C, it is a series RL circuit. If B and C are externally shorted by a wire, and the signal generator is connected between A and D, it becomes a series RC circuit. If the signal generator is connected between A and D without shorting the terminals B and C, it becomes a series RLC circuit, L and C being in series with R. If D is connected to B externally, and the signal generator is connected between A and C, it becomes R in series with L, and C in parallel.

With this simple circuit, the four experiments to be described in the following sections can be carried out with a signal generator.

6.2 Measurement of Self-Inductance of a Coil

1. Aim

To show that the impedance of a coil of resistance \( R_L \) and self-inductance \( L \) varies with frequency as

\[
Z_{\text{coil}} = \left( R_L^2 + 4\pi^2 f^2 L^2 \right)^{1/2},
\]

and to measure the self-inductance of the coil.

2. Principle

A coil with a self-inductance \( L \) and resistance \( R_L \) has an impedance \( Z_{\text{coil}}(\omega) \) given by

\[
Z_{\text{coil}}(\omega) = R_L + j\omega L,
\]

where \( j = \sqrt{-1} \) and \( \omega = 2\pi f \), \( f \) being the frequency of the AC supply. The magnitude of the impedance is \( (R_L^2 + \omega^2 L^2)^{1/2} \). If we connect an AC source across a resistance \( R \) in series with the inductor, then the rms voltage \( V_R \) across \( R \), and \( V_{\text{coil}} \) across the coil will be in the ratio

\[
\frac{V_{\text{coil}}}{V_R} = \frac{|Z_{\text{coil}}|}{R} = \frac{(R_L^2 + \omega^2 L^2)^{1/2}}{R}.
\]

If we measure \( V_{\text{coil}}/V_R \) at different frequencies, a plot of \( (V_{\text{coil}}/V_R)^2 \) vs. \( f^2 \) will give a straight line, the slope of which is given by \( (2\pi^2 LR^2)^{1/2} \). From the slope, one can determine \( L \), knowing \( R \).

It is not necessary to keep the amplitude of the signal constant as one varies the frequency because we are only taking the ratio \( V_{\text{coil}}/V_R \).

3. Apparatus Required

Signal generator, RLC box, and a DMM to measure AC voltage (in the range 2V to three decimal places).

4. Procedure

The experiment is performed as described below.
The two terminals of the signal generator SG are connected to the terminals A and C on the RLC box (Figure 1). The signal generator voltage is applied across the resistance and the coil in series. The output of the signal generator is kept at around 1 volt. A DMM in AC 2 V range, connected between A and B, measures the rms voltage drop $V_R$ across the resistance. The same DMM, connected between B and C, measures the rms voltage drop $V_{coil}$ across the coil. Connected between A and C, the DMM measures $V_{app}$, the total applied voltage. The frequency of the signal is varied between 200 and 2000 Hz in steps of 200 Hz, and $V_{coil}$, $V_R$ and $V_{app}$ are measured. A sample set of readings is shown in Table 1.

### Table 1. Measurement of self-inductance. Resistance $R = 1000$ ohms.

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>$f^2$ (kHz)$^2$</th>
<th>$V_{coil}$ (volts)</th>
<th>$V_R$ (volts)</th>
<th>$V_{app}$ (volts)</th>
<th>$\left(\frac{V_{coil}}{V_R}\right)^2$</th>
<th>$\cos(\phi)$</th>
<th>$\tan(\phi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.04</td>
<td>0.2</td>
<td>0.832</td>
<td>1.00</td>
<td>0.058</td>
<td>0.829</td>
<td>0.675</td>
</tr>
<tr>
<td>400</td>
<td>0.16</td>
<td>0.26</td>
<td>0.82</td>
<td>1.00</td>
<td>0.101</td>
<td>0.61</td>
<td>1.3</td>
</tr>
<tr>
<td>600</td>
<td>0.36</td>
<td>0.342</td>
<td>0.796</td>
<td>1.01</td>
<td>0.185</td>
<td>0.477</td>
<td>1.845</td>
</tr>
<tr>
<td>800</td>
<td>0.64</td>
<td>0.42</td>
<td>0.774</td>
<td>1.00</td>
<td>0.294</td>
<td>0.333</td>
<td>2.831</td>
</tr>
<tr>
<td>1000</td>
<td>1.00</td>
<td>0.492</td>
<td>0.738</td>
<td>1.00</td>
<td>0.444</td>
<td>0.285</td>
<td>3.357</td>
</tr>
<tr>
<td>1200</td>
<td>1.44</td>
<td>0.548</td>
<td>0.71</td>
<td>1.00</td>
<td>0.596</td>
<td>0.251</td>
<td>3.851</td>
</tr>
<tr>
<td>1400</td>
<td>1.96</td>
<td>0.605</td>
<td>0.67</td>
<td>0.99</td>
<td>0.815</td>
<td>0.194</td>
<td>5.057</td>
</tr>
<tr>
<td>1600</td>
<td>2.56</td>
<td>0.66</td>
<td>0.645</td>
<td>1.00</td>
<td>1.047</td>
<td>0.163</td>
<td>6.07</td>
</tr>
<tr>
<td>1800</td>
<td>3.24</td>
<td>0.702</td>
<td>0.612</td>
<td>0.99</td>
<td>1.316</td>
<td>0.138</td>
<td>7.169</td>
</tr>
<tr>
<td>2000</td>
<td>4</td>
<td>0.74</td>
<td>0.575</td>
<td>0.99</td>
<td>1.656</td>
<td>0.124</td>
<td>7.978</td>
</tr>
</tbody>
</table>

Note: A DMM gives the rms value of the AC voltage. The value of the voltage indicated by a cheap DMM will be less than the actual rms value, as the frequency increases in the kHz range. Since we are taking the ratio of voltages, this behavior of the DMM will not introduce any error in the value of the inductance. Figure 2 shows a plot of $\left(\frac{V_{coil}}{V_R}\right)^2$ against $f^2$.

![Figure 2. Plot of $(V_{coil}/V_R)^2$ against $(frequency)^2$ for the coil.](image)
AC Experiments

The linear fit to the points is shown by the continuous line. The slope $\alpha$ of the line is $3.99 \times 10^{-7}$ s$^2$. The self-inductance is given by

$$L = \alpha^{0.5} \left( \frac{R}{2\pi} \right).$$

(4)

In the present case, $R$ is 1 k$\Omega$. So, the coil inductance comes out to be 99 millihenries.

Note that the intercept, according to theory, should be $(R_L/R)^2$. The resistance of the coil is about 1/5 to 1/10 of the resistance $R$ of 1 k$\Omega$. The ratio $(R_L/R)^2$ is of the order of 0.01 to 0.04. This is of the same order as the errors in calculating $(V_L/V_R)^2$. The intercept cannot be measured accurately.

5. Phase Calculation

The voltage across the resistance $R$ is in phase with the current, while the voltage across the inductance leads the current by a phase angle $\phi$ given by

$$\tan \phi = 2\pi f \left( \frac{L}{R_L} \right).$$

(5)

The Argand diagram for this situation is shown in Figure 3 of the previous (Section 6.1). From this diagram, we see that

$$V_{\text{app}}^2 = V_R^2 + V_{\text{coil}}^2 + 2V_R V_{\text{coil}} \cos \phi.$$  

(6)

The total applied voltage is less than the sum of the measured voltages across the resistance $R$ and the coil.

Using the above formula, we can calculate $\cos(\phi)$ from the measured values of $V_{\text{app}}$, $V_R$ and $V_{\text{coil}}$. From this we can obtain $\tan(\phi)$. The values of $\cos(\phi)$ and $\tan(\phi)$ are shown in the last two columns of Table 1. A plot of $\tan(\phi)$ against the frequency $f$ can be fitted to a straight line to give the ratio $L/R_L$. Since $L$ has been measured earlier, one can obtain $R_L$, the resistance of the coil from such a plot.

A plot of the values of $\tan(\phi)$ against the frequency is shown in Figure 3.

![Figure 3](image-url)

**Figure 3.** A plot of $\tan(\phi)$ against frequency.

A linear fit to the data is shown in the figure. The slope $\alpha$ of the graph is 0.00409 s. This should be equated to $2\pi L/R_L$. Taking $L$ to be 99 mH, the calculated value of $R_L$ is 152 ohms.

Note that $\tan(\phi)$ increases with $f$. At high frequency, $\phi$ will approach 90 degrees. Remember that $\tan(\phi)$ increases rapidly with $\phi$, as $\phi$ approaches 90 degrees. So, the error in $\tan(\phi)$ will become very large. That is the reason why the measurements are restricted to frequencies below 2 kHz.
AC Experiments

6. Questions
1. What is the dimension of \( L/R \)?
2. At what frequency will the inductive impedance of the above coil be equal to its resistance? What will the phase angle \( \phi \) be at that frequency?
3. For a given applied AC voltage across the coil, will the current increase or decrease as the frequency is increased?

6.3 Measurement of Capacitance

1. Aim
To verify that the impedance of a capacitance \( C \) varies with frequency as

\[
Z_C = \frac{1}{(2\pi f C)} \tag{1}
\]

and to measure the value of the capacitance.

2. Principle
The impedance of a capacitance \( C \) at a frequency \( f \) is

\[
Z_C = \left(\frac{1}{j}\right) \left(\frac{1}{(2\pi f C)}\right). \tag{2}
\]

The leakage resistance of a capacitor has a very high value, and is in parallel with the capacitance. Its effect on the total impedance can be neglected.

If a resistance \( R \) and a capacitance \( C \) are connected in series, and an AC rms voltage \( V_{app} \) at frequency \( f \) is applied across the combination, the ratio of the rms voltages across \( R \) and \( C \) is given by

\[
\frac{V_R}{V_C} = \frac{R}{|Z_C|} = 2\pi f C R. \tag{3}
\]

If this ratio is measured at different frequencies, and plotted as a function of frequency, one should get a straight line with a slope \( 2\pi CR \). One can determine \( C \) from the slope of the curve, knowing \( R \).

3. Apparatus Required
A signal generator, RLC box and a DMM which can measure AC voltage up to three decimal places in the range of 2V.

4. Procedure

![Connection diagram](image)
**AC Experiments**

Short B and C on the RLC box externally with a wire so that the coil is shorted out. We now have a RC circuit between the terminals A and D. Connect a signal generator output to the two terminals A and D, and set the frequency at 1 kHz. Set the rms voltage of the output to around 1 volt. With a multimeter in the AC 2 volts range, measure the voltage $V_R$ across the resistance $R$ and $V_C$ across the capacitance $C$. Repeat the experiment changing the frequency of the signal generator from 1 to 10 kHz in steps of 1 kHz. It is not necessary to keep $V_{app}$ constant as the frequency is changed, since we are measuring only the ratio $V_R/V_C$. Plot this ratio against $f$. Fit the data to a straight line and obtain the slope $\alpha$. Then $C$ can be calculated from equation (3). A representative set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$V_R$ (volts)</th>
<th>$V_C$ (volts)</th>
<th>$V_R/V_C$</th>
<th>$V_{app}$ (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.190</td>
<td>0.920</td>
<td>0.207</td>
<td>0.940</td>
</tr>
<tr>
<td>2</td>
<td>0.370</td>
<td>0.870</td>
<td>0.425</td>
<td>0.946</td>
</tr>
<tr>
<td>3</td>
<td>0.508</td>
<td>0.800</td>
<td>0.635</td>
<td>0.949</td>
</tr>
<tr>
<td>4</td>
<td>0.616</td>
<td>0.726</td>
<td>0.849</td>
<td>0.955</td>
</tr>
<tr>
<td>5</td>
<td>0.700</td>
<td>0.654</td>
<td>1.070</td>
<td>0.962</td>
</tr>
<tr>
<td>6</td>
<td>0.760</td>
<td>0.591</td>
<td>1.286</td>
<td>0.969</td>
</tr>
<tr>
<td>7</td>
<td>0.805</td>
<td>0.535</td>
<td>1.505</td>
<td>0.976</td>
</tr>
<tr>
<td>8</td>
<td>0.841</td>
<td>0.483</td>
<td>1.741</td>
<td>0.983</td>
</tr>
<tr>
<td>9</td>
<td>0.837</td>
<td>0.425</td>
<td>1.969</td>
<td>0.958</td>
</tr>
<tr>
<td>10</td>
<td>0.857</td>
<td>0.384</td>
<td>2.232</td>
<td>0.959</td>
</tr>
</tbody>
</table>

A plot of $V_R/V_C$ against the frequency in kHz is shown in Figure 2.

![Figure 2](image)

**Figure 2.** Plot of $(V_R/V_C)$ against frequency in kHz.

The slope $\alpha$ of the curve is $2.23 \times 10^{-4}$ s. We can calculate the unknown capacitance value from the equation

\[ C = \frac{\alpha}{(2\pi R)}. \]

In the present case, $R = 1$ kilo-ohm. From this formula, $C$ comes out to be $0.0354 \ \mu$-farad.
**AC Experiments**

Note: Unlike with the inductor, we are working with the capacitor in the range of frequencies 1 to 10 kHz. With the capacitance value in the RLC box, the impedance of the capacitance in this range of frequencies is comparable to the resistance $R$. If we had worked with frequencies below 1 kHz, the impedance of the capacitance would have been high compared to the resistance $R$, and so $V_R$ would become smaller and smaller as the frequency is decreased.

Also note that the rms voltage indicated by the DMM will become smaller and smaller, compared to the actual applied rms voltage, as the frequency is increased. To get an appreciable reading for $V_R$ and $V_C$, one would have to increase the voltage output of the signal generator. Since we are taking the ratio of $V_R$ to $V_C$, this error in the DMM will not affect the ratio.

For a capacitance, the leakage resistance is very high. So, the voltage across the capacitance will lag behind the current (i.e., lag behind $V_R$) by almost exactly 90° at all frequencies. This implies that at all frequencies

$$V_{app} = (V_R^2 + V_C^2)^{1/2}.$$ 

This is checked in Table 2 for the measured values of $V_R$, $V_C$ and $V_{app}$.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$V_R$ (volts)</th>
<th>$V_C$ (volts)</th>
<th>$V_{app}$ (volts)</th>
<th>$(V_R^2 + V_C^2)^{1/2}$ (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.190</td>
<td>0.920</td>
<td>0.940</td>
<td>0.939</td>
</tr>
<tr>
<td>2</td>
<td>0.370</td>
<td>0.870</td>
<td>0.946</td>
<td>0.945</td>
</tr>
<tr>
<td>3</td>
<td>0.508</td>
<td>0.800</td>
<td>0.949</td>
<td>0.948</td>
</tr>
<tr>
<td>4</td>
<td>0.616</td>
<td>0.726</td>
<td>0.955</td>
<td>0.952</td>
</tr>
<tr>
<td>5</td>
<td>0.700</td>
<td>0.654</td>
<td>0.962</td>
<td>0.958</td>
</tr>
<tr>
<td>6</td>
<td>0.760</td>
<td>0.591</td>
<td>0.969</td>
<td>0.963</td>
</tr>
<tr>
<td>7</td>
<td>0.805</td>
<td>0.535</td>
<td>0.976</td>
<td>0.967</td>
</tr>
<tr>
<td>8</td>
<td>0.841</td>
<td>0.483</td>
<td>0.983</td>
<td>0.970</td>
</tr>
<tr>
<td>9</td>
<td>0.837</td>
<td>0.425</td>
<td>0.958</td>
<td>0.939</td>
</tr>
<tr>
<td>10</td>
<td>0.857</td>
<td>0.384</td>
<td>0.959</td>
<td>0.939</td>
</tr>
</tbody>
</table>

If we have a coil and a capacitance in a circuit, the circuit will have a resonance frequency $f_{res}$. By definition, this is the frequency at which the magnitude of the impedance of the inductance of the coil is equal to the magnitude of the impedance of the capacitor. The frequency $f_{res}$ is given by

$$f_{res} = \left(\frac{1}{2\pi}\right) \left(\frac{1}{LC}\right)^{1/2}. \quad (4)$$

Using $L = 99$ mH and $C = 0.0354$ μF, the resonance frequency has the value 2.68 kHz.

5. **Questions**

1. What is the expression for the capacitance of a parallel plate capacitor? Calculate the capacitance of a parallel plate capacitor with surface area of 0.1 m² and distance between the plates of 1 mm.

2. This capacitor is connected in series with a 50 mH inductor. How will the resonance frequency change if the distance between the capacitor plates is increased to 2 mm?

3. Find the impedance of a 0.1 μF capacitor at a frequency of 10 kHz. An AC voltage of 1 V rms is applied to the capacitor. What is the current through the capacitor at 10 kHz? Will this current increase or decrease if the frequency is increased?
6.4 Series and Parallel Resonant Circuits

1. Aim

To show that the impedance of a circuit, in which an inductance $L$ and a capacitance $C$ are in series, is a minimum at the resonance frequency $f_{res}$, and the impedance of a circuit, in which $L$ and $C$ are in parallel, is a maximum at $f_{res}$.

2. Principle

At the resonance frequency $f_{res} = (1/2\pi) (1/LC)^{1/2}$, $\omega L = 1/\omega C$. The impedance of an inductance is $Z_L = R_L + j\omega L$ and of a capacitance is $Z_C = -j\omega C$. When the inductance and capacitance are in series, the total inductance is $Z_L + Z_C$. This has a minimum value $R_L$ at the resonance frequency. At any other frequency the impedance is more than $R_L$.

If the coil and capacitance are connected in parallel, the effective impedance of the combination is

$$Z = \frac{Z_L Z_C}{Z_L + Z_C}.$$ (1)

At the resonance frequency, $Z_L + Z_C$ is a minimum. If $R_L$ were zero, then $Z_L Z_C$ would be independent of frequency, while $Z_L + Z_C$ would be zero at the resonance frequency, making the effective impedance infinite at the resonance frequency. However, $R_L$ is non-zero and small for the coil. This makes the impedance go through a finite, but maximum, value at a frequency close to the resonant frequency.

We verify the above results with the following experiment.

3. Apparatus Required

Signal generator, RLC box and a DMM which can measure AC volts to three decimal places in the range 2 V.

4. Procedure

![Connection diagram for series resonant circuit.](image)

Connect the signal generator output to the two terminals A and D. Set the frequency of the signal generator at 1 kHz. Set the rms amplitude of the output to around 1 volt. Measure the voltage across A and B terminals with a multimeter in the AC 2 volts range. This gives the voltage $V_R$. With the same multimeter measure the voltage across the terminals B and D. This voltage $V_Z$ is the voltage across the series combination of L and C. Repeat the experiment at different frequencies from 1 to 7 kHz. The resonance frequency of the above coil and capacitor has a value of about 2.68 kHz. So, in the range 2 to 4 kHz, take readings at intervals of 200 Hz.

The magnitude of the impedance of the coil and capacitance in series is $(V_Z/V_R) R$, and since $R$ is 1 kilo-ohm, $(V_Z/V_R)$ gives the impedance of the series resonant circuit in kilo-ohms. Plot the graph of $V_Z/V_R$
AC Experiments

against the frequency \( f \). This ratio is \(|Z|/R\), where \( Z \) is the impedance of the series \( L \) and \( C \) combination. The graph will show minimum impedance at a frequency \( f_{\text{min}} \). Compare this frequency with the value of \( f_{\text{res}} = (1/2\pi) \left(1/\sqrt{LC}\right) \).

A representative set of readings is shown in Table 1 for the series resonant circuit.

### Table 1. Impedance of a series resonant circuit.

<table>
<thead>
<tr>
<th>( f ) (kHz)</th>
<th>( V_R ) (volts)</th>
<th>( V_Z ) (volts)</th>
<th>( V_{Z/V_R} ) (volts)</th>
<th>( V_{\text{app}} ) (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.219</td>
<td>0.911</td>
<td>4.160</td>
<td>0.944</td>
</tr>
<tr>
<td>2.0</td>
<td>0.582</td>
<td>0.649</td>
<td>1.115</td>
<td>0.933</td>
</tr>
<tr>
<td>2.2</td>
<td>0.671</td>
<td>0.512</td>
<td>0.763</td>
<td>0.936</td>
</tr>
<tr>
<td>2.4</td>
<td>0.740</td>
<td>0.351</td>
<td>0.474</td>
<td>0.923</td>
</tr>
<tr>
<td>2.6</td>
<td>0.782</td>
<td>0.178</td>
<td>0.228</td>
<td>0.921</td>
</tr>
<tr>
<td>2.8</td>
<td>0.786</td>
<td>0.143</td>
<td>0.182</td>
<td>0.922</td>
</tr>
<tr>
<td>3.0</td>
<td>0.756</td>
<td>0.288</td>
<td>0.381</td>
<td>0.927</td>
</tr>
<tr>
<td>3.2</td>
<td>0.712</td>
<td>0.419</td>
<td>0.589</td>
<td>0.933</td>
</tr>
<tr>
<td>3.4</td>
<td>0.660</td>
<td>0.528</td>
<td>0.800</td>
<td>0.939</td>
</tr>
<tr>
<td>3.6</td>
<td>0.615</td>
<td>0.617</td>
<td>1.003</td>
<td>0.932</td>
</tr>
<tr>
<td>3.8</td>
<td>0.570</td>
<td>0.678</td>
<td>1.190</td>
<td>0.937</td>
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<tr>
<td>4.0</td>
<td>0.528</td>
<td>0.726</td>
<td>1.375</td>
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</tr>
<tr>
<td>4.5</td>
<td>0.442</td>
<td>0.809</td>
<td>1.830</td>
<td>0.951</td>
</tr>
<tr>
<td>5.0</td>
<td>0.372</td>
<td>0.859</td>
<td>2.309</td>
<td>0.974</td>
</tr>
<tr>
<td>5.5</td>
<td>0.332</td>
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<td>0.965</td>
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</tr>
<tr>
<td>6.5</td>
<td>0.265</td>
<td>0.927</td>
<td>3.498</td>
<td>0.974</td>
</tr>
<tr>
<td>7.0</td>
<td>0.240</td>
<td>0.939</td>
<td>3.913</td>
<td>0.978</td>
</tr>
</tbody>
</table>

A plot of \( V_{Z/V_R} \) against the frequency in kHz is shown in Figure 2. From the figure we see that this impedance goes through a minimum between 2.6 and 2.8 kHz. This should be compared with the resonance frequency of the coil and capacitance, that is, 2.68 kHz.

![Figure 2](image)

**Figure 2.** A plot of \( V_{Z/V_R} \) against frequency (in kHz) for the series resonant circuit.

Next connect the terminals B and D together by an external wire and connect the signal generator between A and C. This is shown Figure 3.
As before, set the signal generator frequency at 1 kHz and the rms output at around 1 volt. Measure the voltage between A and B on a multimeter in the AC 2 volts range. This is $V_R$, the voltage across the resistance $R$. With the same multimeter, measure the voltage $V_Z$ between B and C. This is the voltage across the impedance due to $L$ and $C$ connected in parallel. Vary the frequency from 1 to 7 kHz. From 2 to 4 kHz, vary it in intervals of 200 Hz. The graph of $V_Z/V_R$ shows a sharp maximum at a frequency $f_{\text{max}}$. Verify that $f_{\text{max}}$ is nearly equal to $f_{\text{res}} = (1/2\pi) (1/LC)^{1/2}$.

Since $R$ is 1 kilo-ohm, $(V_Z/V_R)$ is the impedance of the parallel resonant circuit in kilo-ohms.

A representative set of data for the parallel resonant circuit is shown in Table 2.

Figure 4 shows the plot of $(V_Z/V_R)$ against the frequency in kilo-Hz for the parallel resonant circuit. We see that the impedance of the parallel resonant circuit goes through a maximum at a frequency around 2.8 kHz, again close to the resonant frequency of 2.68 kHz. The peak is much sharper than the minimum for a series resonant circuit. This is because, in a series resonant circuit, the ratio $V_Z/V_R$ is a minimum. So the values of $V_Z/V_R$ in the neighborhood of the minimum are small. Plotted on a uniform scale over the entire range of frequencies, the minimum appears to be flat.

Table 2. Parallel resonant circuit. $R = 1000$ ohms.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$V_R$ (volts)</th>
<th>$V_Z$ (volts)</th>
<th>$V_Z/V_R$</th>
<th>$V_{\text{app}}$ (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>0.654</td>
<td>0.492</td>
<td>0.752</td>
<td>0.913</td>
</tr>
<tr>
<td>1.2</td>
<td>0.597</td>
<td>0.572</td>
<td>0.958</td>
<td>0.917</td>
</tr>
<tr>
<td>1.4</td>
<td>0.531</td>
<td>0.651</td>
<td>1.226</td>
<td>0.921</td>
</tr>
<tr>
<td>1.6</td>
<td>0.454</td>
<td>0.714</td>
<td>1.573</td>
<td>0.932</td>
</tr>
<tr>
<td>1.8</td>
<td>0.372</td>
<td>0.772</td>
<td>2.075</td>
<td>0.936</td>
</tr>
<tr>
<td>2.0</td>
<td>0.296</td>
<td>0.821</td>
<td>2.774</td>
<td>0.935</td>
</tr>
<tr>
<td>2.2</td>
<td>0.207</td>
<td>0.856</td>
<td>4.135</td>
<td>0.945</td>
</tr>
<tr>
<td>2.4</td>
<td>0.138</td>
<td>0.880</td>
<td>6.377</td>
<td>0.941</td>
</tr>
<tr>
<td>2.6</td>
<td>0.063</td>
<td>0.896</td>
<td>14.222</td>
<td>0.944</td>
</tr>
<tr>
<td>2.8</td>
<td>0.043</td>
<td>0.903</td>
<td>21.000</td>
<td>0.945</td>
</tr>
<tr>
<td>3.0</td>
<td>0.148</td>
<td>0.906</td>
<td>6.122</td>
<td>0.957</td>
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<tr>
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<td>0.165</td>
<td>0.899</td>
<td>5.449</td>
<td>0.948</td>
</tr>
<tr>
<td>3.4</td>
<td>0.195</td>
<td>0.895</td>
<td>4.590</td>
<td>0.948</td>
</tr>
<tr>
<td>3.6</td>
<td>0.263</td>
<td>0.881</td>
<td>3.350</td>
<td>0.949</td>
</tr>
<tr>
<td>3.8</td>
<td>0.329</td>
<td>0.863</td>
<td>2.623</td>
<td>0.949</td>
</tr>
<tr>
<td>4.0</td>
<td>0.377</td>
<td>0.845</td>
<td>2.241</td>
<td>0.950</td>
</tr>
<tr>
<td>4.5</td>
<td>0.473</td>
<td>0.798</td>
<td>1.687</td>
<td>0.948</td>
</tr>
<tr>
<td>5.0</td>
<td>0.549</td>
<td>0.751</td>
<td>1.368</td>
<td>0.950</td>
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<tr>
<td>5.5</td>
<td>0.611</td>
<td>0.703</td>
<td>1.151</td>
<td>0.951</td>
</tr>
<tr>
<td>6.0</td>
<td>0.656</td>
<td>0.660</td>
<td>1.006</td>
<td>0.953</td>
</tr>
<tr>
<td>6.5</td>
<td>0.725</td>
<td>0.632</td>
<td>0.872</td>
<td>0.974</td>
</tr>
<tr>
<td>7.0</td>
<td>0.758</td>
<td>0.596</td>
<td>0.786</td>
<td>0.978</td>
</tr>
</tbody>
</table>
5. Questions

1. A coil of resistance 100 ohms and inductance 100 mH is connected in series with a capacitance of 0.1 μF. What is the approximate resonance frequency of the circuit? What is the impedance of the circuit at this resonant frequency?

2. If the coil and capacitance are connected in parallel what will be the frequency at which the impedance will be a maximum?

3. Given a coil of 50 mH, a signal generator, and an AC ammeter, how will you find the capacitance of an unknown capacitor?

6.5 Passive Filters

1. Introduction

A filter selectively transmits signals over a certain frequency range and blocks signals of other frequencies. The transmission of an ideal (a) low-pass, (b) high-pass, and (c) band-pass filter is shown as a function of frequency in Figures 1(a), (b) and (c) respectively.

An ideal low-pass filter will transmit signals of all frequencies from zero to \( f_{\text{max}} \) without loss, and block signals of all frequencies above \( f_{\text{max}} \). An ideal high-pass filter will block signals of all frequencies below \( f_{\text{min}} \), and will transmit signals of all frequencies above \( f_{\text{min}} \) without any loss. A band-pass filter will transmit signals of all frequencies between \( f_{\text{min}} \) and \( f_{\text{max}} \) without any loss, and will block signals of all other frequencies. An actual filter will not have such abrupt cutoff of transmission.

We shall consider the passive filters shown in Figure 2.

The passive filters are constructed out of resistors and capacitors. In the low-pass filter, one measures the output voltage across \( C_L \). At very low frequencies, the impedance of the capacitance is much higher than the resistance \( R_L \). Then, almost the full input voltage \( V_{\text{in}} \) appears at the output terminals. When the frequency is very high, the impedance of the capacitance is very much smaller than the resistance. So, the output voltage is very low. The transmission coefficient \( T \), defined as the ratio of the output signal to the input signal, varies from nearly unity at very low frequencies to zero at very high frequencies. The roll-off
Figure 1. Transmission characteristics of an ideal (a) low-pass, (b) high-pass, and (c) band-pass filter.

Figure 2. Passive filters: (a) Low-pass; (b) High-pass; and (c) Band-pass.

Frequency $f_{RLP}$ for the low-pass filter is the frequency at which the impedance of the capacitor is equal to the resistance. This happens at a frequency

$$f_{RLP} = \frac{1}{2\pi C_L R_L}.$$  

The roll-off frequency increases as $R_L$ decreases. Since the voltage across the capacitance and the voltage across the resistance are out of phase by $\pi/2$, $V_{out}$ is $V_{in}/\sqrt{2} = 0.707 V_{in}$ at the roll-off frequency $f_{RLP}$. The transmission coefficient $T = 0.707$. This frequency is also called the cut-off frequency. For the same load, the output power will be half the input power.

In the high-pass filter, the output voltage is measured against the resistance $R_H$. At low frequencies, the impedance of the capacitance $C_H$ is very much more than the resistance $R_H$. So, the output voltage across $R_H$ is low. At very high frequencies, the capacitance acts like a short. So, the output voltage across $R_H$ is almost equal to the input voltage. The roll-on frequency $f_{RHP}$ in this case is

$$f_{RHP} = \frac{1}{2\pi C_H R_H}.$$  

To decrease the roll-on frequency, we have to increase the value of $R_H$. For the same reason as with the low-pass filter, the transmission coefficient of a high-pass filter at the roll-on frequency $f_{RHP}$ is 0.707. The roll-on frequency is also called the cut-off frequency.
AC Experiments

For the low-pass filter
\[ T \sim 1 \quad \text{for} \quad f \ll f_{RLP} \]
and
\[ T \sim 0 \quad \text{for} \quad f \gg f_{RLP}. \]

For the high-pass filter
\[ T \sim 0 \quad \text{for} \quad f \ll f_{RHP} \]
and
\[ T \sim 1 \quad \text{for} \quad f \gg f_{RHP}. \]

The band-pass filter is obtained by putting the low- and high-pass filters in series and adjusting \( f_{RHP} \) to be less than \( f_{RLP} \). At any frequency \( f \), the band-pass filter has a transmission coefficient which is the product of \( T_{LP} \) and \( T_{HP} \) at that frequency. Since \( T_{HP} \) decreases as the frequency decreases below \( f_{RHP} \), and \( T_{LP} \) decreases as the frequency increases beyond \( f_{RLP} \), the transmission coefficient \( T_{BP} \) of the band-pass filter reaches a maximum at a frequency between \( f_{RHP} \) and \( f_{RLP} \). In fact, the frequency \( f_{max} \) at which the band-pass filter shows maximum transmission is related to \( f_{RHP} \) and \( f_{RLP} \) as

\[ f_{max} = \left( \frac{f_{RHP}}{f_{RLP}} \right)^{1/2}. \] (3)

2. Passive Filters Box

The low- and high-pass filters are built in a box. The top view of the box is shown in Figure 3. The low-pass filter is on the right and the high-pass filter on the left.

![Figure 3. Top view of the filters box.](image)

On the right are three banana terminals \( L_1, L_2 \) and \( L_3 \). The low-pass filter is connected between these three terminals, the resistance between \( L_1 \) and \( L_2 \), and the capacitance between \( L_2 \) and \( L_3 \). The resistance can be adjusted by the pot \( P_{LP} \). The knob of this pot is projecting to the right of the box. The pot \( P_{LP} \) is set at around 150 ohms. This value of \( R_L \) can be measured with a DMM connected to the terminals \( L_1 \) and \( L_2 \). The capacitance \( C_L \) between the \( L_2 \) and \( L_3 \) terminals has a value of 0.22 \( \mu \)F. This gives a value of \( f_{RLP} \) of about 5 kHz.

The high-pass filter is found on the left of the box. A capacitance of 0.033 \( \mu \)F is connected between the \( H_1 \) and \( H_2 \) terminals on the left of the box. This sets the value of \( C_H \). A pot \( P_{HP} \) of 10 kilo-ohms connected between \( H_2 \) and \( H_3 \) terminals sets the value of \( R_H \). The value of \( R_H \) can be measured between the terminals \( H_2 \) and \( H_3 \). It is set around 5 kilo-ohms so that the roll-on frequency \( f_{RHP} \) is around 1 kHz.

The two terminals \( L_3 \) and \( H_3 \) are to be connected together externally by a wire. The earth of the signal generator must be connected to one of these terminals.

3. Apparatus Required

Signal generator, filters box and a DMM measuring AC volts to three decimal places in the 2 V range.

4. Low-Pass Filter

First, the value of the resistance \( R_L \) between \( L_1 \) and \( L_2 \) is checked and set to around 150 ohms by adjusting the pot \( P_{LP} \).
For studying the transmission characteristics of a low-pass filter, a signal generator is connected to the $L_1$ and $L_3$ terminals on the low-pass side. The frequency of the signal generator is set at 200 Hz and the rms voltage $V_{\text{input}}$ is measured between the $L_1$ and $L_3$ terminals on the LP side with a DMM set in the AC 2 V range. The output voltage $V_{\text{output}}$ of the low-pass filter is measured between $L_2$ and $L_3$ terminals on the LP side with the same voltmeter. The frequency of the signal generator is changed in steps up to 10 kHz and the ratio $V_{\text{output}}/V_{\text{input}}$ is measured at each frequency.

A sample set of data is shown in Table 1.

Table 1. Transmission coefficient of a low-pass filter. $R_L \approx 150$ ohms, $C_L = 0.22 \mu$F; Calculated roll-off frequency $f_{RLP} = 4.82$ kHz.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$\log(f)$</th>
<th>$V_{\text{input}}$ (volts)</th>
<th>$V_{\text{output}}$ (volts)</th>
<th>$T$</th>
<th>$f$ (kHz)</th>
<th>$\log(f)$</th>
<th>$V_{\text{input}}$ (volts)</th>
<th>$V_{\text{output}}$ (volts)</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-0.700</td>
<td>1.001</td>
<td>0.999</td>
<td>0.998</td>
<td>5.0</td>
<td>0.699</td>
<td>1.005</td>
<td>0.697</td>
<td>0.694</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.400</td>
<td>1.001</td>
<td>0.996</td>
<td>0.995</td>
<td>5.5</td>
<td>0.74</td>
<td>1.000</td>
<td>0.654</td>
<td>0.654</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.220</td>
<td>1.000</td>
<td>0.993</td>
<td>0.993</td>
<td>6.0</td>
<td>0.778</td>
<td>1.000</td>
<td>0.630</td>
<td>0.630</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.100</td>
<td>0.999</td>
<td>0.985</td>
<td>0.986</td>
<td>6.5</td>
<td>0.813</td>
<td>0.989</td>
<td>0.590</td>
<td>0.597</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>1.000</td>
<td>0.981</td>
<td>0.981</td>
<td>7.0</td>
<td>0.845</td>
<td>0.990</td>
<td>0.564</td>
<td>0.570</td>
</tr>
<tr>
<td>1.5</td>
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<td>0.96</td>
<td>0.958</td>
<td>7.5</td>
<td>0.875</td>
<td>0.985</td>
<td>0.530</td>
<td>0.538</td>
</tr>
<tr>
<td>2.0</td>
<td>0.301</td>
<td>1.004</td>
<td>0.92</td>
<td>0.916</td>
<td>8.0</td>
<td>0.903</td>
<td>0.987</td>
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<td>0.517</td>
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<tr>
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<td>0.888</td>
<td>0.888</td>
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<td>0.993</td>
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<td>0.85</td>
<td>0.853</td>
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<td>0.995</td>
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<td>0.773</td>
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<td></td>
</tr>
</tbody>
</table>

The transmission coefficient is plotted as a function of the logarithm of the frequency in Figure 4.

![Figure 4](image-url)

**Figure 4.** Transmission coefficient as a function of frequency for the low-pass filter.

The log of the roll-off frequency, when the transmission coefficient is 0.707, is read off from the curve, and is 0.6902. So, $f_{RLP} = 10^{0.6902} = 4.92$ kHz. This agrees well with the calculated value of 4.82 kHz.
5. High-Pass Filter

First, the resistance $R_H$ between the terminals $H_2$ and $H_3$ is measured and set to around 5 kΩ by adjusting the pot $P_{HP}$.

For studying the characteristics of the high-pass filter, the signal generator is connected between the $H_1$ and $H_3$ terminals on the high-pass side. The input voltage is measured between $H_1$ and $H_3$ terminals, while the output voltage is measured between $H_2$ and $H_3$ terminals. The procedure is the same as for the low-pass filter.

A sample set of readings are given in Table 2.

Table 2. Transmission coefficient of a high-pass filter. $R_H = 5 \text{kΩ}$; $C_H = 0.033 \ \mu\text{F}$; Calculated roll-on frequency $f_{RHP} = 0.965 \text{kHz}$.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$V_{\text{input}}$ (volts)</th>
<th>$V_{\text{output}}$ (volts)</th>
<th>$T$</th>
<th>$f$ (kHz)</th>
<th>$V_{\text{input}}$ (volts)</th>
<th>$V_{\text{output}}$ (volts)</th>
<th>$T$</th>
</tr>
</thead>
<tbody>
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<td>0.102</td>
<td>2.4</td>
<td>0.380</td>
<td>0.987</td>
<td>0.920</td>
</tr>
<tr>
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<td>-0.700</td>
<td>1.004</td>
<td>0.202</td>
<td>2.0</td>
<td>0.415</td>
<td>0.985</td>
<td>0.930</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.520</td>
<td>1.002</td>
<td>0.300</td>
<td>2.8</td>
<td>0.447</td>
<td>0.999</td>
<td>0.952</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.400</td>
<td>0.998</td>
<td>0.380</td>
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<td>0.477</td>
<td>0.984</td>
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<tr>
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<td>0.525</td>
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<td>0.602</td>
<td>1.002</td>
<td>0.975</td>
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<td>1.000</td>
<td>0.590</td>
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<td>0.653</td>
<td>1.000</td>
<td>0.980</td>
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<tr>
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<td>0.740</td>
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<td>0.984</td>
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<td>0</td>
<td>0.998</td>
<td>0.730</td>
<td>6.0</td>
<td>0.778</td>
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<td>0.984</td>
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<td>1.2</td>
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<td>1.4</td>
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<td>1.006</td>
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<td>0.845</td>
<td>0.989</td>
<td>0.985</td>
</tr>
<tr>
<td>1.6</td>
<td>0.204</td>
<td>1.002</td>
<td>0.860</td>
<td>8.0</td>
<td>0.903</td>
<td>0.984</td>
<td>0.980</td>
</tr>
<tr>
<td>1.8</td>
<td>0.255</td>
<td>0.995</td>
<td>0.883</td>
<td>9.0</td>
<td>0.954</td>
<td>0.986</td>
<td>0.982</td>
</tr>
<tr>
<td>2.0</td>
<td>0.301</td>
<td>0.993</td>
<td>0.900</td>
<td>10.0</td>
<td>1.000</td>
<td>0.982</td>
<td>0.981</td>
</tr>
<tr>
<td>2.2</td>
<td>0.342</td>
<td>0.991</td>
<td>0.920</td>
<td>11.0</td>
<td>1.000</td>
<td>0.982</td>
<td>0.981</td>
</tr>
</tbody>
</table>

In Figure 5, the transmission coefficient is plotted as a function of the logarithm of the frequency. The roll-on frequency of the HP filter is measured by reading off the frequency from the graph at the point at which transmission is 0.707. This occurs at a frequency $10^{(0.0148)} = 0.966 \text{kHz}$. This agrees with the calculated value.

Figure 5. Transmission coefficient as a function of frequency for a high-pass filter.
6. Band-Pass Filter

To measure the transmission characteristics of the band-pass filter, the signal generator is connected between the L₁ and L₃ terminals on the low-pass side. The terminals L₃ and H₃ are checked to see if they are shorted. If not, they should be connected together. The L₂ terminal on the low-pass side is connected to the H₁ terminal on the high-pass side. The output from the LP filter is now the input to the HP filter. \( V_{\text{input}} \) is measured between L₁ and L₃ terminals on the low-pass side, while \( V_{\text{output}} \) of the band-pass filter is measured between H₂ and H₃ terminals on the high-pass side. The rest of the procedure is the same as before. A sample set of data is given in Table 3.

<table>
<thead>
<tr>
<th>( f ) (kHz)</th>
<th>( \log(f) )</th>
<th>( V_{\text{input}} ) (volts)</th>
<th>( V_{\text{output}} ) (volts)</th>
<th>( f ) (kHz)</th>
<th>( \log(f) )</th>
<th>( V_{\text{input}} ) (volts)</th>
<th>( V_{\text{output}} ) (volts)</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>-0.700</td>
<td>1.003</td>
<td>0.205</td>
<td>0.204</td>
<td>4.0</td>
<td>0.602</td>
<td>0.994</td>
<td>0.748 0.753</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.400</td>
<td>1.005</td>
<td>0.39</td>
<td>0.388</td>
<td>4.5</td>
<td>0.653</td>
<td>0.992</td>
<td>0.705 0.711</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.220</td>
<td>1.002</td>
<td>0.524</td>
<td>0.523</td>
<td>5.0</td>
<td>0.699</td>
<td>0.996</td>
<td>0.675 0.678</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.100</td>
<td>0.998</td>
<td>0.624</td>
<td>0.625</td>
<td>5.5</td>
<td>0.740</td>
<td>1.000</td>
<td>0.643 0.643</td>
</tr>
<tr>
<td>1.0</td>
<td>0</td>
<td>0.996</td>
<td>0.714</td>
<td>0.717</td>
<td>6.0</td>
<td>0.778</td>
<td>1.002</td>
<td>0.620 0.619</td>
</tr>
<tr>
<td>1.5</td>
<td>0.176</td>
<td>1.000</td>
<td>0.810</td>
<td>0.810</td>
<td>6.5</td>
<td>0.813</td>
<td>0.995</td>
<td>0.585 0.588</td>
</tr>
<tr>
<td>2.0</td>
<td>0.301</td>
<td>1.002</td>
<td>0.835</td>
<td>0.833</td>
<td>7.0</td>
<td>0.845</td>
<td>1.001</td>
<td>0.575 0.574</td>
</tr>
<tr>
<td>2.5</td>
<td>0.398</td>
<td>1.005</td>
<td>0.840</td>
<td>0.836</td>
<td>8.0</td>
<td>0.903</td>
<td>0.993</td>
<td>0.510 0.514</td>
</tr>
<tr>
<td>3.0</td>
<td>0.477</td>
<td>1.000</td>
<td>0.820</td>
<td>0.820</td>
<td>9.0</td>
<td>0.954</td>
<td>0.995</td>
<td>0.467 0.469</td>
</tr>
<tr>
<td>3.5</td>
<td>0.544</td>
<td>0.998</td>
<td>0.782</td>
<td>0.784</td>
<td>10.0</td>
<td>1.000</td>
<td>0.986</td>
<td>0.420 0.426</td>
</tr>
</tbody>
</table>

The transmission coefficient is plotted as a function of \( \log(f) \) in Figure 6. Maximum transmission occurs at a frequency \( f_m = 10^{0.384} = 2.42 \) kHz. The calculated value of \( (f_{\text{RLP}}/f_{\text{RHP}})^{0.5} \) is 2.18 kHz. This is in rough agreement with \( f_m \).

Figure 6. Transmission coefficient of a band-pass filter as a function of frequency.
7. Questions

1. To reduce the roll-off frequency of a low-pass filter, should you increase or decrease the resistance keeping the capacitance fixed?

2. To decrease the roll-on frequency of a high-pass filter, should you increase or decrease the resistance keeping the capacitance fixed?

3. If the roll-on frequency of the high-pass section of a band-pass filter is more than the roll-off frequency of the low-pass section, sketch the transmission coefficient as a function of frequency. What would you call such a filter?

6.6 Introduction to AC Wheatstone Bridge

1. Introduction

An AC Wheatstone bridge has impedances $Z_1$ to $Z_4$ connected in the four arms of the Wheatstone bridge, as shown in Figure 1. An AC signal is applied to the points A and C of the bridge. The AC voltage is measured between the pair of points B and D.

![AC Wheatstone Bridge](image)

Figure 1. AC Wheatstone Bridge.

The condition that the bridge is balanced (i.e., no AC voltage appears at the detector DET) is

$$
\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4},
$$

(1)

Impedance $Z$ is a complex number

$$
Z = |Z| \exp(j \phi).
$$

(2)

Two conditions have to be satisfied simultaneously for perfect balance, namely,

$$
\frac{|Z_1|}{|Z_2|} = \frac{|Z_3|}{|Z_4|}
$$

(3)

and

$$
\phi_1 - \phi_2 = \phi_3 - \phi_4.
$$

(4)

Since $Z$ depends on the frequency, the balance will be frequency dependent.

On the other hand, in a DC Wheatstone bridge, all impedances are resistances and only one condition needs to be satisfied, namely,

$$
\frac{R_1}{R_2} = \frac{R_3}{R_4}
$$

(5)

The balance in AC bridges, to be described in the following sections, will generally not be perfect, and we look for a minimum, and not zero, in the detector voltage.

We consider the following three idealized bridges in which the frequency drops out of the balancing condition, and the balance will be independent of the frequency.
If we change one of the impedances away from the balancing condition, a voltage will appear across the detector. The sensitivity of the bridge is the amount of change in the impedance needed to produce a given small amount of unbalanced voltage. The sensitivity will be a maximum (i.e. change in impedance will be a minimum), if the magnitudes of the impedances in the four arms are nearly equal. However, one will not be able to choose impedances to satisfy this condition. But there will be a narrow frequency range in which the disparity of the impedances in the four arms will be a minimum. In this frequency range, the bridge will be most sensitive.

In the idealized Maxwell bridge, we take $Z_1$ and $Z_3$ to be ideal inductances (resistance $R_L = 0$), and $Z_2$ and $Z_4$ to be resistances. In the idealized De Sauty bridge, we take $Z_1$ and $Z_3$ to be ideal capacitances (with infinite leakage resistance), and $Z_2$ and $Z_4$ to be resistances. In the Maxwell–Wien bridge, one of the two impedances, $Z_1$ and $Z_4$, is an ideal inductance and the other is an ideal capacitance. $Z_2$ and $Z_3$ are resistances.

With the low cost AC bridge described in the next section (Section 6.7), all the three bridges can be realized quite easily by making external connections between the banana sockets. The detector can be a DMM in the AC mV range connected to the terminals marked $D_1$, $D_2$. The impedance $Z_2$ is a pot, the resistance of which can be varied to achieve balance.

If one uses a multimeter in the AC 200 mV range, the signal generator can be set to an amplitude of 0.5 to 1 V.

### 6.7 AC Bridges Circuit

![AC bridge circuit](image)

**Figure 1.** AC bridge circuit.

#### 1. Introduction

There are three AC bridge circuits which can be realized with the instrument in Figure 1. These are the Maxwell bridge, the De Sauty bridge and the Maxwell–Wien bridge.

An AC bridge is a Wheatstone bridge with impedances connected in the four arms as shown in Figure 2. The figure is drawn to accord with the symbols on the AC Bridge circuit. $SG_{in}$ and $SG_{out}$ are the two terminals to which the signal generator is to be connected. The terminals marked A and B are shorted with $SG_{in}$, while the terminal C is shorted with $SG_{out}$. Between A and $D_1$ is the impedance $Z_1$. Between $D_1$ and $D_2$ is the DMM in the AC 200 mV range. In the front panel of the bridge, $D_1$ and $D_2$ are connected to two terminals marked DMM. The DMM is to be connected between these two terminals. One can connect any impedance between A and $D_1$. For example, one terminal of a coil is connected to $D_1$ internally and the other end of the coil is connected to a free terminal marked $L_1$. If the terminal A is connected to the terminal marked $L_1$, the coil takes the place of $Z_1$ in Figure 2. In this fashion, by connecting A to the terminals marked $L_1$, $L_1'$, $L_1''$, three different inductances can be connected as $Z_1$. Similarly by connecting A to the terminals marked $C_1$, $C_1'$ or $C_1''$, one may connect three different capacitances as $Z_1$. In the third arm of the bridge circuit, one end of the impedance $Z_3$ is connected internally to $D_2$. By connecting the free end of $Z_3$ to the terminal marked B, one may connect the impedance $Z_3$ in the third arm of the bridge. In the third arm, we may connect inductances $L_3$ or $L_3'$, capacitances $C_3$ or $C_3'$ and resistances $R_3$ or $R_3'$. Similarly by connecting the terminal marked C to $L_4$, $L_4'$, $C_4$, $C_4'$ or $R_4$, $R_4'$, one may connect an inductance, capacitance or resistance...
AC Experiments

as the impedance $Z_4$. The arm $Z_2$ is a variable resistor (pot). This pot is connected between $D_1$ and $C$ when the switches SW$_1$ and SW$_2$ are pressed down. When they are up, the pot is isolated and its resistance can be measured by connecting a DMM between the terminals marked $T_1$ and $T_2$.

External terminals are also provided, marked EXT1, EXT2, EXT3 and EXT4, for connecting external inductances, capacitances or resistances in place of the built-in coils, capacitances and resistances. In this case the terminals $A$, $B$ and $C$ are NOT connected to any other terminals and the switches SW$_1$ and SW$_2$ are in the up position. At the terminals marked EXT2 one connects a dial resistance box and balance is obtained by adjusting the dials. In this way one may use impedances of one’s choice.

Figure 2. Wheatstone bridge drawn to accord with the nomenclature on the AC Bridge circuit.

Three AC bridges can be realized.
(i) By connecting $A$ to $L_1$ ($L_1'$ or $L_1''$), $B$ to $L_3$ (or $L_3'$) and $C$ to $R_4$ (or $R_4'$) one realizes the Maxwell bridge.
(ii) By connecting $A$ to $C_1$ ($C_1'$ or $C_1''$), $B$ to $C_3$ (or $C_3'$) and $C$ to $R_4$ (or $R_4'$) one realizes the De Sauty bridge.
(iii) By connecting $A$ to $L_1$ ($L_1'$ or $L_1''$), $B$ to $R_3$ (or $R_3'$) and $C$ to $C_4$ (or $C_4'$) one realizes the Maxwell–Wien bridge.

Figure 3. Circuit diagram of AC Bridges circuit.

The circuit diagram of the AC Bridges circuit is shown in Figure 3.
2. Troubleshooting

If the bridge does not work, check the following using a DMM in the kilo-ohms range.

1. Check that SG\textsubscript{in}, A and B are connected together.
2. Check that SG\textsubscript{out} and C are connected together.
3. Between D\textsubscript{1} and L\textsubscript{1}, L\textsubscript{1}' and L\textsubscript{1}" one must obtain a resistance of approximately 35, 70, and 105 ohms. Between D\textsubscript{1} and T\textsubscript{1}, resistance should be zero when the switch SW\textsubscript{1} is pressed down.
4. Between D\textsubscript{1} and C, the resistance must vary from zero to 2 kilo-ohms when switches SW\textsubscript{1} and SW\textsubscript{2} are pressed down and the knob marked R\textsubscript{2} is turned.
5. Between D\textsubscript{1} and the black terminal of EXT\textsubscript{1} and between D\textsubscript{1} and the red terminal of EXT\textsubscript{2}, the resistance must be zero.
6. Between D\textsubscript{1} and the top terminal marked DMM, resistance must be zero.
7. Between D\textsubscript{2} and L\textsubscript{3} (L\textsubscript{3}'), the resistance must be approximately 35 (70) ohms.
8. Between D\textsubscript{2} and R\textsubscript{3} (R\textsubscript{3}'), the resistance should be 220 (320) ohms.
9. Between D\textsubscript{2} and the black terminal of EXT\textsubscript{3}, resistance must be zero.
10. Between D\textsubscript{2} and the terminals marked L\textsubscript{4} (L\textsubscript{4}'), resistance must be approximately 35 (70) ohms.
11. Between D\textsubscript{2} and terminals marked R\textsubscript{4} (R\textsubscript{4}'), the resistance must be 220 (440) ohms.
12. Between D\textsubscript{2} and the black terminal of EXT\textsubscript{4}, resistance must be zero.
13. Between D\textsubscript{2} and the lower terminal marked DMM, resistance must be zero.
14. Between terminal A and the red terminal marked EXT\textsubscript{1}, resistance must be zero.
15. Between black terminal of EXT\textsubscript{2} and terminal C, resistance must be zero.
16. Between terminal B and the red terminal of EXT\textsubscript{3}, resistance must be zero.
17. Between terminal C and the red terminal of EXT\textsubscript{4}, resistance must be zero.

This troubleshooting will help you to identify where the problem lies. DO NOT OPEN AND TRY TO RECTIFY THE ERROR. Inform the manufacturer who will rectify the fault.

Often the trouble may arise because the wires are not properly connected to the small banana terminal. First check whether your wires make proper contact with the terminals before you decide to check the AC Bridge circuit.

6.8 Maxwell, De Sauty, and Maxwell–Wien Bridges

1. Introduction

There are three simple AC bridges in which the balancing condition is independent of the frequency. These are the Maxwell, De Sauty and Maxwell–Wien bridges. All the three bridges can be done in about two to three hours, using the AC bridge circuit described in the previous section.

2. Apparatus Required

Signal generator, AC bridge circuit, a DMM to measure AC volts to three decimal places and a DMM to measure resistance up to 2 kilo-ohms.
3. Maxwell Bridge

In this bridge, one compares the self-inductances of two coils \( L_1 \) and \( L_3 \). These coils are connected as shown in Figure 1.

![Figure 1. Maxwell bridge.](image)

The balancing condition is

\[
\frac{\omega L_1}{\omega L_3} = \frac{L_1}{L_3} = \frac{R_2}{R_4},
\]

assuming the inductors to have negligible resistance. If one works at a high frequency, one may neglect the resistance of the coil in comparison with its inductive impedance \( \omega L \). We see that the balancing condition is independent of the frequency.

**Experimental Procedure**

This bridge can be realized by using the AC bridge circuit. Connect the signal generator between the terminals marked SG\(_{\text{in}}\) and SG\(_{\text{out}}\) on the bridge. Adjust the frequency to be about 5 kHz and set an output voltage between 0.5 to 1 V rms. Connect the terminal A to terminal L\(_1\), terminal B to terminal L\(_3\), and terminal C to terminal R\(_4\) on the box. Put the switches S\(_1\) and S\(_2\) on the box down. Connect a multimeter in the AC millivolts range to the terminals marked DMM on the box. Adjust the potentiometer R\(_2\) till the reading on the multimeter is a minimum. Then, put S\(_1\) and S\(_2\) up, and measure the resistance R\(_2\) between the terminals T\(_1\) and T\(_2\) with a different multimeter.

Then, connect terminal C to the terminal marked R\(_4\)' and repeat the operations. Then, connect terminal B to terminal L\(_1\)' and repeat the observations, connecting C to R\(_4\) and R\(_4\)' successively. Then, connect A successively to L\(_1\)', L\(_1\)' and repeat the observations.

While balancing the bridge, the multimeter to measure resistance R\(_2\) should not be connected to the terminals T\(_1\) and T\(_2\).

A sample set of readings are given in Table 1.

<table>
<thead>
<tr>
<th>Connect A to</th>
<th>Connect B to</th>
<th>Connect C to</th>
<th>Value of R(_2) (Ω)</th>
<th>Value of</th>
</tr>
</thead>
<tbody>
<tr>
<td>L(_1)</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>249</td>
<td>L(_1) = 3.6 mH</td>
</tr>
<tr>
<td>L(_1)</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>521</td>
<td>L(_1) = 3.8 mH</td>
</tr>
<tr>
<td>L(_1)'</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>107</td>
<td>L(_1) = 3.6 mH</td>
</tr>
<tr>
<td>L(_1)'</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>211</td>
<td>L(_1) = 3.5 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>443</td>
<td>L(_1)' average = 6.7 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>886</td>
<td>L(_1)' = 6.4 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>210</td>
<td>L(_1)' = 7.1 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>409</td>
<td>L(_1)' = 6.9 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>626</td>
<td>L(_1)&quot; average = 9.3 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>1345</td>
<td>L(_1)&quot; = 9.6 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>266</td>
<td>L(_1)&quot; = 8.9 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4)' = 440 Ω</td>
<td>557</td>
<td>L(_1)&quot; = 9.4 mH</td>
</tr>
<tr>
<td>L(_1)&quot;</td>
<td>L(_3) = 3.3 mH</td>
<td>R(_4) = 220 Ω</td>
<td>626</td>
<td>L(_1)&quot; average = 9.3 mH</td>
</tr>
</tbody>
</table>
AC Experiments

Note: The coils sold in the market as having 3.6 mH inductance do not all have the same inductance. The inductances vary by as much as 20%. Therefore the inductance in your box may have a different value. Please take the above readings as sample readings only.

4. De Sauty Bridge

In this bridge, one compares the capacitances of two condensers $C_1$ and $C_3$. These condensers are connected as shown in Figure 2:

![Figure 2. De Sauty bridge.](image)

The balancing condition is

$$\left(\frac{1}{\omega C_1}\right) \left(\frac{1}{\omega C_3}\right) = \frac{C_3}{C_1} = \frac{R_2}{R_4}$$  \hspace{1cm} (2)

**Experimental Procedure**

Connect the signal generator between the terminals marked SG$_{in}$ and SG$_{out}$ on the bridge. Adjust the frequency to be about 5 kHz and the amplitude between 0.5 V and 1 V. Connect the terminal A to terminal marked $C_1$, terminal B to terminal marked $C_3$ and terminal C to terminal marked $R_4$. Put the switches $S_1$ and $S_2$ down. Connect a multimeter in the AC 200 millivolts range to the terminals marked DMM in the box. Adjust the potentiometer $R_2$ till the reading on the multimeter is a minimum. Put $S_1$ and $S_2$ up, and measure the resistance $R_2$ between the terminals $T_1$ and $T_2$ with another multimeter.

Now, connect terminal C to terminal marked $R_4'$, and repeat the operations. Then, connect terminal marked B to terminal marked $C_3'$, and repeat the operations. Then, connect A successively to terminals $C_1$, $C_1'$ and $C_1''$ and repeat the operations. A sample set of readings is given in Table 2.

<table>
<thead>
<tr>
<th>Connect A to</th>
<th>Connect B to</th>
<th>Connect C to</th>
<th>Value of $R_2$(Ω)</th>
<th>Value of $C_1$</th>
<th>Value of $C_1'$</th>
<th>Value of $C_1''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>$C_3 = 0.1 \mu F$</td>
<td>$R_4 = 220 \Omega$</td>
<td>102</td>
<td>$C_1' = 0.103 \mu F$</td>
<td>$C_1'' = 0.210 \mu F$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_3 = 0.1 \mu F$</td>
<td>$R_4' = 440 \Omega$</td>
<td>206</td>
<td>$C_1' = 0.220 \mu F$</td>
<td>$C_1' = 0.220 \mu F$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_3' = 0.2 \mu F$</td>
<td>$R_4 = 220 \Omega$</td>
<td>196</td>
<td>$C_1' = 0.220 \mu F$</td>
<td>$C_1' = 0.220 \mu F$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$C_3' = 0.2 \mu F$</td>
<td>$R_4' = 440 \Omega$</td>
<td>396</td>
<td>$C_1' = 0.218 \mu F$</td>
<td>$C_1'' = 0.215 \mu F$</td>
<td></td>
</tr>
</tbody>
</table>

$C_1$ average = 0.102 $\mu F$

| $C_1'$    | $C_3 = 0.1 \mu F$ | $R_4 = 220 \Omega$ | 102              | $C_1' = 0.210 \mu F$ | $C_1'' = 0.441 \mu F$ |
|           | $C_3 = 0.1 \mu F$ | $R_4' = 440 \Omega$  | 206              | $C_1' = 0.440 \mu F$ | $C_1'' = 0.440 \mu F$ |
|           | $C_3' = 0.2 \mu F$ | $R_4 = 220 \Omega$  | 95               | $C_1' = 0.455 \mu F$ | $C_1'' = 0.450 \mu F$ |
|           | $C_3' = 0.2 \mu F$ | $R_4' = 440 \Omega$ | 192              | $C_1' = 0.450 \mu F$ | $C_1'' = 0.446 \mu F$ |

$C_1'$ average = 0.215 $\mu F$

$C_1''$ average = 0.446 $\mu F$
5. Maxwell–Wien Bridge

In this bridge, one compares the self-inductance of a coil $L_1$ with the capacitance of a condenser $C_4$. The coil and condenser are connected as shown in Figure 3.

The balancing condition for an ideal inductor and capacitor is

\[
\frac{j\omega L_1}{R_3} = \frac{R_2}{1/j\omega C_4}
\]

or

\[
L_1 = C_4 R_2 R_3.
\]  
(3)

The balancing condition is again independent of frequency.

Experimental Procedure

Connect the signal generator between the terminals marked SG_in and SG_out on the bridge. Adjust the frequency to be about 5 kHz and the amplitude between 0.5 and 1 volt. Connect the terminal A to terminal marked L_1, terminal B to terminal marked R_3 and terminal C to terminal marked C_4 on the box. Put the switches S_1 and S_2 down. Connect a multimeter in the AC 200 millivolts range to the terminals DMM on the bridge. Adjust the potentiometer $R_2$ till the reading on the multimeter is a minimum. Put $S_1$ and $S_2$ up, and measure the resistance $R_2$ between the terminals $T_1$ and $T_2$. Now connect B to terminal marked $R_3'$ and repeat the operations. Then connect A successively to terminals marked $L_1'$ and $L_1''$ and repeat the

<table>
<thead>
<tr>
<th>Connect A to</th>
<th>Connect B to</th>
<th>Connect C to</th>
<th>Value of $R_2$($\Omega$)</th>
<th>Value of $C_4$($\mu$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4$</td>
<td>220</td>
<td>$C_4 = 0.047 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4'$</td>
<td>346</td>
<td>$C_4' = 0.047 \mu$F</td>
</tr>
<tr>
<td>$L_1'$</td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4$</td>
<td>668</td>
<td>$C_4 = 0.046 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4'$</td>
<td>447</td>
<td>$C_4' = 0.047 \mu$F</td>
</tr>
<tr>
<td>$L_1''$</td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4$</td>
<td>916</td>
<td>$C_4 = 0.046 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4'$</td>
<td>601</td>
<td>$C_4' = 0.048 \mu$F</td>
</tr>
</tbody>
</table>

$C_4$ average $= 0.049 \mu$F

<table>
<thead>
<tr>
<th>Connect A to</th>
<th>Connect B to</th>
<th>Connect C to</th>
<th>Value of $R_2$($\Omega$)</th>
<th>Value of $C_4$($\mu$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>$R_3 = 220 \Omega$</td>
<td>$C_4'$</td>
<td>166</td>
<td>$C_4' = 0.098 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4''$</td>
<td>110</td>
<td>$C_4'' = 0.102 \mu$F</td>
</tr>
<tr>
<td>$L_1'$</td>
<td>$R_3 = 220 \Omega$</td>
<td>$C_4'$</td>
<td>314</td>
<td>$C_4' = 0.097 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4''$</td>
<td>209</td>
<td>$C_4'' = 0.100 \mu$F</td>
</tr>
<tr>
<td>$L_1''$</td>
<td>$R_3 = 220 \Omega$</td>
<td>$C_4'$</td>
<td>458</td>
<td>$C_4' = 0.093 \mu$F</td>
</tr>
<tr>
<td></td>
<td>$R_3' = 320 \Omega$</td>
<td>$C_4''$</td>
<td>267</td>
<td>$C_4'' = 0.108 \mu$F</td>
</tr>
</tbody>
</table>

$C_4'$ average $= 0.100 \mu$F
operations. Then connect C to terminal $C_4'$ and repeat the operations. A sample set of readings is given in Table 3.

So $C_4$ is 0.048 microfarad and $C_4'$ is 0.100 microfarad.

6. Questions

1. If the frequency of the AC signal is changed, will the balance of the above bridges be destroyed?

2. Is the sensitivity of the bridge dependent on frequency? When do you think the sensitivity of the bridge will be a maximum?

3. If an inductance is connected instead of a capacitance in the fourth arm of the Maxwell–Wien Bridge, can the bridge be balanced? Justify your answer.

4. Can a steady DC signal be used to balance any of the bridges discussed above?
Chapter 7

Experiments on Dielectric Properties
7.1 Capacitance Measurement Circuit

1. Introduction

The capacitance measurement circuit can be used to measure capacitances up to 250 pF. It gives a DC output in volts proportional to the value of the capacitance. It is useful for measuring the dielectric constants of non-polar liquids with a cylindrical capacitor, and the dipole moment of a polar molecule like acetone.

2. Description of the Instrument

At the lower left corner of the front panel (Figure 1) is the mains switch. When it is pressed down the instrument is activated. On the lower right corner are two banana terminals marked C. The unknown capacitance is connected to these two terminals. In the middle of the front panel are three preset pots marked R₁, R₁' and R₂. Their values are preset in the factory. But one may change the values if one wishes by turning the screws. For a given value of the unknown capacitance, the DC output will vary as these screws are turned. At the top right are two banana terminals. A DMM in the appropriate DC range is connected to these terminals to measure the output.

![Capacitance circuit](image)

Figure 1. Capacitance circuit.

3. Principle of Capacitance Measurement

Inside the instrument there is a capacitance of 2 nF (i.e., 2000 pF). This capacitance is connected in series with a resistance \( R_1 \) that is in the 100 kΩ to 500 kΩ range. The unknown capacitor C is connected to another high resistance \( R_2 \) in the same range. When a DC voltage of 12 V is applied, both capacitors get charged. But the time constants of the two circuits will depend on the products \( C_{\text{std}}R_1 \) and \( C_{\text{unknown}}R_2 \). Since \( C_{\text{unknown}} \) is less than ten times the value of \( C_{\text{std}} \) and \( R_1 \) and \( R_2 \) are nearly of the same order, the time constant \( \tau_{\text{unknown}} \) is less than the time constant \( \tau_{\text{std}} \). So the unknown capacitance charges faster than the standard capacitance. This is shown in Figure 2.
Experiments on Dielectric Properties

![Figure 2](image_url)

**Figure 2.** Unknown capacitor charges faster than standard capacitor. The unknown capacitor reaches 9 V in time $T'$ seconds (AB), while the standard capacitor reaches 9 V in time $T$ seconds (AC). $T > T'$.

While both the capacitors are charging, a fixed current passes through a transistor, and produces a voltage of 10 volts across a 1 kΩ resistor. When the unknown capacitor reaches a voltage of 9 V, a Schmitt trigger circuit switches off the current through the transistor. So current through the transistor flows for a time $T'$, which is proportional to $r_{\text{unknown}}$, i.e., to $C_{\text{unknown}} R_2$. The standard capacitor $C_{\text{std}}$ reaches 9 V at a time $T$ which is longer than $T'$, since $r_{\text{std}}$ is larger than $r_{\text{unknown}}$. The Schmitt triggers discharge both the capacitors when the standard condenser reaches 9 V after a time $T$. The charging process begins again. The voltage across the 1 kΩ resistor appears as a pulse of width $T'$, as shown in Figure 3. This pulse is repeated every $T$ seconds.

![Figure 3](image_url)

**Figure 3.** Voltage pulse across 1 KΩ resistor appears for a time $T'$ in every $T$ seconds.

Thus in 1 second the charge–discharge process takes place at a frequency $1/T$ which is in the range of a few kilohertz. The DMM reads the voltage across the 1 kΩ resistor, averaged over many charge–discharge periods. This average voltage will be $10 T'/T$. $T'$ is proportional to $r_{\text{unknown}}$, i.e., to $C_{\text{unknown}}$ since $R_2$ is fixed. So the output DC voltage is proportional to the unknown capacitance. This is the principle on which the instrument works.

### 4. Capacitance Box

A box is provided with the instrument. This contains three capacitors of 100 pF, 47 pF and 22 pF connected at one end to a common terminal. These are nominal values. When the 100 pF capacitor is connected to the terminals marked C, the DC voltage at the output terminals O/P is about 2 to 2.5 V. The linearity of the readings can be checked by connecting the 47 pF and 22 pF capacitors.

It is recommended that the settings of the resistors $R_1$, $R_1'$ and $R_2$ are NOT altered. They have been set to give a convenient output, which is linear in the capacitance, and which is suitable for measuring the
Experiments on Dielectric Properties

dielectric constant of benzene and dipole moment of acetone with the cylindrical capacitor provided with the instrument.

5. Caution

Do not use the Instrument for Measuring Dielectric Constant of Polar Liquids like Acetone or Water Directly.

7.2 Verification of the Law of Addition of Capacitances

1. Introduction

In a small box three capacitances with nominal values of 100 pF, 47 pF and 22 pF are soldered, respectively, between a red, yellow, and black terminal and a common (green) terminal (Figure 1). Thus, between green and black terminals one has 22 pF, between green and yellow 47 pF, and between green and red 100 pF.

![Figure 1. Capacitance box.](image)

Between black and yellow terminals, the 22 pF and 47 pF are in series. Between black and red, the 22 pF and 100 pF are in series, and between yellow and red, the 47 pF and 100 pF are in series.

If black and yellow terminals are externally connected by a wire, and we measure between green and black (or yellow) terminals, we will have 22 pF in parallel with 47 pF. If red and black are externally connected, and we measure between green and black (or red) terminals, we will have 22 pF and 100 pF in parallel. When yellow and red terminals are externally connected, and we measure between green and yellow (or red) terminals we have 47 pF, in parallel with 100 pF.

2. Apparatus Required

Capacitance circuit described in Section 7.1, capacitance box, and a DMM measuring DC 2 V to three decimal places.

3. Procedure

The principle of working of the capacitance circuit was discussed in Section 7.1. It is sufficient to know that the DC voltage measured at the output terminals of the capacitance circuit is proportional to the capacitance connected to the terminals marked C on the capacitance circuit box.

Connect the terminals marked C on the capacitance circuit described in Section 7.1 to the green and red terminals on the capacitance box shown in Figure 1. Switch on the capacitance circuit, and note the reading on a DMM, in the DC 20 V range, connected to the output terminals of the capacitance circuit. Then, change the connection from the red terminal to the yellow and black terminals on the capacitance box in succession, and note the readings. These three readings will correspond to the nominal values 100, 47 and 22 pF, of the capacitances respectively.

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To measure the capacitances in series, connect the terminals red and yellow on the box to the capacitance circuit and measure the DC multimeter reading. This corresponds to 100 pF in series with 47 pF. Then, change the connections on the box to the pair red and black, and then yellow and black, and take the readings. This measures the capacitance of the series combination of 100 pF and 22 pF, and the series combination of 47 pF and 22 pF, respectively.

Short the yellow and red terminals on the box with a wire. Connect the green and red terminals on the box to the terminals marked C on the capacitance circuit. The DMM reading now corresponds to the parallel combination of 100 and 47 pF capacitances. Short red and black terminals on the box, and connect the red and green terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 100 and 22 pF capacitances. Then short the yellow and black terminals on the box, and connect the green and black terminals to the terminals marked C on the capacitance circuit. This measures the capacitance of the parallel combination of 47 and 22 pF capacitances.

Taking the 100 pF capacitance as standard, and assuming that the output voltage is proportional to the capacitance, calculate the values of the capacitances for the various combinations. This is shown in Table 1.

The 100 pF capacitor gives a DC voltage of 2.2 V. For any other DC voltage \( V \) (in volts) in the table, the corresponding capacitance (in pF) is calculated as

\[
C = \left( \frac{V}{2.2} \right) * 100.
\]  

These are the values given in the column titled Calc. Cap (pF). In the column titled Nominal Value (pF), the value marked on the capacitance is given in rows 1, 2 and 3. The actual value can differ from the nominal value by 10% for these cheap capacitors.

In the fourth, fifth and sixth rows, the actual and nominal values of the capacitances in series are given. The actual value in column 3 is calculated from the measured voltage using equation (1). The nominal value in column 4 is calculated, taking the nominal values of the individual capacitances in the corresponding rows of the table, and using the formula for the series addition of the capacitors, namely,

\[
\frac{1}{C} = \left( \frac{1}{C_1} \right) + \left( \frac{1}{C_2} \right).
\]

Thus, the actual value in row 4 under the last column is calculated from the above equation, taking the nominal values of \( C_1 \) (22 pF) and \( C_2 \) (47 pF), since these two capacitors are in series, when the connection is between black and yellow. The nominal value is 15 pF, which agrees with the value of 14.5 pF, calculated from the DC voltage of 0.31 V using equation (1). Thus, the agreement between the nominal values in rows 4, 5 and 6 with the calculated values, verifies the law of addition of capacitances in series.

In rows 7, 8 and 9, the two capacitances are in parallel. If two capacitances \( C_1 \) and \( C_2 \) are in parallel, the total capacitance \( C \) is

\[
C = C_1 + C_2.
\]

This law is verified by comparing the nominal values in rows 7, 8 and 9 with the values calculated from the measured DC voltages.

<table>
<thead>
<tr>
<th>Connection</th>
<th>DC Voltage (volts)</th>
<th>Calc Cap (pF)</th>
<th>Nominal Value (pF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green &amp; black</td>
<td>0.45</td>
<td>20.5</td>
<td>22</td>
</tr>
<tr>
<td>Green &amp; Yellow</td>
<td>1.1</td>
<td>50</td>
<td>47</td>
</tr>
<tr>
<td>Green &amp; red</td>
<td>2.2</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Black &amp; Yellow</td>
<td>0.31</td>
<td>14.5</td>
<td>15</td>
</tr>
<tr>
<td>Black &amp; Red</td>
<td>0.35</td>
<td>17</td>
<td>18</td>
</tr>
<tr>
<td>Yellow &amp; red</td>
<td>0.7</td>
<td>33.3</td>
<td>32</td>
</tr>
<tr>
<td>Black with Yellow short</td>
<td>1.69</td>
<td>76.8</td>
<td>69</td>
</tr>
<tr>
<td>Between black &amp; Green</td>
<td>2.83</td>
<td>128.6</td>
<td>122</td>
</tr>
<tr>
<td>Red with black short</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Between yellow &amp; green</td>
<td>3.45</td>
<td>156.8</td>
<td>147</td>
</tr>
</tbody>
</table>
7.3 Dielectric Constant of Non-Polar Liquid and Dipole Moment of Acetone

1. Introduction

A neutral molecule will have positive and negative charge distributions, which may not be identical. If the center of charge (CC) of the negative charge distribution in the molecule is displaced from the CC of the positive charge distribution, the molecule is said to have an electric dipole moment. In a highly symmetric molecule, like carbon tetrachloride CC\(_4\) and benzene C\(_6\)H\(_6\), the CC of the positive and negative charges will coincide (for example, in the benzene molecule at the point marked x in Figure 1), and the molecule has no electric dipole moment. In asymmetric molecules like CHCl\(_3\) and acetone, there is a permanent electric dipole moment per molecule.

Figure 1. Structure of the benzene molecule. C is carbon atom and H is hydrogen atom. x marks the center of the molecule.

When an electric field is applied, there will be a shift of the CC of the negative charge relative to the CC of the positive charge. This leads to a dipole moment \( p_{\text{ind}} \), induced in the direction of the field. This leads to the relation

\[ p_{\text{ind}} = \varepsilon_0 \alpha E_{\text{local}}. \]  

(1)

Here, \( \varepsilon_0 \) is the electric permittivity of free space, and has the value \( 8.854 \times 10^{-12} \) farad/m, \( E_{\text{local}} \) is the local electric field at the site of the molecule, and \( \alpha \) is called the electronic polarizability of the molecule. The local electric field is the sum of the applied electric field \( E \) and the electric field arising from the induced dipoles.

In a gas, the density of the molecules is very low. So, the field due to the induced dipoles can be neglected in comparison to the applied field \( E \).

If there are \( N \) molecules of the gas per unit volume, the electric polarization \( P \), which is the dipole moment per unit volume, is

\[ P = Np_{\text{ind}} = \varepsilon_0 N \alpha E. \]  

(2)

The electrical displacement \( D \) is

\[ D = \varepsilon_0 E + P = \varepsilon_0 \varepsilon_r E, \]  

(3)

where \( \varepsilon_r \) is the relative dielectric constant of the material. This is a dimensionless number. So, for a gas

\[ \varepsilon_r = 1 + N\alpha. \]  

(4)

In a liquid, the density is high (several hundred times that of a gas) and the local electric field is the sum of the applied electric field \( E \) and the electric field due to the polarization \( P \) in the medium.

\[ E_{\text{local}} = \frac{E + P}{3 \varepsilon_0}. \]  

(5)

The polarization \( P \) is itself given by

\[ P = \varepsilon_0 N \alpha E_{\text{local}} = \varepsilon_0 N \alpha \left( \frac{E + P}{3 \varepsilon_0} \right). \]  

(6)

Solving for \( P \) in terms of \( E \) we have

\[ P = \frac{\varepsilon_0 N \alpha E}{(1 - N\alpha/3)}. \]  

(7)
Experiments on Dielectric Properties

and
\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} \left[ 1 + \left( \frac{N\alpha}{1 - N\alpha/3} \right) \right] = \varepsilon_0 \varepsilon_r \mathbf{E} \, . \] (8)

From this we get
\[ \frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha}{3} \, . \] (9)

This is called the Clausius–Mossotti relation.

Note that one can obtain the value of the polarizability \( \alpha \) of the molecule from a measurement of the dielectric constant, and knowledge of the density of the liquid. The polarizability has the dimension of volume, and can be written in terms of (nanometer)\(^3\).

2. Molecules with Permanent Dipole Moment: Orientation Contribution to Polarizability

Consider a molecule that has a permanent dipole moment \( \mathbf{p} \), even in the absence of an applied electric field. The moments on different molecules in the material are oriented at random in the absence of the field. This random orientation arises due to thermal agitation. When an electric field is applied, it tries to align the moments in the direction of the field, and this is opposed by thermal energy.

The energy of a dipole in an electric field is
\[ -\mathbf{p} \cdot \mathbf{E}_{\text{local}} = -pE_{\text{local}} \cos(\theta) \, , \] (10)

where \( \theta \) is the angle between \( \mathbf{p} \) and \( \mathbf{E}_{\text{local}} \). \( \mathbf{E}_{\text{local}} \) is the local electric field at the dipole, and is the sum of the applied electric field \( \mathbf{E} \) and the field due to the electric dipoles.

The probability that a dipole is oriented at an angle \( \theta \) to the electric field is proportional to the Boltzmann factor
\[ \exp \left[ -\frac{-pE_{\text{local}} \cos \theta}{kT} \right] \, . \]

The average dipole moment per molecule, in the direction of the local electric field, is
\[ p_{av} = \frac{2\pi}{\int_0^\pi p \cos \theta \exp \left( \frac{p E_{\text{local}} \cos \theta}{kT} \right) \sin \theta d\theta} \, . \] (11)

When \( pE_{\text{local}}/kT \ll 1 \), the above integral is nearly equal to
\[ \cong \left[ \frac{p^2}{3kT} \right] E_{\text{local}} = \varepsilon_0 \alpha_{\text{orien}} E_{\text{local}} \, . \] (12)

\( \alpha_{\text{orien}} \) is called the orientation contribution to the polarizability of the molecule, and \( \varepsilon_0 \) is the dielectric permittivity of vacuum.

The total polarizability \( \alpha \) is the sum of the electronic and orientation polarizabilities for a molecule with a permanent dipole moment \( \mathbf{p} \),
\[ \alpha = \alpha_{\text{el}} + \alpha_{\text{orien}} = \alpha_{\text{el}} + \frac{(p^2/\varepsilon_0)}{3kT} \, . \] (13)

So, the Clausius–Mossotti relation becomes
\[ \frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \frac{N\alpha}{3} = \frac{N[\alpha_{\text{el}} + (p^2/\varepsilon_0)/3kT]}{3} \, . \] (14)

where \( N \) is the number of molecules per unit volume, and \( \varepsilon_r \) is the dielectric constant of the liquid.

The orientation contribution to the polarizability often outweighs the electronic contribution. The dielectric constant is then strongly dependent on temperature, and increases as the temperature is reduced.

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The dielectric constant of any material is a function of frequency. The electronic motions are characterized by frequencies in the ultraviolet or higher. The vibration motions of the atoms in the molecule are in the infrared region. The orientation motions of molecular groups are in the far infrared and microwave region. As the frequency is varied, the dielectric constant shows a variation as shown in Figure 2.

Near the characteristic frequencies of orientation, vibration and electronic motions, the dielectric constant goes through a maximum and a minimum. The region of frequency between the maximum and the neighboring minimum (the region in which the dielectric constant decreases), is called the region of anomalous dispersion.

To calculate the dipole moment, one should work at frequencies low compared to the orientation frequency. A frequency up to a few hundred kilohertz is normally suitable, though in some materials one may see a noticeable dispersion in this region.

3. Apparatus Required

Capacitance measuring circuit described in Section 7.1, cylindrical capacitance in a jar, a DMM measuring to three decimal places in the DC 2 V range, and Analar grade benzene and acetone.

4. Experimental Details

For this measurement, a cylindrical capacitor of 15 cm length is provided. The cylindrical capacitor is suspended in a tall cylindrical graduated plastic jar (of volume 100 ml).

The cylindrical capacitor consists of two coaxial metal tubes of different radii, insulated from each other. Two leads, taken out of the tubes, are connected to banana plugs on the insulating mount on top of the jar (Figure 3). The capacitance of a cylindrical capacitor of length \( l \), radii \( r_1 \) and \( r_2 \) of the inner and outer cylinder respectively, filled with a medium of dielectric constant \( \varepsilon_r \) is

\[
C = \frac{2\pi \varepsilon_0 \varepsilon_r l}{\ln \left( \frac{r_2}{r_1} \right)}.
\]  

(15)

Here \( \ln \left( \frac{r_2}{r_1} \right) \) is the logarithm of \( \frac{r_2}{r_1} \) to the base e.

The cylindrical capacitor provided with the capacitance jar has a capacitance of about 10-30 pF when filled with air (\( \varepsilon_r \approx 1 \)). When it is filled to a height \( l' \) (<\( l \)) with a liquid of dielectric constant \( \varepsilon_r \), the capacitance will have a value

\[
C' (l') = C_{\text{air}} (l - l') + C_{\text{liq}} (l')
\]

\[
= C_{\text{air}} (l) \left[ 1 + \frac{(\varepsilon_r - 1) l'}{l} \right].
\]  

(16)

So, as \( l' \), the length (or height) to which the capacitor is filled with the liquid, is increased, the capacitance will increase linearly.
Experiments on Dielectric Properties

Figure 3. Measuring jar with a cylindrical capacitor inside.

When \( l' = l \), the capacitance is fully filled with the liquid. Then

\[
C'(l) = C_{\text{liq}}(l) = \varepsilon \varepsilon_{r} C_{\text{air}}(l).
\]

So, the dielectric constant of the liquid is

\[
\varepsilon_{r} = \frac{C_{\text{liq}}(l)}{C_{\text{air}}(l)}.
\]

The capacitance of the leads may be a sizeable fraction of the capacitance of the cylindrical capacitor, if the connecting leads are large in diameter, and are close to each other. To reduce the lead capacitance, use thin wires for connection, and separate the wires by a large distance.

Connect the black terminal on the cylindrical capacitance to the ground terminal of the pair, marked \( C \), on the capacitance circuit. Bring the wire from the other terminal of the pair \( C \) close to the red terminal of the cylindrical capacitor. Without connecting it to the red terminal, note the DC reading on the DMM (in the DC 2 V range) connected to the output of the capacitance meter. Let it be \( V_{0} \). Then, connect the lead to the red terminal, and measure the voltage. Let it be \( V_{\text{air}} \). Then, \( V_{\text{air}} - V_{0} \) is proportional to the capacitance of the cylindrical condenser \( C_{\text{air}}(l) \).

Analar grade (this means analytical grade purity) liquid can be filled in the jar through a tube provided in the cap of the capacitor. The liquid that is being transferred must be in a plastic jar with a nozzle which should go into the tube, to avoid spilling of the liquid. If the liquid spills out it may damage the plastic terminals.

Add the non-polar liquid in steps of 10 ml, and note the DC voltage on the DMM after each such addition. Please wait for a minute or two after adding the liquid, for the reading on the DMM to settle down to a steady value. Note the level of the liquid in milliliters from the graduation on the jar, after each addition of the liquid.

Note: There are holes made in the outer cylinder of the capacitance. If there are no holes on the outer cylinder, the insulation caps at the two ends of the capacitor will not allow the liquid to fill the interstitial space between the two cylinders. The holes on the outer cylinder will facilitate the entry of the liquid to fill the interstitial space.

The top hole is a few mm below the top insulation cap. Once the liquid goes above this hole the air that is trapped below the top insulation cap will be compressed and may not allow the liquid to enter the interstices. So, we take readings up to ten mm below the top of the capacitor.
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The jars purchased in the market may be of different cross sections. In some, the top of the cylindrical capacitor may be above the 100 ml mark on the jar. In the others, the top of the cylindrical capacitor may be below the 100 ml mark. In the jar with which the measurements in Table 1 were taken, the top of the cylindrical capacitor was at 90 ml. Readings of the output voltage were taken till the jar was filled up to 80 ml. Note the height of the jar, \( h \), from 0 to 100 ml. Find the height of the top of the cylindrical capacitor from the bottom of the jar. Let it be \( h' \). So, \( h' \) would correspond to \( \frac{h'}{h} \times 100 = \nu \) milliliters of liquid. This is the volume of the liquid needed to fill the capacitor. For the results in Table 1, \( \nu = 90 \) ml.

A graph is plotted between the DC voltage and the volume \( v \) in milliliters of the liquid. A straight line is fitted on the computer to the points, and the equation of the straight line is represented by

\[
V = A + Bv, \tag{19}
\]

where \( V \) is the DC voltage with the liquid for a volume of \( v \) milliliters of liquid, \( A \) is the intercept, and \( B \) is the slope. The reading, corresponding to the situation when the liquid will fill up to the top of the cylindrical capacitor, is obtained from equation (19) by putting \( v = \nu \), i.e.,

\[
V_{\text{liq}} = A + B\nu. \tag{20}
\]

\( V_{\text{liq}} \) includes the contribution \( V_0 \) of the leads. So, the voltage due to the cylindrical capacitance filled with liquid, excluding the capacitance of the leads, will be \( (V_{\text{liq}} - V_0) \).

The dielectric constant of the liquid is

\[
\varepsilon_r = \frac{C_{\text{liq}}(l)}{C_{\text{air}}(l)} = \frac{(V_{\text{liq}} - V_0)}{(V_{\text{air}} - V_0)}. \tag{21}
\]

Table 1 gives a sample set of readings when Analar grade benzene was used in the experiment.

<table>
<thead>
<tr>
<th>( v ) (ml)</th>
<th>( V ) (mV)</th>
<th>( v ) (ml)</th>
<th>( V ) (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>412</td>
<td>50</td>
<td>650</td>
</tr>
<tr>
<td>10</td>
<td>459</td>
<td>60</td>
<td>705</td>
</tr>
<tr>
<td>20</td>
<td>510</td>
<td>70</td>
<td>750</td>
</tr>
<tr>
<td>30</td>
<td>555</td>
<td>80</td>
<td>800</td>
</tr>
<tr>
<td>40</td>
<td>606</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 4 shows a plot of the readings in Table 1. The linear fit to the points is shown. The intercept is 411.2 mV and the slope is 4.85 mV/ml.

The voltage \( V_{\text{liq}} \) when the capacitor is filled with liquid is

\[
V_{\text{liq}} = 411.2 + (4.85 \times 90) = 847.7 \text{ mV} = 0.848 \text{ V}.
\]

The dielectric constant of benzene is

\[
\varepsilon_r = \frac{(V_{\text{liq}} - V_0)}{(V_{\text{air}} - V_0)} \tag{22}
\]

\[= \frac{(0.848 - 0.020)}{(0.412 - 0.020)} = 2.11.\]

For benzene the value of the dielectric constant from the literature is 2.25 at 300 K.
Experiments on Dielectric Properties

Figure 4. Plot of DC voltage against ml of benzene filled.

We now use the Clausius–Mossotti relation

\[
\frac{\epsilon_r - 1}{\epsilon_r + 2} = \frac{N\alpha}{3},
\]

where \(N\) is the number of molecules of the liquid per unit volume, and \(\alpha\) is the electronic polarizability of the molecule.

The molecular weight \(M\) of benzene (\(C_6H_6\)) is 78 gm and its density \(\rho_B\) is 0.899 g/cc. \(M\) grams of benzene contain the Avogadro number \(N_A(6.022 \times 10^{23})\) of molecules. 1cc of benzene has a mass \(m\) of 0.899 g. The number of molecules in 1 cc is

\[
N = 6.022 \times 10^{23} \times \frac{(m/M)}{\rho_B} = 6.94 \times 10^{21} \text{ molecules/cc}.
\]

In SI units, the unit of volume is 1 m\(^3\) = 10\(^6\) cc. So, the number \(N\) of molecules in 1 m\(^3\) is

\[
N = 6.94 \times 10^{21} \times 10^6 = 6.94 \times 10^{27} / \text{m}^3.
\]

Substituting for \(\epsilon_r\) and for \(N\),

\[
\alpha \text{ for benzene} = \frac{3 \times ((2.11 - 1)/(2.11 + 2))}{6.94 \times 10^{27}}
\]

\[
= 0.116 \times 10^{-27} \text{ m}^3 = 0.116 \text{ (nanometer)}^3.
\]

Note that the electronic polarizability of a molecule has the dimensions of volume and roughly gives the size of the molecule.

Note: The experimental values given in Table 1 are for Analar benzene. Benzene is carcinogenic. So it is better to use another non-carcinogenic non-polar liquid for student experiments. One may use Analar cyclohexane. The bottle must be tightly stoppered to prevent contamination by moisture.

5. Dipole Moment of Acetone: Experiment

If we mix a polar liquid \(H\) in small quantities, with a non-polar liquid \(G\), the dielectric constant of the mixture satisfies the relation
Experiments on Dielectric Properties

\[
\frac{(\varepsilon_r - 1)}{(\varepsilon_r + 2)} = \left( \frac{1}{3} \right) \left( N_G \alpha_{elG} + N_H \left( \frac{p_H^2}{3\varepsilon_0 kT} \right) \right).
\] (26)

Here, we have neglected the electronic contribution to the polarizability of the molecule H in relation to its orientation contribution. \(N_G\) and \(N_H\) are, respectively, the number of molecules of liquid G, and liquid H, per cubic meter of the mixture. By adding small quantities of liquid H to liquid G, and varying the relative concentration of G and H, one can plot the left side of equation (26) from the measured dielectric constant of the mixture as a function of the concentration \(N_H\). For low concentrations of liquid H (i.e., a few percent), the variation will be linear. From this plot, one can calculate the dipole moment \(p_H\) of molecule H.

One can also measure the temperature variation of the dielectric constant for a mixture with a given concentration of H in G. A plot of the dielectric constant as a function of \(1/T\) can be used to get the dipole moment \(p_H\).

Acetone is a molecule that has a permanent dipole moment. Its polarizability at low frequencies arises mainly from the orientation contribution of the dipole. To measure the dipole moment of the molecule, mix 100 milliliters of benzene with \(v_{AC}\) milliliters of acetone. For example, we can prepare three mixtures, with \(v_{AC} = 10, 15\) and 20 milliliters of acetone. These mixtures have to be stored in an airtight bottle so that no water is absorbed by the mixtures.

The dielectric constant of each mixture is measured following the procedure described with benzene in the previous subsection.

Let the \(j\)th mixture contain 100 milliliters of benzene and \(v_{jAC}\) milliliters of acetone. Let the dielectric constant of this mixture be \(\varepsilon_{mj}\). Then, we calculate the value of

\[
K_j = \left( \frac{[\varepsilon_{mj-1}]/(\varepsilon_{mj+2})] \ast (\varepsilon_{100} + v_{AC}) - [\varepsilon_B - 1)/(\varepsilon_B + 2)] \ast 100} \right)
\] (27)

for this mixture \(j\). Here, \(\varepsilon_B\) is the dielectric constant of benzene, and \(\varepsilon_{mj}\) is the dielectric constant of the \(j\)th mixture. This value \(K_j\) should be the same for all the mixtures \(j\). Take the average value \(K_{av}\).

Then,

\[
K_{av} = \left( \frac{p_{AC}^2}{9 \varepsilon_0 kT} \right) N_{AC},
\] (28)

where \(N_{AC}\) is the number of molecules per meter\(^3\) of pure acetone, \(k\) is the Boltzmann constant, and \(T\) is the absolute temperature.

Sample readings are given in Table 2.

**Table 2.** Dipole moment of acetone. DC voltage due to leads: \(V_0 = 0.020\) V. DC voltage with the capacitor in air: \(V_{air} = 0.412\) V. Three mixtures were made by mixing 10, 15 and 20 ml of acetone (AC) with 100 ml of benzene (B). The DC voltages for different fillings in ml are given.

<table>
<thead>
<tr>
<th>Milliliters</th>
<th>Mixture 1 (100) cc C(_6)H(_6) + (10) cc acetone (V = 427 + 4.32) (V)</th>
<th>Mixture 2 (100) cc C(_6)H(_6) + (15) cc acetone (V = 447 + 9.89) (V)</th>
<th>Mixture 3 (100) cc C(_6)H(_6) + (20) cc acetone (V = 444 + 13.46) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>427</td>
<td>447</td>
<td>444</td>
</tr>
<tr>
<td>10</td>
<td>512</td>
<td>547</td>
<td>580</td>
</tr>
<tr>
<td>20</td>
<td>591</td>
<td>642</td>
<td>713</td>
</tr>
<tr>
<td>30</td>
<td>680</td>
<td>742</td>
<td>850</td>
</tr>
<tr>
<td>40</td>
<td>755</td>
<td>845</td>
<td>980</td>
</tr>
<tr>
<td>50</td>
<td>843</td>
<td>945</td>
<td>1120</td>
</tr>
<tr>
<td>60</td>
<td>928</td>
<td>1040</td>
<td>1250</td>
</tr>
<tr>
<td>70</td>
<td>1010</td>
<td>1135</td>
<td>1390</td>
</tr>
<tr>
<td>80</td>
<td>1088</td>
<td>1242</td>
<td>1520</td>
</tr>
</tbody>
</table>

As in Figure 4, a graph is drawn, for each mixture, between the voltage and milliliters of liquid, and a straight line is fitted to the equation

\[
V = A + Bv.
\]
Experiments on Dielectric Properties

The fitted equations for the straight line for the three mixtures are given in the third row of the table. From these equations the dielectric constant of the mixtures are evaluated as described in subsection 4.

The dielectric constants of various mixtures and the dielectric constant of pure benzene are given in Table 3. The values of $K$ are given in the last column in the table.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>$\epsilon_r$</th>
<th>$(\epsilon_r-1)/(\epsilon_r+2)$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benzene</td>
<td>2.10</td>
<td>0.268</td>
<td></td>
</tr>
<tr>
<td>Mix1</td>
<td>2.95</td>
<td>0.394</td>
<td>1.65</td>
</tr>
<tr>
<td>Mix2</td>
<td>3.36</td>
<td>0.440</td>
<td>1.59</td>
</tr>
<tr>
<td>Mix3</td>
<td>4.17</td>
<td>0.514</td>
<td>1.74</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td>1.66</td>
</tr>
</tbody>
</table>

The average value of $K = 1.66$.

The molecular weight of acetone (C$_3$H$_6$O) is 58 g. Its density is 0.792 g/cc. So, the number of acetone molecules $N_{AC}$ per m$^3$ is

$$N_{AC} = \left( \frac{6.022 \times 10^{23} \times 0.792}{58} \right) \times 10^6 = 8.49 \times 10^{27}$/m$^3$. \hspace{1cm} (29)$$

Dipole moment of acetone molecule is

$$p = \left( \frac{K}{N_{AC}} \right)^{1/2} \left( 9\epsilon_0 kT \right)^{1/2}. \hspace{1cm} (30)$$

Here,

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m},$$
$$k = 1.36 \times 10^{-23} \text{ J/Kelvin},$$
$$T = 300 \text{ K}. $$

Substituting for the various quantities, we get

$$p = 8.0 \times 10^{-30} \text{ coulomb meter}.$$  

From the literature, the electric dipole moment of acetone molecule is 2.88 Debye (1 Debye = 3.34 $\times$ $10^{-30}$ coulomb meter), or 9.6 $\times$ $10^{-30}$ coulomb meter.

Note: The experiment is intended to show the order of magnitude of dipole moment values in molecules. The electronic charge is of the order of 10$^{-19}$ coulombs, and the relative displacement of the CC of positive charge relative to the CC of negative charge is of the order of a fraction of an angstrom, i.e., of the order of 10$^{-11}$ m. So, the dipole moment of a molecule is of the order of 10$^{-19} \times 10^{-11} = 10^{-30}$ coulomb meter. The dipole moment of Acetone in our experiment is of this magnitude. This value is in reasonable agreement with the actual value of the dipole moment of acetone.

Caution

Never use Water With the Cylindrical Capacitor

6. Questions

1. Is it possible to have a liquid for which the dielectric constant at very low frequencies is less than 1?
2. Can you think of a way of controlling the level of a liquid using its dielectric constant?
3. Derive the formula involving $p$ and $K$ (Equation (8)).
4. Water has a dielectric constant of 80. Do you expect the water molecule to have a large electric dipole moment? Does this have any implication for the structure of water molecule?

5. Does the large dielectric constant of water have any role to play in dissolving salt? Common salt is NaCl and sodium is positively charged and chlorine is negatively charged. Will the Coulomb attraction between the oppositely charged ions increase or decrease as the dielectric constant of the intervening medium is increased?

7.4 Dielectric Properties of Solids Measured with a LCR Meter

1. Introduction

The dielectric properties of a material which are important are (a) the real part of the dielectric constant, \( \varepsilon' \), (b) \( \tan(\delta) \), which is the ratio of the imaginary part, \( \varepsilon'' \), of the dielectric constant to its real part, \( \varepsilon' \), and (c) the dielectric breakdown strength. The electric displacement \( D \) in vacuum in the presence of an electric field \( E \) is

\[
D = \varepsilon_0 E. \tag{1}
\]

Here \( \varepsilon_0 \) is called the dielectric permittivity of free space and has a value \( 8.85 \times 10^{-12} \) farad/m. When the electric field is applied to matter that may be a solid, liquid or gas, the electric displacement increases to a value given by

\[
D = \varepsilon_\varepsilon_0 E = \varepsilon_0 E + P. \tag{2}
\]

Here \( \varepsilon_e \) is called the dielectric constant of the material, and \( P \) is the electric polarization. All materials contain atoms, or molecules which are bound groups of atoms. When an electric field is applied, the positive and negative charge distributions undergo displacements in opposite directions due to the applied electric field. The electric field tries to separate the distributions, and this is opposed by interactions within the atom or molecule. Ultimately the charge distributions are separated by a small distance, proportional to the electric field. The atoms or molecules develop an electric dipole moment given by the product of the magnitude of the charge \( q \) and the charge separation \( u \). This dipole moment is called the induced dipole moment and is proportional to the electric field,

\[
p = a\varepsilon_0 E_{\text{local}}. \tag{3}
\]

\( a \) is called the polarizability of the atom or molecule and has the dimension of volume. These dipoles themselves will produce an electric field which will modify the applied electric field substantially, if the density of the material is high. Then, equation (2) will have to be modified as

\[
D = \varepsilon_0 E_{\text{loc}} + P. \tag{4}
\]

The local electric field \( E_{\text{loc}} \) will differ from \( E \) by the field due to the dipoles. This difference will depend on the symmetry of distribution of neighboring dipoles. If the neighboring dipoles are distributed in an isotropic fashion, the local electric field \( E_{\text{loc}} \) is given by

\[
E_{\text{loc}} = \frac{E + P}{3\varepsilon_0}. \tag{5}
\]

This is true in the case of liquids and gases, but not in general for solids.

If we have a single molecular species in the liquid or gas, the polarization \( P \) is related to the dipole moment \( p \) of the molecule by

\[
P = Np, \tag{6}
\]

where \( N \) is the number of molecules per unit volume. Using equations (2), (4), (5) and (6) we derive a useful relation between the dielectric constant and the polarizability \( a \) given by

\[
\frac{(\varepsilon_e - 1)}{(\varepsilon_e + 2)} = \frac{N a}{3}. \tag{7}
\]
Experiments on Dielectric Properties

This is called the Clausius–Mossotti relation. Any material will convert a small fraction of the electrical energy into heat. So the dielectric constant $\varepsilon_r$ is represented as a complex number

$$\varepsilon_r = \varepsilon' + j\varepsilon''.$$ (8)

where $j = \sqrt{-1}$. If the applied electric field oscillates at a frequency $\nu$, then some electric energy will be converted to heat in the material in each cycle. The ratio of heat energy, produced per cycle, to the electrical energy is given by

$$\tan(\delta) = \frac{\varepsilon''}{\varepsilon'}.$$ (9)

We will measure the values of $\varepsilon'$ and $\varepsilon''$ for three different solid materials at frequencies from 100 Hz to 25 kHz using a LCR meter.

2. Dielectric Cell used

For solids, we use parallel plate capacitor geometry. Here, the material is in the form of either a rectangular or a circular plate or film. This plate or film is sandwiched between two copper plates in a capacitance cell shown schematically in Figure 1. A photograph of the dielectric cell is given in Figure 2.

![Figure 1. Schematic of the capacitance cell.](image)

The description of the cell is as follows. The base plate (1) is a 5 mm thick circular copper plate (diameter approximately 6 cm) with a flat top surface. (2) is a circular copper disc, 2 cm diameter and 3 mm thick, with a small cylindrical pillar about 5 mm height at its center. The pillar has a spherical top (2 mm dia). (3) is a Perspex disc (1.5 cm dia, 1 cm thick). At its bottom there is a hemispherical groove into which the spherical top of the copper plate (2) will fit loosely. The spherical top is fixed to a thin Perspex plate (4) which is fixed to the bottom of (3) with three small screws. A spring (5) (not very stiff) is fixed between the top of the Perspex plate (3) and the bottom of plate (7). When the Perspex plate (3) is pulled up, the spring is compressed. The spring is fixed on the outside of a brass tube (6). This brass tube (6) is fixed at the top to the copper plate (7) which is 5 mm thick and 6 cm dia. The screw (8) is threaded on the outside and works inside a threaded nut (N). At the bottom end it presses into a conical groove on the top surface of the Perspex disc (3). When the Nut N is turned anticlockwise, the screw (8) moves up, and the Perspex disc (3) with the attached copper disc (2) is pulled up compressing the spring (5). We can clamp a dielectric plate between (2) and (1) with this arrangement. The top plate (7) is fixed to the bottom plate (1) by rods (9) with bolts (10). A flexible thin copper lead (11) attached to the copper disc (2) comes through an insulating bush (12) in the top copper plate and is one terminal of the capacitance. The other terminal is the top copper plate itself as it is electrically shorted to the bottom copper plate.

Any solid plate with an area less than the area of the circular disc (2), and with a uniform thickness, can be placed between the disc and the base copper plate. We will then have a parallel plate capacitor, with the solid as dielectric. If the solid specimen covers the full area, the capacitance one measures will be the capacitance of a parallel plate capacitor, with the solid as the dielectric material. If the area of the solid disc is
Experiments on Dielectric Properties

smaller than the area of the disc (2), then we will have a small correction arising from the capacitance of the uncovered area of the disc, with air as the medium. This correction will be small with the specimens used, as the dielectric constants of the solid specimens are high. With the plates of PZT, the solid will protrude beyond the edges of the disc. However the surfaces are metalized. So even if they protrude out, the metalized surface in contact with disc 2 will be an equipotential surface at the potential of disc 2.

![Figure 2. Photograph of the capacitance cell.](image)

A simple equation for a parallel plate capacitance is

\[ C = \frac{\varepsilon \varepsilon_0 A}{d}, \]  

(10)

where \( A \) is the area of the surface of the solid, and \( d \) its thickness. So, a measurement of the capacitance, with knowledge of the dimensions of the specimen, will enable us to calculate the dielectric constant. However, the above expression is valid if the thickness is small compared to the lateral dimensions of the specimen for rectangular or circular plates or films. If this is not true, we will have to apply a correction to the measured dielectric constant. But this correction will not depend on frequency. So the ratio of dielectric constants at two frequencies \( f_1 \) and \( f_2 \) obtained by using formula (10) will be correct.

3. Method of Measurement

The capacitance is measured using the LCR meter SM 6019 manufactured by the Scientific Mes-Technik Pvt Ltd., B-14, Industrial Estate, Pologround, Indore 42015. This meter can measure, among other things, Capacitance and Delta (\( C \) and \( D \)); the capacitance values can be in the range 0.001 pF to 99999 nF and \( D \) (i.e., \( \tan (\delta) \)) in the range 0.0001 to 1. It will indicate the capacitance and \( D \) values at the following frequencies: 100, 120, 250, 500, 1000, 2500, 5000, 7800, 12500, 15,600 and 25,000 Hz.

The first line of the display indicates the capacitance \( C \), and the second line the value of \( D \). The presence of an arrow, to the left of \( C \), in the first line, indicates that the instrument is in an automatic mode. We make measurements in the automatic mode. One can apply a test voltage of either 0.5 volts RMS or 0.05 \( \sqrt{\text{V RMS}} \). The former is denoted by Hi and the latter by Low. This indication appears on the front panel. We will make our measurements in the Hi mode.

There are two cables with clips attached. One clip is marked red and one black. In each cable there are two wires with BNC connectors. Connect the two wires from the red clip to the BNC sockets marked \( H_0 \) and \( H_s \). The two wires from the black clip are attached to the BNC connectors \( L_S \) and \( L_o \). When the instrument is switched on, it will start the measurement at the frequency of 1 kHz. Note the values of \( C \) and \( D \). Then push
Experiments on Dielectric Properties

the button marked Freq on the front panel. This will go to the next frequency in a cycle. So, after 25,000 Hz it will go to 100 Hz. Wait for some time till the readings stabilize and note them. In this way you can cover the entire range of frequencies.

4. Results for Solid Dielectric

We have three types of solid dielectrics for measurement: (1). A film of polyvinylidene fluoride with metalized faces to which leads are attached; (2). Two pellets of ZnO, one sintered at 800 °C and another sintered at 850 °C; (3). Two rectangular parallelepipeds of PZT poled along the thickness.

(i) Polyvinylidene Fluoride

PVDF is a non-reactive thermoplastic fluoropolymer. It is produced by the polymerization of vinylidene fluoride as shown in Figure 3.

![Figure 3. Polymerization of vinylidene fluoride.](image-url)

Piezoelectricity was first observed in PVDF in 1969. In films that were poled (placed in a strong electric field at high temperatures and then removed) the net induced dipole moment was ten times larger than in any other known polymer. It is also ferroelectric.

PVDF can exist in two forms. The fluorine atoms are polar, and the direction along which the dipole moments point are indicated by arrows in Figure 4. The structure of the common α form is shown in

![Figure 4. Arrangement of fluorine and hydrogen atoms in the two common configurations of PVDF. Credit: Wikipedia.](image-url)
Experiments on Dielectric Properties

Figure 4(a). In this form, all the dipoles are not pointing in the same direction. This leads to a dipole moment of about $4 \times 10^{-30}$ coulomb-m per repeat unit. In the second configuration, all the dipole moments are pointing in the same direction leading to a higher dipole moment of about $7 \times 10^{-30}$ C-m per repeat unit. This form is called the $\beta$ form. By subjecting the film in the alpha form to a uniaxial or biaxial strain, and applying a large electric field, one can convert the material from the $\alpha$ to the $\beta$ form, resulting in a large increase in the reversible permanent dipole moment.

Thin films of this material, (thickness about 40 micrometers), coated with silver on both sides, and with leads attached were made available through the kind courtesy of the National Aerospace Laboratory, Bengaluru. The coated area of the film is $30 \times 10$ square mm. Table 1 gives the measured capacitance and tan ($\delta$) for different frequencies using the capacitance meter. A plot of the dielectric constant $\varepsilon'$ vs. frequency is shown in Figure 5, and a plot of dielectric loss $\varepsilon''$ vs. frequency is shown in Figure 6. It is seen that $\varepsilon'$ decreases as the frequency increases, while $\varepsilon''$ increases as the frequency increases. Both behaviors are due to relaxation phenomena.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Capacitance (pF)</th>
<th>Tan ($\delta$)</th>
<th>$\varepsilon'$</th>
<th>$\varepsilon''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>733.2</td>
<td>0.001</td>
<td>11.05</td>
<td>0.011</td>
</tr>
<tr>
<td>120</td>
<td>736.3</td>
<td>0.0037</td>
<td>11.09</td>
<td>0.041</td>
</tr>
<tr>
<td>250</td>
<td>732</td>
<td>0.0147</td>
<td>11.03</td>
<td>0.162</td>
</tr>
<tr>
<td>500</td>
<td>727.4</td>
<td>0.0126</td>
<td>10.96</td>
<td>0.138</td>
</tr>
<tr>
<td>1000</td>
<td>723</td>
<td>0.0158</td>
<td>10.89</td>
<td>0.172</td>
</tr>
<tr>
<td>2500</td>
<td>715.6</td>
<td>0.0206</td>
<td>10.78</td>
<td>0.222</td>
</tr>
<tr>
<td>5000</td>
<td>710.5</td>
<td>0.0257</td>
<td>10.70</td>
<td>0.275</td>
</tr>
<tr>
<td>7800</td>
<td>707.4</td>
<td>0.0319</td>
<td>10.66</td>
<td>0.340</td>
</tr>
<tr>
<td>12500</td>
<td>687.7</td>
<td>0.05</td>
<td>10.36</td>
<td>0.521</td>
</tr>
<tr>
<td>15600</td>
<td>691.6</td>
<td>0.044</td>
<td>10.42</td>
<td>0.453</td>
</tr>
<tr>
<td>25000</td>
<td>685.5</td>
<td>0.05</td>
<td>10.33</td>
<td>0.517</td>
</tr>
</tbody>
</table>

Figure 5. Dielectric constant $\varepsilon'$ of PVDF film.
Experiments on Dielectric Properties

From the online catalog of Goodfellow, Cambridge Ltd., the dielectric constant of a uniaxially oriented film at 1 kHz is 12 and a biaxially oriented film is 11. The corresponding loss factor at 1 kHz for uniaxially stressed film is 0.018, i.e., $\varepsilon''$ is 0.198. These values are in good agreement with our measured values.

PVDF films with favorable piezoelectric and pyroelectric properties find applications as sensors for shock compression. They can be used as actuators. The ferroelectric properties of PVDF can be used in non-volatile memories. Attempts are being made to increase the dielectric constant of these materials by dispersing nano particles of PZT in the film.

(ii) Pellets of Zin Oxide

Zinc oxide crystallizes in the stable state in the hexagonal Wurtzite structure. This structure is shown in Figure 7.

The bond is mainly ionic with Zn in the $2^+$ and O in the $2^-$ states. The point group symmetry of this structure is 6 mm, and it has no center of inversion symmetry. So ZnO is a strong piezoelectric material. It is a very soft material. It has a large direct band gap of 3.4 eV. Because of the large band gap, it can sustain large electric fields before breaking down. Natural ZnO is n-type semiconductor. It can be easily doped with group III elements like Al, Ga and In to produce controlled n-type doping.

Due to its wide band gap it has potential applications as a light emitting diode, when it is combined with GaN. Al-doped ZnO can be used as a transparent electrode, and will be much cheaper than indium tin oxide. When doped with 1 to 10% magnetic ions such as Fe, Mn or Ni, it could become ferromagnetic even at room temperature.
Experiments on Dielectric Properties

Zn atoms are large spheres and oxygen atoms are small spheres. It can therefore have applications in spintronics.

Two different pellets of ZnO were prepared and supplied by Dr. V. Ganesan of UGC-DAE CSR in Indore. These pellets have been sintered at 800 °C and 850 °C. A pellet sintered at 750 °C was found to be weak in strength and broke into fragments.

The pellet is mounted in the dielectric cell with an aluminium pad between the top face of the pellet and the bottom face of the copper disc (2) in the dielectric cell. The screw is turned till the specimen is tightly fixed between the copper disc and the base copper plate.

Whenever $D$ exceeds a value of unity, the LCR meter automatically goes into the $R_p, Q$ mode. This happens at low frequencies for ZnO. The value of $Q$, the quality factor, is shown as negative. The negative sign indicates that it is measuring $Q$ of a capacitor.

\[ |Q| = \frac{1}{D}. \] (11)

The relation between the resistance $R_p$, $|Q|$ and the capacitance $C$ is

\[ C = \frac{|Q|}{(2\pi f R_p)}. \] (12)

So $C$ and $D$ can be calculated from $R_p$ and $Q$.

Tables 2(a) and 2(b) show the measured capacitance and loss for two pellets of ZnO, one sintered at 800 °C and the other sintered at 850 °C.

### Table 2. Dielectric constant and loss of ZnO pellets. (a) Pellet sintered at 800 °C.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$C$ (pF)</th>
<th>$D$</th>
<th>$R_p$ (kΩ)</th>
<th>$Q$</th>
<th>$\varepsilon'$</th>
<th>$\varepsilon''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>(177.8)</td>
<td>35.714</td>
<td>250.7</td>
<td>-0.028</td>
<td>496.3</td>
<td>17725.59</td>
</tr>
<tr>
<td>120</td>
<td>(177.7)</td>
<td>29.412</td>
<td>253.9</td>
<td>-0.034</td>
<td>495.9</td>
<td>14585.15</td>
</tr>
<tr>
<td>250</td>
<td>(160.0)</td>
<td>15.625</td>
<td>254.7</td>
<td>-0.064</td>
<td>446.6</td>
<td>6978.884</td>
</tr>
<tr>
<td>500</td>
<td>(144.8)</td>
<td>8.547</td>
<td>257.4</td>
<td>-0.117</td>
<td>404.0</td>
<td>3452.84</td>
</tr>
<tr>
<td>1000</td>
<td>141.3</td>
<td>4.760</td>
<td>250.7</td>
<td>-0.028</td>
<td>394.3</td>
<td>1876.998</td>
</tr>
<tr>
<td>2500</td>
<td>133.8</td>
<td>2.090</td>
<td>254.7</td>
<td>-0.064</td>
<td>373.4</td>
<td>780.3995</td>
</tr>
<tr>
<td>5000</td>
<td>128.0</td>
<td>1.180</td>
<td>257.4</td>
<td>-0.117</td>
<td>357.2</td>
<td>421.5087</td>
</tr>
<tr>
<td>7800</td>
<td>121.7</td>
<td>0.890</td>
<td>254.7</td>
<td>-0.064</td>
<td>339.6</td>
<td>302.2701</td>
</tr>
<tr>
<td>12500</td>
<td>113.0</td>
<td>0.71</td>
<td>257.4</td>
<td>-0.117</td>
<td>315.4</td>
<td>223.8986</td>
</tr>
<tr>
<td>15600</td>
<td>108.1</td>
<td>0.66</td>
<td>254.7</td>
<td>-0.064</td>
<td>301.7</td>
<td>199.1059</td>
</tr>
<tr>
<td>25000</td>
<td>96.9</td>
<td>0.590</td>
<td>257.4</td>
<td>-0.117</td>
<td>270.4</td>
<td>159.5476</td>
</tr>
</tbody>
</table>

(b) Pellet sintered at 850 °C.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$C$ (pF)</th>
<th>$D$</th>
<th>$R_p$ (kΩ)</th>
<th>$Q$</th>
<th>$\varepsilon'$</th>
<th>$\varepsilon''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>(177.8)</td>
<td>35.714</td>
<td>250.7</td>
<td>-0.028</td>
<td>496.3</td>
<td>17725.59</td>
</tr>
<tr>
<td>120</td>
<td>(177.7)</td>
<td>29.412</td>
<td>253.9</td>
<td>-0.034</td>
<td>495.9</td>
<td>14585.15</td>
</tr>
<tr>
<td>250</td>
<td>(160.0)</td>
<td>15.625</td>
<td>254.7</td>
<td>-0.064</td>
<td>446.6</td>
<td>6978.884</td>
</tr>
<tr>
<td>500</td>
<td>(144.8)</td>
<td>8.547</td>
<td>257.4</td>
<td>-0.117</td>
<td>404.0</td>
<td>3452.84</td>
</tr>
<tr>
<td>1000</td>
<td>141.3</td>
<td>4.760</td>
<td>250.7</td>
<td>-0.028</td>
<td>394.3</td>
<td>1876.998</td>
</tr>
<tr>
<td>2500</td>
<td>133.8</td>
<td>2.090</td>
<td>254.7</td>
<td>-0.064</td>
<td>373.4</td>
<td>780.3995</td>
</tr>
<tr>
<td>5000</td>
<td>128.0</td>
<td>1.180</td>
<td>257.4</td>
<td>-0.117</td>
<td>357.2</td>
<td>421.5087</td>
</tr>
<tr>
<td>7800</td>
<td>121.7</td>
<td>0.890</td>
<td>254.7</td>
<td>-0.064</td>
<td>339.6</td>
<td>302.2701</td>
</tr>
<tr>
<td>12500</td>
<td>113.0</td>
<td>0.71</td>
<td>257.4</td>
<td>-0.117</td>
<td>315.4</td>
<td>223.8986</td>
</tr>
<tr>
<td>15600</td>
<td>108.1</td>
<td>0.66</td>
<td>254.7</td>
<td>-0.064</td>
<td>301.7</td>
<td>199.1059</td>
</tr>
<tr>
<td>25000</td>
<td>96.9</td>
<td>0.590</td>
<td>257.4</td>
<td>-0.117</td>
<td>270.4</td>
<td>159.5476</td>
</tr>
</tbody>
</table>
Experiments on Dielectric Properties

Table 2. (Contd.)

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>C (pF)</th>
<th>D</th>
<th>( R_p ) (k( \Omega ))</th>
<th>Q</th>
<th>( \varepsilon' )</th>
<th>( \varepsilon'' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>(167.8)</td>
<td>(322.581)</td>
<td>29.42</td>
<td>-0.0031</td>
<td>431.3</td>
<td>139122.3</td>
</tr>
<tr>
<td>120</td>
<td>(148.6)</td>
<td>(303.030)</td>
<td>29.47</td>
<td>-0.0033</td>
<td>381.9</td>
<td>115738.5</td>
</tr>
<tr>
<td>250</td>
<td>(118.9)</td>
<td>(181.818)</td>
<td>29.47</td>
<td>-0.0055</td>
<td>305.5</td>
<td>55554.5</td>
</tr>
<tr>
<td>500</td>
<td>(97.4)</td>
<td>(111.111)</td>
<td>29.42</td>
<td>-0.009</td>
<td>250.4</td>
<td>27824.46</td>
</tr>
<tr>
<td>1000</td>
<td>(99.9)</td>
<td>(54.054)</td>
<td>29.5</td>
<td>-0.0185</td>
<td>256.7</td>
<td>13874.5</td>
</tr>
<tr>
<td>2500</td>
<td>(94.6)</td>
<td>(22.831)</td>
<td>29.5</td>
<td>-0.0438</td>
<td>243.1</td>
<td>5549.801</td>
</tr>
<tr>
<td>5000</td>
<td>(93.3)</td>
<td>(11.614)</td>
<td>29.38</td>
<td>-0.0861</td>
<td>239.9</td>
<td>2786.234</td>
</tr>
<tr>
<td>7800</td>
<td>(92.6)</td>
<td>(7.593)</td>
<td>29.05</td>
<td>-0.1317</td>
<td>237.9</td>
<td>1806.337</td>
</tr>
<tr>
<td>12500</td>
<td>(90.6)</td>
<td>(4.878)</td>
<td>28.83</td>
<td>-0.205</td>
<td>232.8</td>
<td>1135.755</td>
</tr>
<tr>
<td>15600</td>
<td>(90.0)</td>
<td>(3.953)</td>
<td>28.69</td>
<td>-0.253</td>
<td>231.4</td>
<td>914.5011</td>
</tr>
<tr>
<td>25000</td>
<td>(87.7)</td>
<td>(2.577)</td>
<td>28.19</td>
<td>-0.388</td>
<td>225.3</td>
<td>580.7702</td>
</tr>
</tbody>
</table>

In Table 2, wherever the capacitance meter gave values in \( R_p, Q \) mode, these values are given. The derived values of capacitance and \( D \), using formulae (12) and (13), are given in brackets.

Figures 8(a) and 8(b) show the variation of the real and imaginary parts of the dielectric constant in the two pellets.

![Graph](image)

**Figure 8.** (a) Variation of the real part of the dielectric constant and (b) imaginary part of the dielectric constant with frequency in ZnO pellets, sintered at two different temperatures.

Note the very large values of the loss as the frequency decreases. This is because of the ionic conductivity of zinc oxide. The dielectric constant of the material is also large at low frequencies. Sintering at a higher temperature increases the loss by one order of magnitude.
Experiments on Dielectric Properties

Both values increase considerably with time of storage. This increase could be due to absorption of moisture, as the pellets were not stored with silica gel.

(iii) PZT Plates

PZT (lead zirconium titanate) bars SP5A were obtained from Sparkler Ceramics Pvt Ltd, Pune, India. These bars are 25 mm × 5 mm × 3 mm, metalized perpendicular to the thickness (3 mm). The capacitance and D readings for one sample are given in Table 3.

Table 3. Dielectric constant and loss of PZT samples (5A) from Sparkler Ceramics.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>C (pF)</th>
<th>D</th>
<th>ε′</th>
<th>ε″</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>861</td>
<td>0.005</td>
<td>2335</td>
<td>11.7</td>
</tr>
<tr>
<td>1000</td>
<td>857</td>
<td>0.0152</td>
<td>2324</td>
<td>35.3</td>
</tr>
<tr>
<td>2500</td>
<td>850</td>
<td>0.0159</td>
<td>2305</td>
<td>36.7</td>
</tr>
<tr>
<td>5000</td>
<td>844.7</td>
<td>0.0168</td>
<td>2291</td>
<td>38.5</td>
</tr>
<tr>
<td>7800</td>
<td>845.8</td>
<td>0.0194</td>
<td>2294</td>
<td>44.5</td>
</tr>
<tr>
<td>12500</td>
<td>840.1</td>
<td>0.0204</td>
<td>2278</td>
<td>46.5</td>
</tr>
<tr>
<td>15600</td>
<td>830.8</td>
<td>0.03</td>
<td>2253</td>
<td>67.6</td>
</tr>
<tr>
<td>25000</td>
<td>847</td>
<td>0.0215</td>
<td>2297</td>
<td>49.4</td>
</tr>
</tbody>
</table>

At 1000 hertz, ε′ is 2324
End correction, 1.297
Corrected ε′, 1792

From the Sparkler Ceramics data sheet, the average capacitance value of these plates is 849 pF and tan (δ) is 0.014. It is not stated at what frequency it was measured. Our values for the capacitance and tan (δ) are in good agreement with this data. The aspect ratio (width/length) of these plates is 0.2. We have to employ end corrections in calculating the dielectric constant. This is discussed in a paper by Hitoshi Nashiyama and Mitsunobu Nakamura, *IEEE Transactions on Components, Packaging and Manufacturing Technology-Part A*, 17, 477, (1994). According to these authors, the measured capacitance in air of the parallel plate, with an aspect ratio b = width/length, will be larger than the simple formula by a factor which depends on b. For b = 0.2 in our case, this factor is 1.297. So, capacitance with vacuum as dielectric will be 1.297ε0A/d = 4.7827 × 10⁻¹³ F. Dividing the measured capacitance at 1 kHz by this number, the dielectric constant ε, comes out to be 1792. The value given in the data sheet of Sparkler Ceramics is 1750.

7.5 Verification of Curie–Weiss Law for the Electrical Susceptibility of a Ferroelectric Material in the Paraelectric State

1. Introduction

The electrical susceptibility χ of a material is the ratio of change in the electric polarization P in the material to the change in the applied electric field E. For an isotropic material, the vector P is in the direction of E and the susceptibility is

\[ \epsilon_0 \chi = \frac{dP}{dE}. \]  (1)

For a linear dielectric material, P is proportional to E, and χ is independent of the field, as long as the field is not very high. This behavior is seen in all materials at high temperatures. Such behavior is called paraelectric, in analogy with paramagnetic behavior. In these materials, there is no electric polarization in the absence of an electric field.

There is a class of solid materials called ferroelectric materials. Below a temperature TC characteristic of a ferroelectric material, the material exhibits an electric polarization, even in the absence of an electric field.
Experiments on Dielectric Properties

Above $T_C$, these materials show paraelectric behaviour. The variation of $P$ with $E$, above and below $T_C$, in the paraelectric and ferroelectric states are shown respectively in Figures 1(a) and 1(b). In the paraelectric state, the material shows no hysteresis, while in the ferroelectric state, the material shows hysteresis. The $P$ vs. $E$ curve in a ferroelectric material is similar to the magnetization vs. magnetic field curve for a ferromagnetic material. Hence, these materials are called ferroelectric. Examples of ferroelectric materials are potassium dihydrogen phosphate ($T_C = 151^\circ$C), triglycine sulphate ($I_C = 47^\circ$C), Rochelle salt (it is ferroelectric between $-18^\circ$C to $24^\circ$C), and barium titanate ($T_C = 128^\circ$C).

![Figure 1](image1.png)

**Figure 1.** Variation of $P$ with $E$ in (a) paraelectric state and (b) ferroelectric state of a ferroelectric material.

Above $T_C$ (i.e., in the paraelectric state), the susceptibility of a ferroelectric material varies strongly with temperature, following the Curie–Weiss law

$$\chi = \frac{C}{(T - \theta)}.$$  \hspace{1cm} (2)

Here, $C$ and $\theta$ are constants characteristic of a given material. $\theta$ has a value close to the transition temperature $T_C$. This law, derived in the mean field approximation of Landau, is valid at temperatures not too close to $T_C$ (i.e., not closer than about $10^\circ$C). Near $T_C$, fluctuations will play a dominant role (see Section 16.1). For a ferroelectric material with a $T_C$ close to room temperature, the susceptibility will have a very high value (of the order of hundreds and thousands) compared to a non-ferroelectric material, like a polymer, for which the susceptibility will be of the order of 1 to 10. The dielectric constant varies with temperature as

$$\varepsilon = 1 + \chi = \frac{1 + C}{(T - \theta)} \approx \frac{C}{(T - \theta)}. \hspace{1cm} (3)$$

Since a ferroelectric material has a large dielectric constant, a capacitance made of such a material will have a relatively small size.

The so-called ceramic capacitances available in the market at low cost are made of ferroelectric materials with $T_C$ around $10^\circ$C to $20^\circ$C. These materials can be used to verify the Curie–Weiss law. The capacitance, being proportional to the dielectric constant, will show a strong dependence on temperature, with the capacitance decreasing as the temperature increases.

On the other hand, one can also get capacitances in the market with a polymer as the dielectric material. The dielectric constant of these polymer materials is not large, and it shows a very weak dependence on temperature. Capacitances with such materials as dielectrics show a weak dependence on temperature.
2. **Aim**

To measure the capacitances of a ceramic capacitor and a polymer capacitor as a function of temperature in the range 40 °C to 130 °C; to show that the capacitance of the ceramic capacitor shows a strong dependence on temperature, while that of a polymeric capacitor shows a weak dependence; and to verify the Curie–Weiss law for the ceramic capacitor.

3. **Apparatus Required**

A regulated DC power supply, temperature indicator/controller, furnace, insert with ceramic and polymeric capacitors, a signal generator and a DMM to measure AC volts to three decimal places in the range of 2 volts.

4. **Experimental Setup**

A ceramic capacitor and a polymer capacitor, each of a nominal value of 0.1 μF, are fixed with Araldite on two faces of an aluminum rod which also carries a Pt100 sensor. Leads from the capacitor are brought to a circular disc carrying eight small banana terminals, as shown in Figure 2.

![Figure 2. Circular terminal block.](image)

The two central unmarked terminals are for the leads of Pt 100 thermometer. The ceramic capacitor leads are connected to the terminals, R₁ (red) and G₁ (green) marked ceramic. Similarly the terminals, R₂ and G₂, marked polymer, are connected to the leads from the capacitor with polymer dielectric. Between G₁ and B₁ (black) is connected a 1 kΩ resistor at the top disc. Similarly, between G₂ and B₂ is connected a 1 kΩ resistor at the disc. Terminals R₁ and R₂ are shorted at the disc (as shown by the dotted line). So also, terminals B₁ and B₂ are shorted. If a signal generator is connected between R₁ and B₁, the same AC voltage is applied to both the RC combinations as shown in Figure 3.

![Figure 3. CA: Ceramic capacitor, RA = 1 kΩ, CB: Polymer capacitor, RB = 1kΩ, SG: Signal generator.](image)

If we measure the voltage, \( V_{CA} \), across R₁ and G₁, and the voltage, \( V_{RA} \), across G₁ and B₁, then

\[
\frac{V_{CA}}{V_{RA}} = \frac{Z_{CA}}{R_A},
\]

(4)
Experiments on Dielectric Properties

where $Z_{CA}$ is the impedance of the capacitor $C_A$, and it is equal to

$$Z_{CA} = \frac{1}{(2\pi f C_A)}.$$  \hfill (5)

So

$$C_A = \frac{1}{(2\pi f \frac{V_{CA}}{V_{RA}} R_A)}.$$  \hfill (6)

Knowing $f$, the frequency of the signal generator, and $R_A$, one can calculate the capacitance $C_A$.

Similarly,

$$C_B = \frac{1}{(2\pi f \frac{V_{CB}}{V_{RB}} R_B)},$$  \hfill (7)

where $V_{RB}$ is the voltage across $G_2$ and $B_2$, and $V_{CB}$ is the voltage across $R_2$ and $G_2$.

Connect the furnace to the regulated power supply. Close the open ends of the ceramic tube of the furnace loosely with plugs of cotton to prevent convection currents. Connect the signal generator terminals to $R_1$ and $B_1$. Set the signal generator frequency to 1 kHz. Set the output voltage at 1 V RMS as measured at the terminals $R_1$ and $B_1$ with a DMM. Connect the two Pt100 terminals to the corresponding terminals on the temperature indicator.

Start with a DC voltage of 7.5 V so that the furnace temperature rises at the rate of 1 to 1.5 degrees/minute. As the temperature increases, increase the DC voltage gradually. At a given value of temperature as indicated on the temperature indicator, measure with a DMM set in AC 2 volts range, the voltages $V_{RA}$, $V_{CA}$, $V_{RB}$ and $V_{CB}$. Carry out the measurements at intervals of 10°C from 40°C to 110°C. A sample set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>Temp (°C)</th>
<th>$V_C$</th>
<th>$V_R$</th>
<th>$Z_C$</th>
<th>$C_A$ (μF)</th>
<th>$V_C$</th>
<th>$V_R$</th>
<th>$Z_C$</th>
<th>$C_B$ (μF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.887</td>
<td>0.409</td>
<td>2168.7</td>
<td>0.0734</td>
<td>0.814</td>
<td>0.533</td>
<td>1527.2</td>
<td>0.1042</td>
</tr>
<tr>
<td>50</td>
<td>0.875</td>
<td>0.284</td>
<td>3080.9</td>
<td>0.0517</td>
<td>0.773</td>
<td>0.507</td>
<td>1524.6</td>
<td>0.1044</td>
</tr>
<tr>
<td>60</td>
<td>0.890</td>
<td>0.232</td>
<td>3836.2</td>
<td>0.0415</td>
<td>0.771</td>
<td>0.508</td>
<td>1517.7</td>
<td>0.1049</td>
</tr>
<tr>
<td>70</td>
<td>0.897</td>
<td>0.191</td>
<td>4696.3</td>
<td>0.0339</td>
<td>0.766</td>
<td>0.507</td>
<td>1510.8</td>
<td>0.1053</td>
</tr>
<tr>
<td>80</td>
<td>0.900</td>
<td>0.164</td>
<td>5487.8</td>
<td>0.029</td>
<td>0.761</td>
<td>0.506</td>
<td>1503.9</td>
<td>0.1058</td>
</tr>
<tr>
<td>90</td>
<td>0.903</td>
<td>0.14</td>
<td>6450</td>
<td>0.0247</td>
<td>0.752</td>
<td>0.511</td>
<td>1471.6</td>
<td>0.1082</td>
</tr>
<tr>
<td>100</td>
<td>0.905</td>
<td>0.129</td>
<td>7015.5</td>
<td>0.0227</td>
<td>0.751</td>
<td>0.517</td>
<td>1452.6</td>
<td>0.1096</td>
</tr>
<tr>
<td>110</td>
<td>0.855</td>
<td>0.106</td>
<td>8066</td>
<td>0.0197</td>
<td>0.701</td>
<td>0.493</td>
<td>1421.9</td>
<td>0.1119</td>
</tr>
</tbody>
</table>

Column 1 gives the temperature reading. Column 2 is the AC RMS voltage reading, in volts, across the ceramic capacitor terminals $R_1$, $G_1$. Column 3 is the AC voltage reading across the resistor terminals $G_1$, $B_1$. Columns 6 and 7 are the voltage readings across $R_2$, $G_2$ and $G_2$, $B_2$ terminals respectively. Columns 4 and 8 are the impedances of the two capacitors, calculated from the voltage readings. Columns 5 and 9 are the values of the two capacitances, calculated using equations (6) and (7).

Figure 4 shows a plot of $1/C_A$ against temperature $T$ for the ceramic capacitor. The linear fit to the points is shown by the black line. The fit is very good.

The intercept $A$ (value of $1/C_A$ at $I = 0$) is $-7.03$ and the slope $B$ is 0.521. So the Curie temperature $\theta$ is given by

$$\theta = -\frac{A}{B} = 13.4 \degree C.$$  

The equation to the straight line is

$$\frac{1}{C_A (C_A \text{ in } \mu F)} = 0.521 (T - 13.4).$$  \hfill (8)
Experiments on Dielectric Properties

Figure 4. A plot of $1/C_A$ vs. temperature in Celsius.

or

$$C_A \text{ (in } \mu \text{F}) = \frac{1.921}{(T - 13.4)}.$$  \hspace{1cm} (9)

The ceramic capacitance (and hence the dielectric constant of the ferroelectric material used in the capacitance) follows the Curie–Weiss law with $\theta = 13.4 \, ^\circ\text{C}$.

Figure 5 shows a plot of $C_A$ and $C_B$ vs. temperature. The black curve represents the fit of $C_A$ to equation (9).

Figure 5. Plot of the capacitance value $C_A$ of the ceramic capacitor and $C_B$ of the polymer capacitor as a function of temperature $T$.

Thus, this experiment demonstrates that

1. The capacitance of a ceramic capacitor decreases rapidly as temperature increases, while the capacitance of a polymer capacitor is relatively insensitive to temperature.
Experiments on Dielectric Properties

2. The dielectric constant of the ferroelectric material used in the ceramic capacitor follows the Curie–Weiss law.

3. The dielectric constant of the material used in the polymer capacitor increases slightly as the temperature is increased.

5. Questions

1. If a ceramic capacitor is used in the tank circuit of the oscillator, how will the frequency vary as the temperature changes, assuming that the inductance in the circuit is independent of temperature.

2. Can a ferroelectric be used as a memory element to represent the binary states 0 and 1?
Chapter 8

Experiments for the Determination of the Physical Constants $k$, $e$ and $h$
8.1 Determination of the Ratio \( k/e \) using a Transistor

1. Introduction

There are four fundamental constants in physics. They are \( e \), the electronic charge, \( k \), the Boltzmann constant, \( h \), the Planck’s constant and \( c \), the velocity of light in vacuum. The determination of the constants individually involves difficult experiments. But one can determine \( k/e \) with reasonable accuracy using a simple experiment.

In a transistor, the collector current \( I_{CE} \) increases exponentially as the base to emitter voltage \( V_{BE} \) increases, according to the relation

\[
I_{CE} = I_f \exp \left( \frac{eV_{BE}}{kT} \right).
\]

Here \( T \) is the absolute temperature and \( I_f \) is a temperature-dependent constant of the given diode. If we measure \( I_{CE} \) as a function of \( V_{BE} \) at room temperature and plot \( V_{BE} \) against \( \ln \left( \frac{I_{CE}}{e} \right) \), we will get a straight line. The slope \( \alpha \) of this line is given by

\[
\alpha = \left( \frac{k}{e} \right) T \quad \text{or} \quad \frac{k}{e} = \frac{\alpha}{T}.
\]

This provides a simple method to measure \( k/e \).

The circuit used is shown in Figure 1.

A battery of 1.5 V is connected to a ten turn 100 k pot. At the variable point it provides a voltage between 0 to 1.5 V to bias the base. The base-emitter voltage is read on the voltmeter V. A DC voltage of 12 V is connected through a resistance of 1 kΩ between the collector and the emitter. The collector current is measured on the milliammeter mA.

The front panel of the \( k/e \) box in which this circuit is built is shown in Figure 2.
Experiments for the Determination of the Physical Constants $k$, $e$ and $h$

The main switch is at the bottom left of the panel. The base-emitter voltage $V_{BE}$ changes when the pot is turned. $V_{BE}$ is measured at the banana terminals marked $V_{BE}$ by connecting a DMM to read DC 2 volts. The collector current, $I_{CE}$ is measured by connecting a DMM to the terminals marked $I_{CE}$ to measure DC micro/milliamps.

2. Procedure

Connect DMM1 to measure DC 2 V to the banana terminals marked $V_{BE}$. Connect DMM2 to measure current in the $\mu$A/mA ranges to the banana terminals marked $I_{CE}$. Switch on the power. Set the pot so that the DMM1 reads a $V_{BE}$ of 0.5 V. Note the current $I_{CE}$ on DMM2. It will be a few $\mu$A. Increase $V_{BE}$ in steps of 0.025 V, and note $I_{CE}$. If $I_{CE}$ goes beyond the $\mu$A range, change the DMM1 to read in mA. Take readings till $V_{BE}$ reaches 0.725 V. Note the room temperature in Kelvin scale.

A sample set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>$V_{BE}$ (volts)</th>
<th>$I_{CE}$ (amps)</th>
<th>$\ln \left( \frac{I_{CE}}{0.500} \right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.500</td>
<td>$1.60 \times 10^{-6}$</td>
<td>$-13.35$</td>
</tr>
<tr>
<td>0.525</td>
<td>$4.10 \times 10^{-6}$</td>
<td>$-12.40$</td>
</tr>
<tr>
<td>0.550</td>
<td>$1.07 \times 10^{-5}$</td>
<td>$-11.45$</td>
</tr>
<tr>
<td>0.575</td>
<td>$2.95 \times 10^{-5}$</td>
<td>$-10.43$</td>
</tr>
<tr>
<td>0.600</td>
<td>$8.16 \times 10^{-5}$</td>
<td>$-9.41$</td>
</tr>
<tr>
<td>0.625</td>
<td>$2.24 \times 10^{-4}$</td>
<td>$-8.40$</td>
</tr>
<tr>
<td>0.650</td>
<td>$6.75 \times 10^{-4}$</td>
<td>$-7.30$</td>
</tr>
<tr>
<td>0.675</td>
<td>$2.31 \times 10^{-3}$</td>
<td>$-6.07$</td>
</tr>
<tr>
<td>0.700</td>
<td>$6.10 \times 10^{-3}$</td>
<td>$-5.10$</td>
</tr>
<tr>
<td>0.725</td>
<td>$1.09 \times 10^{-2}$</td>
<td>$-4.52$</td>
</tr>
</tbody>
</table>

The plot of $V_{BE}$ against $\ln \left( \frac{I_{CE}}{0.500} \right)$ is shown in Figure 3.

The slope $\alpha$ of the graph is 0.0245 $\pm$ 0.0004 V. The value of $k/e$ is

$$\frac{k}{e} = \frac{\alpha}{T} = \frac{0.0245}{297} = \frac{82.6 \mu V}{K}.$$ 

A repeat of the experiment gave a slope of 0.0250 $\pm$ 0.0002 volts. This gives a value of 84.1 $\mu$V/K for $k/e$. The average value is 83.4 $\pm$ 0.8 $\mu$V/K. The actual value of $(k/e)$ is 86.2 $\mu$V/K.
8.2 Determination of Planck’s Constant $h$ using Light Emitting Diodes

1. Introduction

The light emitting diode (LED) is a heavily doped diode which emits light in a narrow band of wavelengths when the forward bias exceeds a value $V_g$, characteristic of the diode. The color of the emitted light depends on the nature of the diode material and its band gap. When a p-n junction is forward biased and the voltage exceeds a value $V_g$, electrons from the n-side come to the conduction band on the p-side and combine with a hole in the valence band. Then, the energy $eV_g = E_g$ (the band gap of the semiconductor) is released as a photon of energy $h\nu$, where $\nu$ is the frequency of the photon and $h$ is the Planck’s constant.

When forward biased, the current–voltage characteristic of a diode varies as shown in Figure 1.

2. Experimental Procedure

In a plastic box, red, yellow, green and blue LEDs are fixed and connected as shown in Figure 2. The p-side of the diodes are connected to four banana terminals marked R, Y, G and B in the figure. The n-sides of all the diodes are connected to the terminal B in the figure. The terminal A on the box is connected to the terminal C on the box through a resistance $R'$ of 1 or 2 k$\Omega$.
A regulated DC power supply (RPS) is connected to the terminals A and B. The positive terminal of the RPS is connected to the positive terminal of DMM1 to read DC current in milliamperes. The common terminal of DMM1 is connected to A. The ground terminal of RPS is connected to terminal B.

We connect C to R. Then, the RPS will apply a forward bias to the red LED, and the current through the LED is measured on DMM1. The forward voltage on the LED is measured on DMM2 connected to the terminals C and B.

We adjust the voltage on the RPS till the forward current measured on DMM1 is 10 mA. Note the forward voltage on DMM2. Change the current from 10 to 2.5 mA in steps of 2.5 mA and note the forward voltage.

Then, connect C to the terminals Y, G and B successively and measure the forward voltage of each diode for different forward currents, varying from 10 to 2.5 mA in steps of 2.5 mA.

A sample set of readings is shown in Table 1. The table is self-explanatory. We take the average of the four readings of the voltage as the value of $V_g$ for the diode. The variation of the voltage from the average value in each column is given in the row marked ±. The deviation from the average is a maximum for the blue LED and amounts to about 4%.

Then, the wavelengths of the light emitted from the LEDs are determined using a spectrograph or a plane grating. The way of measuring the wavelength with a grating is described in Section 12.3. In this experiment, the wavelengths of the red, yellow, green and blue LED emissions, measured with the Holmarc Spectrograph and described in Section 13.1, are 628, 595, 543 and 466 nm, respectively.

Table 2 shows the calculation of Planck’s constant. In column 2, the voltage $V_g$ measured in Table 1 is given for each LED. The energy $eV_g$, obtained by multiplying $V_g$ by the electronic charge of $1.6 \times 10^{-19}$ coulombs, is given in column 3. This energy is in joules. The wavelength $\lambda$, in nm, of emission is given in column 4. The corresponding frequency $\nu$ of emission in Hz is obtained by dividing the velocity of light $c = 3 \times 10^8$ m/s by the wavelength $\lambda$. The frequency is given in hertz in column 5. The Planck’s constant $h$
Experiments for the Determination of the Physical Constants \( k, e \) and \( h \)

in column 6 is calculated by dividing \( eV_g \) by \( v \). It is in joule-seconds. The average value of \( h \) and the error in \( h \) are given in the last row.

**Table 2.** Calculation of Planck’s constant.

<table>
<thead>
<tr>
<th>Color</th>
<th>( V_g ) (volts)</th>
<th>( eV ) (joules)</th>
<th>( \lambda ) (nm)</th>
<th>( v ) (Hz)</th>
<th>( h ) (Js)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Red</td>
<td>1.891</td>
<td>3.03E–19</td>
<td>628</td>
<td>4.77707E+14</td>
<td>6.33E–34</td>
</tr>
<tr>
<td>Yellow</td>
<td>1.945</td>
<td>3.11E–19</td>
<td>595</td>
<td>5.04202E+14</td>
<td>6.17E–34</td>
</tr>
<tr>
<td>Green</td>
<td>2.02</td>
<td>3.23E–19</td>
<td>543</td>
<td>5.52486E+14</td>
<td>5.85E–34</td>
</tr>
<tr>
<td>Blue</td>
<td>2.852</td>
<td>4.56E–19</td>
<td>466</td>
<td>6.43777E+14</td>
<td>7.09E–34</td>
</tr>
<tr>
<td>Average</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.36E–34  ±0.7E–34</td>
</tr>
</tbody>
</table>

The actual value of the Planck’s constant is \( 6.64 \times 10^{-34} \) Js. The observed value is 5% lower than the actual value.

The utility of this experiment is that it gives an approximate value of \( h \) without using a photocell. The experiment is simple to do and inexpensive.

### 8.3 Determination of \( e \) by Shot Noise

**1. Introduction**

When electrons are generated by photo- or thermionic emission, the electrons come out at random. The random generation of electrons follows a Poisson distribution. On an average, if \( \langle N \rangle \) electrons are generated in one second, there are mean square fluctuations about the average value. The root-mean-square fluctuation \( \sqrt{(\langle N \rangle - \langle N \rangle)^2} = \langle N \rangle^{1/2} \) for the Poisson distribution. Since the generation is random, the power spectrum \( S(f) \) of the fluctuation is independent of the frequency \( f \), giving rise to a White Noise called Shot noise.

There is an average current

\[
\langle I \rangle = \langle N \rangle e.
\]  

(1)

If these electrons pass through a resistance \( R \), this current produces an average voltage \( \langle V \rangle = R\langle I \rangle \).

The voltage across \( R \) fluctuates with time. The power spectrum for the mean square voltage fluctuation, \( S_v(f) \) is

\[
S_v(f) = 2eR^2\langle I \rangle.
\]  

(2)

If we have an amplifier with gain \( G(f) \), then the mean square fluctuation of the amplified voltage is

\[
\nu_{RMS}^2 = 2eR^2\langle I \rangle \int G^2(f) \, df
\]  

over the bandwidth \( B \) of the amplifier. If we denote the integral by \( G^2B \), and replace \( R \) by \( \langle V \rangle / \langle I \rangle \), we get

\[
\nu_{RMS}^2 = 2e \left( \frac{\langle V \rangle^2}{\langle I \rangle} \right) G^2B.
\]  

(4)

For a given \( \langle I \rangle \), varying \( R \) will give \( \nu_{RMS} \) proportional to \( \langle V \rangle \). The slope of \( \nu_{RMS} \) vs. \( \langle V \rangle \) is \( \alpha \), which will be a function of \( \langle I \rangle \) for an amplifier with a given \( G^2B \). A plot of \( \alpha^2 \) against \( 1/\langle I \rangle \) will be a straight line with a slope \( \beta \) given by

\[
\beta = 2eG^2B.
\]  

(5)

If we measure \( G^2B \) we can get the value of the electronic charge \( e \). This is the principle of the experiment.
2. **Noise Detector Circuit**

The block diagram of the noise detector circuit is shown in Figure 1.

![Block diagram of the shot Noise detector.](image)

The input to the detector may be a sinusoidal voltage from a signal generator for measuring $G^2B$, or the fluctuating voltage across a resistor. This is amplified about 200 times in a preamplifier. There are three stages of amplification (2, 10 and 10). The output from the preamplifier passes through a band-pass filter. This is a Butterworth filter consisting of two stages of an active high-pass filter, followed by two stages of an active low-pass filter. If there are two stages and each produces an attenuation $\alpha$ at a frequency $f$, the total attenuation of the two stages will be $\alpha^2$, leading to sharp rise of the transmission coefficient at the low frequency end, and sharp decline at the high frequency end of the pass band.

The signal then goes through a final amplifier, which amplifies by a factor of 5, before entering the chip AD 637. This chip will give a DC output, which is the true RMS value of the AC input, *whatever be the waveform of the input*. This output is measured on a DMM.

3. **Front Panel Diagram of the Noise Detector Circuit**

The layout of the noise detector circuit is shown schematically in Figure 2.

![Layout of the noise detector.](image)

In a chassis, three boxes are mounted. In the rear is the split power supply which provides the power to the noise detector and the AD 637 box. The power connections are made through cables with banana plug terminations. The noise detector box is mounted in the front and the box containing AD 637 is mounted between the power supply and the noise detector.

The front panel of the noise detector is shown in Figure 3.

![Front panel of noise detector circuit.](image)
Experiments for the Determination of the Physical Constants \( k, e \) and \( h \)

On the left of the front panel are three banana terminals (red, green and black) marked \(+12\, \text{V}\), \(\text{Gnd}\) and \(-12\, \text{V}\) respectively. These are connected to the corresponding banana terminals on the split power supply in the rear of the chassis. Slightly to the right of the center is the BNC connector marked O/P. The amplified AC output of the noise detector appears at this BNC connector. This has to be connected to the input of the AD 637 box. There is a BNC connector on the right top of the box marked INPUT. This is to be connected to the corresponding BNC connector on the noise source to be described below. There is a DPDT switch SW1 below the BNC connector. When calibrating the noise detector, this switch is put down. When measuring the external noise, this switch is put up. There is an RCA socket below SW1. During calibration, the signal generator output is connected to this RCA terminal. Inside the box there is a potential divider which feeds 1/1000 of the input voltage of the signal generator to the broadband amplifier. There is a second DPDT switch SW2 at the top right corner of the front panel. During calibration, the switch should be put down to read the RMS input from the signal generator. It is put up when the amplified output of 1/1000 of the signal generator voltage is to be read. During measurement of external noise, this switch is in the Up position.

The front panel of the AD 637 box is shown in Figure 4.

![Figure 4. Front panel of AD 637 box.](image)

On the left, there are three small banana plugs (red, green and black) marked \(+12\, \text{V}\), \(\text{Gnd}\) and \(-12\, \text{V}\) respectively. They are connected to the split power supply terminals. In the center, there is an RCA socket marked INPUT. This should be connected to the BNC socket marked O/P at the center of the front panel of the noise detector shown in Figure 3. On the right are two small banana terminals marked DC O/P. These are connected to measure the amplified RMS voltage from the output of the AD 637 chip.

4. Calibration

We have to measure the gain of the system as a function of frequency, when we give a sinusoidal AC input. The AC output of a signal generator, producing sine waves, is connected to the RCA socket marked SG, on the front panel of the noise detector circuit (Figure 3). This signal is attenuated by a potential divider by a factor of 1000. A 1 volt RMS AC signal from the signal generator will then generate an AC input of 1 mV RMS. The RMS value \( V \) of the signal generator input is measured on the DC multimeter connected to the AD637 box when the switch SW2 is put in the Down position. When the switch SW2 is pushed up, the DC multimeter connected to the banana terminals on the AD 637 box will measure the RMS amplified output \( v_{\text{RMS}} \) of the noise detector circuit. When it amplifies 1/1000-th part of the signal generator input. The ratio of the RMS output to 1/1000 of the input value gives the gain \( G \) of the broadband amplifier. Thus, one can measure the gain as a function of frequency. We measure the AC input signal from the signal generator, and the output of the noise detector amplifier on the same meter, connected to the AD 637 output. In this way, we avoid any differences that may arise using two different meters for measuring the AC input and output signals.

Table 1 gives the frequency \( f \), \( V_{AC} \) from the signal generator in volts, and \( v_{\text{RMS}} \) at the output. The gain \( G \) is given by the formula

\[
G = \left( \frac{v_{\text{RMS}}}{V_{AC}} \right) \times 1000.
\]

The first column of the table gives the frequency of the signal generator output.
Experiments for the Determination of the Physical Constants \(k, e \text{ and } h\)

**Table 1.** \(G^2\) as a function of frequency.

<table>
<thead>
<tr>
<th>(f) (kHz)</th>
<th>(V) (volts)</th>
<th>(v_{\text{RMS}}) (volts)</th>
<th>(G)</th>
<th>(G^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>1.031</td>
<td>0.080</td>
<td>7.76 \times 10^4</td>
<td>6.02 \times 10^5</td>
</tr>
<tr>
<td>1.0</td>
<td>1.014</td>
<td>0.365</td>
<td>3.60 \times 10^6</td>
<td>1.30 \times 10^7</td>
</tr>
<tr>
<td>1.5</td>
<td>1.012</td>
<td>0.880</td>
<td>8.70 \times 10^5</td>
<td>7.56 \times 10^6</td>
</tr>
<tr>
<td>2.0</td>
<td>1.007</td>
<td>1.410</td>
<td>1.40 \times 10^3</td>
<td>1.96 \times 10^6</td>
</tr>
<tr>
<td>2.5</td>
<td>1.002</td>
<td>1.806</td>
<td>1.80 \times 10^3</td>
<td>3.25 \times 10^6</td>
</tr>
<tr>
<td>3.0</td>
<td>1.000</td>
<td>2.107</td>
<td>2.11 \times 10^3</td>
<td>4.44 \times 10^6</td>
</tr>
<tr>
<td>3.5</td>
<td>0.999</td>
<td>2.302</td>
<td>2.30 \times 10^3</td>
<td>5.31 \times 10^6</td>
</tr>
<tr>
<td>4.0</td>
<td>0.999</td>
<td>2.427</td>
<td>2.43 \times 10^3</td>
<td>5.90 \times 10^6</td>
</tr>
<tr>
<td>4.5</td>
<td>0.998</td>
<td>2.474</td>
<td>2.48 \times 10^3</td>
<td>6.15 \times 10^6</td>
</tr>
<tr>
<td>5.0</td>
<td>0.998</td>
<td>2.491</td>
<td>2.50 \times 10^3</td>
<td>6.23 \times 10^6</td>
</tr>
<tr>
<td>5.5</td>
<td>0.997</td>
<td>2.468</td>
<td>2.48 \times 10^3</td>
<td>6.13 \times 10^6</td>
</tr>
<tr>
<td>6.0</td>
<td>0.996</td>
<td>2.419</td>
<td>2.43 \times 10^3</td>
<td>5.90 \times 10^6</td>
</tr>
<tr>
<td>6.5</td>
<td>0.995</td>
<td>2.363</td>
<td>2.37 \times 10^3</td>
<td>5.64 \times 10^6</td>
</tr>
<tr>
<td>7.0</td>
<td>0.993</td>
<td>2.298</td>
<td>2.31 \times 10^3</td>
<td>5.36 \times 10^6</td>
</tr>
<tr>
<td>7.5</td>
<td>0.994</td>
<td>2.228</td>
<td>2.24 \times 10^3</td>
<td>5.02 \times 10^6</td>
</tr>
<tr>
<td>8.0</td>
<td>0.993</td>
<td>2.157</td>
<td>2.17 \times 10^3</td>
<td>4.72 \times 10^6</td>
</tr>
<tr>
<td>8.5</td>
<td>0.993</td>
<td>2.091</td>
<td>2.11 \times 10^3</td>
<td>4.43 \times 10^6</td>
</tr>
<tr>
<td>9.0</td>
<td>0.992</td>
<td>2.027</td>
<td>2.04 \times 10^3</td>
<td>4.18 \times 10^6</td>
</tr>
<tr>
<td>9.5</td>
<td>0.992</td>
<td>1.971</td>
<td>1.99 \times 10^3</td>
<td>3.95 \times 10^6</td>
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<tr>
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<td>0.999</td>
<td>1.902</td>
<td>1.90 \times 10^3</td>
<td>3.62 \times 10^6</td>
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<td>1.849</td>
<td>1.85 \times 10^3</td>
<td>3.43 \times 10^6</td>
</tr>
<tr>
<td>11.0</td>
<td>1.004</td>
<td>1.809</td>
<td>1.80 \times 10^3</td>
<td>3.25 \times 10^6</td>
</tr>
<tr>
<td>11.5</td>
<td>1.010</td>
<td>1.776</td>
<td>1.76 \times 10^3</td>
<td>3.09 \times 10^6</td>
</tr>
<tr>
<td>12.0</td>
<td>1.013</td>
<td>1.732</td>
<td>1.71 \times 10^3</td>
<td>2.92 \times 10^6</td>
</tr>
<tr>
<td>12.5</td>
<td>1.000</td>
<td>1.675</td>
<td>1.68 \times 10^3</td>
<td>2.81 \times 10^6</td>
</tr>
<tr>
<td>13.0</td>
<td>1.001</td>
<td>1.639</td>
<td>1.64 \times 10^3</td>
<td>2.68 \times 10^6</td>
</tr>
<tr>
<td>13.5</td>
<td>1.001</td>
<td>1.615</td>
<td>1.61 \times 10^3</td>
<td>2.60 \times 10^6</td>
</tr>
<tr>
<td>14.0</td>
<td>1.003</td>
<td>1.577</td>
<td>1.57 \times 10^3</td>
<td>2.47 \times 10^6</td>
</tr>
<tr>
<td>14.5</td>
<td>1.005</td>
<td>1.558</td>
<td>1.55 \times 10^3</td>
<td>2.40 \times 10^6</td>
</tr>
<tr>
<td>15.0</td>
<td>1.005</td>
<td>1.532</td>
<td>1.52 \times 10^3</td>
<td>2.32 \times 10^6</td>
</tr>
<tr>
<td>15.5</td>
<td>1.012</td>
<td>1.515</td>
<td>1.50 \times 10^3</td>
<td>2.24 \times 10^6</td>
</tr>
<tr>
<td>16.0</td>
<td>1.016</td>
<td>1.491</td>
<td>1.47 \times 10^3</td>
<td>2.15 \times 10^6</td>
</tr>
<tr>
<td>16.5</td>
<td>1.016</td>
<td>1.477</td>
<td>1.45 \times 10^3</td>
<td>2.11 \times 10^6</td>
</tr>
<tr>
<td>17.0</td>
<td>1.016</td>
<td>1.463</td>
<td>1.44 \times 10^3</td>
<td>2.07 \times 10^6</td>
</tr>
<tr>
<td>17.5</td>
<td>1.016</td>
<td>1.445</td>
<td>1.42 \times 10^3</td>
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<td>1.93 \times 10^6</td>
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<td>1.274</td>
<td>1.23 \times 10^3</td>
<td>1.51 \times 10^6</td>
</tr>
<tr>
<td>24.0</td>
<td>1.041</td>
<td>1.233</td>
<td>1.18 \times 10^3</td>
<td>1.40 \times 10^6</td>
</tr>
<tr>
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<td>1.190</td>
<td>1.15 \times 10^3</td>
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<td>1.105</td>
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<td>1.14 \times 10^6</td>
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<td>1.071</td>
<td>1.03 \times 10^3</td>
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<td>1.034</td>
<td>1.028</td>
<td>9.94 \times 10^2</td>
<td>9.88 \times 10^5</td>
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<td>0.987</td>
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<td>9.15 \times 10^5</td>
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<td>8.93 \times 10^2</td>
<td>7.97 \times 10^5</td>
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<tr>
<td>34.0</td>
<td>0.995</td>
<td>0.800</td>
<td>8.04 \times 10^2</td>
<td>6.46 \times 10^5</td>
</tr>
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<td>36.0</td>
<td>0.994</td>
<td>0.721</td>
<td>7.25 \times 10^2</td>
<td>5.26 \times 10^5</td>
</tr>
</tbody>
</table>
Experiments for the Determination of the Physical Constants $k$, $e$ and $h$

The second column of the table gives the RMS input voltage from the signal generator, given at the input of the noise detector. For this measurement, switch SW2 should be down. The third column gives the RMS voltage of the amplified output of 1/1000 of the signal generator input signal. For this measurement, switch SW2 must be up. The fourth column gives the gain $G$ calculated from the formula above. The fifth column gives $G^2$, the square of the gain.

A plot of $G^2$ against $f$ is shown in Figure 5.

![Figure 5](5.png)

The area under the curve gives $G^2B$.

5. **Shot Noise Source**

An LED is operated with a 9V battery and a potential divider. The voltage applied to the LED can be ramped up slowly by turning a POT. The brightness of the LED increases as the voltage across it increases. The light from the LED falls on a Motorola phototransistor. This is also driven by a 9V battery. The transistor is in series with a 10 kΩ resistor and a 100 kΩ pot. The resistance connected to the transistor is constant at 110 kΩ. But the noise voltage is measured across a part of the resistor, and the value of this part can be changed by turning the pot. Thus, we can vary the resistance (or equivalently $\langle V \rangle$) across which noise is measured, keeping the current through the circuit constant. The DC voltage $\langle V \rangle$ will be blocked by a 0.47 microfarad capacitor. But the noise voltage will go through and will be the input to the noise detector.
Experiments for the Determination of the Physical Constants $k$, $e$ and $h$

The front panel diagram of the noise source is shown in Figure 6.

There are four SPDT switches SW1 to SW4. SW1 switches on the power to the LED, and SW2, the power to the Motorola phototransistor, when the switches are down. SW3 should be up for shot noise measurements. The LED voltage is adjusted by POT1. The brightness of the LED increases when POT1 is turned right. This is seen by an increase in the phototransistor current. The phototransistor is connected to a 10 kΩ resistor and the two ends of a 100 kΩ pot (POT2). A DMM connected to the banana terminals marked DMM measures the DC voltage in the 2 volt range. When SW4 is put down, the DMM measures the DC voltage across 10 kΩ. So the average current $\langle I \rangle$ is this DC voltage divided by 10 kΩ. The voltage across the movable contact of POT2, relative to ground, is measured on the DMM when SW4 is up. If $\langle I \rangle$ is the current, and the voltage across the moveable contact is $\langle V \rangle$, we are measuring the shot noise across a resistance $R = \langle V \rangle / \langle I \rangle$. This variable voltage for a given current $\langle I \rangle$ appears at the banana connector marked DMM. When the switch SW3 is up, this voltage with noise appears at the BNC connector marked O/P. O/P is connected to the BNC connector marked INPUT on the front panel of the noise detector (see Figure 3).

The current $\langle I \rangle$ can be varied from 10 to 70 microamps. We work in the range of 15 to 50 microamps. The current can be measured by measuring the voltage across 10 kΩ. The variable resistance can be set at values of 20 kΩ to 80 kΩ in steps of 10 kΩ, by measuring $\langle V \rangle$ for a given $\langle I \rangle$. Thus, we set $\langle I \rangle$, then change $\langle V \rangle$ in steps and measure $v_{RMS}$ at the output of the noise detector. A set of readings are shown in Table 2.

<table>
<thead>
<tr>
<th>(I) (μA)</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>20000</td>
<td>300</td>
<td>27</td>
<td>400</td>
<td>29</td>
</tr>
<tr>
<td>30000</td>
<td>450</td>
<td>36</td>
<td>600</td>
<td>37</td>
</tr>
<tr>
<td>40000</td>
<td>600</td>
<td>43</td>
<td>800</td>
<td>45</td>
</tr>
<tr>
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<tr>
<td>60000</td>
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<td>54</td>
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<tr>
<td>80000</td>
<td>1200</td>
<td>65</td>
<td>1600</td>
<td>68</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(I) (μA)</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
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<td>31</td>
<td>800</td>
<td>31</td>
</tr>
<tr>
<td>30000</td>
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<td>50</td>
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<tr>
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<tr>
<td>70000</td>
<td>2450</td>
<td>72</td>
<td>2800</td>
<td>74</td>
</tr>
<tr>
<td>80000</td>
<td>2800</td>
<td>83</td>
<td>3200</td>
<td>83</td>
</tr>
</tbody>
</table>
A plot of $v_{RMS}$ against $\langle V \rangle$ for a current of $35 \mu A$ is shown in Figure 7.

We get a good straight line with an error in slope of about 2%. For every current from 15 to $50 \mu A$ we get similar straight lines with different slopes, with an error in slope within 3%.

We now collect the slopes $\alpha$ for different currents $\langle I \rangle$ in Table 3.

<table>
<thead>
<tr>
<th>$\langle I \rangle$ ($\mu A$)</th>
<th>$1/\langle I \rangle$ ($10_6 A$)</th>
<th>Slope $\alpha$</th>
<th>Error</th>
<th>$\alpha^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>0.067</td>
<td>0.041</td>
<td>0.001</td>
<td>0.00168</td>
</tr>
<tr>
<td>20</td>
<td>0.050</td>
<td>0.032</td>
<td>0.001</td>
<td>0.00102</td>
</tr>
<tr>
<td>25</td>
<td>0.040</td>
<td>0.030</td>
<td>0.001</td>
<td>0.00090</td>
</tr>
<tr>
<td>30</td>
<td>0.033</td>
<td>0.026</td>
<td>0.001</td>
<td>0.00068</td>
</tr>
<tr>
<td>35</td>
<td>0.029</td>
<td>0.024</td>
<td>0.001</td>
<td>0.00058</td>
</tr>
<tr>
<td>40</td>
<td>0.025</td>
<td>0.021</td>
<td>0.004</td>
<td>0.00045</td>
</tr>
<tr>
<td>45</td>
<td>0.022</td>
<td>0.019</td>
<td>0.003</td>
<td>0.00036</td>
</tr>
<tr>
<td>50</td>
<td>0.020</td>
<td>0.017</td>
<td>0.004</td>
<td>0.00030</td>
</tr>
</tbody>
</table>

Column 1 gives the average current in microamps. Column two gives $1/\langle I \rangle$. Column 3 gives the slope $\alpha$ and column 4 is the error in slope $\alpha$. Column 5 gives $\alpha^2$. We plot $\alpha^2$ against $1/\langle I \rangle$. This plot is shown in Figure 8. The linear variation of $\alpha^2$ with $1/\langle I \rangle$ is a clear signature that what we are measuring is Shot Noise. The slope $\beta$ of this graph is $2.83 \times 10^{-8}$ amps. The error in slope is about 5%.

\[ \beta = 2.83 \times 10^{-8} \text{ A} \]

Figure 8. Plot of $(\text{slope } \alpha)^2$ against $1/\langle I \rangle$. 

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Using this value of \( \beta \) and the value of \( G^2B \), we get

\[
e = \frac{\beta}{(2 \times G^2B)} = \frac{2.83 \times 10^{-8}}{(2 \times 8.7 \times 10^{10})} = 1.63 \times 10^{-19} \text{ coulombs}
\]

with an error of 5%.

A second measurement gave \( G^2B \) as \( 8.72 \times 10^{10} \text{ Hz} \) and slope \( \beta \) as \( 2.82 \times 10^{-8} \text{ amps} \). This gives \( e = 1.62 \times 10^{-19} \text{ coulombs} \) with an error of 5%.

The literature value of \( e \) is \( 1.60 \times 10^{-19} \text{ coulombs} \), which agrees with our value within the limits of error of the experiment. This shows that the methodology of the experiment is correct, and the experiment is successful.
Chapter 9
Experiments in Magnetism
9.1 Introduction to Magnetism

1. Introduction

When an external magnetic intensity field $H$ is applied to a material, the material develops a magnetization $M$, proportional to $H$.

$$ M = \mu_0 \chi_M H. \quad (1) $$

Here, $\mu_0$ is the magnetic permeability of vacuum and, in SI units, is equal to $4\pi \times 10^{-7}$ Henry/meter, and $\chi_M$ is a dimensionless number, called the magnetic susceptibility of the material. A plot of $M$ vs. $H$ is a straight line. In the absence of $H$ there is no magnetization. A material in this state is said to be paramagnetic.

If we consider an atomic picture in which atoms have magnetic moments, these moments are randomly oriented in the paramagnetic state. When the temperature of a paramagnetic material is reduced, the susceptibility $\chi_M$ increases following the Curie–Weiss relation

$$ \chi_M = \frac{C}{T - \theta}. \quad (2) $$

Here, $C$ and $\theta$ are constants characteristic of the material. If $\theta$ is positive, the susceptibility will diverge when $T = \theta$. The material develops a spontaneous magnetization in the absence of a field, at a temperature $T_C$ close to $\theta$. If we represent identical atoms strung along a linear lattice, in the atomic picture where the atoms are strung along a linear chain, the magnetic moments of all atoms point in the same direction as shown in Figure 1.

![Figure 1](image1.png)

Figure 1. A linear ferromagnetic chain of atoms.

If $\theta$ is negative, the susceptibility will not diverge. But below a temperature $T_C$ close to $\theta$, the magnetic moments of adjacent atoms will be aligned anti-parallel as shown in Figure 2. This is called anti-ferromagnetic ordering.

![Figure 2](image2.png)

Figure 2. A linear anti-ferromagnetic chain of atoms.

Unlike in the paramagnetic case, the material has no net magnetization below the temperature $T_C$, though the magnetic moments are aligned in an ordered fashion.

If the adjacent atoms on the chain have different magnetic moments, the anti-ferromagnetic ordering leads to the situation shown in Figure 3.

![Figure 3](image3.png)

Figure 3. A linear ferrimagnetic chain of atoms.

Since the magnetic moments of the two atoms are different, there will be a net magnetization in this linear chain, even though the neighboring magnetic moments point in opposite directions. This type of ordering is called ferrimagnetism and the materials are called ferrites.
These different types of ordering that one observes are due to the exchange interaction between the atoms, which will be explained below.

2. Exchange Interaction

Atoms contain electrons that occupy different quantum mechanical (QM) states. In a QM state, the electron has an orbital angular momentum \( l \) and a spin angular momentum \( s \). The orbital angular momentum is associated with a magnetic moment \( m_{\text{orb}} \) and the spin angular momentum is associated with a magnetic moment \( m_{\text{spin}} \). The relation between \( l \) and \( m_{\text{orb}} \) is

\[
m_{\text{orb}} = \left( \frac{eh}{2m} \right) l.
\]

Similarly, the relation between \( m_{\text{spin}} \) and \( s \) is

\[
m_{\text{spin}} = 2 \left( \frac{eh}{2m} \right) s.
\]

Here, \( e \) is the electron charge in coulombs, \( m \) the electron mass in kg, \( h \) is the Planck’s constant divided by \( 2\pi \) in joule-second. The magnetic moment is in SI units.

The quantity \( \left( \frac{eh}{2m} \right) / \frac{\hbar}{2} \) is called the Bohr magneton, \( \mu_B \). Its value is \( 9.27 \times 10^{-24} \) joule/tesla. Note the presence of the factor 2 in the numerator of equation (4).

There are many electrons in an atom. The atom as a whole will have a total orbital angular momentum \( L \), which in the commonly occurring L-S coupling scheme, is the vector sum of the orbital angular momentum of all the electrons, i.e.,

\[
L = \sum_j l_j.
\]

Similarly the total spin angular momentum \( S \) of the atom is given by

\[
S = \sum_j s_j.
\]

The total angular momentum \( J \) of the atom is given by

\[
J = L + S.
\]

Associated with this total angular momentum \( J \), the atom has a magnetic moment \( m_J \) given by

\[
m_J = g_J \mu_B J.
\]

where \( g_J \) is a number between 1 and 2, called the Landé splitting factor.

The electrons obey the Pauli exclusion principle, which states that no two electrons can occupy a given quantum state characterized by a complete set of quantum numbers. Each electron in a quantum state, specified by a collection of quantum numbers, is represented by a wave function, which is a product of a function of its position relative to the nucleus and another function of its spin \( s \), that is,

\[
\phi_n(r) \zeta_v(s).
\]

The spin index \( v \) which is the projection of \( s \) along the \( z \)-axis, can only take two values \( \pm \frac{1}{2} \).

Consider an atom \( \alpha \) with an electron in a state with a quantum number \( n \), at a distance \( r_\alpha \) from its nucleus, and in the spin state \( \nu \). Take its nearest neighbor atom \( \beta \) in the one-dimensional chain shown in Figure 1. Consider an electron of atom \( \beta \), at a distance \( r_\beta \) from its nucleus with a quantum number \( n' \) and in the spin state \( \nu' \). The total wave function for the pair of electrons will be a product of two wave functions \( \phi_n(r_\alpha) \zeta_v(s_\alpha) \) and \( \phi_{n'}(r_\beta) \zeta_{\nu'}(s_\beta) \).

But the two electrons cannot be distinguished one from the other. We can put electron 2 on the nucleus of \( \alpha \) and electron 1 on the nucleus of \( \beta \), giving a product of two functions \( \phi_n(r_\alpha) \zeta_v(s_\beta) \) and \( \phi_{n'}(r_\beta) \zeta_{\nu'}(s_\alpha) \). This wave function will also have the same energy. Since the two states are degenerate in energy, any linear combination of the two wave functions will also be a state with the same energy. The Pauli exclusion principle demands that the total wave function of the atom should be anti-symmetric when the two electrons are interchanged. So, the total wave function of the atom will be either
Experiments in Magnetism

\[
\Psi_{n,v,v',\nu'}(1, 2) = \left( \frac{1}{\sqrt{2}} \right) \{ \phi_n(r_{1a}) \phi_{n'}(r_{2b}) + \phi_n(r_{2a}) \phi_{n'}(r_{1b}) \} \\
\times \left( \frac{1}{\sqrt{2}} \right) \{ \xi_v(s_1) \xi_v(s_2) - \xi_v(s_2) \xi_v(s_1) \} \\
\times \left( \frac{1}{\sqrt{2}} \right) \{ \xi_{v'}(s_1) \xi_{v'}(s_2) + \xi_{v'}(s_2) \xi_{v'}(s_1) \} \\
\]  \hspace{1cm} (10a)

or

\[
\Phi_{n,v,v',\nu'}(1, 2) = \left( \frac{1}{\sqrt{2}} \right) \{ \phi_n(r_{1a}) \phi_{n'}(r_{2b}) - \phi_n(r_{2a}) \phi_{n'}(r_{1b}) \} \\
\times \left( \frac{1}{\sqrt{2}} \right) \{ \xi_v(s_1) \xi_v(s_2) + \xi_v(s_2) \xi_v(s_1) \} \\
\times \left( \frac{1}{\sqrt{2}} \right) \{ \xi_{v'}(s_1) \xi_{v'}(s_2) - \xi_{v'}(s_2) \xi_{v'}(s_1) \}. \\
\]  \hspace{1cm} (10b)

The spin indices \( v \) and \( v' \) can only take two values \( \pm \frac{1}{2} \). The factor \( 1/\sqrt{2} \) takes care of the normalization of the total wave function.

We see that both wave functions will change sign if we interchange 1 and 2, and so satisfy the Pauli exclusion principle, namely, the total wave function should be anti-symmetric under an interchange of the two electrons.

In the wave function \( \Psi_{n,v,v',\nu'}(1, 2) \), the space part is symmetric while the spin part is anti-symmetric with respect to the interchange of the two electrons. The total spin \( S = 0 \), and it has only one \( z \)-component \( m_z = 0 \), since \( v \) can only be equal to \( -v' \). If \( v \) is the same as \( v' \), the spin part of the wave function will become zero. So, \( \Psi_{n,v,v',\nu'}(1, 2) \) corresponds to a state where the two electrons have parallel spin.

In the wave function \( \Phi_{n,v,v',\nu'}(1, 2) \), the space part of the wave function will change sign when the electrons 1 and 2 are interchanged. But the spin part will not change sign. The spin part of the wave function corresponds to a total spin \( S = 1 \). Such a spin will have three components along the \( z \)-axis, namely, \( m_z = \pm 1, 0 \) or \( -1 \). The corresponding spin parts of the wave functions will be \( v = v' = +1/2 \), \( v = -v' = -1/2 \) and \( v = v' = -1/2 \).

The expectation value of the Coulomb interaction between the two electrons will be

\[
\iint \Psi_{n,v,v',\nu'}^*(1, 2) \left[ \frac{e^2}{|r_1 - r_2|} \right] \Psi_{n,v,v',\nu'}(1, 2) \, dr_1 \, dr_2, \\
\]  \hspace{1cm} (11)

with a similar expression for the state \( \Phi_{n,v,v',\nu'}(1, 2) \). Here, the integration over the electron coordinates \( r_1 \) and \( r_2 \) also involves summation over the two spin states, \( v, v' = \pm 1/2 \).

When we substitute for \( \Psi_{n,v,v',\nu'}(1, 2) \) from (10a) or for \( \Phi_{n,v,v',\nu'}(1, 2) \) from (10b) in equation (11), we will have two types of integrands in the space part of the integration, namely,

\[
\iint \phi_n^*(r_{1a}) \phi_{n'}^*(r_{2b}) \left[ \frac{e^2}{(|r_1 - r_2|)} \right] \phi_n(r_{2b}) \phi_{n'}(r_{1a}) \, dr_1 \, dr_2, \\
\]  \hspace{1cm} (12a)

and

\[
\iint \phi_n^*(r_{1a}) \phi_{n'}^*(r_{2b}) \left[ \frac{e^2}{(|r_1 - r_2|)} \right] \phi_n(r_{2b}) \phi_{n'}(r_{1a}) \, dr_1 \, dr_2. \\
\]  \hspace{1cm} (12b)

Since \( \phi_n^*(r_{1a}) \phi_n(r_{1a}) e \) is the charge density of electron 1 at the point \( r_1 \) and \( \phi_{n'}^*(r_{2b}) \phi_{n'}(r_{2b}) e \) is the charge density of electron 2 at the point \( r_2 \), the term (12a) gives the average Coulomb repulsion energy between the two electrons. This is always positive.

But in (12b), we get the product \( e \phi_n^*(r_{1a}) \phi_{n'}(r_{1a}) \). Note that the subscript is \( n \) in \( \phi_n^*(r_{1a}) \) and is \( n' \), different from \( n \), in \( \phi_{n'}(r_{1a}) \). The product \( e \phi_n^*(r_{1a}) \phi_{n'}(r_{1a}) \) cannot be interpreted as the charge density of electron 1. So the term (12b) which arises from the interchange of the two electrons is called the exchange energy. This term can be positive or negative. Both the terms (12a) and (12b) will depend on \( R_{\beta\alpha} \), which is the distance between two neighbouring nuclei, since

\[
|r_1 - r_2| = |r_{1a} - r_{2b} - R_{\beta\alpha}|, \\
\]  \hspace{1cm} (13)
as seen from Figure 4.
In Figure 4, $\alpha$ and $\beta$ are the positions of the nuclei, 1 and 2 are the positions of the electrons. $R_{\beta\alpha}$ is the vector distance of the nucleus $\beta$ from the nucleus $\alpha$.

So, the exchange energy will depend on $R_{\beta\alpha}$, and the sign can change when this distance increases. The total interaction energy between the two electrons is

$$E_{\text{inter}} = E_{\text{Coul}} - (\pm J (R_{\beta\alpha})),$$

where $J(R_{\beta\alpha})$ is the integral (12b). If $J(R_{\beta\alpha})$ is positive, the parallel orientation of the spins is favored, and we get a ferromagnetic arrangement of the magnetic moments as shown in Figure 1. If $J$ is negative, an anti-parallel arrangement of the magnetic moments will be favored, as shown in Figures 2 and 3.

The variation of exchange energy with inter-nuclear separation is shown in Figure 5 for the transition metals. This is called the Bethe–Salpeter curve. We see that Co has the highest positive exchange interaction. This situation explains the ordering of the magnetic moments of paramagnetic metal ions in insulating materials, since the discussion does not assume the presence of free electrons.

In a metal, the electrons in the conduction band are not localized on an atom, as we assumed in the earlier discussion. They occupy all energy values up to the Fermi level $E_F$, at absolute zero. Each energy level has an electron with spin up and an electron with spin down. When a magnetic field is applied, the energy levels of the spin up and spin down electrons will shift, because the electron has a magnetic moment. Because of this difference in energy, there will be more number of electrons with magnetic moments aligned in the direction of the field, and less number of electrons with magnetic moments aligned opposite to the direction of the field. This gives rise to Pauli paramagnetic susceptibility given by

$$\chi_p = 2\mu_0 N(E_F)\mu_B^2.$$

Here, $\mu_0$ is the permeability of free space = $4\pi \times 10^{-7}$ H/m and $N(E_F)$ is the number density of states at the Fermi level.

In a transition metal like iron, the d band is narrow and this gives a large density of states $N(E_F)$ at the Fermi level. So, the Pauli paramagnetic susceptibility is large. Stoner took the Weiss model of an internal field, in which the magnetization $\mathbf{M}$ produces an internal field

$$H_i = n_S M,$$

where $n_S$ is the Stoner parameter. The internal field $H_i$ causes the magnetization $\mathbf{M}$ to align with the external magnetic field $\mathbf{H}$, as shown in Figure 6.
where \( n_s \) is a constant of proportionality.

This internal field arises due to the exchange interaction. It adds to the applied field in producing the magnetization. This leads to an effective susceptibility which is given by

\[
\chi = \frac{\chi_p}{1 - n_s \chi_p}.
\]

(17)

When \( n_s \chi_p \) becomes equal to 1, the susceptibility will diverge.

This will happen at the Curie temperature \( T_C \). When this happens, the down spin energy levels will be spontaneously lowered, relative to the up spin energy levels, even in the absence of an external field. In the paramagnetic case shown in Figure 6(a), there are the same number of electrons with up spin as there are with down spin, up to the Fermi level in the absence of an applied magnetic field. The up spin electrons have magnetic moments pointing in one direction, while the magnetic moments of the down spin electrons are pointing in the opposite direction. So there is no net magnetization. On the other hand, in Figure 6(b), there are more down spin electrons up to the Fermi level than there are up spin electrons, even in the absence of an applied magnetic field. So, the material shows a net spontaneous magnetization.

Stoner stated the criterion \( n_s \chi_p > 1 \) in terms of the exchange interaction and the number density of states at the Fermi level. This is called the Stoner criterion. In ferromagnetic metals like Fe, Ni and Co, this criterion is satisfied below a certain temperature and the metals show ferromagnetic behavior.

3. **B–H Curve in Ferromagnetic and Ferrimagnetic Materials**

In a ferromagnetic material, the magnetic induction field \( B \) is not a linear function of the magnetic field \( H \). The magnetic induction field, for a given \( H \), depends on the previous history of the specimen. The curve of \( B \) vs. \( H \) is shown in Figure 7.

![Figure 7. B–H curve for a ferromagnetic metal.](image)

When the magnetic field \( H \) is very large in the positive or negative direction, the magnetic induction field \( B \) saturates at a value \( \pm B_s \), called the saturation magnetic field. At any given value of the magnetic field \( H \), there are two values for \( B \), one, while the magnetic field \( H \) is decreasing, and another, while the magnetic field \( H \) is increasing.

Thus, \( B \) depends on the history, and the phenomenon is called hysteresis. If the magnetic field is reduced from a maximum value to zero, the magnetic induction field does not go to zero. It has a value \( \pm B_r \), depending on whether the magnetic field \( H \) is brought to zero from a positive value, or from a negative value. The value of \( B_r \) is called the remnant magnetic induction field. To make the magnetic induction field \( B \) zero, one has to apply a magnetic field \( \pm H_c \) (+’ when the magnetic field is increasing and ‘−’ when it is decreasing). This field \( H_c \) is called the coercive field. The values of \( B_r \) and \( H_c \) are characteristics of a ferro- or ferri-magnetic material. A material with a small \( H_c \) is called a soft magnetic material, while one with a large \( H_c \) is called a hard magnetic material. If the magnetic material is subjected to an AC magnetic field \( H \), the area enclosed by the B–H curve gives the amount of heat generated per cycle per unit volume in the material. So, hysteresis leads to wastage of electrical energy as heat.

Examples of ferromagnetic metals at room temperature are Fe, Ni and Co. The disadvantage of metals is their relatively high electrical conductivity. In AC applications, eddy currents will be introduced in metals,
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and this will result in wasteful heat production. The larger the frequency the larger the heat power wasted. Eddy currents can be reduced by laminating the material, and coating each lamina with an insulating coating.

Ferrites are ferri-magnetic materials which also show a high saturation induction $B_s$, a remnant magnetization $B_r$ and a coercive field $H_c$. But they have high electrical resistivity and so they have very low eddy current losses. We shall give a brief introduction to ferrites because they find many commercial applications.

4. Ferrites

There are three types of ferrites: those having the spinel structure, those having a hexagonal structure and those having the garnet structure. We will only discuss the spinel structure ferrites here.

The spinels have the chemical formula $XY_2O_4$ where X and Y are metal atoms. The structure is shown in Figure 8.

![Figure 8. Structure of spinel.](image)

The structure is cubic. There are two types of sites, A and B, within the cube. A is coordinated tetrahedrally with oxygen while B is coordinated octahedrally. There are 64 tetrahedrally coordinated sites, and 32 octahedrally coordinated sites within a unit cell. Of these, 8 tetrahedrally coordinated and 16 octahedrally coordinated sites are occupied by ions of the metal atoms. If all X ions are in tetrahedral sites, and all Y ions are in octahedral sites, the structure is called a normal spinel. In the inverse spinel structure, eight Y ions occupy the eight A sites, and the 16 B sites are occupied randomly by the remaining 8 X ions and 8 Y ions. The ions, in the neighborhood of a given ion, exert an electric field on the electron belonging to the given ion. This large crystal field splits the degenerate energy levels of the free ion. The crystal field is so strong that it locks the orbital angular momentum vector of a metal ion in a fixed direction. The application of an external magnetic field cannot reorient the orbital angular momentum. So, there is no contribution of the orbital magnetic moment to the magnetic properties of the material. We say the orbital momentum is quenched. On the other hand, the crystal field has no effect on the spin angular momentum, which can reorient its direction in an applied magnetic field. So, the magnetic moment of the ion effectively comes from the spin angular momentum.

The cations (X or Y) are separated by large distances due to the intervening oxygen ion. So, the direct exchange interaction between two cations is weak. However, a strong anti-ferromagnetic exchange interaction arises due to the superexchange mechanism via the oxygen ion. In the ground state $S = 0$, the oxygen ion has no magnetic moment. In the excited state, one of the $2p$ electrons can spend some time with the neighboring cation. This results in the oxygen acquiring a non-zero spin which can interact through the direct exchange mechanism with the other neighboring cation. In effect, this is an indirect exchange mechanism between two cations, mediated by the intervening oxygen ion. This is called superexchange interaction. Its sign and strength between two cations will depend on the bond lengths and the bond angles. There are some semi-empirical rules which serve as a guide to determine the relative strengths and the
signs of the superexchange interaction between two cations. If we label by $J_{ij}$ the superexchange interaction between cation at site $i$ and cation at site $j$, then all the superexchange interactions $J_{AA}$, $J_{AB}$ and $J_{BB}$ are negative in the spinel structure and their relative strengths are $|J_{AB}| >> |J_{BB}| >> |J_{AA}|$.

Because of this, the magnetic moments of the ions at the A and B sites are oriented in opposite directions. Take the case of magnetite which is $\text{Fe}^{2+}\text{Fe}^{3+}\text{O}_4^{2-}$. In this material, the tetrahedral A sites are occupied by $\text{Fe}^{3+}$ ions. The octahedral B sites are occupied either by $\text{Fe}^{3+}$ or by $\text{Fe}^{2+}$. In the ferrimagnetic state, the magnetic moments of $\text{Fe}^{3+}$ on the tetrahedral sites cancel the magnetic moments of an equal number of $\text{Fe}^{3+}$ ions on the octahedral sites. The magnetic moment of the unit cell comes from the eight $\text{Fe}^{2+}$ ions on the B sites which are aligned parallel to each other. This is shown in Figure 9.

![Figure 9](image-url) Ordering of magnetic moments in the ferromagnetic state of magnetite.

If we substitute some of the magnetic ions by non-magnetic ones, we can change the properties of the ferrite. For example, ferrites containing nickel or manganese which are magnetic atoms, and zinc a non-magnetic atom, are soft ferrites with a low coercive field. They can be used as transformer cores. Their hysteresis loss is low. Because of their high resistivity, eddy current losses are also low. We will study the $B$–$H$ loop of one such soft ferrite. They find extensive applications in RF transformers and switched-mode power supplies.

Hard ferrites have a very high coercive field. They can be used as permanent magnets. They are useful in microwave devices (strontium ferrite), as permanent magnets in loud speakers and in magnetic recording (barium ferrite). These have a hexagonal crystal structure with high values of magneto-crystalline anisotropy. This makes it difficult to move the domain walls and hence they are good materials to make permanent magnets.

For further information one must consult books on ferrites.

### 9.2 $B$–$H$ Curve in a Hard Ferromagnetic Material, and in a Soft Ferrite

1. **Introduction**

   In the previous section, we gave a brief introduction to magnetism in materials. In this section, we will describe an experiment to record $B$–$H$ loops for a hard magnetic material and for a soft ferrite.

   The $B$–$H$ loop of a ferro- or ferri-magnetic material is sketched in Figure 1.

2. **Setup for $B$–$H$ curve**

   One may use an AC method with an oscilloscope for observing the $B$–$H$ loop of a material. But to illustrate various physical principles involved, we have preferred to use a DC method with an integrator and multimeter to record the $B$–$H$ loop.

   There are four segments in the $B$–$H$ curve. In segment I, we go from the point $A_1$ to various points on the section $A_1A_2$. In this segment, we decrease the magnetic field $H$ from a maximum value to zero, by decreasing the current which always remains positive.
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Figure 1. $B$–$H$ loop for a ferro- or ferri-magnetic material.

In segment II, we change the magnetic field from a positive maximum value to a negative value by reversing the current. We increase the magnitude of the magnetic field $H$ in steps till the point $A_3$, where the magnetic induction field is saturated in the opposite direction.

In segment III, we start from a negative maximum magnetic field $H$ at the point $A_3$, and reduce its magnitude till it becomes zero at $A_4$. The magnetic field remains negative throughout.

In segment IV, we again reverse the magnetic field starting from negative maximum value to a positive value, which is increased in steps to reach point $A_1$.

This is achieved with the setup shown in Figure 2.

Figure 2. Front panel of the $B$–$H$ curve setup box.

Inside the box, the magnetic material is in the form of a torus. SWG 26 copper wire is wound around the torus with an average number of 13 turns per centimeter. The hard magnetic material is an iron rod of 6 mm diameter which is used in civil engineering construction. Since this material has a high coercive field, we have to produce a large $H$ field with a moderate current. The maximum current we will use is 0.8 amps. So, the primary coil is made of three layers each having 13 turns per centimeter. A secondary coil of 100 turns is wound on the primary coil.

We also have a sample of a soft ferrite, supplied by Cosmo Ferrites. This is also in the form of a ring. But, its cross-sectional area is nearly six times the cross-sectional area of the iron rod. Its coercive field is low. So, we have wound only one layer of copper wire, again 13 turns per centimeter. We will operate this with a maximum current of about 0.4 A. Because of the large cross-sectional area of the ferrite, each turn of the secondary coil will be linked with a large flux. So, the number of turns in the secondary coil is reduced to 20, so that the integrator does not reach saturation during operation.

The primary coil, connected in series with a nichrome wire of about 3 ohms resistance, is connected to banana terminals at the back of the box. This is connected to a DC power supply through a rheostat with a resistance of about 5 ohms.

On the front panel, there are three DPDT switches A, B and C. There are three RCA sockets marked AMP, SEC and RH, respectively.

The primary coil circuit is shown in Figure 3.

A regulated power supply provides a DC voltage at the top terminals of the DPDT switch A. When the switch A is up, this voltage $V$ appears at the central terminals of the switch A. When the switch A is toggled down, the voltage at the central terminals of switch A is reversed. The voltage at the central terminals of A is connected to the central terminals of switch B. When switch B is up, this voltage is applied to the primary coil and sends a current through the primary coil in one direction. A DMM in the DC 10 A range, connected to the RCA socket AMP on the front panel, measures the DC current through the primary coil. A plug pattern resistance box from 1 to 1000 ohms is connected to the RCA socket marked RH on the
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Figure 3. Circuit diagram in the $B-H$ curve setup.

front panel. When switch B is put down, the DC voltage is applied through the resistance box to the middle terminals of switch C. When C is up, the DC voltage sends a current through the primary coil in series with the resistance in the resistance box. The direction of the current through the primary coil is the same as when the switch B is up. But the magnitude of the current is reduced because of the resistance in the box. When switch C is down, the primary coil is connected in the reverse direction. So, the current flows in the reverse direction.

The secondary coil is connected to the RCA socket marked SEC.

3. Principle

When a current $I$ passes through the primary coil, a magnetic intensity field $H = NI$ is generated in the rod. $N$ is the number of turns per meter on the primary coil.

$$N = nL.$$ 

Here $n$ is the number of turns/m in each layer, and $L$ is the number of layers in the primary coil. In the case of the steel rod, the number of turns $n$ per meter of the primary coil is 1300 and $L = 3$. So, the magnetic intensity field $H$ for a current $I$ is 3900 ampere-turns/m. In the case of the ferrite material, the primary coil has the same number of turns per meter, but the number of layers $L$ is 1. So, $N$ for the ferrite case is 1300/m.

By reversing the current, the magnetic field direction can be reversed. This magnetic field $H$ produces a magnetic induction field $B$ in the torus. This induction field will be larger, the larger the magnetic permeability of the material of the torus. Since the magnetic material has no free ends, there is no demagnetizing field due to free ends, as one would have with a straight coil.

A secondary coil of $n_s = 100$ turns of the same wire, is wound on the primary coil on one part of the torus of the iron rod. For the ferrite material the number of turns of the secondary coil $n_s$ is 20. For a magnetic induction field $B$, the flux $\Phi$ linked with the secondary coil is given by

$$\Phi = n_s \phi = n_s BA.$$ 

Here, $\phi$ is the flux linked with one turn of the secondary coil, and $A$ is the area of cross section of the secondary coil. $B$ changes when the magnetic field $H$ changes, and so the flux linked with the secondary coil also changes. An emf $v$ is induced in the secondary coil given by

$$v = -\frac{d\Phi}{dt} = -n_s \frac{d\phi}{dt}.$$ 

To measure the flux change when the magnetic field is changed from $H_1$ to $H_2$, we have to find the integral of $v$ over the time needed for the change in magnetic field. For this we developed the integrator, described below.


4. Integrator

Any time-varying pulse of voltage $v(t)$ can be integrated by the integrator. The output will be a DC voltage proportional to the integrated value of $v(t)$.

The front panel of the integrator is shown in Figure 4.

![Figure 4. Front panel diagram of the integrator circuit.](image)

The MAINS switch is in the lower left corner. The input signal, from the secondary coil in the $B-H$ curve experiment, is connected to the RCA socket marked INPUT. There are two toggle switches. If the switch S2 at the right is put in the position marked FOR, the input signal is directly connected to the input of the integrator chip shown in Figure 5. If the switch is put in position marked REV, the input signal is reversed before it is connected to the integrator chip. Since we use a peak detector in the final stage, an output will be obtained only for one polarity of the input signal. If the input signal polarity is opposite, the reversing switch helps to change the polarity of the input to the integrator, so that a positive voltage appears at the peak detector. The second toggle switch S1 is used to short the output of the peak detector, when it is put in the position marked SHORT. When we need to take a measurement this switch should be put in the position marked OPEN. For the $B-H$ curve experiment, the band switch BS is put in the position BH. For calibrating a search coil with a solenoid, the band switch is put in the position SC. An offset adjust potentiometer is used to adjust the offset to zero, when the toggle switch S1 is in position SHORT. The output is measured by a DMM to read DC voltage to two decimal places. The DMM is connected to the banana terminals marked DMM.

The operation of the integrator is explained below.

The pulsed voltage that is to be integrated is connected to the integrator input. The integrator circuit is shown in Figure 5.

The voltage pulse $v(t)$ produces a current

$$I(t) = \frac{v(t)}{R}.$$  

(3)
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This charges the capacitor \( C \). If the voltage across the capacitor is \( V(t) \), then

\[
\frac{C dV}{dt} = I = \frac{v(t)}{R}.
\]

So,

\[
\left( \frac{1}{CR} \right) \int_{i}^{f} v(t) \, dt = V.
\]

The subscript ‘i’ refers to the initial value and the subscript ‘f’ to the final value. The change in voltage of the capacitor is proportional to the integral of the pulsed input voltage.

For the band switch in the BH position, \( C = 47 \mu F \) and \( R = 220 \) ohms. For the band switch in the SC position, \( C = 1 \mu F \) and \( R = 220 \) ohms. In the SC position, the signal is initially amplified before it is applied to the integrator.

The output is amplified in two stages of amplification, each amplifying the integrator output by a factor of 2.2. Finally the amplified output is fed to a peak detector shown in Figure 6.

The output will be a DC voltage corresponding to the peak value of the input, provided the input is of the proper sign. If the input is of the improper sign, the reversing toggle switch S2 can be put in the position REV to make the input to the peak detector of the proper sign. The condenser of the peak detector will discharge slowly. After taking the reading of the output, the switch S1 is put in the position SHORT (Figure 4) to short the condenser, and discharge it completely. When this switch is put in the position OPEN (Figure 4), the peak detector is ready to measure again.

Putting \( v(t) = -d\Phi/dt \) in equation (5), we get

\[
V = \frac{n_s (\phi(I_{\text{init}}) - \phi(I_{\text{final}}))}{CR}.
\]

Here, \( \phi(I_{\text{init}}) \) is the flux linked with one turn of the secondary coil at the initial current \( I_{\text{init}} \), and \( \phi(I_{\text{final}}) \) is the flux linked with one turn of the secondary coil at the final current \( I_{\text{final}} \). Thus, the peak voltage across the capacitor is proportional to the change in flux.
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The peak amplified voltage $V_p$ is recorded on a multimeter connected to the banana terminals marked DMM on the front panel of the integrator.

*If $H$ is increased in the primary coil, the sign of the input voltage pulse to the integrator is negative. So, no voltage will be recorded on the meter, as the diode in the peak detector will conduct only in the forward direction. By putting the toggle switch S2 in the position REV, the input signal to the integrator is reversed, and $V_p$ becomes positive, and will go through the diode to charge the capacitor. But in noting the value, we have to put a negative sign before the reading.*

5. Apparatus Required

A DC power supply, $B–H$ curve setup, integrator, one resistance box of 1 to 1000 ohms which can carry 1 A current, one rheostat of 10 ohms and of 2A rating, a DMM M1 to measure current in DC 10A range, and another DMM M2 to measure DC volts to 2 decimal places in the DC 20 volts range.

6. Procedure

The connection diagram for the $B–H$ curve experiment is shown in Figure 7.

![Connection diagram for $B–H$ curve experiment.](image)

**Figure 7.** Connection diagram for $B–H$ curve experiment.

Legends:

PS: Regulated Power Supply; BH: BH curve setup; INT: Integrator; ☐ Toggle switches; ○ Banana sockets; ⊙ RCA Sockets; M1 DMM 10A DC; M2 DMM 20V DC; ⊙ Offset pot; RH: Rheostat; RB: Resistance box.

Connect a DC power supply through the rheostat to the banana plugs at the back of the $B–H$ curve setup. Set the rheostat around 5 ohms. Connect a DMM M1 in DC 10 A range to the RCA socket marked AMP on the $B–H$ curve box. Connect the plug pattern resistance box RB to the RCA socket marked RH on the front panel of the $B–H$ curve setup. Connect the RCA socket marked SEC on the $B–H$ curve box to the RCA socket marked INPUT on the integrator. Connect a DMM M2 in the DC 20 V range to the banana plugs marked DMM on the integrator. Set the band switch on the integrator to the position $B–H$.

There are four parts to the $B–H$ curve. The way to measure the four parts is described below.

1. Part 1 is $A_1$ to $A_2$ (Figure 1): In this part, we vary the current from $+I_{max}$ at $A_1$ to a lower positive value on the curve $A_1 A_2$. The switch settings on the $B–H$ curve box for this region are;
   - A UP; C UP; B DOWN. Switch S2 on the integrator in the FORWARD position.
   - Adjust the current to 0.8 A when all the plug keys in the box RB are plugged in. Put switch S1 on the integrator to the position SHORT and adjust the offset pot so that the reading on M2 is zero. Then, put switch S1 in position OPEN. The integrator is ready to measure.
   - To change the current from $+I_{max}$ to $+I$ ($I < I_{max}$), pull out the 20 ohms key plug on the resistance box. Note the maximum output voltage on M2, connected to the output of the integrator. Note the final current $I$ on M1.
   - Then, push switch S1 on the integrator to position SHORT. Again, adjust the offset pot, if necessary, so that M2 reads zero.

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Plug in the 20 ohm key to restore the current to 0.80 A. Push the shorting switch S1 on the integrator to the position OPEN. Then, pull out the 50 ohm key plug on the resistance box. Note the maximum reading on M2, and the final current on M1.

In this fashion one can change the current from $I_{\text{max}}$ to a final positive value $I$, down to zero, and note the M2 readings.

2. Part 2 is A$_2$ to A$_3$ (Figure 1): In this part we vary the current from $+I_{\text{max}}$ to $-I$ ($0 < I < I_{\text{max}}$).

The switch positions are the following:

A UP; C DOWN; B is pressed from UP to DOWN. Switch S2 on the integrator in the FORWARD position.

Put shorting switch S1 on the integrator to SHORT. Put B down. Adjust the negative current to a desired value by pulling out appropriate plugs from the resistance box RB. Then, push B up to get a current of $+I_{\text{max}}$. Put switch S1 to position OPEN. Adjust the offset pot so that the reading on the DMM connected to the integrator is zero. Now, push B down so that the current changes from $+I_{\text{max}}$ to $-I$. Note the maximum output voltage on M2.

Repeat this operation, changing $-I$ in steps of 0.05A, till one reaches $-I_{\text{max}}$ by plugging in all the keys in resistance box.

3. Part 3 is A$_3$ to A$_4$ (Figure 1): Here the current is changed from $-I_{\text{max}}$ to $-I$ ($0 < I < I_{\text{max}}$).

The switch settings are

A DOWN; C UP; B DOWN. Switch S2 on the integrator to the REVERSE position. All the keys in resistance box are plugged in to get the maximum current $-I_{\text{max}}$.

Put the switch S1 in position SHORT, and adjust the offset pot so that M2 shows zero reading. Then put switch S1 in position OPEN.

As in step 1, pull out the 20 ohm key plug and note the maximum reading on M2 and the current on M1. Repeat this process till the current $I$ becomes zero.

4. Part 4 is A$_4$ to A$_1$ (Figure 1): Here the current is changed from $-I_{\text{max}}$ to $+I$ ($0 < I \leq I_{\text{max}}$).

The switch settings are

A DOWN; C DOWN; B pressed from UP to DOWN. Switch S2 on the integrator in the REVERSE position.

To adjust the final current, put switch S1 on the integrator in position SHORT. Put B down, and adjust the resistance box till M1 shows the desired final current. Then, push B up. Put switch S1 in position OPEN, and then, push B down. Note the maximum reading on M2, and the final current on M1.

Note: When switch A is down and the current is changed from $-I_{\text{max}}$, the secondary coil voltage gives a negative pulse. The integrated value of this pulse will not pass through the diode in the peak detector. So we put switch S2 on the integrator in the REVERSE position to reverse the input voltage. The output voltage read by M2 will always be positive. But when S2 is in the REVERSE position, we enter the voltage in the table with a minus sign.

We note the readings of the current and voltage in tabular form as shown in Table 1. The table is divided into four parts. The first two parts are under the heading, Current decreasing from $+I_{\text{max}}$ (+0.8A). These contain the readings for the part A$_1$A$_2$A$_3$ of the $B$--$H$ curve. The second two parts of the table are under the heading, Current increasing from $-I_{\text{max}}$ (0.8A). These trace the part A$_3$A$_4$A$_1$ of the $B$--$H$ curve.
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A sample set of readings are given in Table 1.

Table 1. B–H Curve for hard iron.

<table>
<thead>
<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_i ) (volts)</th>
<th>( \Delta \phi ) (10(^{-7}) webers)</th>
<th>( \phi ) (10(^{-5}) webers)</th>
<th>B (teslas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>UP</td>
<td>0.80</td>
<td>312</td>
<td>0</td>
<td>0</td>
<td>2.99</td>
<td>0.79</td>
</tr>
<tr>
<td>B</td>
<td>DN</td>
<td>0.52</td>
<td>2028</td>
<td>0.01</td>
<td>2.14</td>
<td>2.97</td>
<td>0.78</td>
</tr>
<tr>
<td>C</td>
<td>UP</td>
<td>0.29</td>
<td>1131</td>
<td>0.09</td>
<td>19.20</td>
<td>2.80</td>
<td>0.74</td>
</tr>
<tr>
<td>S2</td>
<td>Forward</td>
<td>0.16</td>
<td>624</td>
<td>0.25</td>
<td>53.40</td>
<td>2.35</td>
<td>0.65</td>
</tr>
<tr>
<td>PULL PLUGS</td>
<td>0.09</td>
<td>351</td>
<td>0.30</td>
<td>64.10</td>
<td>2.20</td>
<td>0.58</td>
<td></td>
</tr>
<tr>
<td>FROM RB</td>
<td>0.04</td>
<td>156</td>
<td>0.43</td>
<td>91.90</td>
<td>2.07</td>
<td>0.55</td>
<td></td>
</tr>
</tbody>
</table>

Current increasing from \(-I_{\text{max}}\) (0.8 A); Part 3.

<table>
<thead>
<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_i ) (volts)</th>
<th>( \Delta \phi ) (10(^{-7}) webers)</th>
<th>( \phi ) (10(^{-5}) webers)</th>
<th>B (teslas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Down</td>
<td>-0.8</td>
<td>-1677</td>
<td>0</td>
<td>0</td>
<td>-3.15</td>
<td>-0.83</td>
</tr>
<tr>
<td>B</td>
<td>Down</td>
<td>-0.43</td>
<td>-1677</td>
<td>-0.11</td>
<td>-24.00</td>
<td>-2.91</td>
<td>-0.77</td>
</tr>
<tr>
<td>C</td>
<td>Down</td>
<td>-0.29</td>
<td>-1131</td>
<td>-0.18</td>
<td>-38.00</td>
<td>-2.77</td>
<td>-0.73</td>
</tr>
<tr>
<td>S2</td>
<td>Reverse</td>
<td>-0.09</td>
<td>-351</td>
<td>-0.39</td>
<td>-83.00</td>
<td>-2.32</td>
<td>-0.61</td>
</tr>
<tr>
<td>PULL PLUGS</td>
<td>-0.04</td>
<td>-156</td>
<td>-0.43</td>
<td>-92.00</td>
<td>-2.23</td>
<td>-0.59</td>
<td></td>
</tr>
<tr>
<td>FROM RB</td>
<td>0</td>
<td>0</td>
<td>-0.44</td>
<td>-94.00</td>
<td>-2.21</td>
<td>-0.58</td>
<td></td>
</tr>
</tbody>
</table>

172
The first two columns of the table tracing the parts 1 and 2 give the switch positions. The third column gives the maximum current $+I_{\text{max}}$ and the other set currents $+I$ or $-I$ on the branch $A_1A_2A_3$ of the $B-H$ curve. The fourth column is the magnetic intensity field $H$ corresponding to the current $I$. Since the primary coil has 3900 turns/m, $H$ is obtained by multiplying the current in column three by 3900. Column 5 gives the peak output voltage as read on M2 when the current is changed from $+I_{\text{max}}$ to $+I$ by pulling plugs from RB or $-I$ by pressing switch B down. The first row in this column will be zero as it will correspond to no change in current. From this, the flux change, $(\phi(I_{\text{max}}) - \phi(I)) = \Delta \phi$, is obtained by dividing the voltage in column 5 by $4.84 \, \text{n}$/$CR = (4.84 \times 100)/(47 \, \mu\text{F} \times 220 \, \text{ohms})$. This flux change is in webers and is in column 6. The last reading in this column corresponds to a change $((\phi(I_{\text{max}}) - \phi(-I_{\text{max}})) = 2 \phi(I_{\text{max}})$. Column 7 gives
the flux \( \phi(I) \). The first row in this column is obtained by dividing the last value in column 6 by 2. The other rows in column 7 are obtained by subtracting from the first value in column 7 the corresponding \( \Delta \phi \) value in column 6. Thus, we obtain in column 7 the flux values at various currents. Column 8 gives the magnetic induction field \( B(I) \). This is obtained by dividing the flux value \( \phi(I) \) in column 7 by \( A \), the area of cross section of the secondary coil, which is \( 0.38 \times 10^{-4} \) m\(^2\). Thus, we have obtained \( B \) as a function of \( H \) for the part \( A_1A_2A_3 \) of the \( B-H \) curve.

For the part \( A_3A_4A_1 \), the calculations are shown in the corresponding columns in the table above tracing part 3 and 4. Now, we are increasing the current from \( -I_{\text{max}} \). Since we have put the switch \( S_2 \) on the integrator in position REV for this part, the output voltage must be taken with a negative sign. The flux \( \phi(I_{\text{max}}) \) which is in the first row of column 7 for the part \( A_3A_4A_1 \), is (1/2) of the last value at the bottom of column 6. The flux at any other value of current is obtained by subtracting the value in column 6 from this value.

The curve of \( B \) vs. \( H \) is shown in Figure 8.

A broad hysteresis curve is seen, indicating that the material is a hard ferromagnet. From the figure, we get \( H_C = (1140 + 1044)/2 = 1092 \) amp.turns/m, \( B_r = (0.551 + 0.583)/2 = 0.567 \) T. The area of the loop is \( 3390 \) J/m\(^3\). This is the energy loss per cycle due to magnetic hysteresis in the material. If the loop had been a square extending from \(-H_c\) to \(+H_c\) along the X–axis and from \(-B_r\) to \(+B_r\) along the Y-axis, the area of the loop would be \( 4B_rH_c = 2477 \) J/m\(^3\). The actual area under the loop is 137% of this value. In fact, the product \( 4B_rH_c \) roughly characterizes the hysteresis loss in a material.

For example, if a current of 0.80 A at 50 Hz flows through the torus, the heat generated per second would be \( 3390 \times 50 = 169500 \) W/m\(^3\). This will heat up the specimen. The higher the frequency, the higher the heat generated by hysteresis loss. This is in addition to the eddy current loss in the material. To reduce hysteresis loss in a power transformer, we use a soft ferromagnetic core with a low \( B_rH_c \) value, and the eddy current loss is minimized by making the core out of laminated sheets.

The experiment was repeated with a torus with the soft ferrite material. The set of readings are given in Table 2.

For the soft ferrite, the number of turns per meter in the primary coil is 1300. The number of turns of the secondary coil is 20. The cross section of the torus is rectangular with a breadth of 18.1 mm and a width of 15 mm. So, area of cross section is \( 2.72 \times 10^{-4} \) m\(^2\). The material saturates at a current of 0.4 A.

### Table 2. \( B-H \) curve for soft ferrite (Cosmo ferrite).

<table>
<thead>
<tr>
<th>Primary turns/m, ( N )</th>
<th>1300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary turns, ( n_s )</td>
<td>20</td>
</tr>
<tr>
<td>CR Value</td>
<td>0.01034</td>
</tr>
<tr>
<td>Amplification</td>
<td>4.84</td>
</tr>
<tr>
<td>Area of secondary coil, ( A )</td>
<td>( 2.72 \times 10^{-4} ) m(^2)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_p ) (volts)</th>
<th>( \Delta \phi ) (10(^{-7}) webers)</th>
<th>( \phi ) (10(^{-5}) webers)</th>
<th>( B ) (teslas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>UP</td>
<td>0.400</td>
<td>520.0</td>
<td>0</td>
<td>0</td>
<td>8.06</td>
<td>0.30</td>
</tr>
<tr>
<td>B</td>
<td>DN</td>
<td>0.328</td>
<td>426.4</td>
<td>0</td>
<td>0</td>
<td>8.06</td>
<td>0.30</td>
</tr>
<tr>
<td>C</td>
<td>UP</td>
<td>0.278</td>
<td>361.4</td>
<td>0</td>
<td>0</td>
<td>8.06</td>
<td>0.30</td>
</tr>
<tr>
<td>S2</td>
<td>Forward</td>
<td>0.190</td>
<td>247.0</td>
<td>0</td>
<td>0</td>
<td>8.06</td>
<td>0.30</td>
</tr>
<tr>
<td>PULL</td>
<td>PLUGS</td>
<td>0.125</td>
<td>162.5</td>
<td>0.03</td>
<td>32.0</td>
<td>7.74</td>
<td>0.29</td>
</tr>
<tr>
<td>FROM</td>
<td>RB</td>
<td>0.074</td>
<td>96.2</td>
<td>0.08</td>
<td>85.5</td>
<td>7.21</td>
<td>0.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.033</td>
<td>42.9</td>
<td>0.22</td>
<td>235.0</td>
<td>5.71</td>
<td>0.21</td>
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<tr>
<td></td>
<td></td>
<td>0.017</td>
<td>22.1</td>
<td>0.26</td>
<td>278.0</td>
<td>5.29</td>
<td>0.19</td>
</tr>
</tbody>
</table>
### Experiments in Magnetism

**Table 2. (Contd.)**

Current decreasing from +0.4 A.

<table>
<thead>
<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_p ) (volts)</th>
<th>( \Delta \phi ) ( \times 10^{-7} ) webers</th>
<th>( \phi ) ( \times 10^{-5} ) webers</th>
<th>( B ) (teslas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>UP</td>
<td>-0.012</td>
<td>-15.6</td>
<td>0.53</td>
<td>566.00</td>
<td>2.40</td>
<td>0.09</td>
</tr>
<tr>
<td>C</td>
<td>DN</td>
<td>-0.017</td>
<td>-22.1</td>
<td>0.58</td>
<td>620.00</td>
<td>1.87</td>
<td>0.07</td>
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<tr>
<td>S2</td>
<td>Forward</td>
<td>-0.029</td>
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<td>694.00</td>
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<td>B</td>
<td>UP to DN</td>
<td>-0.039</td>
<td>-50.7</td>
<td>0.77</td>
<td>823.00</td>
<td>-0.16</td>
<td>0.00</td>
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<td>-3.47</td>
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<td></td>
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<td>-4.97</td>
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<td>1.36</td>
<td>1450.00</td>
<td>-6.46</td>
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<td></td>
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<td>-0.21</td>
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<tr>
<td></td>
<td></td>
<td>-0.239</td>
<td>-310.7</td>
<td>1.39</td>
<td>1480.00</td>
<td>-6.78</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>-0.321</td>
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<td>1.47</td>
<td>1570.00</td>
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<tr>
<td></td>
<td></td>
<td>-0.4</td>
<td>-520</td>
<td>1.51</td>
<td>1610.00</td>
<td>-8.07</td>
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</table>

Current increasing from -0.4 A.

<table>
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<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_p ) (volts)</th>
<th>( \Delta \phi ) ( \times 10^{-7} ) webers</th>
<th>( \phi ) ( \times 10^{-5} ) webers</th>
<th>( B ) (teslas)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>DN</td>
<td>-0.400</td>
<td>-520</td>
<td>0</td>
<td>0</td>
<td>-7.96</td>
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<tr>
<td>B</td>
<td>DN</td>
<td>-0.289</td>
<td>-375.7</td>
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<tr>
<td>C</td>
<td>UP</td>
<td>-0.248</td>
<td>-322.4</td>
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<tr>
<td>S2</td>
<td>Reverse</td>
<td>-0.164</td>
<td>-213.2</td>
<td>0</td>
<td>0</td>
<td>-7.96</td>
<td>-0.29</td>
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<tr>
<td>PULL</td>
<td>PLUGS</td>
<td>-0.105</td>
<td>-136.5</td>
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<td>FROM</td>
<td>RB</td>
<td>-0.060</td>
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<td>-16.9</td>
<td>-0.31</td>
<td>-331</td>
<td>-4.65</td>
<td>-0.17</td>
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Current increasing from -0.4 A.

<table>
<thead>
<tr>
<th>Switch</th>
<th>Position</th>
<th>( I ) (amps)</th>
<th>( H ) (amp.turns/m)</th>
<th>( V_p ) (volts)</th>
<th>( \Delta \phi ) ( \times 10^{-7} ) webers</th>
<th>( \phi ) ( \times 10^{-5} ) webers</th>
<th>( B ) (teslas)</th>
</tr>
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<tbody>
<tr>
<td>A</td>
<td>DN</td>
<td>0.007</td>
<td>9.1</td>
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<td>-0.12</td>
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<td>DN</td>
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<td>-2.40</td>
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<td>Reverse</td>
<td>0.016</td>
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<td>UP to DN</td>
<td>0.021</td>
<td>27.3</td>
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<td>0.031</td>
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<td></td>
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<td>0.125</td>
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<td>0.173</td>
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<td></td>
<td>0.235</td>
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<td>-1550.00</td>
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<td>0.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.400</td>
<td>520</td>
<td>-1.49</td>
<td>-1590.00</td>
<td>7.96</td>
<td>0.29</td>
</tr>
</tbody>
</table>
The $B-H$ curve is shown in Figure 9.

From the curve we see that the remnant induction is $(0.12+0.15)/2 = 0.135$ tesla. The coercive field is $(52+52)/2 = 52$ amp.turns/m. The area of the loop is $82.9 \text{ J/m}^2$. Compare this with the coercive field of the steel rod. This is very low. The area of the loop is very small. This material is good for being used as cores of transformers.

Because we are using a plug pattern resistance box to change the current, the current can be only changed by discrete amounts. This is a serious limitation in the case of the soft ferrites because the rapid change in $B$ occurs at very small values of the current.

Dr Ramesh has made an arrangement so that the current can be changed accurately at small values of current. With this arrangement we get a better curve for the soft ferrites. This arrangement is not yet commercially available.

6. Questions

1. Calculate the heat generated by hysteresis per second for a Cosmo Ferrite sample of 0.1 $\text{m}^3$ volume if 50 cycles AC is used. If the mass of the material is 500 grams and its specific heat is 0.8 cal/gm per °C, calculate the rate of rise in temperature of the sample assuming no loss of heat.

2. Of the two materials discussed here, which material will you prefer for (i) making a permanent magnet, and (ii) for making a transformer core?
Chapter 10

Experiment in Thermal Relaxation
Time of a Serial Light Bulb
10.1 Measurement of Thermal Relaxation Time of a Serial Light Bulb

1. Introduction

Relaxation phenomena occur very commonly in physics. Take a spherical ball of radius \( a \) moving with an initial velocity \( v(0) \) in a medium with viscosity \( \eta \). The equation of motion for the ball is

\[
m \left( \frac{dv}{dt} \right) + 6\pi \eta av = 0.
\]

Its velocity will decrease exponentially with time as

\[
v(t) = v(0) \exp \left( -\frac{t}{\tau} \right).
\]

It will come to rest eventually. The relaxation time \( \tau \) is given by

\[
\tau = \frac{m}{6\pi \eta a}.
\]

Here is another example. A sample is connected to a bath at temperature \( T_0 \). It is heated to a temperature \( T \). Then heat is cut off. The sample will cool, and its temperature \( T(t) \), as a function of time \( t \), satisfies the equation

\[
ms \frac{dT}{dt} + E(T - T_0) = 0.
\]

Here, \( m \) is the mass of the sample, \( s \) its specific heat and \( E \) the constant in Newton’s law of cooling. The second term in the above equation represents the heat lost in one second by the sample to the bath. The solution to this equation is

\[
T(t) = (T - T_0) \exp \left( -\frac{t}{\tau} \right) + T_0.
\]

The temperature of the sample will approach the temperature of the bath exponentially with a characteristic relaxation time \( \tau \) given by

\[
\tau = \frac{ms}{E}.
\]

A third example is the following. An electric field \( E \) is applied to a polar dielectric medium like acetone. This creates an electric polarization \( P \) in the medium. When the electric field is switched off at time \( t = 0 \), the polarization decays to zero exponentially with time. There is a characteristic relaxation time associated with this process. The mechanism of relaxation here is more complicated than in the other two cases considered above.

This is also true of magnetization created in a paramagnetic material on the application of a magnetic field. When the magnetic field is switched off, the magnetization in the specimen decreases exponentially to zero with a characteristic relaxation time.

Relaxation is a phenomenon characteristic of dissipative mechanical, thermal, electric and magnetic systems. If \( X \) is the material property, which relaxes with a characteristic relaxation time \( \tau \) in the absence of a ‘force’, it satisfies the equation

\[
\frac{dX}{dt} + \frac{X}{\tau} = 0.
\]

Let us now consider forcing the system at a frequency \( f \). For the ball moving in a viscous medium, this is achieved by applying a mechanical force \( F \exp (i\omega t) \), where

\[
\omega = 2\pi f.
\]

In the case of the thermal system, forcing is done by applying a periodic heat \( Q = Q_0 \exp (i\omega t) \) to the system. In the dielectric example, forcing is done by applying a periodic electric field \( E \exp (i\omega t) \). In the magnetic example, forcing is done by applying a magnetic field \( B \exp (i\omega t) \). A relaxation system forced by a simple harmonic force will satisfy the equation

\[
\frac{dX}{dt} + \frac{X}{\tau} = F \exp (i\omega t).
\]

The solution to this equation is
Experiment in Thermal Relaxation Time of a Serial Light Bulb

\[ X(t) = \left[ \frac{F \tau}{(1 + i\omega \tau)} \right] \exp(i\omega t). \] (10)

The response of the system is also simple harmonic with the same frequency \( f \). But the response is not in phase with the force. If we write

\[ \frac{F \tau}{(1 + i\omega \tau)} = A(\omega) \exp(-i\phi), \] (11)

then

\[ A(\omega) = \frac{F \tau}{(1 + \omega^2 \tau^2)^{1/2}}, \] (12)

and

\[ \tan(\phi) = \omega \tau. \] (13)

For a fixed amplitude of ‘force’ \( F \) the amplitude \( A(\omega) \) of the response decreases as the frequency increases, as shown by equation (12). The amplitude, as the frequency tends to zero, is \( A(0) = F \tau \). If \( \omega \tau >> 1 \), the amplitude \( A(\omega) \) at frequency \( \omega \) will decrease as

\[ A(\omega) = \frac{A(0)}{\omega \tau}. \] (14)

The phase lag \( \phi \) varies as (13) and approaches \( \pi/2 \) at very high frequencies.

2. Theory of the Experiment

Let us consider a serial light bulb. We pass a current

\[ I = I_0 + I_1 \sin \omega t \] (15)

through the bulb, where \( I_1 \) is small compared to \( I_0 \). If we choose \( I_0 \) and \( I_1 \) suitably, and the frequency \( f \) of the oscillating current is small (about 10 Hz), we can see the bulb blinking. Let \( m \) be the mass of the filament and \( s \) its specific heat. The bulb will lose heat through the thermal conductance of the support connecting the filament to the cap of the bulb, and through radiation. We model the heat loss by Newton’s law of cooling. We measure the temperature difference \( \theta \) of the filament over that of the cap, which is at the ambient temperature \( T_0 \). The heat power input to the filament is

\[ Q = I^2 R = \left( I_0^2 + \left( \frac{1}{2} \right) I_1^2 \right) R + 2I_0I_1R \sin(\omega t) - \left( \frac{1}{2} \right) I_1^2 R \cos(2\omega t), \] (16)

i.e.,

\[ Q = Q_0 + Q_1 \sin \omega t - Q_2 \cos 2\omega t. \] (17)

The temperature \( \theta \) of the filament, above the temperature \( T_0 \), satisfies the equation

\[ ms \frac{d\theta}{dt} + E\theta = Q_0 + Q_1 \sin(\omega t) - Q_2 \cos(2\omega t), \] (18)

where \( E \) is the constant in Newton’s law of cooling.

Writing \( E/ms = 1/\tau \), \( Q_j/ms = \Theta_j \), the above equation can be written as

\[ \frac{d\theta}{dt} + \frac{\theta}{\tau} = \Theta_0 + \Theta_1 \sin \omega t - \Theta_2 \sin 2\omega t. \] (19)

Integrating this equation from \( t = 0 \), when \( \theta = 0 \), we get

\[ \theta(t) = \Theta_0 \left[ 1 - \exp \left( \frac{-t}{\tau} \right) \right] + \left( \frac{\Theta_1}{\left[ 1 + \omega^2 \tau^2 \right]^{1/2}} \right) \sin \left( \omega t + \phi_1 \right) \]

\[ - \left( \frac{\Theta_2}{\left[ 1 + 4\omega^2 \tau^2 \right]^{1/2}} \right) \sin \left( 2\omega t + \phi_2 \right). \] (20)
Experiment in Thermal Relaxation Time of a Serial Light Bulb

where
\[ \tan(\phi_1) = \omega r, \quad \text{and} \quad \tan(\phi_2) = 2\omega r. \]  \hspace{1cm} (21)

If \( I_1/I_0 = 0.1 \), then \( \Theta_1/\Theta_0 \approx 0.2 \) and \( \Theta_2/\Theta_0 \approx 5 \times 10^{-3} \). So, the second harmonic signal is less than 2.5% of the first harmonic. We shall neglect the second harmonic term.

The intensity of the radiation emitted from the filament will vary as \( T^a \). The exponent \( a \) has a value close to four. So the intensity of the emitted light will vary as
\[ I_L = I_{L0} + I_{L1} \sin(\omega t), \]  \hspace{1cm} (22)

where \( I_L \) will be proportional to
\[ \Theta_0^a + \alpha \left( \frac{\Theta_1}{\Theta_0} \right) \Theta_0^{a-1} \left[ \frac{1}{(1 + \omega^2 \tau^2)^{1/2}} \right] \sin(\omega t + \phi_1). \]

We pick this intensity fluctuation with a photodiode circuit in the linear mode. The photodiode BPW 34 will give an output voltage proportional to the intensity of the light. The RMS AC output voltage \( V_{\text{out}} \) of the photodetector is measured with a multimeter.

The frequency \( f \) of the AC signal is changed, keeping the AC voltage across the bulb constant. The resulting set of measurements of the photodiode AC output voltage \( V_{\text{AC}}(f) \) is fitted to a formula
\[ V_{\text{AC}}(f) = \frac{V(0)}{[1 + (2\pi f \tau)^2]^{0.5}}. \]  \hspace{1cm} (23)

The value of \( V(0) \) and \( \tau \) are found as described below.

3. Thermal Relaxation Setup

The photodiode is BPW 34. The photodiode circuit is shown in Figure 1.

![Figure 1. Photodiode circuit.](image-url)

The blinking light source is a 3 V serial light bulb. The connection to the light bulb is as shown in Figure 2.

The front panel diagram of the relaxation setup is shown in Figure 3.

A resistance of 6 ohms (1 W) and a resistance of 2 ohms (1 W) are connected in series with the bulb and a 5 V regulated DC supply, as shown in Figure 2. When the switch SW2 is pressed down, the DC voltage across the 6 ohm resistance can be measured by DMM2, in the DC 2 V range, connected to the terminals \( T_1 \) and \( T_2 \). This voltage, divided by 6, gives the DC current through the bulb. When the switch SW1 is in the LC position (LC stands for low current), the bulb glows a dull red. One can connect an external plug pattern resistance box (0.1 to 10 ohm) across the two ohm resistor, and can vary the current through the bulb by adjusting the resistance in the box. The plug pattern box is connected to the terminals marked RB. When the switch SW1 is in the HC (high current) position, it shorts the resistance of 2 ohms. The current through the bulb is then a maximum and the bulb glows brightest. The distance of the photodiode from the bulb is adjusted by turning the screw N. The photodiode output DC voltage is measured by another multimeter DMM1, at the terminals marked PD O/P. This should be about 4.5 to 5 V in the high current position of SW1.
Experiment in Thermal Relaxation Time of a Serial Light Bulb

**Figure 2.** Connections for relaxation experiment.

**Figure 3.** Front panel diagram of thermal relaxation setup.
A signal generator is connected to the terminals marked SG. It sends a current through two 50 ohm resistances and two 1000 μF capacitors connected in series with the bulb. This AC current is superposed on the DC current through the bulb. If the frequency is low, one can see the bulb blinking. The RMS AC voltage across 50 ohms can be measured by DMM2 in the AC 2 volts range, by throwing the switch SW2 upwards to the position marked AC. The output of the signal generator is adjusted so that the RMS AC voltage across the 50 ohm resistor as measured by DMM2 is 1 volt. The AC current is about 10 to 20% of the DC current. The AC frequency is changed, keeping the AC voltage across the 50 ohm resistor constant, by adjusting the signal amplitude knob on the signal generator. The AC current causes the temperature of the filament in the bulb to oscillate about a mean temperature, determined by the DC current. This produces a fluctuating AC output of the photodiode at the frequency of the AC current. The RMS value of the fluctuating voltage is measured by DMM1, set to measure AC 200 millivolts. The frequency of the AC voltage is varied in steps and the AC output voltage is measured.

4. Apparatus Required

Thermal relaxation setup box, a signal generator, two DMMs which will measure DC volts, AC volts in the 200 mV range.

5. Procedure

(i) AC Part

Switch on the thermal relaxation setup. Put the DPDT switch SW1, at the right top, to position HC. Put the DPDT switch SW2, at the right bottom, to position DC. Connect DMM2 between the banana terminals marked T₁ and T₂ on the right. Set it in DC 2 volts range to measure the voltage to three decimal places. In our experiment, the meter showed a reading around 1.8 V. This was the DC voltage across 6 ohms in series with the bulb. If another meter, DMM1, is connected between terminals marked PD O/P on the top left of the panel, the meter will show an amplified photodiode output. The distance between the photodiode and the bulb can be changed by releasing the locking nut and turning the screw N at the center of the panel. The reading on the meter will increase as the photodiode is brought closer to the bulb, and it will decrease as the photodiode is moved farther from the bulb. Adjust the screw so that the DC output of the photodiode is between 4 and 5 volts in magnitude. Lock the nut.

Connect a signal generator between the two banana plugs marked SG on the bottom left. The lower of the two banana terminals should be connected to the earth of the signal generator. Set a frequency of about 20 Hz on the signal generator, and the output voltage around 2 V AC. Put the DPDT switch SW2 at the bottom right corner to AC. The meter DMM2 should now be set to read AC 2 volts. The meter will read the AC voltage across 50 ohms, in series with the bulb. Adjust the output amplitude of the signal generator so that the meter reads 1 V AC. The RMS value of the AC current through the bulb is 1/50 = 0.020 A. Adjust the frequency to 20 Hz by turning the frequency adjusting knob on the signal generator. Again, check that the AC voltage read by DMM2 is 1 V. Then read the AC voltage at the terminals marked PD O/P using DMM1 in the AC 200 mV mode. Note this reading. Adjust the AC frequency from the signal generator to be 22 Hz. Check that the AC voltage read by DMM2 is 1 V. Measure the AC output voltage of the photodiode on DMM1. Repeat these measurements as the frequency increases from 20 to 100 Hz and as the frequency decreases from 100 to 20 Hz. The average signal output in mV is taken at each frequency.

\[ \text{Always keep the AC voltage across } T₁T₂ \text{ terminals to be } 1 \text{ V at each frequency.} \]

Table 1 shows a sample set of readings. The table is self-explanatory. Column 4 gives the average of the two values of the AC photodiode output in columns 2 and 3. Column 5 gives \( 1/V_{\text{ACave}} \).

For the serial light bulbs, the relaxation time is of the order of 0.1 s. So, even at the lowest frequency of 20 Hz, \( 2\pi f \tau \) is about 12, and \((2\pi f \tau)^2\) is more than unity in the denominator of equation (23). We may neglect unity in the denominator, and \( V_{\text{ACave}} \) should then be proportional to \( 1/f \). In Figure 4 we plot \( 1/V_{\text{ACave}} \) against the frequency.
Experiment in Thermal Relaxation Time of a Serial Light Bulb

Table 1. Thermal relaxation measurement AC part. DC voltage across 6 ohms: 1.862 V. AC output of photodiode: 5.2 V. AC voltage across 50 ohms: 1.0 V.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{AC, f \text{ increasing}}$ (mV)</th>
<th>$V_{AC, f \text{ decreasing}}$ (mV)</th>
<th>Average (mV)</th>
<th>$1/V_{AC, \text{ ave}}$</th>
<th>$V_{AC}$ (mV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>117</td>
<td>119</td>
<td>118</td>
<td>0.008</td>
<td>113</td>
</tr>
<tr>
<td>22</td>
<td>108</td>
<td>106</td>
<td>107</td>
<td>0.009</td>
<td>103</td>
</tr>
<tr>
<td>24</td>
<td>100</td>
<td>102</td>
<td>101</td>
<td>0.010</td>
<td>94</td>
</tr>
<tr>
<td>26</td>
<td>92</td>
<td>92</td>
<td>92</td>
<td>0.011</td>
<td>87</td>
</tr>
<tr>
<td>28</td>
<td>83</td>
<td>85</td>
<td>84</td>
<td>0.012</td>
<td>81</td>
</tr>
<tr>
<td>30</td>
<td>81</td>
<td>81</td>
<td>81</td>
<td>0.012</td>
<td>75</td>
</tr>
<tr>
<td>35</td>
<td>65</td>
<td>67</td>
<td>66</td>
<td>0.015</td>
<td>65</td>
</tr>
<tr>
<td>40</td>
<td>58</td>
<td>58</td>
<td>58</td>
<td>0.017</td>
<td>56</td>
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<td>45</td>
<td>52</td>
<td>52</td>
<td>52</td>
<td>0.019</td>
<td>50</td>
</tr>
<tr>
<td>55</td>
<td>47</td>
<td>47</td>
<td>47</td>
<td>0.021</td>
<td>41</td>
</tr>
<tr>
<td>60</td>
<td>38</td>
<td>38</td>
<td>38</td>
<td>0.026</td>
<td>38</td>
</tr>
<tr>
<td>70</td>
<td>33</td>
<td>33</td>
<td>33</td>
<td>0.030</td>
<td>32</td>
</tr>
<tr>
<td>80</td>
<td>28</td>
<td>28</td>
<td>28</td>
<td>0.036</td>
<td>28</td>
</tr>
<tr>
<td>90</td>
<td>26</td>
<td>26</td>
<td>26</td>
<td>0.038</td>
<td>25</td>
</tr>
<tr>
<td>100</td>
<td>23</td>
<td>23</td>
<td>23</td>
<td>0.043</td>
<td>23</td>
</tr>
</tbody>
</table>

Figure 4. Plot of $1/V_{AC, \text{ ave}}$ against frequency.

The slope of this graph, $2\pi\tau/V(0)_{RMS}$, is $4.39 \times 10^{-1}$ s/V. The AC voltages measured on the DMM are RMS voltages. To find $\tau$, we should find $V(0)_{RMS}$. This is done by doing a DC experiment described below.

(ii) DC Part

Remove the signal generator connections. Put the DPDT switch SW1 on the right top to LC. Connect a plug pattern resistance box (0.1 to 10 ohms) to the terminals marked RB. Put the bottom DPDT switch SW2 to position DC. Connect DMM2 in the DC 2 volts mode to the terminals $T_1$ and $T_2$. Pull out 0.1 ohm from the plug pattern box. Measure the voltage $V$ on DMM2. The current through the bulb is now $(V/6)$ A. Measure the output DC voltage $V_{DC,PD}$ of the photodiode on DMM1. Repeat the measurement as the resistance in the box is increased in small steps of 0.1 or 0.2 ohms, till the PD output voltage comes down to about less than half the value for the maximum current when switch SW2 is put in HC mode.

A sample set of readings is given in Table 2.
Experiment in Thermal Relaxation Time of a Serial Light Bulb

Table 2. DC photodiode output as a function of DC current through the bulb.

<table>
<thead>
<tr>
<th>DC voltage across 6Ω (volts)</th>
<th>$I$ (amps)</th>
<th>$I^2$</th>
<th>DC output of PD (volts)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.843</td>
<td>0.307</td>
<td>0.094</td>
<td>3.459</td>
</tr>
<tr>
<td>1.837</td>
<td>0.306</td>
<td>0.094</td>
<td>3.321</td>
</tr>
<tr>
<td>1.817</td>
<td>0.303</td>
<td>0.092</td>
<td>2.858</td>
</tr>
<tr>
<td>1.771</td>
<td>0.295</td>
<td>0.087</td>
<td>1.993</td>
</tr>
<tr>
<td>1.761</td>
<td>0.294</td>
<td>0.086</td>
<td>1.824</td>
</tr>
<tr>
<td>1.742</td>
<td>0.290</td>
<td>0.084</td>
<td>1.559</td>
</tr>
<tr>
<td>1.728</td>
<td>0.288</td>
<td>0.083</td>
<td>1.331</td>
</tr>
<tr>
<td>1.720</td>
<td>0.287</td>
<td>0.082</td>
<td>1.232</td>
</tr>
<tr>
<td>1.704</td>
<td>0.284</td>
<td>0.080</td>
<td>1.045</td>
</tr>
<tr>
<td>1.696</td>
<td>0.283</td>
<td>0.080</td>
<td>0.953</td>
</tr>
</tbody>
</table>

The first column gives the DC voltage measured at terminals $T_1$ and $T_2$. This voltage divided by 6 is the current $I$ through the bulb, given in the second column. The third column gives the square of $I$. The fourth column is the DC voltage measured at the terminals PD O/P. A plot of the photodiode voltage against $I^2$ is given in Figure 5.

![Figure 5](image.png)

Figure 5. Plot of DC output voltage of photodiode against the DC current squared through the bulb.

The equation to this line is

$$V_{DC\,PD} = A + BI^2,$$

where $A$, the intercept, is $-13.06$ and $B$, the slope, is 174.1.

This equation may be written as

$$V_{DC\,PD} = B \left( I^2 + \frac{A}{B} \right) = 174.1 \left( I^2 - 0.075 \right).$$

Written this way, the equation shows that a minimum current is required through the bulb for the photodiode to respond.

In the AC experiment, the RMS AC current through the bulb is $1/50 = 0.02$ A. So, the peak amplitude of AC current is $1.414 \times 0.02 = 0.028$ A. This is superposed on the DC current of $1.862/6 = 0.310$ A. Superposition of AC with DC currents leads to a maximum current of $0.310 + 0.028 = 0.338$ A through the
Experiment in Thermal Relaxation Time of a Serial Light Bulb

bulb, and to a minimum current of 0.310 – 0.028 = 0.282 A through the bulb. As the frequency tends to zero, these currents will produce an output of the photodiode of 6.83 V and 0.78 V respectively, when substituted in equation (24). In the limit of low frequencies (f tending to zero), this excursion of the photodiode output voltage, (6.83 – 0.78) V, is equal to 2V(0)_{peak}. So, V(0)_{peak} = 3.02 V. The DMMs always measure RMS AC voltages. In the AC part of the experiment, we measure RMS AC voltages on the multimeter. We must find V(0)_{RMS}, which is equal to V(0)_{peak} \times 0.707. This means that the RMS value, V_{RMS}(0), in the slope of the AC experiment, is 3.02 \times 0.707 = 2.14 V. Then \(\tau\) can be calculated from the slope of the line in Figure 4. The slope is 2\pi\tau/V(0)_{RMS} = 4.39 \times 10^{-2} \text{s/V}, and V_{RMS}(0) = 2.14 V. So, \(\tau\) comes out to be 0.15 s.

Using the values of V_{RMS}(0) and \(\tau\) and the Debye formula

\[
V(f) = \frac{V_{RMS}(0)}{[1 + 4\pi^2f^2\tau^2]^{0.5}},
\]

we calculate the values of V at the frequencies of measurement. These values are shown in the last column of Table 1 under V_{AC cal} in mV. Figure 6 shows the experimental values of V_{AC} as a function of frequency. The values in the last column of Table 1, calculated from the Debye formula, are plotted as a continuous curve. The agreement is good.

Figure 6. Comparison of experiment with the Debye Formula.

Note:

Different makes of the serial light bulbs have different values of the relaxation time. The maximum current through the bulb, when switch SW1 is in the HC position, will also be different. So the above readings must be taken as a sample only. For most of the bulbs, the relaxation time is in the range 0.08 to 0.1 s.

Though the signal generator can go down to 1 Hz, stable output readings are obtained only above 10 Hz. So, we started the measurements from 20 Hz.

Dr Ramesh has built a signal generator producing sine waves down to 1 millihertz. The experiments can be done down to very low frequencies, and we may directly measure the value of V(0), without resorting to a DC experiment. He has also measured the phase \(\phi\) and plotted it as a function of frequency. It gives a straight line from which the value of \(\tau\) can be found. Results obtained with his signal generator are in agreement with the values found by the method described in this experiment. This signal generator going down to millihertz is not produced commercially.
Experiment in Thermal Relaxation Time of a Serial Light Bulb

6. Questions

1. Give examples of relaxation in physics.
2. A capacitance $C$ is connected in series with a resistance $R$. A DC voltage $V$ is applied at time $t = 0$. How will the voltage across the capacitor change with time? On what factors does the relaxation time depend?
3. There are two thermal systems. The first system has a large thermal capacity and is connected to a thermal reservoir at temperature $T$ through a weak link. The second system has a low thermal capacity and is connected to the reservoir with a strong thermal link. Which system will relax faster and why?
Chapter 11

Experiments in Phase Sensitive Detection
11.1 Principle of Phase Sensitive Detection

1. Introduction

Amplifiers are used to amplify a weak AC signal. However, the amplifier also amplifies noise. Amplifiers have a bandwidth of several kilohertz. Usually, noise is present over a range of frequencies, while the signal is at a single frequency. If the signal is very weak, amplification alone will not enable us to pick out the signal against noise.

There are different sources of noise in an amplifier. Flicker noise is present in all electronic instruments. Its power spectrum varies inversely with the frequency. This noise becomes a problem only if one works at very low frequencies. Then, we have noise due to electromagnetic pickup from running motors, tube lights and so on. Such a noise has a peak in the power spectrum only at the frequency of the motor or the frequency of the mains. Such noise can be reduced by electromagnetic shielding. The third source of noise is thermal noise. This is of thermodynamic origin and cannot be avoided. Its power spectrum extends over all frequencies. So it is called white noise.

If we have a resistance \( R \) through which a current \( I \) is flowing, the voltage across the resistance will vary randomly about the average value \( V_0 = IR \). The mean square fluctuation of the voltage is defined by \( \langle (V - V_0)^2 \rangle \), where the average is taken over a long period of time. If we measure the noise over a bandwidth \( W \), the mean square voltage will be

\[
\langle (V - V_0)^2 \rangle = 4k_BTWR.
\]  

To reduce this noise, we either reduce the temperature \( T \) or reduce the bandwidth \( W \) of the amplifier. For detecting very weak signals at room temperature, we have to effectively reduce the bandwidth to a few hertz, though the amplifier has a large bandwidth in kilohertz range. This is achieved by Phase Sensitive Detection.

2. Signal and Reference

Let us have an AC sinusoidal cause. For example, the cause may be the current through the primary coil of a mutual inductance. It may be a light signal whose intensity is modulated periodically. This cause will have an effect. It will have the same frequency as the cause, but may differ in phase from the cause. The effect in the case of mutual inductance is the induced emf in the secondary coil. This will be 90° out of phase with the current in the primary coil, but it will have the same frequency as the current. The effect will be weak, of the order of a few microvolts or nanovolts. If we merely amplify the weak signal a million times, we will also amplify the noise. In an ordinary amplifier, the noise is collected over a wide frequency range and so may even swamp the weak signal due to the cause.

To overcome the noise vis-à-vis the weak effect signal, we take a reference signal, which is derived from the cause signal and is in phase with it. For example, in the mutual inductance, a resistance is connected in series with the primary coil, so that the current through the primary also passes through the resistance \( R \). The voltage across the resistance will be the reference signal. By choosing \( R \) suitably, one may generate a reference signal of a few tens of millivolts or more.

Figure 1 shows the weak effect signal and Figure 2 shows the reference signal.

![Figure 1](image)

Figure 1. Weak signal as a function of time \((t/T)\), where \( T \) is the period (note: y-axis is in \( \mu V \)).
Experiments in Phase Sensitive Detection

Figure 2. Reference voltage as a function of time \((t/T)\), where \(T\) is the period (note: y-axis is in mV).

Figure 3. Amplified output \(V_{\text{out}}\) when the reference signal is fed to the comparator.

In this example, it is seen that the weak signal is advanced in phase relative to the reference signal by an angle \(\phi = \pi/6\).

We may write

\[ v_{\text{signal}} = v_0 \sin(\omega t + \phi) \]  

and

\[ V_{\text{ref}} = V_0 \sin(\omega t). \]  

Let us say these are the signal and reference voltages after some amplification in our circuit.

3. Working of Phase Sensitive Detector Chip AD 630

These signals are then fed to the phase sensitive detector chip AD 630 made by Analog Devices. In this chip, there are two identical amplifiers. One is a direct amplifier. This means that the output is in phase with the input. The other is an inverting amplifier, i.e., the output is 180° out of phase with the input. There is an electronic switch operated by a comparator. The comparator senses the reference signal. When the reference signal is positive (i.e., for \(0 < t < \frac{T}{2}\), in Figure 1), the comparator connects the weak signal \(v_{\text{signal}}\) to the direct amplifier, so that the output of the amplifier is

\[ V_{\text{out}} = +\alpha v_{\text{signal}} ; \quad 0 < t < \frac{T}{2}. \]  

When the reference signal is negative (i.e., for \(t\) between \(T/2\) and \(T\)), the weak signal is connected to the inverting amplifier, so that the output voltage is

\[ V_{\text{out}} = -\alpha v_{\text{signal}} ; \quad \frac{T}{2} < t < T. \]  

Figure 3 shows the amplified signal \(V_{\text{out}}\) assuming a total amplification of 1000 (including the amplification at the amplifier preceding the AD 630 chip).
Experiments in Phase Sensitive Detection

From Figures 1 and 2, we see that, due to the phase difference \( \phi \) between the signal and reference, the sign of the signal changes ahead of the sign of the reference. So, part of the signal between 0 and \( T/2 \) is negative. Similarly, between \( T/2 \) and \( T \), most of the signal is negative and a part is positive. But, now the signal goes through the inverting amplifier in AD 630 and comes out with its sign changed. So, the output signal from AD 630 is positive for most of the period but there is a small negative portion as shown in Figure 3.

The output signal will have an average DC part, superposed on an AC part with components at frequencies \( \omega \), \( 2\omega \) . . . . If we have a low-pass filter at the output of the AD 630 chip, it will bypass the AC components to earth, and we will get a DC output \( V_{DC} \). We may calculate the DC output from the equation

\[
V_{DC} = \frac{1}{T} \left\{ \int_{0}^{T/2} \alpha v_{0} \sin(\omega t + \phi) \, dt - \int_{T/2}^{T} \alpha v_{0} \sin(\omega t + \phi) \, dt \right\} = \left( \frac{4\alpha v_{0}}{2\pi} \right) \cos(\phi)
\]

(6)

(7)

remembering that \( \omega T = 2\pi \).

If we introduce a circuit to phase shift the reference signal and then send the phase-shifted reference signal to the comparator in AD 630, the output DC voltage will vary as the phase shift is increased from zero. It will reach a maximum when the reference signal is phase shifted by \( \frac{\phi}{2} \), since the phase-shifted reference signal will now be in phase with the weak signal \( v_{signal} \). Then, the output voltage will look like the one shown in Figure 4. The average DC voltage \( V_{DC} \) will then reach a maximum value \( 2\alpha v_{0}/\pi \).

That is why we call this technique ‘phase sensitive detection’ and the amplifier a lock-in amplifier, because we make the weak signal lock in phase with the phase-shifted reference to give maximum DC output voltage. If we measure on an oscilloscope how much we have to shift the phase of the reference signal to get maximum DC output, we get the phase difference between effect and cause.

![Figure 4](image)

**Figure 4.** Amplified output signal when the reference signal is phase shifted by \( \phi \) and fed to AD 630 chip.

Suppose we have noise at frequency \( \omega' \) different from the frequency \( \omega \) of the reference signal. Suppose the integration time of the low-pass filter is large (several times the period of the reference signal). Then we can show that the contribution of this noise to \( V_{DC} \) drops rapidly as the difference \( \delta \omega \) between \( \omega' \) and \( \omega \) increases. If \( \tau \), the integration time, is large compared to the period of the reference signal, then, only noise frequencies differing from \( \omega \) by \( n/\tau \) (where \( n \) is a small number) will make a contribution to \( V_{DC} \). This means that the effective bandwidth of the lock-in amplifier is \( n/\tau \). If the integration time is 1 second, this implies
that the effective bandwidth $W$ is a few Hz. So, one can understand how thermal noise and other noises are suppressed by the lock-in amplifier.

Phase sensitive detection is a very important signal processing technique which enables us to measure very weak AC signals in the presence of noise. We will describe two simple experiments to illustrate the power of this technique. This technique is also used in detecting the ferromagnetic to paramagnetic transition in materials using AC susceptibility.

4. Lock-in Amplifier

Figure 5 shows the lock-in amplifier that we have made for student experiments. This lock-in amplifier can work in the frequency range 100 Hz to 3 kHz. An AC signal, of 200 $\mu$V RMS amplitude, will produce a DC output of about 1 V.

Figure 5. Lock-in amplifier.

The Mains switch is at the top left corner of the front panel. When the switch is pressed down, the lock-in amplifier is powered, and an indicator light in the switch comes on. On the right half of the front panel, there are a number of RCA sockets. The bottom-most socket is labelled SIG GEN. A signal generator can be connected to this socket for calibration of the lock-in amplifier. Above the SIG GEN RCA socket, we have two other sockets marked REF and REF'. These can be connected to a two-channel oscilloscope to display the reference signal, and the phase-shifted reference signal, respectively.

The reference signal is phase shifted by turning the knob of a pot marked PHASE SHIFT ADJUST. The RCA socket marked SIGNAL, can be connected to an oscilloscope to show the amplified small AC signal with noise. The RCA socket marked PIN 13 shows the output at PIN 13 of the chip AD 630. As the phase shift adjust knob is turned, the pattern of the output at PIN 13, as seen on an oscilloscope, changes. When the phase-shifted reference is in phase with the signal, the output at PIN 13 looks like the output of a full-wave rectifier. The RCA pin marked OUTPUT, is connected to a DMM in an appropriate DC range. This measures the amplified DC output from PIN 13 of the Lock-in chip.

There are two toggle switches. When both are put towards CAL mode, one can calibrate the lock-in as explained in Section 11.2. When an external signal is to be measured, the toggle switches are put towards EXT. In this mode, the external AC signal should be connected to the two banana terminals (red and black) at the bottom of the front panel. The external reference signal should be connected to the RCA socket marked EXT REF.

5. Block Diagram of the Lock-in Amplifier Circuit

A block diagram of the lock-in amplifier circuit is shown in Figure 6. All the front panel controls are marked on the diagram. The thick line gives external reference wiring. The broken line gives the wiring for the signal generated internally in the calibration mode, when a signal generator is connected to the RCA socket marked SIG GEN. The internal calibration circuit consists of three resistances, 220 k$\Omega$, 10 $\Omega$ and 5 k$\Omega$ in series across which the AC voltage from the signal generator is applied. The
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voltage across the 10 Ω provides the internally generated signal used for calibration. The voltage across the 5 kΩ provides the internally generated reference signal. When the switch SW1 is put in the position CAL, the internally generated signal is connected to the signal amplifier. Similarly, when the switch SW2 is put to the position marked CAL, the internally generated reference signal is connected to the reference amplifier.

![Schematic diagram of the lock-in amplifier circuit.](image)

When measurements on external systems have to be done, the switches SW1 and SW2 are put in the positions marked EXT. The signal generated from the external circuit is connected to the red and black banana terminals marked EXT SIGNAL. Similarly, the reference signal from the external circuit is connected to the RCA socket marked EXT REF. When the switches SW1 and SW2 are in the EXT position, the external signal is connected to the signal amplifier, and the external reference to the reference amplifier. The signal coming out of the signal amplifier is fed to PIN 1 of AD 630 chip.

The reference signal after amplification, goes to a phase shifter. Phase shifting is achieved by an RC network. R is a pot, which can be adjusted by turning a knob marked PHASE SHIFT. The maximum phase shift that one can achieve is given by the product of frequency f, C and R. At frequencies below 1 kHz, one has to use a larger capacitance to achieve a phase shift close to 180° at the maximum resistance in the pot. At frequencies greater than 1 kHz, the use of the same capacitor will cause the phase to change rapidly as the resistance in the pot is increased. To make the phase change more gradual in the frequency range 1 to 10 kHz, a smaller capacitance is used. The switch SW3 selects the capacitor. At frequencies below 1 kHz the switch is put in the up position, and at frequencies greater than 1 kHz the switch is put in the down position.

The reference signal after phase shifting, is connected to PIN 9 of the AD 630 chip. Leads are taken from the input of the phase shifter and from PIN 9 of AD 630 to two RCA sockets marked REF and REF', respectively. By connecting REF and REF' to two channels of an oscilloscope, one can see how much the phase is shifted on turning the pot. When the signal to be measured and the phase-shifted reference signal match in phase, the output at PIN 13 of AD 630 looks like that of a full-wave rectifier. A lead is taken from PIN 13 to an RCA socket marked PIN 13 on the front panel. By connecting it to an oscilloscope, one may see how the output at PIN 13 varies as the phase of the reference signal is shifted. This output of AD 630 is connected to a low-pass filter (LPF), and then to an output amplifier. The output of this amplifier is
connected to an RCA socket marked OUTPUT. A DMM in the appropriate DC V range, connected to this socket, measures the DC output of the lock-in amplifier.

A lead is taken to an RCA socket, marked SIGNAL on the front panel. By connecting it to an oscilloscope, one can see the trace of the signal.

At frequencies greater than about 3 kHz, there could be a distortion in the reference signal as seen at REF or REF$. However, the lock-in will measure only the first harmonic component of the signal, independent of the distortion. For student experiments this distortion does not matter.

If there is a problem in the lock-in amplifier, please send it to the manufacturer for repair.

### 11.2 Calibration of the Lock-in Amplifier

1. **Introduction**

   In the previous section we described a lock-in-amplifier. Before using the lock-in amplifier for experiments, one must calibrate the lock-in amplifier. Calibration means measuring the maximum output DC voltage of the lock-in amplifier for a small known AC signal voltage, as a function of the frequency of the signal.

2. **Apparatus Required**

   Signal generator, lock-in amplifier, two-channel oscilloscope, and a DMM to measure DC volts to three decimal places in the 2 V range.

3. **Procedure**

   1. Connect a signal generator to the RCA socket marked SIG GEN at the bottom of the front panel of the LIA (lock-in amplifier). The current drawn from the signal generator will be very low. There is no need for a power amplifier.

   2. Throw the switches SW1 and SW2 to the position marked CAL. SW1 must point to the right and SW2 to the left. Then the internal calibration circuit is connected to the LIA chip.

   3. Set the frequency of the signal generator at 500 Hz. Adjust the amplitude $V_{pp}$ of the signal generator at about 8 V. This is the peak-to-peak voltage. Then RMS voltage will be

      $$V_{app} = V_{RMS} = \frac{V_{pp}}{2\sqrt{2}}.$$  \hspace{1cm} (1)

      Inside the lock-in amplifier, this voltage $V_{app}$ will be applied to three resistances in series, namely 220 kΩ, 10 Ω and 5 kΩ. The voltage across 10 Ω will be $v_{signal}$. Hereafter, when we refer to any AC voltage, it will be RMS voltage.

      $$v_{signal} = \frac{V_{app} \times 10}{225 \times 10^3} = 44.4V_{app}$ \text{μV}.$$  \hspace{1cm} (2)

      The reference voltage is the voltage across the 5 kΩ resistor, one end of which is grounded.

   4. Put the switch SW3 up. Turn the phase shifter potentiometer (POT) knob to the right extreme (resistance is zero and phase shift is zero). This pot phase shifts the reference voltage across the 5 kΩ.
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5. Connect the RCA sockets, marked REF and REF', to the two channels of an oscilloscope. There will be two sinusoidal traces at the frequency of the signal generator. These are amplified reference signals, one before phase shifting, and the other after phase shifting. Since the phase shift is zero when the phase adjusting pot is at the extreme right, the two traces will be superposed exactly indicating zero phase shift.

6. Connect a DMM in the DC 2 V range to the RCA socket marked OUTPUT. Note the DC volts on this DMM. This DC voltage will be positive. Turn the phase shifter potentiometer knob anti-clockwise and observe what happens to the signal coming from REF' relative to the signal coming from REF on the oscilloscope screen. You will see that the signal from REF' moves to the right relative to the signal from REF. This indicates a phase shift between the two signals. Note that as you turn the knob, the DC voltage on the multimeter comes down. Turn the knob and adjust to the position when the DC voltage on the multimeter is zero. You will see that the maxima of the REF' signal now occur at the positions of zeroes of the REF Signal. This indicates a phase shift of 90°. You can check this by feeding the two signals to the X- and Y-plates of the oscilloscope, and noting that the Lissajous pattern is a circle on the oscilloscope. (There is a slight distortion of the reference signal after phase shifting. So, the circle will be slightly distorted.) If the knob is turned further, the phase shift increases beyond 90°, and the DC voltage reading becomes negative. When the knob of the phase shift potentiometer is to the left end, the phase shift is nearly (but less than) 180°. The DMM indicates the largest negative value.

In the calibration circuit, the signal and reference voltages are derived from the potential differences across two resistances in series. They are in phase. So, the DMM voltage is largest when the phase shift is zero.

7. Connect the RCA socket marked PIN 13 to the oscilloscope and see how the output of the lock-in chip changes as the phase shifter knob is turned. As the knob is turned, the output at PIN 13 develops a larger and larger negative part.

These observations show that the lock-in amplifier (i) gives a DC output for an AC input, (ii) the DC output is a maximum when the signal and the phase-shifted reference signal going into the comparator are in phase, and (iii) the DC output varies as \( \cos(\phi) \), where \( \phi \) is the phase difference between the weak signal and the phase-shifted reference signal.

Turn the phase adjustment potentiometer knob to the right extreme, and wait till the DMM connected to the output socket of the lock-in amplifier shows a steady DC value. Note the reading \( V_{DC} \) on the DMM.

Change the output voltage \( V_{pp} \) of the signal generator from about 8.5 V to zero in steps. Note the corresponding values of \( V_{DC} \) on the DMM. The phase shifting pot knob is always at the right extreme position.

Change the frequency of the AC signal in steps of 0.5 kHz from 0.5 kHz to 3 kHz, and for each frequency measure \( V_{DC} \) as a function of \( V_{pp} \).

Sample readings are shown in Table 1.

<table>
<thead>
<tr>
<th>( f ) (Hz)</th>
<th>500</th>
<th>1000</th>
<th>1500</th>
<th>2000</th>
<th>2500</th>
<th>3000</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{pp} ) (V)</td>
<td>( V_{signal} ) (mV)</td>
<td>( V_{DC} ) (V)</td>
<td>( V_{DC} ) (V)</td>
<td>( V_{DC} ) (V)</td>
<td>( V_{DC} ) (V)</td>
<td>( V_{DC} ) (V)</td>
</tr>
<tr>
<td>3</td>
<td>47.1</td>
<td>0.260</td>
<td>0.257</td>
<td>0.253</td>
<td>0.251</td>
<td>0.255</td>
</tr>
<tr>
<td>4</td>
<td>62.8</td>
<td>0.346</td>
<td>0.342</td>
<td>0.338</td>
<td>0.335</td>
<td>0.340</td>
</tr>
<tr>
<td>5</td>
<td>78.5</td>
<td>0.433</td>
<td>0.428</td>
<td>0.422</td>
<td>0.419</td>
<td>0.425</td>
</tr>
<tr>
<td>6</td>
<td>94.2</td>
<td>0.519</td>
<td>0.513</td>
<td>0.506</td>
<td>0.503</td>
<td>0.510</td>
</tr>
<tr>
<td>7</td>
<td>109.9</td>
<td>0.606</td>
<td>0.599</td>
<td>0.591</td>
<td>0.587</td>
<td>0.595</td>
</tr>
<tr>
<td>8</td>
<td>125.6</td>
<td>0.693</td>
<td>0.684</td>
<td>0.675</td>
<td>0.671</td>
<td>0.680</td>
</tr>
</tbody>
</table>
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Figure 1. $V_{DC}$ in V against $v_{signal}$ in $\mu$V for 500 Hz.

In converting $V_{pp}$ into $v_{signal}$, we use the conversion factor

$$v_{signal} = \left( \frac{V_{pp}}{2\sqrt{2}} \right) \times 44.4,$$

where $V_{pp}$ is in volts and $v_{signal}$ is in $\mu$V. $V_{pp}$ is the peak-to-peak output of the signal generator and $v_{signal}$ is the RMS voltage across 10 $\Omega$ in the internal resistance network.

Figure 1 shows the calibration curve for 500 Hz.

From this figure, we see that the lock-in amplifier output in DC volts is proportional to the signal. The linear fit has a slope of $(5.35 \pm 0.022) \times 10^3$. Similar fits are done at all the other frequencies and the slopes are collected in Table 2.

<table>
<thead>
<tr>
<th>$f$ (kHz)</th>
<th>$\alpha \times 10^3$</th>
<th>Devn. $\times 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5.35</td>
<td>0.022</td>
</tr>
<tr>
<td>1.0</td>
<td>5.32</td>
<td>0.048</td>
</tr>
<tr>
<td>1.5</td>
<td>5.26</td>
<td>0.087</td>
</tr>
<tr>
<td>2.0</td>
<td>5.21</td>
<td>0.108</td>
</tr>
<tr>
<td>2.5</td>
<td>5.46</td>
<td>0.140</td>
</tr>
<tr>
<td>3.0</td>
<td>5.48</td>
<td>0.240</td>
</tr>
</tbody>
</table>

We see from Table 2 that the amplification factor can be taken to be a constant with a value $(5.35 \pm 0.15) \times 10^3$ independent of frequency.

If we want to measure the RMS value of an unknown AC signal, we measure the maximum DC value this signal will produce on adjusting the phase shift pot. If this value is $V_{DC}$, then the unknown RMS AC signal voltage is

$$v_{signal} = \frac{V_{DC}}{\alpha},$$

where $\alpha$ is the amplification factor obtained by a calibration of the lock-in amplifier.

The amplification factor may vary slightly from lock-in amplifier to lock-in amplifier. The above readings are sample readings for the lock-in amplifier used in this experiment.

11.3 Measurement of Mutual Inductance with Lock-in Amplifier

1. Introduction

When two coils are placed side by side and an AC current is passed through one coil (called the primary), an AC voltage at the same frequency is induced in the other coil (called the secondary). If the primary current
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varies as

\[ I = I_0 \sin(2\pi ft), \]  

where \( f \) is the frequency in hertz, the emf induced in the secondary is

\[ V = -M \frac{dl}{dt} = -2\pi M f I_0 \cos(2\pi ft) \]
\[ = -2\pi M f I_0 \sin\left(2\pi ft + \frac{\pi}{2}\right). \]  

So,

(1) The phase difference between the primary current and the induced emf is \( \pi/2 \);
(2) The secondary emf is proportional to the amplitude \( I_0 \) of the primary current at a given frequency \( f \);
(3) The induced emf is proportional to the frequency \( f \).

In this experiment, all these factors will be verified using the lock-in amplifier.

2. The Mutual Inductance Coil

![Figure 1. Mutual inductance coil.](image)

On an insulating former, a primary coil of about 20 turns is wound using insulated copper wire, which can carry a current of about a few milliamperes. \( W_p \) and \( W_s \) are the widths of the primary and secondary coils, respectively. \( D \) is the distance from the center of the primary coil to the center of the secondary coil (Figure 1).

There are three banana terminals, red, yellow and black, at primary end of the coil. A 4.7 kΩ resistor and the primary coil are connected between the red and yellow banana terminals. Between the yellow and black terminals, a hundred ohm resistor is connected. When a signal generator is connected between the red and black terminals, and an RMS AC voltage of \( V_{\text{RMS}} \) volts is applied, an RMS current flows through the primary coil. This current is given by

\[ I_{\text{RMS}} = \frac{V_{\text{RMS}}}{R_{\text{Total}}} = \frac{V_{\text{RMS}}}{4.8 \times 10^3} \text{ A}. \]  

The resistance of the primary coil, which is of the order of 1 Ω, is neglected in the denominator compared to 4800 Ω. The voltage across 100 Ω will be used as the reference signal. The signal generator ground must be connected to the black terminal on the primary side of the coil box, and the other terminal of the signal generator must be connected to the red banana terminal on the primary side of the coil box.
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On the same former, a secondary coil of about 100 turns is wound, at a distance of a few centimeters from the primary coil. The terminals of the secondary coil are brought to two banana terminals on the other end of the insulating former. The emf generated by the mutual inductance of the coils will appear at these terminals, and will be measured by the lock-in amplifier.

3. Apparatus Required

Signal generator, lock-in amplifier, two-channel oscilloscope, mutual inductance coil, and a DMM to measure DC output of lock-in amplifier to two decimal places in DC 20 V range.

4. Procedure

Since we are measuring an external signal, the switches SW1 and SW2 on the LIA front panel should be set to the positions marked EXT. The switch SW1 will point to the left and the switch SW2 to the right.

The red terminal on the signal generator is connected to the red terminal, and the black terminal on the signal generator to the black terminal, on the primary coil side of the insulating former. Connect the yellow and black terminals on the primary coil side of the insulating former to the RCA socket marked EXT REF on the front panel of the LIA.

Connect the red terminal on the secondary side of the insulating former to the red banana terminal marked EXT SIG on the LIA. Connect the black terminal on the secondary side of the insulating former to the black banana terminal marked EXT SIG on the front panel of the LIA.

Connect a DMM in the DC 20 V range to the RCA socket marked OUTPUT on the front panel of the LIA. Connect the two RCA sockets marked REF and REF' on the front panel of the LIA to the two channels of an oscilloscope.

Switch on the signal generator. Set the peak-to-peak amplitude $V_{pp}$ on its panel meter to 11 volts, and the frequency to 400 Hz. Switch on the lock-in amplifier and the oscilloscope. Put the toggle switch SW3 up. You will see two sinusoidal traces at the frequency of 400 Hz on the oscilloscope. When the phase-shift-adjust pot on the LIA is to the extreme right, the two traces will be in phase. Note that the DMM reading is small. Now turn the phase-adjust pot to the left. The trace of REF' will shift relative to the trace of REF, and the DMM reading will increase. The sign of the DMM reading will be negative. The DMM takes some time to reach a steady value. So, turn the phase-adjusting pot little by little and wait for the DMM to reach a steady reading. The magnitude of the reading on the DMM will increase, and it will reach a maximum at one position of the phase-adjusting pot. A further turning of the phase-adjusting pot will reduce the magnitude of the reading. Keep the phase-adjusting pot at the position when you get the maximum negative reading on the DMM. On the oscilloscope screen, we see that the phase difference between the two traces is 90°. To show this more dramatically, press the XY button on the oscilloscope. Now one sees the Lissajous figure due to the signals REF and REF'. When the DMM shows the maximum value, this figure on the oscilloscope screen is a slightly distorted circle. The reference signal is in phase with the current. A maximum output of the LIA indicates that the phase-shifted reference signal is in phase with the mutual inductance emf. The Lissajous figure shows that the phase-shifted reference signal is 90° out of phase with the reference signal. So, the mutual inductance emf is 90° out of phase with the primary current.

Note the DMM reading at this position of the pot. Keeping the frequency of the signal generator the same, change the amplitude from 11 to 3 V in steps of 2 V. There is no need to adjust the phase-shift pot every time, at a given frequency. Once the phase shift is adjusted for an output of 11 V to get a circle as the Lissajous figure on the oscilloscope screen, changing the amplitude only changes the radius of the circle, but does not change its shape.

Having taken the readings of the DMM for different signal generator amplitudes $V_{pp}$ at 400 Hz, repeat the experiment at different frequencies in steps of 200 Hz. When the frequency is 1 kHz or more, switch SW3 will have to be pressed down. When the frequency is changed, the phase-shift pot needs to be adjusted to get the maximum magnitude for the DMM reading. At this maximum magnitude, the Lissajous figure will be a distorted circle.

A sample set of readings is given in Table 1. In this table, we give the values of the output voltage $V_{pp}$ of the signal generator, and the DC output voltage $V_{DC}$ of the lock-in amplifier.

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**Table 1.** Mutual inductance (all readings of voltage are in volts).

<table>
<thead>
<tr>
<th>( f ) (Hz)</th>
<th>( V_{pp} )</th>
<th>( V_{RMS} )</th>
<th>( V_{DC} )</th>
<th>( V_{DC} )</th>
<th>( V_{DC} )</th>
<th>( V_{DC} )</th>
<th>( V_{DC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>1.06</td>
<td>0.21</td>
<td>0.32</td>
<td>0.43</td>
<td>0.53</td>
<td>0.64</td>
<td>0.76</td>
</tr>
<tr>
<td>600</td>
<td>1.77</td>
<td>0.35</td>
<td>0.53</td>
<td>0.70</td>
<td>0.90</td>
<td>1.07</td>
<td>1.24</td>
</tr>
<tr>
<td>800</td>
<td>2.48</td>
<td>0.50</td>
<td>0.75</td>
<td>1.01</td>
<td>1.25</td>
<td>1.50</td>
<td>1.77</td>
</tr>
<tr>
<td>1000</td>
<td>3.18</td>
<td>0.65</td>
<td>0.91</td>
<td>1.30</td>
<td>1.63</td>
<td>1.98</td>
<td>2.27</td>
</tr>
<tr>
<td>1200</td>
<td>3.89</td>
<td>0.81</td>
<td>1.20</td>
<td>1.64</td>
<td>2.04</td>
<td>2.46</td>
<td>2.81</td>
</tr>
<tr>
<td>1400</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1600</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 2.** DC output of LIA vs. AC voltage applied to primary circuit.

**Table 2.** Slope \( \alpha = \frac{dV_{DC}}{dV_{RMS}} \) against frequency.

<table>
<thead>
<tr>
<th>( f ) (Hz)</th>
<th>Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>0.212</td>
</tr>
<tr>
<td>600</td>
<td>0.303</td>
</tr>
<tr>
<td>800</td>
<td>0.427</td>
</tr>
<tr>
<td>1000</td>
<td>0.530</td>
</tr>
<tr>
<td>1200</td>
<td>0.644</td>
</tr>
<tr>
<td>1400</td>
<td>0.726</td>
</tr>
<tr>
<td>1600</td>
<td>0.868</td>
</tr>
</tbody>
</table>

The column marked \( V_{pp} \) gives the peak-to-peak AC output of the signal generator. The column marked \( V_{RMS} \) is the RMS value of the amplitude given by

\[
V_{RMS} = \frac{V_{pp}}{2\sqrt{2}}.
\]

The column marked \( V_{DC} \) is the maximum DC output of the lock-in amplifier, when the Lissajous figure on the oscilloscope is a circle.

For each frequency, plot \( V_{DC} \) against \( V_{RMS} \). These plots are shown in Figure 2. The slopes of these curves at different frequencies are given in Table 2.
Figure 3. Plot of slopes of $V_{DC}$ vs. $V_{RMS}$ curves in Table 2 vs. frequency.

The slopes $\alpha$ of $V_{DC}$ vs. $V_{RMS}$ curves in Table 2 are plotted against frequency in Figure 3. This curve is linear with a slope $\beta = 5.41 \times 10^{-4}$ s.

Figure 2 shows that the mutual inductance emf is proportional to the current through the primary coil at a fixed frequency. Figure 3 shows that for a given change of current through the primary coil, the change in mutual inductance voltage is proportional to the frequency of the AC current.

The slope $\beta$ of the graph in Figure 3 must be equal to

$$\beta = \frac{2\pi \alpha M}{R},$$

where $M$ is the mutual inductance of the coil, $\alpha$ is the amplification factor of the LIA, and $R$ is the resistance in the primary circuit ($4.8 \, k\Omega$). The value of $\alpha$ for the LIA is $5.35 \times 10^3$ from Section 11.2.

Putting these values in formula (4), we get

$$M = 77.3 \, \mu H.$$  

This is the value of the mutual inductance of the first coils which were made. The mutual inductance will depend on the distance between the primary and secondary coils. For the coils which are currently manufactured, the mutual inductance has a value of about $110 \, \mu H$.

You learn from this experiment that:

1. The mutual inductance emf is $90^\circ$ out of phase with the current in the primary coil.
2. The emf is proportional to the current in the primary coil.
3. The emf is proportional to the frequency.

5. Questions

1. If the number of turns in the secondary coil is increased, will the mutual inductance increase or decrease? Give a reason.
2. Keeping the number of turns constant in the primary coil, we increase the width $W_p$ of the primary coil. Will the mutual inductance increase, decrease or remain a constant? Give a reason for your answer.
3. If the distance $D$ between the primary and secondary coils is increased, will the mutual inductance increase or decrease? Give a reason.
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4. If the frequency of the signal is decreased, what will happen to the induced emf in the secondary for the same current in the primary?

5. To get a DC signal voltage of 1 volt from the lock-in amplifier, what should be the current through the primary coil in the mutual inductance setup at a frequency of 500 Hz?

11.4 Measurement of Low Resistance using a Lock-in Amplifier

1. Introduction

Measurement of low resistance (resistance much less than an ohm) with the DC technique would require a high current. Also, since the voltage developed across the resistance will be small, it will be affected by noise when it is amplified.

The AC technique using a lock-in amplifier provides a solution to the problem. This is illustrated by the following experiment.

2. Low Resistance Box

The low resistance box is shown in Figure 1. In this box, a resistance of 4.7 kΩ (1/4 W) is connected in series with a low resistance of the order of 0.1 Ω, and a 100-Ω (1/4 W) resistor. The free end of the 4.7 kΩ resistor is connected to the red banana terminal, and the free end of the low resistance to the middle terminal (green). A 100-Ω resistor is connected to the middle terminal (green) and the end terminal (in the above box, this terminal is yellow). For measuring the low resistance, a four terminal arrangement is used. Potential leads across the low resistance come to the RCA socket marked SIGNAL on the left top of the box. The voltage across the 100-Ω resistor serves as the reference signal. This reference signal can be taken between the middle and end banana terminals on the box, or from the RCA socket marked REFERENCE at the right top of the box.

![Figure 1. Low resistance box.](image)

The low resistance is made of insulated thin copper wire (SW 42). One free end of the wire is soldered to the 4.7 kΩ resistor and the other free end to the 100-Ω resistor. Two potential leads are soldered on the copper wire between the current leads. The free ends of the potential leads are soldered to the terminals of the RCA socket marked SIGNAL.

The 4.7 kΩ resistor limits the current through the wire to a few hundred microamperes when the signal generator output is connected to the terminals of the box.

3. Apparatus Required

Signal generator, lock-in amplifier, two-channel oscilloscope, low resistance box and one DMM to measure output of lock-in amplifier to two decimal places in the DC 20 V range.
4. Procedure

The output terminals of the signal generator are connected to the two end banana terminals on the box. The reference signal, taken from the RCA socket marked REFERENCE, is connected to the RCA socket marked EXT REF on the front panel of the lock-in amplifier. The RCA socket marked SIGNAL on the box is connected to the banana terminals marked EXT SIG on the lock-in amplifier front panel. Connect the socket marked OUTPUT on the front panel of the LIA to a DMM in the DC 20 V range. Put the switches SW1 and SW2 on the LIA to the positions marked EXT and the switch SW3 up.

Switch on the signal generator. Adjust its frequency to be about 200 Hz and its output peak-to-peak amplitude to 11 V ($V_{pp}$). Switch on the lock-in amplifier. A DC voltage will appear on the DMM which will reach a positive maximum value as the phase-adjust pot is turned to the right extreme position. Note the value of this maximum DC voltage $V_{DC}$. Change the signal generator output $V_{pp}$ in steps of 2 V down to 3 V and note the voltage $V_{DC}$. Repeat the experiments at frequencies 400, 600, 800 Hz.

A voltage output $V_{pp}$ corresponds to an AC RMS voltage $V_{RMS}$ equal to

$$V_{RMS} = \frac{V_{pp}}{2\sqrt{2}}. $$ \hspace{1cm} (1)

At each frequency, plot a graph between $V_{DC}$ against $V_{RMS}$. Fit the points to a straight line and get the slope $\beta = \frac{dV_{DC}}{dV_{RMS}}$. When the signal generator voltage is changed by $dV_{RMS}$, $dI_{RMS}$, the change in the RMS current through the low resistance is given by

$$dV_{RMS} = R dI_{RMS}, \hspace{1cm} \text{(2)}$$

where $R$ is the total resistance in the primary circuit (4.8 kΩ). The output DC signal $V_{DC}$ is proportional to the voltage $V_r$ across the low resistance $r$. When the current through the low resistance is changed by $dI_{RMS}$, the voltage $V_r$ changes by $dV_r$, where

$$dV_r = r dI_{RMS}. \hspace{1cm} \text{(3)}$$

This causes a change in $V_{DC}$ by $dV_{DC}$. given by

$$dV_{DC} = adV_r = ar dI_{RMS} = \left(\frac{ar}{R}\right) dV_{RMS}. \hspace{1cm} \text{(4)}$$

So,

$$\frac{dV_{DC}}{dV_{RMS}} = \left(\frac{ar}{R}\right). \hspace{1cm} \text{(5)}$$

To ensure that the contribution to $V_r$ from any residual inductance of the low resistance is small, we work at low frequencies and we check that the slope $dV_{DC}/dV_{RMS}$ is independent of the frequency.

A set of sample readings is shown in Table 1.

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{pp}$</td>
<td>$V_{RMS}$</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
<td>$V_{DC}$</td>
</tr>
<tr>
<td>3</td>
<td>1.06</td>
<td>0.18</td>
<td>0.18</td>
<td>0.21</td>
</tr>
<tr>
<td>5</td>
<td>1.77</td>
<td>0.30</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>7</td>
<td>2.48</td>
<td>0.41</td>
<td>0.41</td>
<td>0.46</td>
</tr>
<tr>
<td>9</td>
<td>3.18</td>
<td>0.53</td>
<td>0.53</td>
<td>0.58</td>
</tr>
<tr>
<td>11</td>
<td>3.89</td>
<td>0.65</td>
<td>0.64</td>
<td>0.70</td>
</tr>
</tbody>
</table>
A plot of $V_{DC}$ against $V_{RMS}$ for 400 Hz is shown in Figure 2. From the linear fit, the slope $\beta$ is found to be 0.163. The slopes for all four frequencies are collected in Table 2.

**Table 2.**

<table>
<thead>
<tr>
<th>$f$ (Hz)</th>
<th>Slope $\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.163</td>
</tr>
<tr>
<td>400</td>
<td>0.163</td>
</tr>
<tr>
<td>600</td>
<td>0.164</td>
</tr>
<tr>
<td>800</td>
<td>0.167</td>
</tr>
</tbody>
</table>

We see that the slope $\beta$ is nearly constant, around $0.165 \pm 0.002$. From this average slope, we calculate the low resistance $r$ from the formula (5). In that formula, $a = 5.35 \times 10^3$ for the lock-in amplifier used, and $R = 4.8$ k$\Omega$.

Substituting these values in formula (5), we find

$$r = 0.147 \pm 0.002 \Omega.$$  

The value of the small resistance will vary from box to box.

5. **Questions**

1. If you do not use voltage leads, but measure the voltage at the points where the two ends of the low resistance are soldered to 4.7 k$\Omega$ and 100 $\Omega$, will you get the correct value of the low resistance? Justify your answer.

2. What is the heat dissipated in the low resistance when $V_{RMS}$ is 2 V?

3. If the residual inductance of the low resistance is 1 $\mu$H, what will be the contribution of the inductance to the voltage $V_t$ at a frequency of 10 kHz?
Chapter 12

Experiments in Optics
12.1 Diffraction Experiments

1. Introduction

A commercial setup, made by The Holmarc Company, is used here for performing diffraction experiments. The setup consists of a one-meter long optic bench with holders, a laser diode source (power 3.5 mW), a detector (photodiode or transistor) mounted on a translation stage with a current measuring device, and diffracting objects such as single slits (50 and 100 microns width), double slits, a grating, and a pinhole.

In this experiment, we will measure the intensity of diffraction as a function of the transverse position of the detector for (a) a single slit and (b) a double slit, and fit it to theoretical formulae for Fraunhofer diffraction.

We will also measure the intensity as a function of the transverse position of the detector for a diffraction grating, and calculate the number of lines per centimeter of the grating.

Diffraction phenomena can be classified as belonging to Fraunhofer or Fresnel types. In the former, the incident wavefront and diffracted wavefronts are planar, i.e., the source sends a parallel beam of light, and the screen is at an infinite distance from the diffracting object. In the Fresnel type, the source and/or the detector are at finite distances from the diffracting object. Theoretically, it is easier to calculate the diffracted intensity at an angle $\theta$ to the incident light in Fraunhofer diffraction.

If we use a laser source, the laser beam consists of parallel rays to a very good approximation. So, the incident wavefront is planar. If the detector is at a distance large compared to the width of the diffracting object, we may apply the Fraunhofer diffraction formulae to calculate the diffracted intensity as a function of angle $\theta$.

2. Fraunhofer Diffraction at a Single Slit

![Figure 1. Fraunhofer diffraction at a single slit.](image)

AB is a single slit of width $w$ on which parallel rays of light of wavelength $\lambda$ from a laser diode are incident (see Figure 1). Consider the diffracted rays at an angle $\theta$ to the incident direction. Drop a perpendicular from A on the diffracted ray through B to meet it at C. Then AC is a plane wavefront. From this wavefront every ray travels the same distance to the screen. The ray BE travels an extra distance $BC = w \sin(\theta)$ compared to the ray AD, and so has a phase difference $\phi = (2\pi/\lambda) w \sin(\theta)$. Rays between AD and BE will have a phase difference which will vary continuously from 0 to $\phi$. To get the resultant amplitude at an angle $\theta$, we have to superpose the amplitudes of all the rays. The resultant amplitude $A(\theta)$ is given by

$$A(\theta) = A(0) \frac{\sin(\pi w \sin(\theta)/\lambda)}{(\pi w \sin(\theta)/\lambda)}.$$  \hspace{1cm} (1)

The intensity in the direction $\theta$ is proportional to the square of this amplitude. So the intensity $I(\theta)$ will be given by
Here, \( I(0) \) is the intensity of the direct light reaching the detector.

When \( \theta \) approaches zero, (i.e., for the direct beam), the quantity in the flower bracket approaches the value unity. As \( \theta \) increases and reaches a value such that \( w \sin \theta = \lambda \), the rays coming from the edges of the slit have a path difference of \( \lambda/2 \). We may divide the slit into two halves, such that the rays coming from corresponding points in the two halves have a path difference of \( \lambda/2 \). They interfere destructively to give zero intensity.

A second peak in intensity will appear at a value of \( \theta \), such that \( w \sin (\theta)/\lambda \approx 3/2 \). However, the intensity of this peak will be \( 1/(3\pi/2)^2 \) (about 5%) of the intensity of the direct beam. The diffraction pattern will show a principal maximum at \( \theta = 0 \), and a series of subsidiary maxima at larger angles, with the maxima decreasing progressively in intensity. On either side of the principal maximum, there is a minimum when \( w \sin(\theta) = \lambda \). The shadow of the slit at the detector is not sharp, and does not have the same width \( w \) as the slit.

3. Diffraction at a Double Slit

Consider a double slit, each of width \( w \), and separated by a distance \( d \). Parallel rays of wavelength \( \lambda \) are incident normally on the double slit and are diffracted at an angle \( \theta \). This is shown in Figure 2.

Each slit has a width \( AB = A'B' = w \). The distance between the center of the first slit and the center of the second slit is \( d \). So \( AA' = BB' = d \). Each slit will produce amplitude \( A(\theta) \) at angle \( \theta \) given by

\[
A(\theta) \sin \left( \frac{\pi w \sin(\theta)/\lambda}{\pi w \sin(\theta)/\lambda} \right)
\]

as seen in the last subsection. If we take two rays from corresponding points in the two slits (say at \( A \) and \( A' \) or \( B \) and \( B' \)), the path difference \( AC \) between the two rays is \( d \sin(\theta)/\lambda \). The phase difference between the two rays is \( 2\pi d \sin(\theta)/\lambda \). We have to add two amplitudes with this phase difference. So, the intensity will be modulated by the factor

\[
2 \left\{ 1 + \cos \left( \frac{2\pi d \sin(\theta)}{\lambda} \right) \right\}.
\]
Experiments in Optics

The intensity \( I(\theta) \) for diffraction at a double slit is given by

\[
I(\theta) = I(0) \left\{ \frac{\sin(\pi w \sin(\theta)/\lambda)}{(\pi w \sin(\theta)/\lambda)} \right\}^2 \times 2 \left\{ 1 + \cos \left( \frac{2\pi d \sin(\theta)}{\lambda} \right) \right\}. \tag{4}
\]

Whenever \( d \sin(\theta)/\lambda = n \) (\( n \) is an integer), we will get a maximum intensity. Whenever \( d \sin(\theta)/\lambda = (n+1/2) \), we get zero intensity. Thus, we will get nearly equally spaced bright and dark fringes, but the intensity of the bright fringes will keep decreasing as the angle \( \theta \) increases.

4. Diffraction at a Grating of \( N \) Slits

Let us have a series of \( N \) equidistant slits with a spacing \( d \). Each slit has a width \( w \). Then, the diffracted rays from corresponding points in adjacent slits will have a path difference \( d \sin(\theta) \). If this is equal to \( n\lambda \), then rays from corresponding points in all the slits will be in phase. We will get an intensity \( N^2 I(\theta) \), where \( I(\theta) \) is the intensity at an angle \( \theta \) due to a single slit. We will get many bright peaks in the diffraction pattern, corresponding to positive and negative integral values of \( n \). The intensity of diffraction at an angle \( \theta \) will now be

\[
I(\theta) = I(0) \left\{ \frac{\sin(\pi w \sin(\theta)/\lambda)}{(\pi w \sin(\theta)/\lambda)} \right\}^2 \times \left\{ \frac{\sin(N\pi d \sin(\theta)/\lambda)}{\sin(\pi d \sin(\theta)/\lambda)} \right\}^2. \tag{5}
\]

We will have subsidiary minima and maxima. However, the subsidiary maxima will have intensities of the order of \( 1/N^2 \) of the principal maxima, and so will not be visible. The first minimum, neighboring a maximum of order \( n \), will occur when \( d \sin(\theta)/\lambda = 1/nN \). As the total number of lines \( N \) (i.e., as the total width of the grating) increases, the principal maxima become narrower and narrower. So, we say that the resolving power of the grating (i.e., its ability to see the principal maxima of two neighboring wavelengths as distinct) increases as the width of the grating on which the light falls increases, and the order of diffraction increases.

5. Experiment

There is an optic bench of 1 m length. At one end we mount a laser diode which emits light at 650 nm wavelength. The power of the laser is only 3.5 mW.

CAUTION: DO NOT LOOK AT THE LASER LIGHT DIRECTLY. IT WILL DAMAGE YOUR EYE.

We can see the scattered light when the beam falls on a screen. At the other end of the optic bench there is a photodetector with a very small pinhole. This detector is mounted in a holder and can be moved perpendicular to the length of the optic bench, with a micrometer screw. One full turn of the screw corresponds to a displacement of 0.5 mm. The holder can be moved along the bench and fixed at any position. The detector height can be adjusted.

A. Diffraction at a Single Slit

A slit of 100 micron width is placed in a holder, and put in front of the source near it. The slit should be mounted vertical as judged by the eye. Its position on the optic bench can be noted. Using the screws behind the mount for the laser source, adjust the light to fall on the slit at its center.

The detector is kept at a distance of about 800 mm from the slit. On the detector you will see a bright patch of light and some fainter patches. Connect the detector to the current meter and switch on the current meter. Adjust the micrometer screw to a value of about 16 mm. You may find the bright patch to one side of the center of the detector. Use the screws on the source mount to adjust the position of the bright patch so that the current meter (in the microampere position) shows a maximum reading.

Now move the detector to a position of 6 mm on the micrometer scale. Then move it backwards (i.e., reading on the micrometer scale should increase) in steps of one turn of the screw (i.e., change in distance of 0.5 mm), and note the current reading on the meter. Carry out this procedure till the micrometer scale indicates 26 mm.

A sample set of readings is given in Table 1.
Table 1. Diffraction at a Single Slit.

<table>
<thead>
<tr>
<th>x (mm)</th>
<th>x - x₀ (mm)</th>
<th>Det. curr. (µA)</th>
<th>(πw/λ) * (x - x₀)/D</th>
<th>I(0) * (sinZ/Z)^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
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<td>-5.89</td>
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</tr>
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</tr>
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<td>-4.08</td>
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</tr>
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<td>10.25</td>
<td>0.4</td>
<td>6.19</td>
<td>0</td>
</tr>
</tbody>
</table>

Column 1 gives the micrometer reading. We see that the maximum detector current occurs for x = 15.5 and 16 mm. So, the center of the principal maximum is at x₀ = (15.5+16)/2 = 15.75 mm. Column 2 gives (x - x₀). Column 3 gives the detector current in microamperes. Column 4 gives the value of (πw/λ) * (x - x₀)/D, where w is the width of the slit (100 microns), λ is the wavelength of light (650 nm), and D is the distance of the detector from the slit. As D is large compared to the micrometer traverse, we
Experiments in Optics

may approximate the angle of diffraction $\theta$ by $\tan(\theta) \approx \sin(\theta) = (x - x_0)/D$. The last column gives the theoretical intensity as a function of $x$ from equation (2). Here, $I(0)$ is the maximum detector current for the direct beam, which from the above table is 60.6 $\mu$A.

Figure 3 shows the plot of the detector current (column 3) vs. $(x - x_0)$ (column 2). The continuous curve shows the plot of the values in the last column.

![Figure 3](image)

**Figure 3.** Plot of detector current vs. position for diffraction at a single slit. Continuous curve gives the fit equation (2).

We see that the fit is good.

### B. Diffraction at a Double Slit

Instead of the single slit, mount the double slit. The rest of the procedure is same. A sample set of readings is given in Table 2.

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>Det. curr. ($\mu$A)</th>
<th>$Z_1 = (2\pi d/\lambda) * (x - x_0)/D$</th>
<th>$Z_2 = (\pi w/\lambda) * (x - x_0)/D$</th>
<th>Intensity $I$</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
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<td>18.78</td>
<td>3.76</td>
<td>3.35</td>
</tr>
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<td>0.3</td>
<td>17.15</td>
<td>3.43</td>
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<td>15.51</td>
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<td>2.78</td>
<td>0.93</td>
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<td>2.45</td>
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<td>8.98</td>
<td>1.80</td>
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</tr>
<tr>
<td>16</td>
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<td>7.35</td>
<td>1.47</td>
<td>36.00</td>
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<td>1.14</td>
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<td>0.98</td>
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<td>0.82</td>
<td>4.79</td>
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<td>14.75</td>
<td>13</td>
<td>3.27</td>
<td>0.65</td>
<td>0.00</td>
</tr>
</tbody>
</table>
The first two columns do not need explanation. The third column gives $Z_1 = (2\pi d/\lambda) * (x - x_0)/D$, and the fourth column gives $Z_2 = (\pi w/\lambda) * (x - x_0)/D$. In both, we have used the approximation $\sin(\theta) \approx \tan(\theta) = (x - x_0)/D$. The last column gives the intensity $I(x) = I_{\text{max}} \left( \frac{\sin(Z_2)}{Z_2} \right)^2 \cos^2(Z_1/2)$.

The values of $d$ and $w$ are adjusted to give the best fit. We find $d = 250 \, \mu\text{m}$ and the width comes out to be $100 \, \mu\text{m}$. The fit is good as seen in Figure 4.

![Double slit diffraction pattern](image-url)
C. Diffraction Grating

The diffraction grating is mounted in place of the double slit on the optic bench. One will see a series of bright spots on the detector cover. If necessary, rotate the grating so that the spots are in a horizontal line. The rest of the procedure is the same as in the other two cases.

A sample set of data is given in Table 3.

<table>
<thead>
<tr>
<th>$x$ (mm)</th>
<th>$(x - x_0)$ (mm)</th>
<th>Det. curr. ($\mu$A)</th>
<th>$x$ (mm)</th>
<th>$(x - x_0)$ (mm)</th>
<th>Det. curr. ($\mu$A)</th>
</tr>
</thead>
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<td>10.00</td>
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<td>10.25</td>
<td>-3</td>
<td>6.3</td>
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<td>-7.75</td>
<td>8.8</td>
<td>10.50</td>
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<td>-2.5</td>
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<td>-7.00</td>
<td>44.2</td>
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<td>18.2</td>
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<td>10</td>
<td>11.50</td>
<td>-1.75</td>
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<td>11.75</td>
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<td>5630</td>
</tr>
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<tr>
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<td>14.00</td>
<td>0.75</td>
<td>60.6</td>
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<tr>
<td>9.25</td>
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<td>14.25</td>
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<td>14.50</td>
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<td>7.9</td>
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<td>1.75</td>
<td>6.7</td>
<td>19.50</td>
<td>6.25</td>
<td>3.4</td>
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<td>19.75</td>
<td>6.50</td>
<td>4.3</td>
</tr>
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<td>20.25</td>
<td>7.00</td>
<td>64.4</td>
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<td>22.8</td>
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<td>7.25</td>
<td>91.5</td>
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<td>16.25</td>
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<td>40</td>
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<td>16.50</td>
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<td>7.75</td>
<td>7</td>
</tr>
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<td>21.25</td>
<td>8.00</td>
<td>2.4</td>
</tr>
<tr>
<td>17.00</td>
<td>3.75</td>
<td>3.7</td>
<td>21.50</td>
<td>8.25</td>
<td>2.6</td>
</tr>
<tr>
<td>17.25</td>
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<td>11.8</td>
<td>21.75</td>
<td>8.50</td>
<td>2.8</td>
</tr>
<tr>
<td>17.50</td>
<td>4.25</td>
<td>1703</td>
<td>22.00</td>
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<td>2.8</td>
</tr>
<tr>
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<td>22.25</td>
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<td>9.75</td>
<td>200</td>
</tr>
<tr>
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<td>5.50</td>
<td>37</td>
<td>23.25</td>
<td>10.00</td>
<td>52</td>
</tr>
<tr>
<td>19.00</td>
<td>5.75</td>
<td>6.8</td>
<td>23.50</td>
<td>10.25</td>
<td>8</td>
</tr>
<tr>
<td>19.25</td>
<td>6.00</td>
<td>4.2</td>
<td>23.75</td>
<td>10.50</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24.00</td>
<td>10.75</td>
<td>1.6</td>
</tr>
</tbody>
</table>

A plot of the detector current as a function of $(x - x_0)$ is shown in Figure 5.

We see the $n = 1$ and $n = -1$ order in this grating. They occur when $(x - x_0) = \pm 4.73$ mm. Using the equation

$$d \sin(\theta) = n\lambda$$

and $\sin(\theta) = (x - x_0)/D$, we find $d = 0.010$ cm. The manufacturer gives a value for $d = 0.01$ cm.
12.2 Dispersive Power of a Prism

1. Introduction

The refractive index $n$ of the material of a prism varies with the wavelength $\lambda$ of the light. If the wavelength of light is much longer than the absorption wavelength of the material, the refractive index varies as

$$n = \frac{B + C}{\lambda^3},$$

where $B$ and $C$ are material constants. This is called the Cauchy dispersion formula. The dispersive power of a prism is the rate of change of refractive index with wavelength. Differentiating the Cauchy formula with respect to $\lambda$ we get

$$\text{Dispersive power} = \frac{dn}{d\lambda} = \frac{-2C}{\lambda^3}.$$  \hfill (2)

Note that the dispersive power is negative, i.e., the refractive index increases as the wavelength decreases. The prism will have a lower refractive index for red light as opposed to blue light, because red light has a longer wavelength than blue light. This is the common behavior of all materials at wavelengths much longer compared to the absorption wavelengths of the material.

2. Procedure

We use a mercury lamp which gives the following prominent spectral lines as in Table 1.

<table>
<thead>
<tr>
<th>Colour of the line</th>
<th>Wavelength $\lambda$ (nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow 1</td>
<td>579</td>
</tr>
<tr>
<td>Yellow 2</td>
<td>577</td>
</tr>
<tr>
<td>Green</td>
<td>546.1</td>
</tr>
<tr>
<td>Blue</td>
<td>435.8</td>
</tr>
<tr>
<td>Violet 1</td>
<td>407.6</td>
</tr>
<tr>
<td>Violet 2</td>
<td>404.6</td>
</tr>
</tbody>
</table>

Figure 5. Diffraction pattern for a grating.
3. **Adjustment of the Spectrometer**

Pull out, or push in, the eyepiece of the telescope on the spectrometer so that the cross-wires are clearly seen. Turn the telescope on the spectrometer to a distant object, and adjust the focusing screw so that, on looking through the eyepiece, the object is focused on the cross-wire. Then turn the telescope so that it is in line with the collimator. Put a mercury lamp in front of the collimator so that its light falls on the collimator slit. Looking through the telescope, adjust the screw changing the distance between the collimator slit and collimator lens, so that the slit is sharply focused at the cross-wire of the telescope. The collimator is now adjusted to send parallel rays. Adjust the width of the collimator slit so that the image of the slit is narrow. Note the direct reading $R_{\text{dir}}$ on the vernier scale attached to the telescope on the spectrometer table.

4. **Measuring the Angle of the Prism**

Now put the prism on the prism table and adjust the leveling screws on the prism table to make it level.

![Diagram](image)

**Figure 1.** To measure the angle of the prism.

Turn the prism table so that the vertical edge of the prism is symmetrically situated with respect to the collimator lens. This is shown in Figure 1. Turn the telescope to the right until the image of the slit reflected by the face EF of the prism is seen at the cross-wire. Note the reading $R_{\text{Rt}}$ on the vernier of the spectrometer table. Then turn the telescope to the left so that the image of the slit reflected by the face EG of the prism is seen on the cross-wire. Note the reading $R_{\text{Lt}}$ on the vernier. The angle $A$ of the prism is

$$A = \frac{1}{2} \left( \text{difference between readings } R_{\text{Rt}} \text{ and } R_{\text{Lt}} \right).$$

The left and right readings of the vernier were $120^\circ 12'$ and $240^\circ 18'$. So, the angle of the prism is $60^\circ 03'$.

5. **Measuring the Angle of Minimum Deviation**

Turn the prism table so that the rays from the collimator are incident at an angle on the face EG. This is shown in Figure 2.

The incident parallel polychromatic rays are refracted at the two faces EG and EF of the prism. Rays belonging to different colors are deviated through different angles. If the telescope is turned to the left, one sees a series of colored images of the slit, starting with yellow and proceeding to violet as the telescope is turned more to the left. As one turns the prism table clockwise, the colors move to the right, stop at one position of the prism, and then move back to the left. Keep the prism at the position where the lines move to the right, and start to turn back. This is the position of minimum deviation. In this position, the angle $D$ between the incident light and the parallel rays reaching the telescope after refraction at the prism faces, is the least value. This angle of minimum deviation will vary from color to color. For each color, find the minimum deviation position. Adjust the image of the slit in that color to be on the cross-wire.
of the eyepiece by moving the telescope and note the reading $R_{MD}$ on the vernier of the spectrometer table.

The refractive index for any one color is

$$n = \frac{\sin((A + D)/2)}{\sin(A/2)}, \quad (2)$$

where $A$ is the angle of the prism, and $D$ is the angle of minimum deviation for light of that color. Calculate $n$ for different wavelengths and plot $n$ vs. $1/\lambda^2$.

A sample set of readings is shown in Table 2. Note that the readings of the angles are converted to decimal fractions of a degree. A reading of $30^\circ 12' 57.25$ would be $30.2$°. In calculating the refractive index, the angles are converted to radians. $1^\circ$ is equal to $0.01745$ radians.

**Table 2.** Refractive index of the material of the prism as a function of wavelength.

<table>
<thead>
<tr>
<th>Wavelength $\lambda$ (nanometers)</th>
<th>$R_{MD}$</th>
<th>$A$ (degrees)</th>
<th>$D$ (degrees)</th>
<th>$(A + D)/2$ (degrees)</th>
<th>Ref. index</th>
<th>$[1/\lambda^2]$ (m$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>579</td>
<td>122°58'</td>
<td>57.25</td>
<td>57.3</td>
<td>58.625</td>
<td>1.7075</td>
<td>2.98 × 10$^{12}$</td>
</tr>
<tr>
<td>577</td>
<td>122°55'</td>
<td>57.3</td>
<td>58.65</td>
<td>58.075</td>
<td>1.7080</td>
<td>3.00 × 10$^{12}$</td>
</tr>
<tr>
<td>546.1</td>
<td>122°53'</td>
<td>57.83</td>
<td>58.915</td>
<td>58.725</td>
<td>1.7128</td>
<td>3.35 × 10$^{12}$</td>
</tr>
<tr>
<td>435.8</td>
<td>119°35'</td>
<td>60.63</td>
<td>60.315</td>
<td>60.005</td>
<td>1.7375</td>
<td>5.27 × 10$^{12}$</td>
</tr>
<tr>
<td>407.8</td>
<td>117°53'</td>
<td>62.33</td>
<td>61.165</td>
<td>61.745</td>
<td>1.7520</td>
<td>6.01 × 10$^{12}$</td>
</tr>
<tr>
<td>404.6</td>
<td>117°39'</td>
<td>62.57</td>
<td>61.285</td>
<td>61.745</td>
<td>1.7540</td>
<td>6.11 × 10$^{12}$</td>
</tr>
</tbody>
</table>

Figure 3 shows a plot of $n$ vs. $1/\lambda^2$.

A linear fit to the data give the values of $B$ and $C$ as $1.6640$ and $1.449 \times 10^{-14}$ /m$^2$. So

$$n(\lambda) = \frac{1.6640 + 1.449 \times 10^{-14}}{\lambda^2}.$$
12.3 Spectrum Analysis with a Diffraction Grating

1. Introduction

In a grating we have periodic rulings with a spacing \(d\), expressed in meters. The number \(N\) of such rulings per \(m\) of the grating, is

\[
N = \frac{1}{d}. \tag{1}
\]

When parallel light of wavelength \(\lambda\) falls on a grating, each of the transparent regions on the grating diffracts the light. The telescope collects the rays, which are diffracted at an angle \(\theta\) to the incident rays, and brings them to a focus. This is shown in Figure 1.

\[\text{Figure 1. AB is the diffraction grating. Incident parallel rays normal to the grating are diffracted through an angle } \theta \text{ and are brought to a focus by the objective of the telescope.}\]
The phase difference between rays diffracted from corresponding points in the adjacent slits of the grating is \( d \sin (\theta) \). If this is an integer \( n \), times the wavelength \( \lambda \), then all the rays from corresponding points in all the transparent regions diffracted through the angle \( \theta \) will be in phase, and one will get maximum intensity at the focal point of the telescope. For a monochromatic beam of wavelength \( \lambda \), constructive interference will occur at an angle \( \theta_1 \) for \( n = 1 \), at an angle \( -\theta_1 \) for \( n = -1 \), an angle \( \theta_2 \) for \( n = 2 \), at an angle \( -\theta_2 \) for \( n = -2 \), and so on. Thus, we see many bright images of the slit on either side of the incident beam. \( n = 1 \) gives the first order diffraction image, \( n = 2 \) the second order diffraction image, and so on. The larger the diffraction angle, the higher the order. If the incident beam contains many different wavelengths, each of the wavelengths produces diffraction images. We see that the shorter the wavelength of the light, the smaller is the angle \( \theta_n \) for the \( n \)-th order diffraction beam. One should note that violet light will be diffracted at a smaller angle than red light. The order of colors from the direction of the incident beam will be from violet to red, as the diffraction angle increases.

On the other hand, in a prism, red is deviated less than violet. So, the angle of deviation increases as one goes from red to violet. The order is opposite to that in the grating.

In this experiment, we will determine the wavelength of the different lines of mercury light, by measuring the diffraction angles \( \theta_1 \) in the order \( n = 1 \), and \( \theta_{-1} \) in the order \( n = -1 \).

2. Procedure

The procedure for adjusting the lens to produce a collimated beam of rays is the same as for the case of the prism. This was described in the previous section.

The grating has to be set perpendicular to the incident rays. First, set the telescope to receive direct rays from the collimator and note the reading \( R_D \) on the vernier scale. Then, rotate the telescope to the left by an angle 90° and fix it. Mount the grating in the given mount on the prism table. Then, rotate the prism table till the image of the slit, reflected by the grating, appears on the cross-wire of the eyepiece of the telescope. Note the prism table reading, if it has a separate scale. Then, rotate the prism table through 45° so that the grating is normal to the incident rays from the collimator, and fix the prism table.

(If the prism table does not have a separate scale, adjust the prism table so that the grating appears to the eye to be normal to the axis of the collimator. View the green line of mercury in the order \( n = 1 \) by turning the telescope clockwise. Fix the cross-wire on the green line, and take the scale reading on the telescope table. Then, rotate the telescope anticlockwise to see the green line in the \( n = -1 \) order. The angle measured from the incident beam direction will be the same, if the orientation of the grating is correct. If the grating is misoriented, the two angles will be different. Turn the prism table slightly so that the two angles become the same, within 30° of arc of the telescope scale reading. Now the misorientation of the grating will be less than 15° of arc.)

After this adjustment do not touch the prism table.

Measure the angles \( \theta_1 \) for the different colors, starting from the yellow lines and moving towards the violet. The telescope will be turned from left to right. Proceed further and take the readings \( \theta_{-1} \) for the \( n = -1 \) order staring from violet to yellow. In the grating obtained from Holmarc, we see only \( n = 1 \) and \( n = -1 \) order for all the lines of the mercury spectrum.

A sample set of readings is given in Table 1.

<table>
<thead>
<tr>
<th>Mercury lines</th>
<th>( \theta_1 ) (deg)</th>
<th>( \theta_{-1} ) (deg)</th>
<th>( \theta_{av} ) (deg)</th>
<th>( \theta_{calc} ) (m)</th>
<th>Actual ( \lambda ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yellow1</td>
<td>35.67</td>
<td>35</td>
<td>35.34</td>
<td>5.78E–07</td>
<td>5.79E–07</td>
</tr>
<tr>
<td>Yellow2</td>
<td>35.55</td>
<td>34.9</td>
<td>35.23</td>
<td>5.77E–07</td>
<td>5.77E–07</td>
</tr>
<tr>
<td>Green</td>
<td>33.5</td>
<td>32.8</td>
<td>33.15</td>
<td>5.47E–07</td>
<td>5.46E–07</td>
</tr>
<tr>
<td>Blue</td>
<td>26</td>
<td>25.67</td>
<td>25.84</td>
<td>4.36E–07</td>
<td>4.36E–07</td>
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<tr>
<td>Violet2</td>
<td>24</td>
<td>23.75</td>
<td>23.88</td>
<td>4.05E–07</td>
<td>4.05E–07</td>
</tr>
</tbody>
</table>

The wavelengths of the different lines are within about 1 nm from the correct result.


Experiments in Optics

12.4 Resolving Power of a Prism and Grating

1. Introduction

Light is an electromagnetic wave. All waves undergo diffraction at an obstacle, and spread around. A parallel beam of light of wavelength $\lambda$ falls on a slit of width $w$. This situation is shown in Figure 1.

![Figure 1](image)

Figure 1. Parallel beam of wavelength $\lambda$ falls normally on a slit of width $w$.

The intensity $I(\theta)$ for an angle of diffraction $\theta$ is given by

$$I(\theta) = I(0) \left( \frac{\sin(x)}{x} \right)^2.$$  \hspace{1cm} (1)

Here

$$x = \frac{\pi w \sin(\theta)}{\lambda},$$ \hspace{1cm} (2)

and $I(0)$ is the intensity of the direct beam.

This intensity distribution, as a function of $\theta$, is shown in Figure 2.

![Figure 2](image)

Figure 2. Intensity distribution as a function of the diffraction angle $\theta$.

The intensity is maximum at $\theta = 0$ and decreases on both sides of the direct beam. The intensity goes to zero at an angle $\pm \theta_{\text{min}}$ for which $w \sin \theta_{\text{min}} = \lambda$. \hspace{1cm} (3)

The path difference of rays coming from two extreme edges of the slit, for a diffraction angle $\theta$, is $w \sin(\theta)$. At $\theta_{\text{min}}$, this path difference is $\lambda$. We may divide the slit into two equal halves. The waves, coming from the corresponding points in the two halves, will differ in path by $\lambda/2$ at $\theta_{\text{min}}$, and will interfere destructively. So, the total amplitude becomes zero, and the intensity is a minimum.

Suppose we have two sources, with an angular separation $\xi$, sending out parallel rays of the same wavelength through the slit. Each source will produce its own diffraction pattern, and the total intensity will be the sum of the intensities due to each source. We show in Figure 3, the total intensity curves for three different angular separations, namely (i) $\xi < \theta_{\text{min}}$, (ii) $\xi = \theta_{\text{min}}$, and (iii) $\xi > \theta_{\text{min}}$. In these figures, the black
**Figure 3(i).** Intensity distribution of the diffraction patterns due to two sources of the same wavelength, and $\xi < \theta_{\min}$.

**Figure 3(ii).** Intensity distribution of the diffraction patterns due to two sources of the same wavelength, and $\xi = \theta_{\min}$.

**Figure 3(iii).** Intensity distribution of the diffraction patterns due to two sources of the same wavelength, and $\xi > \theta_{\min}$. 

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and red curves are the intensities produced by two different sources of the same wavelength, separated by an angle \( \xi \). The blue curve is the sum of the intensities.

From these figures, it is clear that a well-defined minimum occurs between the two maximum values in the total intensity curve only when \( \xi \geq \theta_{\text{min}} \). At \( \xi = \theta_{\text{min}} \), the first minimum of one diffraction pattern (black) falls exactly under the principal maximum of the second pattern (red).

So we have the Rayleigh criterion for the diffraction images of two sources to be seen as distinct:

The two diffraction images will be resolved if the angular separation of the principal maxima of the two images equals the angle at which the first minimum of each pattern is separated from its principal maximum.

2. Resolving Power of a Prism

Figure 4 shows the two extreme rays which pass through the slit in front of the equilateral prism \( P \) and enter the objective \( O \) of the telescope. In the minimum deviation position the rays pass through the prism parallel to its base.

Let us say that the principal maximum for wavelength \( \lambda \) occurs at the set angle. The ray \( EBE' \) travels entirely through air for which the refractive index \( n \) may be taken as 1. The other extreme ray travels the distance \( B'B'' \) in the prism. The refractive index of the prism is \( n \) and its dispersive power is \( \frac{dn}{d\lambda} \).

The path difference between the extreme rays for wavelength \( \lambda \) is

\[
\text{PD for } \lambda = nB'B'' - EBE'.
\]

For the wavelength \( \lambda + \Delta \lambda \) travelling in the same direction, the path difference is

\[
\text{PD for } \lambda + \Delta \lambda = \left(n + \left[\frac{dn}{d\lambda}\right] \Delta \lambda\right) B'B'' - EBE'.
\]

If we get the image of the slit for the wavelength \( \lambda \) in the objective of the telescope for this setting, the PD for \( \lambda \) in equation (4) is zero.

Light of wavelength \( \lambda + \Delta \lambda \) will produce an image of the slit at a slightly different angle than the light of wavelength \( \lambda \). If we have to distinguish these two images, the principal maximum of the diffraction pattern due to wavelength \( \lambda \) should fall on the first minimum of the diffraction pattern due to wavelength \( \lambda + \Delta \lambda \). The first minimum of the diffraction pattern due to \( \lambda + \Delta \lambda \) corresponds to a path difference of the extreme rays (equation 5) of \( \lambda + \Delta \lambda \). So subtracting equation (4) from equation (5), and equating the difference to \( \lambda + \Delta \lambda \), we get

\[
B'B'' \left(\frac{dn}{d\lambda}\right) \Delta \lambda = \lambda + \Delta \lambda \approx \lambda.
\]

\( \Delta \lambda \) is small compared to \( \lambda \), and represents the minimum difference in wavelengths for which the images of the slit at the different wavelengths can be seen differently.

The resolving power of the prism is defined as

\[
\frac{\lambda}{\Delta \lambda} = \left[ \frac{dn}{d\lambda}\right].
\]
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Here \( t = B'B'' \). The entire width of the light beam traverses the prism with the base BB'.

Note that \( t \) is the length of travel of the extreme ray \( B'B'' \) through the prism. This is determined by the width \( w \) of the slit through which the incident beam passes. It is not the length of the base of the prism used.

In the experiment, we use mercury light as the source. There are two close wavelengths in the yellow, with wavelengths \( \lambda \) and \( \lambda + \Delta \lambda \) of 577 and 579 nm. With the equilateral prism of glass commercially available, the base of the prism is large enough to see the images of the slit produced at these two wavelengths as distinct.

We put a slit in the path of the incident beam after the collimator lens. The slit should be normal to the beam. The slit should be kept wide open to start with, so that the images of the slit produced by the two wavelengths are seen as distinct. Then the slit is closed. As the width of the slit decreases, the two images of the slit start broadening. At one value of the slit width, the two images just merge, and we see a single patch of light, and not two sharp images of the slit. At this point the slit is removed, and its width determined by a traveling microscope.

We must express \( t \), the extra length travelled in the prism by one of the extreme rays over the other, in terms of the width \( w \) of the slit. Figure 5 shows the path of the rays at minimum deviation position for the wavelength \( \lambda \).

![Figure 5. Path of the rays through the prism at minimum deviation position for wavelength \( \lambda \).](image)

In this position ray \( B'B'' \) is parallel to the base of the prism. From Figure 5, at the minimum deviation position, the angle \( FB'B'' \) is

\[
\text{angle } FB'B'' = \frac{D}{2},
\]

where \( D \) is the angle of minimum deviation.

So

\[
\text{angle } FB'B = 60 - \left( \frac{D}{2} \right)
\]

since the prism is equilateral, i.e., angle \( BB'B'' \) is 60°. So, the

\[
\text{angle } BB'E = 30 + \left( \frac{D}{2} \right).
\]

The width \( EB' \) is \( w \), the width of the slit. So

\[
B'B = BB'' = t = \frac{w}{\cos (30 + D/2)}.
\]

The angles are expressed in degrees here.

If the two yellow lines just merge for the width \( w \) of the slit, the resolving power of the prism is

\[
RP = t \left( \frac{dn}{d\lambda} \right)_{\text{yellow}} = \left[ \frac{w}{\cos (30 + D/2)} \right] \left( \frac{dn}{d\lambda} \right)_{\text{yellow}}.
\]
In Section 12.2 we found the dispersive power of the prism to be given by the formula

\[ n(\lambda) = \frac{1.664 + 1.449 \times 10^{-14}}{\lambda^2}. \] (13)

We also found the angle of minimum deviation \( D \) for the 577 nm yellow line of mercury to be \( 57^\circ 18' \).

The width of the slit when the images at the two wavelengths merged came out to be 1.27 mm. From the angle of minimum deviation \( D \) for the yellow lines, one calculates \( t \) of the prism at the limit of resolution, to be 2.6 mm. The magnitude of the dispersive power for the yellow line, calculated from (13), is \( 1.54 \times 10^5 \) m. So the resolving power is

\[ \text{RP} = 2.6 \times 10^{-3} \times 1.54 \times 10^5 = 392. \] (14)

\( \lambda/\Delta\lambda \) for the yellow lines of mercury is \( 578/2 = 289 \). Thus the resolving power is nearly equal to the value of \( \lambda/\Delta\lambda \). Different people’s eyes can distinguish minimum and maximum intensities differently. So the resolving power measured by different people, for the same two wavelengths, will differ. But the measured values will nearly be equal to \( \lambda/\Delta\lambda \).

3. Resolving Power of a Grating

In a grating we have periodic rulings with a spacing \( d \). The number \( N \) of such rulings per unit width of the grating is

\[ N = \frac{1}{d}. \] (15)

When parallel light of wavelength \( \lambda \) falls normally on a grating, each of the transparent regions on the grating diffracts the light. The telescope collects the rays which are diffracted at an angle \( \theta \) to the incident rays and brings them to a focus. This is shown in Figure 6.

![Figure 6](image_url)

**Figure 6.** AB is the diffraction grating. Incident parallel rays normal to the grating are diffracted through an angle \( \theta \) and are brought to a focus by the objective lens.

The phase difference between rays diffracted from corresponding points in the adjacent slits of the grating is \( d \sin(\theta) \). If this is an integer \( n \) times the wavelength \( \lambda \), then all the rays from corresponding points in all the transparent regions diffracted through the angle \( \theta \) will be in phase. Then one will get maximum intensity at the focal point of the telescope. For a monochromatic beam of wavelength \( \lambda \), constructive interference will occur at an angle \( \theta_1 \) for \( n = 1 \), at an angle \( -\theta_1 \) for \( n = -1 \), at an angle \( \theta_2 \) for \( n = 2 \), at an
angle $-\theta_2$ for $n = -2$, and so on. Thus we see many bright images of the slit on either side of the incident beam. $n = 1$ gives the first order diffraction image, $n = 2$ the second order diffraction image and so on. The larger the diffraction angle, the higher the order. If the incident beam contains many different wavelengths, each of the wavelengths produces diffraction images.

Suppose we have two close wavelengths $\lambda$ and $\lambda + \Delta\lambda$ falling on the grating. Each wavelength will produce an image in the $n$-th order at slightly different angles of diffraction. If the primary maximum of the diffraction image due to wavelength $\lambda + \Delta\lambda$ falls on the first minimum of the diffraction image due to $\lambda$, the two images will be seen as distinct peaks in intensity. If the width of the grating is $w$, then the two extreme rays from the grating will have a path difference $w \sin \theta$. If $w \sin \theta = (N'n + 1)\lambda$, then the first minimum of the diffraction image of $n$-th order due to $\lambda$ will occur at the angle $\theta$. Here $N'$ is the total number of lines in the grating in the width $w$. The $n$-th order principal maximum of $\lambda + \Delta\lambda$ will also occur at this angle for the two images to be just resolved. So,

$$d \sin \theta = n(\lambda + \Delta\lambda).$$

(16)

$$N' = \frac{w}{d}.$$  

(17)

$$w \sin \theta = N'n \sin \theta = N'n(\lambda + \Delta\lambda) = (N'n + 1)\lambda.$$  

(18)

Therefore,

$$\frac{\lambda}{\Delta\lambda} = nN'.$$  

(19)

So the resolving power of the grating is proportional to the total number of lines $N'$ in the width of the grating, and the order $n$ of diffraction.

4. Procedure

The resolving power of a grating is much higher than that of a prism. So to determine the resolving power of a grating we use a sodium lamp. This gives two yellow lines at 589.0 and 589.6 nm. In a prism with a side about 5 cm long, you will not see the two wavelengths as separate. But with a grating with 10,000 lines per cm and of about 5 cm width, the two lines will be seen as clearly separate even in the $\pm 1$ order.

Fix the telescope on the yellow lines of sodium in the first order. Mount a slit of adjustable width on the collimator. Adjust the slit width till the two yellow lines appear to merge into a single broad line. When the slit is reduced in width, the intensity of the lines will go down, and fixing this position requires care. Remove the slit and measure its width $w$ on a traveling microscope.

Repeat the measurement for the yellow lines in the $n = -1$ order. Let the width be $w'$. Take the average of $w$ and $w'$. Let this be $w_{av}$. Then the width of the grating from which rays are received by the telescope through the slit is $w_{av}$. The number $N'$ of lines in the grating of width $w_{av}$ is

$$N' = w_{av}N.$$  

(20)

Compare this with $\lambda/\Delta\lambda$. Sample data are given in Table 2.

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Resolving power of the grating.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number $N'$ of lines of grating per cm: 10,000.</td>
<td></td>
</tr>
<tr>
<td>$n = 1$ order. Width $w$ of slit: 0.098 cm.</td>
<td></td>
</tr>
<tr>
<td>$n = -1$ order. Width $w''$ of slit: 0.103 cm.</td>
<td></td>
</tr>
<tr>
<td>Average width $w_{av} = 0.101$ cm.</td>
<td></td>
</tr>
<tr>
<td>Number $N'$ of lines in width $w_{av} = 1010$.</td>
<td></td>
</tr>
<tr>
<td>$\lambda/\Delta\lambda = 589/0.6 = 981$.</td>
<td></td>
</tr>
</tbody>
</table>

We see that $N'$ is close to $\lambda/\Delta\lambda$.  

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1. Introduction

Light is a transverse electromagnetic wave. In an isotropic medium, the velocity of propagation of light is independent of the direction of propagation. We define the wave vector of a light wave as the vector \( \mathbf{k} \) which has the magnitude \( 2\pi/\lambda \), where \( \lambda \) is the wavelength of the wave. The direction of \( \mathbf{k} \) is the direction of propagation of the wave. If \( \nu \) is the frequency of the wave, the phase velocity \( v_p \) of the wave is \( v_p = \omega/k \), where \( \omega = 2\pi\nu \). The phase velocity is the velocity with which the phase of the wave propagates.

Generally, light propagates as a group of waves with a slight spread in frequency. The velocity with which the group propagates is the velocity with which the energy in the group propagates. The group velocity \( v_g \) is given by \( \partial\omega/\partial k \). If the phase velocity in a material is the same at all frequencies, the medium is said to be non-dispersive. Vacuum is such a medium. In such a non-dispersive medium, the phase velocity is equal to the group velocity. For vacuum this velocity is \( c = 2.998 \times 10^8 \) meters per second. All media, other than vacuum, are dispersive. In such media, \( \omega \) does not vary linearly with \( k \). So the phase velocity is different from the group velocity. The refractive index of a medium for light of wavelength \( \lambda \) is the ratio of the velocity of light in vacuum to the phase velocity of light (of this wavelength) in the medium. The refractive index is denoted by the symbol \( n \) and is greater than 1, except in the close vicinity of an absorption wavelength of the medium. For an isotropic medium, \( n \) is independent of the direction of propagation of the light.

In a light wave, there are the electric displacement vector \( \mathbf{D} \) and the magnetic induction field vector \( \mathbf{B} \) varying with position and time. These two fields are in a plane perpendicular to \( \mathbf{k} \). That is why light is called a transverse electromagnetic wave. Associated with the electric displacement \( \mathbf{D} \) is the electric field \( \mathbf{E} \). With the magnetic induction field \( \mathbf{B} \) is associated the magnetic intensity field \( \mathbf{H} \). \( \mathbf{D} \) and \( \mathbf{B} \) are in a plane perpendicular to the propagation vector \( \mathbf{k} \). \( \mathbf{D} \) and \( \mathbf{B} \) are also mutually orthogonal. In an isotropic medium \( \mathbf{E} \) is parallel to \( \mathbf{D} \), and \( \mathbf{H} \) is parallel to \( \mathbf{B} \). This will not be so in an anisotropic medium.

In the plane perpendicular to the direction of propagation, we may choose two unit vectors \( \mathbf{i} \) and \( \mathbf{j} \) which are mutually perpendicular. Then the electric displacement vector \( \mathbf{D} \) can be written as

\[
\mathbf{D} = D_i \mathbf{i} + D_j \mathbf{j}. \tag{1}
\]

If \( D_i = 0 \), the displacement vector is in the direction of \( \mathbf{i} \) and if \( D_j = 0 \), it is in the direction \( \mathbf{j} \). These two states are said to be two independent states of polarization of the light. In general, the phase velocity of light may depend on (i) direction of propagation, (ii) direction of polarization for a given direction of propagation, and (iii) wavelength of the light.

2. Isotropic, Uniaxial and Biaxial Crystals

In an isotropic medium, the phase velocity of light does not depend on the direction of propagation or on the direction of polarization. It depends only on the wavelength of light. There is a unique refractive index \( n \) which depends only on wavelength. Examples of such materials are glass, liquids such as water, or crystals which belong to the cubic system like diamond, sodium chloride or calcium fluoride.

In a uniaxial material, there is a unique direction of propagation for which the refractive index does not depend on the direction of polarization. For any other direction of propagation, the velocity depends on the state of polarization. Light polarized perpendicular to the plane containing the direction of the unique axis and the direction of propagation, will travel with a phase velocity independent of the direction of propagation. The refractive index for this state of polarization is called \( n_o \), and the corresponding light ray is called the ordinary ray. For light polarized in a direction not perpendicular to the unique axis, the refractive index will vary with the direction of propagation. The corresponding ray is called the extraordinary ray. For light propagating perpendicular to the unique axis, the polarization of the extraordinary ray will be parallel to the unique axis and the corresponding refractive index \( n_e \) is called the extraordinary refractive index. For any direction of propagation inclined to the unique axis, the extraordinary refractive index will have a value between \( n_o \) and \( n_e \). Light propagation in a uniaxial crystal is completely determined by the
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two values \( n_o \) and \( n_e \) of the refractive index of the material. Crystals belonging to the tetragonal, trigonal or hexagonal crystal systems are optically uniaxial. The optic axis (i.e., the direction of propagation for which the refractive index is \( n_o \) for both states of polarization) is along the four-fold, three-fold or six-fold axis of symmetry, respectively, of the crystal. Examples of uniaxial crystals are rutile (tetragonal TiO\(_2\)), calcite (trigonal CaCO\(_3\)) and quartz (hexagonal SiO\(_2\)).

In a biaxial material the phase velocity in any direction of propagation is dependent on the state of polarization. There are three principal refractive indices for such a material corresponding to three mutually perpendicular directions of propagation \( X', Y' \) and \( Z' \). These indices are labeled \( n_1, n_2 \) and \( n_3 \). Crystals belonging to orthorhombic, monoclinic and triclinic systems are, in general, biaxial. Light propagation in these crystals is very complicated. New phenomena like internal and external conical refraction arise in these materials. For a discussion of these phenomena one should refer to text books in optics.

3. States of Polarized Light

The electric displacement vector \( \mathbf{D} \) at a point \( \mathbf{r} \) in a light wave, propagating along a direction \( \mathbf{k} \), can be written in general as

\[
\mathbf{D}(t) = (D_1 \mathbf{i} + D_2 \exp(i\phi) \mathbf{j}) \exp(i\omega t). \tag{2}
\]

Here \( \mathbf{i} \) and \( \mathbf{j} \) are two orthogonal unit vectors in the plane perpendicular to the direction of propagation. We may now define different states of polarized light.

a) Unpolarized Light

For unpolarized light, \( D_1 = D_2 \), and \( \phi \), the phase difference between the displacements along \( \mathbf{i} \) and \( \mathbf{j} \), varies randomly with time. The intensity of the light beam will be proportional to \( 2D^2 \), where \( D \) is the value of \( D_1 = D_2 \).

Light from a thermal source like a sodium or mercury vapour lamp, or sunlight, is unpolarized.

b) Linearly Polarized Light

If \( \phi = 0 \) (i.e., if the two displacements along \( \mathbf{i} \) and \( \mathbf{j} \) vibrate in phase) then the light is linearly polarized. The displacement vector oscillates in the \( \mathbf{ij} \)-plane along a line making an angle \( \xi \) with the \( \mathbf{i} \)-axis is given by

\[
\tan(\xi) = \frac{D_2}{D_1}. \tag{3}
\]

Such linearly polarized light is shown in Figure 1(a).

If \( \phi = \pi \) (i.e., if the two displacements along \( \mathbf{i} \) and \( \mathbf{j} \) vibrate 180° out of phase) then also the light is linearly polarized. Now the displacement vector oscillates in the \( \mathbf{ij} \)-plane along a line making an angle \( \xi \) with the \( \mathbf{i} \)-axis given by

\[
\tan(\xi) = \frac{-D_2}{D_1}. \tag{4}
\]

If \( D_2 = 0 \), the light is polarized along the \( \mathbf{i} \)-axis. If \( D_1 = 0 \), the light is polarized along the \( \mathbf{j} \)-axis. If \( \phi = 0 \) and \( D_1 = D_2 \), the light is polarized in a direction making 45° to the \( \mathbf{i} \)-axis. If \( \phi = \pi \) and \( D_1 = D_2 \), the light beam is polarized in a direction making 135° to axis \( \mathbf{i} \). The intensity of the light beam is proportional to \( (D_1^2 + D_2^2) \).

c) Elliptically Polarized Light

If \( \phi \) is different from zero or \( \pi \), and \( D_1 \neq D_2 \neq 0 \), then light is elliptically polarized. The tip of the displacement vector goes around an ellipse in one period. The directions of the principal axes of the ellipse are in general inclined to the unit vectors \( \mathbf{i} \) and \( \mathbf{j} \). The angle of inclination and the ratio of the semi-principal axes of the ellipse will depend on \( \phi, D_1 \) and \( D_2 \). The intensity of the light beam is proportional to the sum of the squares of the semi-minor and semi-major axes of the ellipse. Such elliptically polarized light is shown in Figure 1(b).
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If $\phi = \pi/2$ or $3\pi/2$, the principal axes of the ellipse are along $i$ and $j$. The ratio of the semi-principal axes is $D_1/D_2$. If $D_1 < D_2$ the minor axis is along $i$. If $D_1 > D_2$ the minor axis is along $j$. If $\phi = \pi/2$, the tip of vector $\mathbf{D}$ goes around the ellipse in a clockwise direction. This is shown in Figure 1(c). If $\phi = 3\pi/2$, the tip of vector $\mathbf{D}$ goes around the ellipse in an anti-clockwise direction. This is shown in Figure 1(d).

In these cases the intensity of the light beam is proportional to $D_1^2 + D_2^2$.

d) Circularly Polarized Light

This is a special case of elliptically polarized light when $\phi = \pi/2$ or $3\pi/2$ and $D_1 = D_2 = D$. Here the tip of the displacement vector moves on a circle of radius $D$. The tip goes around the circle clockwise when $\phi = \pi/2$ (Figure 1(e)) and anti-clockwise when $\phi = 3\pi/2$ (Figure 1(f)).

![Figure 1(a–f). Different states of polarized light.](image)

4. Polarizer and Quarter-Wave Plate

To produce different states of polarized light from unpolarized light, we use devices known as a polarizer and a quarter-wave plate.

There are different types of polarizers. The least expensive polarizer is a Polaroid sheet. This is a transparent sheet coated with a material. This material will completely absorb light polarized in one direction, and allow light polarized in a perpendicular direction to go through. The direction of polarization of the light, which is transmitted with maximum intensity, is called the axis of the polarizer. If unpolarized or polarized light falls on this Polaroid sheet, it will allow the component of the light with the displacement vector parallel to its axis to pass through. The light coming out of the sheet will be linearly polarized, with the displacement vector parallel to its axis. This is how linearly polarized light is produced.

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If we have a second Polaroid sheet placed after the first one, in the path of the light beam, no light will be transmitted if the axis of the second sheet is perpendicular to the axis of the first sheet. The two Polaroid sheets are said to be crossed.

To produce elliptically or circularly polarized light from an incident linearly polarized beam, one uses a quarter-wave plate. This is made of a uniaxial crystal like quartz. A plate is cut with the optic axis parallel to the surface of the plate. If linearly polarized light falls on the plate, and the displacement vector in the incident light makes an angle \( \theta \) with the optic axis of the plate, the displacement vector \( \mathbf{D} \) will be resolved into two components: \( D_1 = D \cos(\theta) \) parallel to the optic axis, and \( D_2 = D \sin(\theta) \) perpendicular to the optic axis. If the incident beam is normal to the plate, both components will travel normal to the plate, within the plate. The first component will travel as the extraordinary ray with refractive index \( n_e \), and the second component will travel as the ordinary ray with refractive index \( n_o \). When they come out, they will have a phase difference

\[
\phi = \left( \frac{2\pi}{\lambda} \right) (n_e - n_o) t. \tag{5}
\]

Here \( t \) is the thickness of the plate and \( \lambda \) is the wavelength of light. In general the emergent light will be elliptically polarized with its principal axes at an angle to the optic axis of the plate.

If we choose the thickness \( t \) such that \( \phi = \pm \pi/2 \), the emergent light will be elliptically polarized with one of its principal axes parallel to the optic axes of the plate. The plate is called a Quarter Wave Plate (QWP) for the wavelength \( \lambda \). A QWP for wavelength \( \lambda \) will not act as a QWP for a different wavelength \( \lambda' \).

If we use a QWP for the incident wavelength \( \lambda \), and orient it so that its optic axis makes an angle \( \theta = \pi/4 \) to the polarization direction of the incident light, then the emergent light will be circularly polarized.

Thus using a polarizer and a QWP, we can produce light in different states of polarization from incident unpolarized light.

5. Analysis of Polarized Light

How do we analyze light to find its state of polarization? This can be done by using a Polaroid. We place the analyzer Polaroid in the path of the beam and find the intensity variation as the Polaroid is rotated.

If the intensity of the light coming out of the analyzer does not change as the analyzer is rotated, the incident beam is either unpolarized or circularly polarized. To distinguish between the alternatives, we need a QWP designed for the wavelength of light used. We place this QWP in the path of the light beam before the analyzer. If the analyzer is now rotated, and the intensity is found to remain constant, the incident light is unpolarized. A circularly polarized incident beam will emerge from a QWP as a linearly polarized beam. When the analyzer is rotated, the intensity will vary from a maximum to zero.

If the intensity of the light beam is viewed through an analyzer, and the intensity varies from a maximum to a minimum value as the analyzer is rotated, the incident beam is elliptically polarized. When the transmitted intensity is a maximum, the axis of the analyzer Polaroid is parallel to the major axis of the ellipse. The minor axis of the ellipse is perpendicular to this direction. The length of the major axis of the ellipse is proportional to the square root of the maximum intensity. The length of the minor axis of the ellipse is proportional to the square root of the minimum intensity. When the Polaroid is rotated through an angle \( \psi \) from the setting for maximum intensity, the intensity \( I(\psi) \) of the light transmitted through the Polaroid varies as

\[
I(\psi) = I_{\text{max}} \cos^2(\psi) + I_{\text{min}} \sin^2(\psi). \tag{6}\]

If the light is linearly polarized, then \( I_{\text{min}} = 0 \). No light comes through, because the axis of the analyzer is at right angles to the polarization direction of the incident light. If we turn the analyzer through an angle \( \eta \) from this crossed direction, the intensity \( I(\eta) \) of the transmitted light will vary as

\[
I(\eta) = I_{\text{max}} \sin^2(\eta). \tag{7}\]

In the next section we describe the experiment for the production and analysis of linear and elliptically polarized light.
12.6 Experiment for Production of Linear and Elliptically Polarized Light

1. Introduction

In the previous section an introduction to different states of polarized light was given. In this section we will describe an experiment to produce linearly and elliptically polarized light, and analyze the light so produced quantitatively.

For this purpose we use a 5 mW laser diode emitting green light at 550 nm wavelength, a diode detector to measure the intensity of the light, two Polaroid sheets on a rotatable mount to serve as a polarizer and analyzer, and a quarter-wave plate for wavelength 589 nm. These items were procured from Holmarc Opto-Mechatronics Company. Since the laser wavelength is different from the wavelength for which the QWP is designed, the QWP will not produce a phase difference of $\pi/2$ for the components of the electric field of the laser light, resolved along the fast and slow axes of the QWP. The phase difference $\phi$ produced by the QWP at the laser wavelength will be

$$\left(\frac{\pi}{2}\right)\left(\frac{589.3}{550}\right) = 1.683 \text{ radians.}$$

All the components, namely the laser, polarizer, QWP, analyzer and detector are mounted on an optic bench.

2. Linearly Polarized Light

Place the laser, the polarizer, the analyzer and the detector in this sequence on the optic bench. Switch on the laser and adjust the alignment screws so that the laser light falls on the detector. Turn the analyzer on the rotatable mount till the detector current is zero in the microamp range. At this position, the polarizer and analyzer are crossed, i.e., the axis of the analyzer is at right angles to the axis of the polarizer.

From this position, rotate the analyzer in steps of 10°, and note the detector current. Do this till the analyzer is rotated through 360°. A sample set of data is given in Table 1.

<table>
<thead>
<tr>
<th>Setting</th>
<th>$\theta$</th>
<th>$I(\mu A)$</th>
<th>$\sin^2\theta$</th>
<th>$I_{\text{max}}\sin^2(\theta)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>275</td>
<td>0</td>
<td>0.1</td>
<td>0.0000</td>
<td>0.0</td>
</tr>
<tr>
<td>265</td>
<td>−10</td>
<td>44</td>
<td>0.0302</td>
<td>54.3</td>
</tr>
<tr>
<td>255</td>
<td>−20</td>
<td>157.8</td>
<td>0.1170</td>
<td>210.6</td>
</tr>
<tr>
<td>245</td>
<td>−30</td>
<td>500</td>
<td>0.2500</td>
<td>450.0</td>
</tr>
<tr>
<td>235</td>
<td>−40</td>
<td>700</td>
<td>0.4132</td>
<td>743.8</td>
</tr>
<tr>
<td>225</td>
<td>−50</td>
<td>1000</td>
<td>0.5869</td>
<td>1056.3</td>
</tr>
<tr>
<td>215</td>
<td>−60</td>
<td>1300</td>
<td>0.7500</td>
<td>1350.1</td>
</tr>
<tr>
<td>205</td>
<td>−70</td>
<td>1600</td>
<td>0.8830</td>
<td>1589.5</td>
</tr>
<tr>
<td>195</td>
<td>−80</td>
<td>1800</td>
<td>0.9699</td>
<td>1745.8</td>
</tr>
<tr>
<td>185</td>
<td>−90</td>
<td>1800</td>
<td>1.0000</td>
<td>1800.0</td>
</tr>
<tr>
<td>175</td>
<td>−100</td>
<td>1700</td>
<td>0.9698</td>
<td>1745.7</td>
</tr>
<tr>
<td>165</td>
<td>−110</td>
<td>1500</td>
<td>0.8830</td>
<td>1589.4</td>
</tr>
<tr>
<td>155</td>
<td>−120</td>
<td>1200</td>
<td>0.7499</td>
<td>1349.9</td>
</tr>
<tr>
<td>145</td>
<td>−130</td>
<td>900</td>
<td>0.5867</td>
<td>1056.1</td>
</tr>
<tr>
<td>135</td>
<td>−140</td>
<td>600</td>
<td>0.4131</td>
<td>743.6</td>
</tr>
<tr>
<td>125</td>
<td>−150</td>
<td>21</td>
<td>0.0301</td>
<td>54.2</td>
</tr>
</tbody>
</table>

The first column in the table gives the angular reading of the rotatable mount of the analyzer. The second column gives the rotation $\theta$ in degrees from the crossed analyzer setting. The third column gives the detector current $I$ in microamps. The fourth column gives $\sin^2(\theta)$. The last column gives $I_{\text{max}}\sin^2(\theta)$. 

The first column in the table gives the angular reading of the rotatable mount of the analyzer. The second column gives the rotation $\theta$ in degrees from the crossed analyzer setting. The third column gives the detector current $I$ in microamps. The fourth column gives $\sin^2(\theta)$. The last column gives $I_{\text{max}}\sin^2(\theta)$. 

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Since the analyzer is in the crossed position when the transmitted intensity is zero, the light coming from the polarizer has its electric vector perpendicular to the analyzer axis. When the analyzer is turned through an angle $\theta$, the incident electric displacement vector makes an angle $(90-\theta)$ with the axis of the analyzer. So the component of the electric displacement vector parallel to the analyzer axis is $D \sin(\theta)$ and the intensity is $I_{\text{max}} \sin^2(\theta)$, where $I_{\text{max}}$ is the maximum detector current when $\theta = 90$ (i.e., when the axes of the polarizer and analyzer are parallel). Figure 1 shows the intensity as a function of $\theta$, with the continuous curve showing the plot of $I_{\text{max}} \sin^2(\theta)$. This plot shows how the intensity varies with the angle of rotation of the analyzer for linearly polarized light.

![Figure 1](image)

Figure 1. Variation of transmitted intensity as a function of the angle of rotation from the crossed position of the analyzer for linearly polarized light.

3. Elliptically Polarized Light

Interpose the quarter-wave plate between the polarizer and analyzer in the crossed position. Rotate the quarter wave plate till the detector shows zero current. Then one axis of the QWP is parallel to the axis of the polarizer, and the other axis is perpendicular to it. In this case, light emerges from the QWP with its state of polarization unchanged. Now rotate the QWP through some angle $\phi$ from this position. The light emerging from the QWP will be elliptically polarized, with its principal axes at an angle to the polarizer and analyzer axes. Now rotate the analyzer in steps of 10° through 360°, and note the detector reading for each position of the analyzer. A sample set of data is given in Table 2.

<table>
<thead>
<tr>
<th>Table 2. Analysis of elliptically polarized light.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Polarizer setting : 164 degrees</td>
</tr>
<tr>
<td>Analyzer setting in crossed position: 172 deg.</td>
</tr>
<tr>
<td>QWP setting to get null intensity: 225 deg.</td>
</tr>
<tr>
<td>QWP turned to 235 deg.</td>
</tr>
<tr>
<td>$\Theta_0$ 270 deg.</td>
</tr>
<tr>
<td>$I_{\text{max}}$ 12.7 mA</td>
</tr>
<tr>
<td>$I_{\text{min}}$ 0.2 mA</td>
</tr>
</tbody>
</table>

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Table 2. (Contd.)

<table>
<thead>
<tr>
<th>Anal. Setting</th>
<th>$\Theta - \Theta_0$ (degrees)</th>
<th>I (milliamps)</th>
<th>$I_{\text{max}} \cos^2 \beta$ (milliamps)</th>
<th>$I_{\text{max}} \sin^2 \beta$ (milliamps)</th>
<th>$I_{\text{cal}}$ (milliamps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>172</td>
<td>−98</td>
<td>0.9</td>
<td>0.2462</td>
<td>0.1961</td>
<td>0.4423</td>
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<tr>
<td>182</td>
<td>−88</td>
<td>0.2</td>
<td>0.0154</td>
<td>0.1998</td>
<td>0.2152</td>
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<tr>
<td>192</td>
<td>−78</td>
<td>0.3</td>
<td>0.5487</td>
<td>0.1914</td>
<td>0.7401</td>
</tr>
<tr>
<td>202</td>
<td>−68</td>
<td>1</td>
<td>1.7818</td>
<td>0.1719</td>
<td>1.9538</td>
</tr>
<tr>
<td>212</td>
<td>−58</td>
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<td>0.1438</td>
<td>3.7098</td>
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<tr>
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<td>5.7963</td>
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<tr>
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<td>7.8859</td>
<td>0.0758</td>
<td>7.9617</td>
</tr>
<tr>
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<td>9.1</td>
<td>9.9007</td>
<td>0.0441</td>
<td>9.9448</td>
</tr>
<tr>
<td>252</td>
<td>−18</td>
<td>10.7</td>
<td>11.4872</td>
<td>0.0191</td>
<td>11.5063</td>
</tr>
<tr>
<td>262</td>
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<td>12.7</td>
<td>12.4540</td>
<td>0.0039</td>
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<td>272</td>
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<td>0.0002</td>
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<tr>
<td>282</td>
<td>12</td>
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<td>12.1510</td>
<td>0.0086</td>
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<td>0.0281</td>
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<td>312</td>
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<tr>
<td>332</td>
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<td>3.7</td>
<td>2.7987</td>
<td>0.1559</td>
<td>2.9547</td>
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<tr>
<td>342</td>
<td>72</td>
<td>1.8</td>
<td>1.2124</td>
<td>0.1809</td>
<td>1.3933</td>
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<tr>
<td>362</td>
<td>92</td>
<td>0.2</td>
<td>0.0155</td>
<td>0.1998</td>
<td>0.2153</td>
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<tr>
<td>372</td>
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<td>0.4</td>
<td>0.5493</td>
<td>0.1914</td>
<td>0.7407</td>
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<tr>
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<td>1.7828</td>
<td>0.1719</td>
<td>1.9547</td>
</tr>
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<td>5.7977</td>
</tr>
<tr>
<td>412</td>
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<td>6.5</td>
<td>7.8872</td>
<td>0.0758</td>
<td>7.9630</td>
</tr>
<tr>
<td>422</td>
<td>152</td>
<td>8.1</td>
<td>9.9018</td>
<td>0.0441</td>
<td>9.9459</td>
</tr>
<tr>
<td>432</td>
<td>162</td>
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<td>11.4880</td>
<td>0.0191</td>
<td>11.5071</td>
</tr>
<tr>
<td>442</td>
<td>172</td>
<td>11.1</td>
<td>12.4544</td>
<td>0.0039</td>
<td>12.4582</td>
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<tr>
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<td>182</td>
<td>10.7</td>
<td>12.6844</td>
<td>0.0002</td>
<td>12.6847</td>
</tr>
<tr>
<td>462</td>
<td>192</td>
<td>10.4</td>
<td>12.1504</td>
<td>0.0087</td>
<td>12.1591</td>
</tr>
<tr>
<td>472</td>
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<td>10.9167</td>
<td>0.0281</td>
<td>10.9448</td>
</tr>
<tr>
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<td>9.1322</td>
<td>0.0562</td>
<td>9.1884</td>
</tr>
<tr>
<td>492</td>
<td>222</td>
<td>6.8</td>
<td>7.0121</td>
<td>0.0896</td>
<td>7.1017</td>
</tr>
<tr>
<td>502</td>
<td>232</td>
<td>4.6</td>
<td>4.8121</td>
<td>0.1242</td>
<td>4.9363</td>
</tr>
<tr>
<td>512</td>
<td>242</td>
<td>3</td>
<td>2.7976</td>
<td>0.1559</td>
<td>2.9535</td>
</tr>
<tr>
<td>522</td>
<td>252</td>
<td>1.5</td>
<td>1.2116</td>
<td>0.1809</td>
<td>1.3925</td>
</tr>
<tr>
<td>532</td>
<td>262</td>
<td>0.6</td>
<td>0.2454</td>
<td>0.1961</td>
<td>0.4416</td>
</tr>
</tbody>
</table>

Column 1 gives the setting of the analyzer. Column 3 gives the detector current in milliamp. From the readings we note that the maximum detector current $I_{\text{max}}$ is 12.7 milliamps, and the minimum detector current $I_{\text{min}}$ is 0.2 milliamps. The maximum current occurs for the analyzer setting between 265 and 275°. When the detector current is a maximum, the axis of the analyzer is parallel to the major axis of the elliptically polarized light. Call this setting $\Theta_0$. At a setting $\Theta$ of the analyzer, the major axis of the ellipse makes an angle $\beta = (\Theta - \Theta_0)$ with the analyzer axis. This is given in column 2. The intensity of the light passing through the analyzer should vary as
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\[ I_{\text{cal}} = I_{\text{max}} \cos^2(\beta) + I_{\text{min}} \sin^2(\beta). \]

Columns 4, 5 and 6 give the values of \( I_{\text{max}} \cos^2(\beta), I_{\text{min}} \sin^2(\beta) \) and \( I_{\text{cal}} \) using the values of \( I_{\text{max}}, I_{\text{min}} \) and \( \Theta_0 \) given in the table. Figure 2 compares the measured intensity \( I \) with the value calculated from the above formula.

![Figure 2](image)

**Figure 2.** Comparison of the transmitted intensity as a function of the setting \( \Theta \) of the analyzer for an elliptically polarized light. The parameters of the fit are \( \Theta_0 = 270^\circ, I_{\text{max}} = 12.7 \text{ mA}; I_{\text{min}} = 0.2 \text{ mA}. \)

Thus the major axis of the ellipse is parallel to the axis of the analyzer set at 270°. The ratio of the semi-minor to the semi-major axis of the ellipse is \( b/a = (I_{\text{min}}/I_{\text{max}})^{0.5} = (0.2/12.7)^{1/2} = 0.125. \)

We repeat the experiment by setting the QWP at 240°, 250° and 275°. The values of \( \Theta_0 \) and \( (b/a) \) are given in Table 3 for the different settings of the quarter-wave plate.

<table>
<thead>
<tr>
<th>QWP Setting</th>
<th>( \Theta_0 )</th>
<th>( b/a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>235</td>
<td>270</td>
<td>0.125</td>
</tr>
<tr>
<td>240</td>
<td>282</td>
<td>0.198</td>
</tr>
<tr>
<td>250</td>
<td>302</td>
<td>0.327</td>
</tr>
<tr>
<td>275</td>
<td>365</td>
<td>0.415</td>
</tr>
</tbody>
</table>

The parametric equation for an ellipse referred to its principal axes is

\[ X' = a \cos(\eta), \]
\[ Y' = b \sin(\eta). \]  \hspace{5cm} (1)

If \( X' \) and \( Y' \) are calculated for all values of \( \eta \) from 0 to \( 2\pi \) and the points \((X', Y')\) are plotted, they lie on an ellipse with semi-principal axes \( a \) and \( b \). \( \Theta_0 - \alpha \) gives the angle which \( X' \) makes with the fixed \( i \) axis. Here, \( \alpha \) is the setting of the analyzer when it is crossed with the polarizer. In this case \( \alpha = 172^\circ \). We can get \((X, Y)\) from \((X', Y')\) using the transformation equations

\[ X = X' \cos(\Theta_0 - \alpha) - Y' \sin(\Theta_0 - \alpha), \]
\[ Y = X' \sin(\Theta_0 - \alpha) + Y' \cos(\Theta_0 - \alpha). \]  \hspace{5cm} (2)
The ellipses plotted thus are shown in Figures 3 (a) to (d).

Figure 3(a–d). Ellipses of the polarized light for different settings of the QWP.

These pictures show that as the QWP settings are changed, the major axis of the ellipse rotates clockwise and the ratio \((b/a)\) increases.
Chapter 13

Experiments in Spectroscopy
13.1. Luminescence

1. Introduction

When some materials are excited by friction, pressure, heat, chemical reaction, irradiation with light, passage of a current or irradiation with electrons, they emit light. This phenomenon is called luminescence. Depending on the excitation mechanism, luminescence is known by different names as shown in Table 1.

<table>
<thead>
<tr>
<th>Excitation</th>
<th>Name</th>
<th>Examples of materials</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Friction</td>
<td>Tribo-luminescence</td>
<td>Quartz; Sugar; Europium dibenzyol trimethyl ammonium</td>
<td>Impact sensors</td>
</tr>
<tr>
<td>Pressure</td>
<td>Piezo-luminescence</td>
<td>Piezoelectric materials doped with rare earths ZnS:Mn$^{2+}$; SrAl$<em>2$O$<em>4$Eu$^{2+}$; K(Na$</em>{0.5}$Nb$</em>{0.5}$)$_2$:Sm$^{3+}$; LaNbO$_3$:Pr$^{3+}$</td>
<td>Stress sensors; Mechanodriven displays</td>
</tr>
<tr>
<td>Heat</td>
<td>Thermo-luminescence</td>
<td>CaSO$_4$: Ti or Cu (Prior irradiation with high energy ions) 7LiF: Mg, Cu, P</td>
<td>Radiation dosimetry; Archaeological dating</td>
</tr>
<tr>
<td>Chemical</td>
<td>Chemi-luminescence</td>
<td>White phosphorous oxidizing in moist air producing a green glow; Luminol(C$_6$H$_7$N$_3$O$_2$) with H$_2$O$_2$ in the presence of Fe or Cu</td>
<td>Gas analysis; Assay of bio- molecules</td>
</tr>
<tr>
<td>Irradiation with light</td>
<td>Photo-luminescence</td>
<td>Iodine vapour; Dyes; Direct band gap semi conductors</td>
<td>Band gap studies; Defects in solid state</td>
</tr>
<tr>
<td>Passage of current</td>
<td>Electro-luminescence</td>
<td>Light emitting diodes</td>
<td>Lighting; Displays</td>
</tr>
<tr>
<td>Irradiation with electrons</td>
<td>Cathodo-luminescence</td>
<td>Semiconductors and other materials</td>
<td>Geology; Microscopy</td>
</tr>
</tbody>
</table>

If the luminescence dies out within a few pico- or nano-seconds after the removal of the excitation, it is called fluorescence. On the other hand, a system which is excited may come down to a metastable state with a long lifetime before returning to the ground state emitting radiation. In such a case it is called phosphorescence. The lifetime in the metastable state may be many milliseconds. So, the light emission persists after the excitation is switched off.

We will consider here only photo- and electro-luminescence.

2. Photoluminescence

As an example we will consider a dye molecule. In a molecule, there are electronic energy levels, vibrational energy levels within each electronic level, and rotational energy levels within each vibration level. There is a ground state electronic level (called $S_0$ here) with its associated vibrational levels shown in Figure 1. There are excited electronic levels which will have different multiplicities (like singlet, triplet, etc). These are shown as levels $S_1$ and $S_2$ in Figure 1.

In Figure 1, vibrational states associated with each electronic state are shown as equally spaced thin lines. Note that the spacing between adjacent vibrational levels in the three electronic states is different. This corresponds to different vibration frequencies of the atoms in the three electronic states.
Figure 1. Electronic ground state $S_0$ and excited states $S_1$ and $S_2$ are shown as thick lines. Vibrational states in each electronic state are shown as thin lines.

The dipole selection rule permits a transition between $S_0$ and $S_1$, but not between $S_0$ and $S_2$. Upward arrows from the ground vibrational level in $S_0$, to other vibrational levels in $S_1$, shown on the right in the figure, represent absorption transitions. Light is absorbed in a band in a molecule, spread over a range of wavelengths starting with a long wavelength cutoff.

We should remember that the potential energy as a function of internuclear coordinates will be different for the different electronic states. For example Figure 2 shows the potential energy as a function of internuclear distance for the molecule $I_2$ for the ground state $X$ and excited electronic states, $A$ and $B$.

Figure 2. Potential energy as a function of internuclear distance for the ground X and excited states A and B of the molecule $I_2$. 

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Experiments in Spectroscopy

The equilibrium internuclear distance is at the minimum of the potential energy. We see that the minimum of the potential energy function, for different electronic states, occurs at different internuclear distances. This internuclear distance increases as the electronic excitation energy increases. So the vibration frequency becomes less as the electronic excitation energy becomes more.

The excited molecule can come down from a given vibrational state $v'$ in the electronic state $S_1$, to any vibrational state $v''$ in the ground electronic state $S_0$. This gives the fluorescence band. Since the atom stays in the excited vibrational state of $S_1$ for a time of a few hundred picoseconds, fluorescence emission occurs immediately on absorption. Fluorescence transitions are shown as downward arrows, from $v'$ in $S_1$ to different vibrational states in $S_0$, immediately to the left of the absorption transitions in Figure 1.

An electron, excited by absorption of light from the ground vibrational state of $S_0$ to an excited vibrational state of $S_1$, may come down to the ground electronic state $S_0$ by a different mechanism. It may either come to a vibrational state $v''$ of the excited electronic state $S_2$ by an allowed electronic transition in a molecule or a solid, or by exchange of energy with a phonon in a solid. A phonon is a quantum of energy associated with a mode of vibration of the solid lattice. Since dipole selection rule forbids transition from $S_2$ to $S_0$, this electron will stay in the vibrational state $v''$ of $S_2$ for a long time, of the order of milliseconds, before coming down to the ground state by quadrupole transition. In this case, the emission of light follows absorption with a time delay of several milliseconds. This is called phosphorescence.

The electronic transitions occur very fast compared to the period of vibration of the atoms in the molecule. This has the following consequence. When the electronic transition takes place, the nuclear distance does not change. So the atom makes a vertical transition from one vibronic energy level to another. This is shown in Figure 2 for the diatomic iodine molecule. The transition probability depends on the overlap of the wave functions in the two levels. For example, the maximum of the ground state wave function occurs at the internuclear distance $r_X$ for the iodine molecule in the vibronic ground state $(X, v'' = 0)$. When an upward transition occurs from $(X, v'' = 0)$ to the excited vibronic state $(B, v')$, we see that the overlap of the ground state wave function with the excited state wave function will be small, for small values of $v'$, because the internuclear separation $r_B$ is much greater than $r_X$. The overlap becomes appreciable only when $v'$ is large (about 25). So, only the transitions to vibronic energy levels $v' = 25$ to 35 have appreciable transition probabilities and contribute to absorption.

The Franck–Condon principle states that

(i) in a transition between two electronic states, the nuclear coordinates remain the same, and

(ii) the transition probability depends on the overlap of the wave functions in the ground and excited states.

The absorption spectrum will therefore show a number of nearly equally spaced bands starting with a minimum frequency. The absorption spectrum of anthracene is shown in Figure 3. It is spread over a wavelength range of 390 to 300 nm.

![Figure 3](image-url)
The excited molecule will come down from the excited vibronic level \((S_1,v')\) to a ground vibronic level \((S_0,v'')\) in a few pico- to nano-seconds. This is shown as a downward transition in Figure 1. The maximum energy of the emitted photon occurs for \(v'' = 0\) and the energy decreases as the value of \(v''\) increases. So the fluorescence spectrum starts at a frequency \(\nu_0\) and shows peaks at decreasing frequencies. The fluorescence spectrum of anthracene is shown as the blue line. *The fluorescence band always occurs towards the longer wavelength side of the absorption band.*

Fluorescence can be exhibited by some molecules, and by impurity atoms in a crystal. Photo-luminescent paints incorporate zinc sulphide which gives a green fluorescence, or strontium aluminate which fluoresces in the red. These paints are used to paint traffic signs and kerbs of roads.

### 3. Electro-Luminiscence

In this phenomenon light is emitted when a current passes through the material, or when a large voltage is applied across a capacitor with the luminescent material as the dielectric.

The most common example of an electroluminescent device is the light-emitting diode. Here we have a p–n junction diode in which the n-side is heavily doped. The energy level diagram of the p–n junction diode is shown in Figure 4.

![Figure 4](image)

**Figure 4.** (a) p–n+ junction; (b) forward biased to emit light.

On the left in Figure 4(a), we show a p–n+ junction without bias. The conduction band on the n+ -side is depressed relative to the conduction band on the p-side, forming a junction potential. If the junction is sufficiently forward biased, as shown in Figure 4(b), the conduction band on the n+ -side rises to the same level or higher than the conduction band on the p-side. The electrons flow from the conduction band on the n+ -side to the conduction band on the p-side. Some of the electrons make a downward transition from the CB to the VB on the p-side to combine with a hole. The energy \(E_g\) (band gap energy) is released as a photon with frequency \(\nu\), where \(\hbar \nu = E_g\).

There are two types of semiconducting materials called (a) direct band gap and (b) indirect band gap materials. The difference is shown in the plot of energy \(E\) vs. wave vector \(k\) for these two materials in Figure 5.

![Figure 5](image)

**Figure 5.** Direct and indirect band gap materials.

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Experiments in Spectroscopy

In the direct band gap materials, the minimum of the valence band and the maximum of the conduction band occur at the same wave vector \( k = 0 \) as shown in Figure 5(a). The momentum states of the electron and the hole are the same, and the transition is a momentum-conserving transition with the emission of light. Examples of such materials are GaAs, InP, etc. In the indirect band gap materials, the minimum of the conduction band and the maximum of the valence band occur at different \( k \) values (Figure 5(b)). In this case, a transition of the electron from the conduction to the valence band involves a change in momentum. This momentum has to be carried away by a phonon. Examples of indirect band gap materials are silicon and germanium. Direct band gap materials have a higher efficiency in converting electrical energy into light energy than the indirect band gap materials.

The color of the light emitted in an LED depends on the band gap. The larger the band gap, the shorter the wavelength of light emitted. Blue LEDs were made in the 1980s with epitaxially grown GaN with p-doping. By the 1990s bright blue LEDs were available made with InGaN quantum wells, sandwiched between thick layers of GaN. The Nobel Prize in Physics for 2014 was awarded to Nakamura, Amano and Akasaki for the development of blue LEDs. Using AlGaN it is possible to make ultraviolet LEDs.

White LEDs can be made in two different ways. One is by having three color LEDs emitting in the red, green and blue, and mixing the colors. The other method is to take a blue or UV LED with a phosphor coating. The emitted light from the phosphor comes in a broad spectrum approximating white light.

![Figure 6](#) The spectrum from a phosphor-coated LED emitting blue light from InGaN LED and the broad spectrum coming from Ce:YAG phosphor coating (from Wikipedia).

A major problem in light-emitting diodes is the high refractive index of the semiconductor materials. Light coming from the junction, at an angle greater than the critical angle, will be reflected back into the material. To avoid this, the diode is encased in a moulded plastic cover. This plastic cover has a graded refractive index, and so allows light to come out of the chip at all angles.

The LED emits more lumens of light per watt of electric power than the conventional filament electric bulb. Power LEDs which consume a few watts of power, and producing 70 to 100 lumens/watt, are commercially produced. The conventional filament bulb has an efficiency of about 15 lumens/watt. Table 2 gives the available colors of LED materials in use.
### Table 2. Available colors of LED and materials used. (from Wikipedia).

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<tbody>
<tr>
<td>Infrared</td>
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<td>$\Delta V &lt; 1.63$</td>
<td>Gallium arsenide (GaAs), Aluminium gallium arsenide (AlGaAs)</td>
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<td>$610 &lt; \lambda &lt; 760$</td>
<td>$1.63 &lt; \Delta V &lt; 2.03$</td>
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<td>Blue</td>
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<td>$2.48 &lt; \Delta V &lt; 3.7$</td>
<td>Zinc selenide (ZnSe), Indium gallium nitride (InGaN), Silicon carbide (SiC) as substrate Silicon (Si) as substrate—under development</td>
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<td>Pink</td>
<td>Multiple types</td>
<td>$\Delta V \approx 3.3$</td>
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<td>White</td>
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<td>$2.8 &lt; \Delta V &lt; 4.2$</td>
<td><strong>Cool/Pure White:</strong> Blue/UV diode with yellow phosphor, <strong>Warm White:</strong> Blue diode with orange phosphor</td>
</tr>
</tbody>
</table>
Experiments in Spectroscopy

Though the cost of LEDs is high, their low power consumption, and long life (50,000 hours) make them commercially viable.

4. Spectrograph

We use a spectrograph manufactured by the Holmarc Mechatronics Company to record the spectrum of luminescent material. This spectrograph is compact and easy to use. It is a Czerny spectrograph with a holographic grating. The spectrum is recorded on an array of 3648 pixels of a CCD camera. The pixels cover a range of wavelengths from 180 nm to 1054 nm. The spectrograph has a resolution of 1 nm. The exposure time of the pixels can be changed, by computer control, from 0 to 1000 ms in three ranges. In the first range we go from 0 to 10 ms, in the second up to 100 ms and in the third up to 1000 ms. We can also set the spectrograph to grab a certain number of frames of the spectrum, and average the intensity values over the frames. The resulting spectrum is displayed on the computer screen. One can record the spectrum continuously or one can display a single image. Two spectra can be overlaid, which feature is especially useful for transmission or absorption measurements. One can display intensity as a function of the pixel number, or as a function of wavelength. The conversion from pixel number to wavelength is done by calibrating the spectrometer with the known wavelengths of mercury spectrum, and using either a Hartman regression formula or a polynomial regression. The coefficients of the regression are stored in a wavelength calibration file. The display can be changed from pixels to wavelengths on the control panel of the spectrum analyzer. The wavelengths of peaks in a spectrum can then be found by pressing a button called Peaks and setting the intensity of the base line in a dialog box. The wavelength values are accurate to 0.1 nm in blue, to 0.3 nm in green, and 0.5 nm in red.

The slit is opened by turning the screw clockwise. As the screw is turned clockwise, one feels a pressure on the fingers at one point. Turning the screw further opens the slit. This is the only adjustment to make.

To take a spectrum, arrange the source in front of the slit. Start with the slit closed. Start the spectrum analyzer program stored in the computer. To start with, set an exposure time of 5 ms on the control panel, if the light is bright. Click the start button. The dark spectrum appears as a horizontal base line with counts around 1000. Then open the slit slowly by turning the screw. As the slit is opened the intensity increases. At one point, the spectrum appears on the computer screen. If the slit is opened too wide the spectrum saturates. Adjust the slit width and the exposure time, so that a good clean spectrum is seen. Then press the start/stop button. The spectrum disappears from the screen. Then press the button for a single spectrum. The spectrum appears on the screen. It can be stored in a folder as a Holmarc file. The file can be exported to an Excel file, which gives the intensity as a function of the wavelength.

5. To Study the Absorption and Fluorescence Spectrum of Rhodamine B and Fluorescein

Rhodamine B and Fluorescein are two dyes commonly used in fluoroscopy. The dyes have the chemical structures shown in Figures 7(a) and (b).

![Figure 7. Molecular structure, chemical formula and molecular weight of Rhodamine B and Fluorescein.](image)

**Caution:** These chemicals can stain the hands and clothes. So wear gloves and see that the dye does not fall on the clothes. They are also carcinogenic if ingested.

Prepare millimolar solutions (0.479 gms of Rhodamine B and 0.332 gm of fluorescein per liters of water) of these dyes in 100 milliliters of water, and store in glass flasks with ground glass stoppers. Label them appropriately.
Experiments in Spectroscopy

Since the position of the fluorescent peak will depend on the pH, prepare buffer solutions of citric acid and disodium hydrogen phosphate. Citric acid anhydrous has a molecular weight of 192.1 gm and disodium hydrogen phosphate (Na$_2$HPO$_4$·2H$_2$O) has a molecular weight of 178 gm. Prepare 100 cc of 0.2 molar solutions of citric acid anhydride (0.384 g per liter of water) and Na$_2$HPO$_4$·2H$_2$O (0.356 gm per liter of water), and store in two glass flasks closed with ground glass stoppers. By mixing these two solutions in different ratios, one can obtain different pH values in the mixture.

Fill the cuvette in the spectrometer to three fourths of its height with distilled water. Use an ink filler to fill the water or the solutions into the cuvette. Put the cuvette in the slot before the spectrometer. Switch on the halogen lamp and place it in front of the cuvette, so that the light from the lamp passes through the cuvette and falls on the slit of the spectrograph. Take a spectrum and store it as a Holmarc file. Then add a drop of the 0.1 molar Rhodamine B solution to the water. Microliters of the solution will be enough to stain the water in the cuvette. The water in the cuvette will appear purple in color. Now take the spectrum again. Figure 8(a) shows the two spectra.

Figure 8. (a) Raw spectra with pure distilled water, and distilled water colored with a drop of 0.1 millimolar solution of Rhodamine B. (b) Absorption spectrum of Rhodamine B from the curves in Figure (a).

The data on the two curves are exported to Excel files and the percentage absorption at each wavelength is calculated from the formula

\[ \% \text{absorption} = \frac{[I(1) - I(2)]}{I(1)} \times 100. \]

Here \( I(1) \) and \( I(2) \) are the intensities of curves (1) and (2) in Figure 8(a) at the same wavelength. Curve 1 is the curve with water and curve 2 is the curve with water, colored with Rhodamine. The absorption curve calculated thus is shown in 8(b). The curve shows a broad absorption with a peak at 557.7 nm.

To observe fluorescence, we must send the exciting radiation (green laser light at 533 nm) and observe the fluorescent light in a direction perpendicular to the exciting beam. Since the cuvette has only two transparent sides, it is unsuitable for fluorescence experiments. So we transfer the liquid in the cuvette to a circular test tube. This test tube is fixed in front of the slit and the green laser light traverses the solution transversely, at a height which is the same as the height of the spectrometer slit from the table. The fluorescence spectrum is obtained on the computer screen and stored as a Holmarc file. This is exported as an Excel file and the graph is plotted on Origin. The fluorescence graph is shown in Figure 8(c).
In Figure 8(c) we see the green laser light scattered by the solution as a sharp line. To the long wavelength side of this line we see the fluorescence spectrum. The peak wavelength of the fluorescence spectrum is at 588 nm. Figure 8(d) shows the absorption and fluorescence emission spectra of Rhodamine B in water taken by Terpetschnig et al. (http://www.iss.com/resources/research/technical_notes/PC1_PolarizationStandards.html) They give the values of the peak in the absorption spectrum at 554 nm which agrees with the value 558 nm in the present study. They used a xenon lamp giving a continuous spectrum for exciting the fluorescence, and reported the fluorescence peak at 579 nm. All the wavelengths in the xenon lamp, within the range of the absorption of Rhodamine, will give fluorescence with intensities depending on the intensity distribution of the xenon lamp. Also, the fluorescence efficiency varies with the excitation and emission wavelengths. Their fluorescence spectrum is a superposition of the fluorescent spectra due to different excitation wavelengths and so its peak position will be slightly different from the peak position found in the present study.

Figures 9(a) and (b) show the absorption curves for fluorescein in water.
Experiments in Spectroscopy

The absorption peak occurs at 489 nm while the emission peak, when excited by a blue LED with wavelength at 458 nm, is 516 nm with subsidiary peaks at 548 and 573 nm. From the internet data the peak absorption wavelength is 485 nm, and the fluorescence peak occurs at 512 nm. The agreement is good.

Figure 9. Absorption spectra (a) and (b), and fluorescence spectrum (c) of fluorescein dissolved in water. (d) Data from internet.

6. Electroluminescence

We study the emission spectra of red, yellow, green, blue and white LEDs. The first four LEDs are mounted on a PCB so that we can apply a variable voltage to each LED by turn. We switch on the power supply and connect each of the LEDs one after the other. The LED is put in front of the slit and the spectrum of each LED is taken. For the white LED we use an LED torch to get the spectrum. The spectra of the LEDs are shown in Figures 10(a–c).

We see that the white LED has a peak at 453 nm in the blue and a broad emission in the 500 to 700 nm range. The spectrum looks like the one in Figure 6. The peak at 453 nm is due to the blue emission from the InGaN semiconductor LED, and the broad spectrum from 500 to 700 nm is probably due to a fluorescent coating. The blue LED emits a fairly sharp line at 466 nm and can be used to excite fluorescence.

Figure 10(b) shows the fluorescence peaks due to the green LED.

Figure 10(c) shows the electroluminescent spectra of red and yellow LEDs. These are of low power.
Figure 10. (a) Electroluminescent spectrum of white and blue LEDs.

Figure 10. (b) Electroluminescent spectrum of green LED.

Figure 10. (c) Electroluminescent spectra of red and yellow LEDs.
Chapter 14

Experiments in Non-Linear Dynamics
14.1 Feigenbaum Circuit

1. Introduction

Consider the population of a species whose growth with time from year to year is represented by the equation

\[ Z_{n+1} = r Z_n - s Z_n^2. \]  

Here, \( Z_n \) is the population in the year \( n \), \( r \) is its rate of growth, and \( s \) is the rate of decay due to the sharing of the scarce resources among the population. Given \( Z_1 \), the population in year 1, \( r \) and \( s \), this equation will predict how the population will vary from year to year.

We may use a normalized population by substituting

\[ Y_n = \left( \frac{s}{r} \right) Z_n. \]

Equation (1) then becomes

\[ Y_{n+1} = r Y_n (1 - Y_n). \]

The condition on \( Y_n \) is that it should be positive. So, \( Y_n \) must always be a positive number between 0 and 1.

Let us see how this equation behaves for \( Y_1 = 0.2 \) for different values of \( r \). First, we choose \( r = 1.22 \), and calculate on a computer the values of \( Y_n \) for \( n \) up to 72. It is convenient to plot the result in a polar diagram taking 1 year to equal an angle of 30°. We show such a plot in Figure 1. We see that, as \( n \) increases, the population moves along a tighter and tighter spiral approaching the limiting value \( Y_\infty = 1 - 1/r \), which is 0.18180330551464600000. This limiting value is called the attractor of the nonlinear equation.

Since the equation is deterministic, one would expect a single limit point or attractor for all values of \( r \). Let us do the calculation for \( r = 3.0 \). The calculation shows that for odd years we get one limiting value, and for even years we get a different limiting value.

We plot, in Figure 2(a), the evolution of population for the odd years starting from \( n = 3 \) and going up to \( n = 71 \). This is a spiral like the one shown in Figure 1.

In Figure 2(b), we plot \((1 - Y_n)\) for even years from 4 to 72. (Note: If we plot \( Y_n \) it will spiral out to reach a constant radius as \( n \) tends to infinity. By plotting \(1 - Y_n\) against \( n \) we get an inward spiral which tends to a point as \( n \) tends to infinity. This illustrates clearly the idea of an attractor.)

From these figures, we see that the single attractor for \( r = 1.22 \) has bifurcation to two attractors at \( r = 3.0 \).
Experiments in Non-Linear Dynamics

Figure 2(a). Variation of $Y_n$ in the odd years from 3 to 71. $Y_1 = 0.2$, $r = 3$.

Figure 2(b). Variation of $1-Y_n$ in the even years from 4 to 72. $Y_1 = 0.2$, $r = 3$.

We see that each attractor occurs once in two years. We call this *period doubling*. As $r$ is increased further, the two attractors will bifurcate successively into 4, 8, 16, ... attractors. Figure 3 shows the plot of $Y_n$ as a function of $n$ for 72 years, for $r = 4$. We see that $Y_n$ takes values between 0 and 1 randomly. If we had calculated for $n$ tending to infinity, we would have had values of $Y_n$ distributed between 0 and 1 and filling the unit circle. We say we have reached *chaos*.
Figure 3. Plot of $Y_n$ as a function of $n$ from 1 to 72 years. $Y_1 = 0.2$, $r = 4$.

In all the above figures, the starting value $Y_1$ is 0.2. We see that the deterministic behaviour for $r = 1.22$ goes to chaotic behaviour for $r = 4$ through a series of bifurcations of the attractors, at critical values of $r$. This is one of the routes taken by a deterministic nonlinear system, when it goes from ordered to chaotic behaviour.

Table 1 gives the values of $Y_n$ for different values of $r$ as calculated on a computer up to the first three bifurcations. When $r$ is 3.84 we see that there are only three values of $Y_n$. There are only three bifurcations. The even number of bifurcations collapses to an odd number. With an increase in $r$, the three bifurcations become six and so on.

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The bifurcation diagram up to the first three bifurcations is shown in Figure 4.
2. Feigenbaum Circuit

The Feigenbaum circuit performs the iteration

\[ X_{n+1} = rX_n \left( 10 - \frac{X_n}{10} \right), \quad (4) \]

where \( r \) is a parameter with a value from 1 to 4, and \( X_n = 10Y_n \) is a positive number between 0 and 10.

As \( r \) is varied, the value of \( X \) undergoes a series of bifurcations before it reaches chaos at \( r = 4 \).

The circuit described here is taken from “Simple Experiments in Chaotic Dynamics” by Keith Briggs, *American Journal of Physics*, Vol. 55, 1083–1089, (1987), with a few minor modifications. The principle of the circuit is described by the block diagram shown in Figure 5.

![Block diagram of the Feigenbaum circuit](image)

**Figure 5.** Block diagram of the Feigenbaum circuit.

The parameter \( r \) is the voltage at the variable point of a potentiometer, to which 5 volts are applied at both ends. If the switch S is pushed up, this voltage \( r \) appears at the input of the multiplier \( M_1 \), and serves as the starting value \( X_0 \) of the iteration. A voltage \((10-X)\) is generated within the box, when \( X \) is the voltage at the input of multiplier one. The output of multiplier one is

\[ Q = \frac{X (10 - X)}{10}. \quad (5) \]

This output is the input to the second multiplier \( M_2 \), and \( r \) is the other input to \( M_2 \). The output of multiplier \( M_2 \) is

\[ Y = r \left( \frac{X}{10} \right) \left( 1 - \left( \frac{X}{10} \right) \right). \quad (6) \]

During the iteration process, the sample-and-hold chip \( S_1 \) receives this value of \( Y \), holds it for a short time, till it receives the next value of \( Y \) from \( M_2 \). When the next value of \( Y \) is received, it passes the old value of \( Y \)
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to the second sample-and-hold chip $S_2$. This chip also retains the value till it receives the next value from $S_1$ during the iteration. It then passes the old value of $Y$ to the amplifier $A$, which amplifies this value ten times. This new value of $X = 10Y$ comes to the switch $S$. If $S$ is pressed down, the iteration starts. The iteration takes place around 250 times in one second. The value of $r$ can be read on a DMM. The iterated value of $X$, after much iteration, can be seen on a storage oscilloscope.

The front panel of the Feigenbaum circuit is shown in Figure 6.

![Figure 6. Front panel of Feigenbaum circuit.](image)

In the above diagram, $M$ is the Mains switch. $P$ is a ten-turn potentiometer which sets the value of $r$. This value of $r$ can be measured by connecting the RCA socket marked $R$, to a DMM in the 20 V range. The inputs, and output, at multiplier $M_1$ can be measured by connecting the red prong of a DMM to the corresponding banana terminal, and the black prong to the banana terminal marked $Gd$ on the front panel. The inputs and output of multiplier $M_2$ can be measured similarly. The value of $X$, after much iteration, can be measured on a storage oscilloscope connected to the RCA socket marked $X$.

One must first learn the functions of the different controls on the oscilloscope.

We connect the terminal marked $X$ to an oscilloscope. For each value of $r$, the oscilloscope will give a horizontal line with the value of $X_n = 10Y_n$, as $n$ tends to infinity. For a given value of $r$, the oscilloscope will show one horizontal line if there is one value of the attractor. As $r$ is increased, this line will become double at a critical value of $r$, showing that there are two attractors and a bifurcation has taken place. As $r$ is further increased, each of these lines will become double. When $r$ reaches a value 4, the whole screen of the oscilloscope from 0 to 10 volts will be filled with dots showing that the limiting value of $X_n$ is chaotic.

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3. Experimental Procedure

The procedure is written for a GW Instek storage oscilloscope. For other oscilloscopes one should study the controls of the oscilloscope and use them appropriately.

Connect the RCA socket marked X to channel 1 of the two channel storage GW INSTEK oscilloscope. Connect the RCA socket marked R to a DMM in the DC 20 V range. Set the scale on channel 1 to 2 V.

Adjust the pot so that the value of r is 2 V as read on the DMM. Put SPDT switch SW in position O, so that the starting value of X is r. The output at socket X will be r (10–r)/10 at the end of the first iteration. On the oscilloscope screen you will see a horizontal line. Put the switch SW down to positions S to start the iteration. The horizontal line will shift up. Press the button marked Cursor on the oscilloscope front panel. Two yellow lines will appear on the screen. Press button X™Y on the right side of the screen, if the lines are vertical. The lines will become horizontal. Press the select button Y1 on the side of the screen. Then turn the knob marked Variable, and adjust the Y1 cursor to coincide with the trace of X on the screen. On the screen the Y1 value will be indicated. Note this value of X. Increase r in steps of 0.2 V, and note the value of X on the oscilloscope for each value of r. Near r = 3 volts, the trace on the oscilloscope becomes double, indicating that bifurcation has started.

Near r = 3.43, each of the lines on the screen will bifurcate. Note the value r1 of r at which the first bifurcation occurs. Now, change r in steps of 0.1 V, and measure the two values of X by moving the cursor Y1 to the two traces of X successively. Near r = 3.43, each of the lines on the screen will bifurcate. Note the value r2 of r at which this happens. Change r in steps of 0.02V, and for each value of r, note the four values of X. When r is around 3.53V, one of the traces will show bifurcation. Note the value r3 of r at which this third bifurcation occurs. One will not be able to measure the values of X as r is increased further, because bifurcations will occur rapidly, and the differences between the values of X will be masked by noise. Change r to 3.84 V. Then you will see only three traces on the screen. Note the three values of X. As r approaches 4, you see dots on the screen occupying all values between X = 0 to X = 10 V. This indicates chaos.

The bifurcation diagram, constructed from the measured values of X as a function of r up to the third bifurcation, is shown in Figure 7.

\[ X_{n+1} = 10^r Y_n (1-Y_n) \]

where \( X_n = 10^r Y_n \)

Iterated value of X

Parameter r

Figure 7. Bifurcation diagram measured with the Feigenbaum circuit.

The values r1, r2 and r3, at which the first, second and third bifurcations occur are 2.988, 3.434 and 3.526 V, respectively. The ratio \((r_2 - r_1)/(r_3 - r_2)\) comes out to be 4.80 in agreement with the theoretical value of 4.703.
14.2 Chua’s Circuit for Non-Linear Dynamics

1. Introduction

Chua’s circuit is shown in Figure 1. This circuit is taken with minor modifications from ‘Robust Op-amp realization of Chua’s circuit’: Michael Peter Kennedy, http://www.physics.smu.edu/scalise/chaoscircuit.pdf.

![Chua's Circuit](image)

Figure 1. Chua’s Circuit.

The circuit consists of four linear circuit elements L, C₁, C₂, and R, and a nonlinear negative resistor NLR. The nonlinear resistor has the current–voltage characteristic shown in Figure 2.

![I–v Characteristic](image)

Figure 2. I–v characteristic of the nonlinear resistor NLR.

Note that the current decreases as the voltage is increased. This is characteristic of a negative resistance. The slope of the curve is different in different ranges of voltage. This is nonlinear behavior.

The current through the NLR as a function of the voltage is given by the relation:

For $v < -v_B$, \[ i_{NLR} = -m_0 (v + v_B) + m_1 v_B. \] (1)

For $-v_B < v < v_B$, \[ i_{NLR} = -m_1 v. \] (2)

For $v > v_B$, \[ i_{NLR} = -m_0 (v - v_B) - m_1 v_B. \] (3)

Here, $m_0$ and $m_1$ are positive quantities, with $m_1 > m_0$. They have the dimensions of electrical conductance (siemens).

There are three potential drops, namely, $v_{C_2}$ across the capacitance $C_2$, $v_R = (v_{C_2} - v_{C_1})$ across the resistance and $v_{NLR} = v_{C_1}$ across the nonlinear resistance. The current through the inductance is $i_L$, and the current through the nonlinear negative resistance is $i_{NLR}$. These quantities are related by the following three differential equations:

\[ -L \frac{di_L}{dt} = v_{C_2}, \] (4)

\[ \frac{C_2}{R} \frac{dv_{C_2}}{dt} = i_L - \frac{(v_{C_2} - v_{C_1})}{R}, \] (5)
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\[
\frac{C_1 \text{d}v_{C_1}}{\text{d}t} = \left(\frac{v_{C_2} - v_{C_1}}{R}\right) - i_{\text{NLR}}. \tag{6}
\]

When \( R \) is more than a certain value, the system is quiescent (i.e., it is not oscillating). \( v_{C_1} \) and \( v_{C_2} \) are zero. When \( R \) is reduced, oscillations start. The system shows a variety of nonlinear behavior. The behavior can be seen by connecting \( v_{C_2} \) to the Y-plates of an oscilloscope, and \( v_{C_1} \) to the X-plates of an oscilloscope.

Nonlinear dynamical behavior, leading from order to chaos, is seen in many deterministic nonlinear mechanical and electrical systems. Chua’s circuit provides a simple experimental demonstration of the different types of behavior.

2. Construction of a Negative Resistance Element

Ordinary resistors are always positive and are dissipative. Negative resistance devices can be constructed using diodes, transistors, etc. A simple way of realizing negative resistance with an operational amplifier is shown in Figure 3.

![Figure 3](Figure 3. Realizing a negative resistance with an op-amp.)

When a voltage \( V \) is applied to the terminals of this circuit, the current \( I \) through the circuit is

\[
I = -\left(\frac{1}{R_3}\right) V \tag{7}
\]

when \( R_2 = R_1 \), and when \( V \) lies between \(-V_B\) and \(+V_B\). The \( I-V \) characteristic is shown in Figure 4. \( +V_B \) and \(-V_B \) are called the breakpoints. \( V_B \) is related to the saturation voltage of the op-amp.

![Figure 4](Figure 4. I-V characteristic for a negative resistance circuit using an op-amp.)

Using two op-amps in parallel, with different \( V_B \) values, and different \( R_1(=R_2) \) values, we can have the nonlinear characteristic shown in Figure 2 over a range of voltage.

3. Front Panel of Chua’s Circuit

Chua’s circuit box is shown in Figure 5. A diagram of the front panel of Chua’s circuit is also shown in the figure.
The front panel is divided into two parts. The left part is marked Chua’s circuit, and has all the terminals and controls for doing this experiment. The right part is a split power supply giving +8, 0 and −8 volts. This can be used as a split power supply for other electronic experiments.

The main switch is at the back of the cabinet. The switch SW1 can be put in the O or S position. In position O, the NLR is not connected to the rest of the circuit. In this position one may connect a DC power supply to the banana terminals marked A and B through a milliammeter, and measure the $I–V$ characteristic of the NLR. One can measure, at terminals marked I, the voltage across a 10 ohms resistor in series with the NLR. If an ammeter is not available, this voltage measures the current $I$ through the NLR.

The main switch at the back of the cabinet must be on, when measuring the $I–V$ characteristic of the NLR.

When the switch SW1 is put in position S, the NLR is connected to the rest of the Chua circuit. At that time the power supply should be disconnected from A and B.

The voltage across $C_1$ is measured between the terminal marked O1 and the terminal marked COM. Similarly, the voltage across $C_2$ is measured between the terminal marked O2 and the terminal marked COM. These two voltages must be connected to two channels of an oscilloscope put in the XY mode.

The terminals marked R serve to measure the resistance of the potentiometer.

### 4. Apparatus Required

Chua’s circuit box, a regulated power supply, one DMM to measure current in mA, one DMM to measure DC voltage up to 20 V and a two channel oscilloscope.

### 5. Procedure

a) To Measure the $I–V$ Characteristic of the Nonlinear Resistor Device Made of Two op-amps

Switch SW1 is put in position O. A regulated power supply is connected to the terminals A and B marked VS, through a DMM in series to measure up to DC 3000 $\mu$A. When positive of the DC supply is
Experiments in Non-Linear Dynamics

connected to A and negative to B, the voltage applied to the nonlinear resistive device is \( +V \), where \( V \) is the voltage of the DC source. If the connections at A and B are reversed, the voltage applied is \( -V \) volts.

Table 1 shows a sample set of readings of \( I \) vs. \( V \) when the voltage is varied from \( -6.5 \) V to \( +6.5 \) V.

<table>
<thead>
<tr>
<th>( V_{DC} ) (volts)</th>
<th>( I ) (( \mu )A)</th>
<th>( V_{DC} ) (volts)</th>
<th>( I ) (( \mu )A)</th>
<th>( V_{DC} ) (volts)</th>
<th>( I ) (( \mu )A)</th>
<th>( V_{DC} ) (volts)</th>
<th>( I ) (( \mu )A)</th>
</tr>
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<td>2.5</td>
<td>(-1388)</td>
<td>6.5</td>
<td>113</td>
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</table>

(If a microammeter is not available, one can measure the DC voltage at the terminals marked I. This gives the voltage across a 10 ohms resistor. The current is voltage divided by 10. If required, the voltage across the terminals can be connected to a DC differential amplifier which is put in the amplification range 10.)

Figure 6 shows the \( I-V \) characteristic of the nonlinear device.

Figure 6. \( I-V \) characteristic of the nonlinear device.
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There are five regions in the curve, marked I to V on the figure. In regions I ($V < -5.3$ V) and V ($V > 5.3$ V), the device has a positive resistance with a slope of about $+2.2 \times 10^{-3}$ ohm$^{-1}$. The device shows a negative resistance behavior between $-5$ and $+5$ volts. This part is divided into three regions, II, III and IV. In regions II and IV, the slope of the negative resistance curve is $-4.18 \times 10^{-4}$ ohm$^{-1}$ (expected value according to circuit parameters is $-4.55 \times 10^{-4}$ ohm$^{-1}$), and in region III, the slope is $-7.71 \times 10^{-4}$ ohm$^{-1}$ (expected value from circuit parameters is $-7.67 \times 10^{-4}$ ohm$^{-1}$). Region III lies between $-1$ to $+1$ volt.

b) To See the Nonlinear Dynamics of the System as the Resistance $R$ is Varied

Remove the power supply connection to A and B (and the DC Differential Amplifier DDA connection to I, if the current is measured across 10 $\Omega$). Put the switch SW1 in position S. Now, the nonlinear resistive device is connected to the rest of the circuit in Figure 1. O2 measures the voltage across $C_2$ in Figure 1, relative to ground (terminal marked COM in Figure 5). O1 measures the voltage across $C_1$ relative to the ground. Connect O2 to channel 1 of a dual channel oscilloscope, and O1 to channel 2 of the oscilloscope. The ground of the oscilloscope is connected to the terminal marked COM on the front panel. The potentiometer (marked POT) knob on the front panel can be turned right to decrease the resistance from about 1.4 k$\Omega$ to 0 $\Omega$. The resistance can be measured at the terminals marked R, when the power to the Chua circuit is turned off.

(The oscilloscope used in this work is the 25 MHz storage oscilloscope from GW Instek with the input probes set at $\times 1$)

Put channels 1 and 2 in the AC coupling mode. Set channels 1 and 2 in the scale 500 mV. Put the oscilloscope in the XY mode. The oscilloscope screen shows a dot at the center when the pot is at the extreme left (i.e., resistance of the pot is a maximum). This indicates that the circuit is not oscillating.

Turn the pot to the right slowly. At one point of the pot, a loop appears on the oscilloscope screen. This is shown in Figure 7(a). Turn off the power to the Chua circuit and measure the resistance of the pot by connecting a DMM, in the 2000 ohms range, to the terminals marked R on the front panel. The resistance is 1.01 k$\Omega$. The loop grows as the resistance is reduced slowly from this value. The appearance of the loop indicates that voltages across $C_1$ and $C_2$ are oscillating with a single frequency, but with a difference in phase. If we go from the XY to the Main tab on the oscilloscope, the two voltages are seen as a function of time. Two sinusoidal traces appear. The frequency of the sine waves is about 3 kHz.

![Figure 7(a)](image)

Figure 7(a). Appearance on the oscilloscope screen in the XY mode when the resistance of pot is 1.1 k$\Omega$.

As the resistance is decreased, the loop becomes double at a resistance of approximately 0.975 k$\Omega$. This is shown in Figure 7(b). This indicates that the system is oscillating at two periods, close to each other. The period of oscillation has bifurcated. The loop is repeated every second cycle indicating period doubling. If we go to the depiction of two voltages as a function of time, we see a distortion of the pure sine wave.
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Figure 7(b). Appearance of a double loop in the XY mode on the oscilloscope screen when resistance is 0.975 kΩ.

As the resistance is further decreased, the separation of the two loops grows. At a resistance of about 0.963 kΩ, four loops appear. This is shown in Figure 7(c). This indicates further doubling of the two periods. Bifurcation has occurred again. One of the routes to chaos from order in a deterministic system is generally through such a bifurcation process. We now see that the points representing \((v_{C1}, v_{C2})\) wander over a larger region of the phase space.

Figure 7(c). Appearance of four loops at a resistance of 0.963 kΩ.

As the resistance is reduced further, the protuberance extends farther and farther as shown in Figure 7(d). This is called a Rössler-type attractor.

Figure 7(d). Rössler type attractor.
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With a further decrease in resistance, the figure on the oscilloscope changes to the one shown in Figure 7(e).

![Image of Figure 7(e)](image)

**Figure 7(e).** Double scroll attractor.

This is called a double scroll attractor. For part of the time, the voltages fluctuate about one DC point \((v_{C1}, v_{C2})\), and then suddenly switch to fluctuating about another DC point \((v_{C1}', v_{C2}')\).

Finally, as the resistance is reduced further, we get a large figure as shown in Figure 7(f).

![Image of Figure 7(f)](image)

**Figure 7(f).** Large limit cycle.

This corresponds to the large limit cycle corresponding to outer segments of the V–I characteristic of the nonlinear resistor.

Figures 7(a to f) appearing on the oscilloscope screen can be photographed with a mobile phone.

In the Feigenbaum circuit, the attractor is a point. In the Chua circuit, the attractor is a curve. Thus, we have seen how the attractor, which is now a curve, changes as the nonlinearity is changed.
Chapter 15

Experiments in Materials Science
15.1 Energy Band Gap of Silicon Using a Silicon Diode

1. Introduction

In a crystal, in which the atoms are arranged in a periodic array, the potential energy of an electron varies periodically. The energy levels of an electron moving in such a periodic potential is split into allowed and forbidden bands of energy. In a metal, the conduction band is partially filled, while all energy bands with lower energy are completely filled. Since closely spaced unoccupied energy levels are present in a metal, a small electric field causes the electrons to move to higher energy levels, and this leads to good electrical conductivity of a metal. In a semiconductor, the highest occupied band is completely filled, and the unfilled next higher band is called the conduction band. At a finite temperature, thermal energy raises some of the electrons near the top of the valence band to the bottom of the conduction band. The energy separation between the top of the valence band and the bottom of the conduction band is called the band gap $E_g$ of a semiconductor. The number of electrons excited to the conduction band when $kT < E_F$ (Fermi level, which is equal to $E_g/2$ for a lightly doped semiconductor) is proportional to $\exp\left(-\frac{E_g}{2kT}\right)$.

In a diode, made up of a junction between n-type and p-type semiconductors, a current will flow when a voltage is applied. If a positive potential is applied to the p-side of the junction, and a negative potential to the n-side, the junction is forward biased. In this case, the forward current flowing from the p-side to the n-side will increase with the potential difference $V$ across the junction according to the relation

$$I_{\text{for}}(V) = I_0 \left[\exp\left(\frac{eV}{\eta kT}\right) - 1\right] \cong I_0 \exp\left(\frac{eV}{\eta kT}\right).$$

Here, $I_0$ is the temperature dependent reverse saturation current of the diode. At room temperature, $kT$ will be about 0.02 eV. $\eta$, called the non-ideality factor, will vary from diode to diode, and has a value between 1.4 and 2. If $V$ is even as small as 0.3 V, $eV/\eta kT$ will be between 7.5 and 11, and $\exp(eV/\eta kT) \gg 1$. So we are justified in neglecting unity in comparison with $\exp(eV/\eta kT)$. A plot of $\ln(I)$ against the forward voltage $V$ will have a slope $e/\eta kT$. Knowing $T$, from the slope $\alpha$ we can find the non-ideality factor $\eta$ for the diode.

If we now reverse the voltage to $-V$ (i.e., the n-side of the junction is now positive and the p-side negative, and the junction is reverse biased), the current will become

$$I_{\text{rev}} = I(-V) = I_0 \left[\exp\left(-\frac{eV}{\eta kT}\right) - 1\right] \cong -I_0,$$

since $\exp(-eV/kT)$ for large $V$ (above a few hundred millivolts) becomes very much less than 1. $I_{\text{rev}}$ is called the reverse saturation current as it is independent of $V$.

The reverse saturation current is made up of two contributions: the diffusion current due to the difference in concentrations of electrons and holes on the two sides of the junction, and a thermal excitation current due to the creation of electrons and holes by thermal energy. $I_{\text{rev}}$ will vary with temperature as

$$I_{\text{rev}} \propto \frac{1}{T^\alpha} \exp\left(-\frac{E_g}{\eta kT}\right),$$

where $\epsilon$ has a value between 1.5 and 3. If we measure the reverse saturation current at different temperatures, a plot of $[\ln(I_{\text{rev}}) - \epsilon \ln(T)]$ vs. $1/T$ will give a straight line with a slope $\beta = -E_g/\eta k$. Knowing $\eta$ from the slope $\alpha$ of $\ln(I_{\text{rev}})$ vs. $V$ at room temperature, and taking $k$ to be $86.6 \times 10^{-6}$ electron volt/K, one can calculate the energy band gap in electron volts (eV). This is the principle of the experiment.

2. Apparatus Required

Regulated DC power supply, temperature indicator, furnace, insert for band gap of a semiconductor, a variable voltage box, and a DC multimeter reading DC voltages in the 2 V and 200 mV range.
3. Experimental Procedure

A silicon diode (1N4148) is pasted with Araldite on one face of an aluminium rod, of 1 cm square cross section and 5 cm length. The two leads are insulated from one another and from the aluminium rod by Teflon tape. A Pt100 resistance is fixed inside a hole in the Al rod. (This rod also contains a coil of thin enamel coated copper wire of resistance about 25 ohms, so that one may measure the TCR of copper also). Electrical leads for the resistance thermometer, for the 1N4148 diode, (and for the copper coil), are brought to a circular holder and connected to banana terminals as shown in Figure 1.

![Figure 1. Connection diagram at the circular holder.](image)

The connection diagram at the circular holder is shown in Figure 1. The leads from the Pt100 thermometer are connected to terminals 1 and 8. (The leads from the copper coil are connected to terminals 2 and 3.) A 2 kilohm resistor is connected between terminals 4 and 5 and is at room temperature. The leads from the 1N4148 diode are connected to terminals 5 and 6, so that it is in the forward direction between 5 and 6 (i.e., if the positive side of the voltage is connected to 5 and the negative side to 6, the junction will be subject to a forward bias). A resistance of 470 kΩ is connected between terminals 6 and 7 and is at room temperature. The aluminium rod with the Pt100 thermometer, (copper coil,) and the 1N4148 diode hangs inside a furnace from the holder.

3(a) Measuring the Forward Current as a Function of the Forward Voltage at Room Temperature to Determine the Non-Ideality Parameter $\eta$  

The terminals 1 and 8 of the Pt100 are connected to a temperature indicator. When the indicator is switched on, it indicates the temperature of the aluminium block in °C. It will be at room temperature to start with.

A box for obtaining a variable voltage is shown in Figure 2.

![Figure 2. Variable voltage box – top panel.](image)

Inside the box, there is a 9V battery. Between the two terminals A (red) and B (black), we will get a voltage of 9 V. Between terminals C (red) and D (black) we get a variable voltage between 0 to 1.5 V. The voltage is adjusted by turning the knob of the ten turn pot P. Switch S1 switches on the 9V supply, while switch S2 switches on the 0 to 1.5 V supply.

To measure the forward current vs. voltage through the 1N4148 diode, connect the terminal C on the box (Figure 2) to the terminal 4 on the circular holder (Figure 1), and the terminal D on the box to the terminal 6 on the circular holder. The voltage across CD is applied to a 2 kΩ resistor and the 1N4148 diode (in the forward direction) in series. Adjust the pot P so that the voltage measured across terminals 5 and 6
is 0.3 V, as measured by a DMM. This is the forward voltage across the diode. Now measure the voltage across 4 and 5 terminals on the circular holder. That is the voltage across 2 kΩ, that is proportional to the forward current through the diode. Change the forward voltage from 0.3 to 0.5 volts in steps of 0.025 volts by turning the knob of the pot P. For each forward voltage, measure the voltage across the 2 kΩ resistor. These readings are taken at room temperature shown by the temperature indicator, connected to terminals 1 and 8 on the circular holder.

Table 1 shows a sample set of readings. The table is self-explanatory. A plot of ln(v) against the forward voltage $V_{for}$ is shown in Figure 3.

<table>
<thead>
<tr>
<th>$V_{for}$ (volts)</th>
<th>v (mV) across 2k</th>
<th>ln(v)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>3.1</td>
<td>1.131</td>
</tr>
<tr>
<td>0.326</td>
<td>5.0</td>
<td>1.609</td>
</tr>
<tr>
<td>0.351</td>
<td>8.4</td>
<td>2.128</td>
</tr>
<tr>
<td>0.375</td>
<td>14.1</td>
<td>2.646</td>
</tr>
<tr>
<td>0.400</td>
<td>23.6</td>
<td>3.161</td>
</tr>
<tr>
<td>0.425</td>
<td>39.6</td>
<td>3.679</td>
</tr>
<tr>
<td>0.45</td>
<td>65.7</td>
<td>4.185</td>
</tr>
<tr>
<td>0.475</td>
<td>110.6</td>
<td>4.706</td>
</tr>
<tr>
<td>0.500</td>
<td>183.7</td>
<td>5.213</td>
</tr>
</tbody>
</table>

Figure 3. Plot of ln(v) vs. forward voltage for 1N4148 diode.

We see that the linear fit to the data is very good. The slope $\alpha$ of the curve is 20.6. The room temperature at which data were taken was 26.4 °C i.e., 299.5 K. From this, the non-ideality factor $\eta$ for the diode is

$$\eta = \frac{e}{(akT)} = 1.87.$$  

Here $k/e$ is in $eV/K$ and has a value $86.6 \times 10^{-6}$.

A repeat of the experiment with the same diode gave an identical value. The experiment was repeated for a different 1N4148 diode twice, and the values of the non-ideality factor were 1.87 and 1.88. Thus, the value of the non-ideality factor is very reproducible.

3(b) Variation of Reverse Saturation Current $I_{rev}$ with Temperature

For this experiment, the terminal A on the box is connected to the terminal 7 on the circular holder. The terminal B on the box is connected to terminal 5 on the circular holder. Now, the diode is reverse biased, terminal 6 being positive and terminal 5 being negative. The reverse saturation current flows through the 470 kΩ resistor connected between terminals 6 and 7. The voltage across 6 and 7 will be proportional to the reverse saturation current.
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A DC power supply is connected to the furnace. The top of the furnace is lightly closed with a cotton plug to prevent convection currents of air. The voltage on the power supply is set at 7.5 to 8 V. The temperature of the aluminium rod, as shown on the temperature indicator, goes up. The voltage on the power supply is adjusted so that the temperature goes up by 1 to 1.5 degrees centigrade per minute. The voltage $v$ across the 470 kΩ resistor is measured, using a DMM, for every five degree rise in temperature, up to about 100 °C. A sample set of readings on the same diode is shown in Table 2.

Table 2. Reverse Saturation current of 1N4148 at different temperatures. $v$ in mV is the voltage across 470 kΩ proportional to $I_{rev}$.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$T$ (K)</th>
<th>$1/T$ (K⁻¹)</th>
<th>$v$ (mV)</th>
<th>$\ln(v)−1.5\ln(T)$</th>
<th>$\ln(v)−3\ln(T)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.0</td>
<td>303.1</td>
<td>0.00330</td>
<td>4.9</td>
<td>−6.982</td>
<td>−15.553</td>
</tr>
<tr>
<td>35.2</td>
<td>308.3</td>
<td>0.00324</td>
<td>7.7</td>
<td>−6.555</td>
<td>−14.152</td>
</tr>
<tr>
<td>45.2</td>
<td>318.3</td>
<td>0.00314</td>
<td>17.6</td>
<td>−5.777</td>
<td>−14.421</td>
</tr>
<tr>
<td>49.5</td>
<td>322.6</td>
<td>0.00310</td>
<td>24.9</td>
<td>−5.450</td>
<td>−14.114</td>
</tr>
<tr>
<td>56.0</td>
<td>329.1</td>
<td>0.00304</td>
<td>41.4</td>
<td>−4.971</td>
<td>−13.665</td>
</tr>
<tr>
<td>60.0</td>
<td>333.1</td>
<td>0.00300</td>
<td>56.5</td>
<td>−4.678</td>
<td>−13.391</td>
</tr>
<tr>
<td>65.0</td>
<td>338.1</td>
<td>0.00296</td>
<td>82.0</td>
<td>−4.328</td>
<td>−13.063</td>
</tr>
<tr>
<td>75.1</td>
<td>348.2</td>
<td>0.00287</td>
<td>162.9</td>
<td>−3.686</td>
<td>−12.465</td>
</tr>
<tr>
<td>79.2</td>
<td>352.3</td>
<td>0.00284</td>
<td>214.0</td>
<td>−3.431</td>
<td>−12.227</td>
</tr>
<tr>
<td>84.8</td>
<td>357.9</td>
<td>0.00279</td>
<td>308.3</td>
<td>−3.089</td>
<td>−11.909</td>
</tr>
<tr>
<td>90.4</td>
<td>363.5</td>
<td>0.00275</td>
<td>426.0</td>
<td>−2.789</td>
<td>−11.632</td>
</tr>
<tr>
<td>95.5</td>
<td>368.6</td>
<td>0.00271</td>
<td>562.0</td>
<td>−2.533</td>
<td>−11.397</td>
</tr>
<tr>
<td>100.0</td>
<td>373.3</td>
<td>0.00268</td>
<td>718.0</td>
<td>−2.307</td>
<td>−11.190</td>
</tr>
</tbody>
</table>

The first column gives the temperature $T$ in °C. The second column gives the temperature $T$ in Kelvin. The third column gives $1/T$ (in Kelvin⁻¹). The fourth column gives the voltage $v$ (in mV) across 470 kΩ. This voltage is proportional to $I_{rev}$. The fifth column gives $\ln(v)−1.5\ln(T)$. The sixth column gives $\ln(v)−3\ln(T)$. We have calculated $\ln(v)−\epsilon\ln(T)$ for the two extreme values of $\epsilon$, namely 1.5 and 3.

Figure 4 shows a plot of $\ln(v)−1.5\ln(T)$ against $1/T$ and Figure 5 shows a plot of $\ln(v)−3\ln(T)$ against $1/T$.

![Figure 4. Plot of \(\ln(v)−1.5\ln(T)\) against \(1/T\).](image-url)
We get a good linear fit to the data with a slope $\beta = -7458$ in Figure 4, and $\beta = -7049$ in Figure 5. The band gap is calculated from

$$E_g = -\beta \eta k.$$  

Using the value of $\eta$ from Section 3(a) and $k = 86.6 \times 10^{-6}$ eV/k, the band gap comes out to be 1.21 eV for $\epsilon = 1.5$ (Figure 4) and 1.14 eV for $\epsilon = 3$ (Figure 5). Measurement with other 1N4148 diodes gave values within $\pm 2\%$ of one another. Thus we get reproducible values for the band gap.

The actual band gap of silicon is 1.1 eV. The present results show that the value for $\epsilon = 3$ is in better agreement with the actual value, than the value for $\epsilon = 1.5$. We may conclude that the value of $\epsilon$ for the 1N4148 diode is close to 3.

### 4. Questions

1. What is the meaning of doping? If the sample is heavily doped what happens to the energy band picture?
2. What is the meaning of Fermi energy?
3. The electron volt eV is a unit of energy. Express 1 eV in joules.

### 15.2 Integral Method for Evaluating the Seebeck Coefficient of Constantan as a Function of Temperature

1. **Seebeck Analyzer: General Description**

   The Seebeck-emf analyzer has been specially designed to study the temperature characteristics of standard and non-standard thermocouples. Some typical experiments that can be carried out using this equipment are:

   a) Calibration of any thermocouple against a standard thermocouple viz., chromel–alumel thermocouple.

   b) Study of phenomena like ‘Neutral temperature’ and ‘Inversion temperature’ in non-standard thermocouples like iron–copper and iron–silver.
c) Experimental verification of the ‘Law of Intermediate Metals’.

d) Measurement of ‘Absolute Seebeck Coefficient’ of metallic samples in wire form as a function of temperature through an integral method.

2. System

The block diagram of the system to measure Seebeck-emf of various thermocouples as a function of temperature is given in Figure 1. The system essentially consists of the following.

1. High temperature furnace capable of reaching temperatures up to 800 °C.

![Figure 1. Block diagram of Seebeck-emf analyzer.](image-url)
3. Operating Instructions

1. Before turning on the equipment, make the following connections. The output at the front panel of the AC power source is connected to the furnace. Never connect the furnace socket directly to mains.

2. The furnace assembly has a 4–way terminal strip at the top cover for easy connection of thermocouples to the measuring system. The same arrangement can be used to verify the ‘Law of Intermediate Metals’. The following color code identifies the thermocouples coming out of the thermowell arrangement.

   Reference thermocouple
   Chromel (+ve leg) → Red sleeves.
   Alumel (−ve leg) → Blue sleeves.

3. Make the appropriate connections between the thermocouple wires and the measuring instrument using the supplied interconnecting cables. The chromel–alumel thermocouple leads are connected to ‘Channel 1’ at the back panel of the differential amplifier. The leads of the experimental thermocouple which in the present case happens to be copper–constantan are connected to ‘Channel 2’ located at the back panel.

4. Turn on the Mains of the equipment. Allow for a warm-up period of 5 minutes before starting the measurements. The ‘Tare’ push-in switch is operated once to null the offset in the differential amplifier. Typically the readings will be within ±0.001 mV.

4. Measurement of Absolute Seebeck Coefficient of Constantan as a Function of Temperature

Method

The chromel–alumel and copper–constantan thermocouples are kept in close proximity in a cell and the thermowell assembly is then inserted into the center of the furnace. The heating coil has a nominal resistance of 6Ω. Typically, 40 to 50 watts are sufficient to attain temperatures around 500 °C at the center of the furnace. The thermo-emf developed across chromel–alumel (which is a standard thermocouple), which is fed to Channel 1, is amplified using an instrumentation amplifier (gain=100) and the amplified signal is digitized and read in a Digital Panel Meter directly in millivolts. Similarly the thermo-emf developed across the copper–constantan thermocouple which is fed to Channel 2 can be read directly by shifting the TC selector switch to position labeled 2.

Normally, thermocouple voltages are measured with the reference junction held at 0 °C. These values are tabulated in the NBS tables and used to read out the temperature. In the present experimental arrangement, the concept of a single junction thermocouple is used where it is understood that the reference junction is at room temperature. Hence to use the NBS tables, a voltage value corresponding to the room temperature has to be added to the measured voltage from the single junction thermocouple. This can be carried out by measuring the room temperature on a thermometer, and the thermo-emf corresponding to this temperature can be read off from the tables.

Typically for a chromel–alumel thermocouple, room temperature of 25 °C corresponds to 1.0 mV. For example, a 7 mV read from single junction chromel–alumel thermocouple corresponds to 8 mV (reference junction at 0 °C) and the temperature as read by NBS tables is 196.58 °C.

Similarly for the experimental copper–constantan thermocouple, an emf of the order of 1 mV corresponding to the room temperature of 25 °C has to be added to the measured value.
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The entire experiment up to 400 °C can be carried out in about 50 minutes. It is also worth mentioning that the special thermowell arrangement facilitates fast equilibration, and readings can be taken during the heating run without waiting for the steady state.

Sample Results

Seebeck-emf of copper–constantan is measured against the temperature as read through the NBS tables for chromel–alumel thermocouple. The typical plot of this data is given in Figure 2.

![Figure 2. Seebeck-emf of copper–constantan thermocouple vs. temperature.](image)

The thermo-emf $E$ is related to the absolute Seebeck coefficient of copper, $S_{Cu}$, and that of constantan, $S_{Const}$, through the integral relation

$$E = \int_{0}^{T} [S_{Cu} - S_{Const}] \, dT.$$  \hspace{1cm} (1)

Here $[S_{Cu} - S_{Const}]$ is referred to as the ‘Relative Seebeck Coefficient’ of the copper–constantan thermocouple.

Thus, slope of the thermo-emf versus temperature curve gives the relative Seebeck coefficient of the copper–constantan thermocouple. This quantity can thus be experimentally evaluated as a function of temperature covering the range 25 °C to 400 °C (Figure 3).

$S_{Cu}$, through Thomson effect studies, has been experimentally determined and can be approximated through a linear relation, viz.,

$$S_{Cu} = CT + D, \quad C = 4.989 \times 10^{-3} \frac{\mu V}{({^\circ}C)^2}, \quad D = 1.7 \frac{\mu V}{^\circ C}. \hspace{1cm} (2)$$

The experimental determination of the relative Seebeck coefficient of the copper–constantan thermocouple coupled with the data on absolute Seebeck coefficient of copper, thus provides a method of evaluating the absolute Seebeck coefficient of constantan.

Figure 4 presents the variation with temperature of the Seebeck coefficient of constantan along with that of copper.

The main results that emerge out of these experimental studies are the following.

1. The absolute Seebeck coefficient of constantan is negative and its magnitude is large, of the order of $-40 \mu V/^\circ C$ near ambient temperature and increases to nearly $-60 \mu V/^\circ C$ near 400 °C.
2. The relative Seebeck coefficient, which is the sensitivity of the copper–constantan thermocouple, exhibits a marked increase in its magnitude over the temperature range 0–400 °C.

3. The Seebeck-emf of the copper–constantan thermocouple exhibits a strong nonlinear behavior with respect to temperature. A polynomial of 6th order is required to fit the experimental data.

4. Among the alloys, Constantan has the highest negative Seebeck Coefficient and thus forms the negative leg of standard thermocouples like Iron–Constantan and Chromel–Constantan. The integral method of determining the absolute Seebeck coefficient is simple and can be used for any metallic system which can be processed in wire form.

Primarily, constantan is composed of nickel and copper. It contains 55% copper and 45% nickel. The large negative value of the Seebeck coefficient in this material is related to the electronic band structure of this alloy where both s-type and d-type states coexist near the Fermi energy. The gradual filling up of the holes in the d-band due to alloying of nickel with copper leads to a situation wherein the Fermi energy of the alloy system probes the strongly decreasing density of states of the d-band. This leads to a large negative value for the Seebeck coefficient of constantan.
15.3 Differential Method to Measure Absolute Seebeck Coefficients of n and p Type Bismuth Telluride (Bi$_2$Te$_3$)

1. Introduction

Semiconductors offer a unique thermoelectric advantage in the sense that their Seebeck coefficients are orders of magnitude higher than that in metals. The sign of the absolute Seebeck coefficient in a semiconductor is determined by the sign of the majority charge carriers. An n-type semiconductor is characterized by a negative Seebeck coefficient, while a p-type semiconductor exhibits a positive Seebeck coefficient. Thus, from an experimental determination of the Seebeck coefficient, one can distinguish between n-type and p-type semiconductors.

Apart from the Hall coefficient studies, the sign of the Seebeck coefficient provides a convenient tool to decipher whether the majority carriers are electron-like or hole-like.

2. Preparation of Thermoelectric Materials by Mechanical Alloying

The ability of direct thermal to electrical energy conversion of a thermoelectric (TE) material has received enormous attention in the past two decades considering its potential application in eco-friendly solid state energy harvesting. In the TE family, bismuth telluride (Bi$_2$Te$_3$) and its alloys are the most widely used TE materials around room temperature. Unidirectional crystal growth methods such as Bridgman method, Travelling Heater Method (THM), etc., are widely used for their preparation. The unidirectional grown materials are inclined to cleavage fracture due to their weak van der Waals bond between hexagonally stacked five-atomic layers, Te–Bi–Te–Bi–Te.

Powder metallurgical methods, which produce randomly oriented polycrystalline microstructure and grain-strengthening effect to avoid incidental cleavage failure, are commonly used to improve their mechanical property. Conventional powder metallurgical process is devalued for the preparation of TE materials by its high-energy consumption steps and impurity contamination. Mechanical Alloying (MA) by high-energy ball milling is a powerful powder processing method. It has been applied to prepare powders of various alloys and compounds in quite a short time. Mechanical alloying method has also proved to be a versatile alternative to other processing routes in preparing nano-structured materials with a broad range of chemical compositions. Although there are numerous techniques to produce nanostructured materials, this method has become more popular due to its simplicity, capacity for large-scale production and cost effectiveness. MA process is greatly affected by a number of factors that play a very important role in the preparation of homogeneous materials. The properties of the milled powders (particle size distribution, degree of disorder or amorphization, and final stoichiometry) depend on the milling conditions. A very large number of grain boundaries are generated when mechanical alloying is employed to prepare nano-size powder. Low dimension and nanostructure enhance the thermoelectric performance of a material in a significant fashion.

Bi$_2$Te$_3$ with stoichiometric composition was prepared by high-energy ball milling using planetary ball mill Fritsch Pulverisette 6. The elemental granules of bismuth (Bi) and tellurium (Te) with 4N purity were crushed down to pieces smaller than 10 mm. The starting materials were weighed according to the composition and then subjected to mechanical alloying in planetary ball mill under inert atmosphere. The bigger particles were milled to a fine powder by high-energy ball milling at the speed of 450 rpm using a hardened tungsten carbide vessel and grinding balls. Prepared powder of Bi$_2$Te$_3$ was analyzed by X-ray diffraction (XRD) and surface electron microscopy for phase formation and microstructural study. Later it was hot pressed at 700 K and 50 megapascal pressure using cylindrical graphite die.

3. Differential Method of Measuring the Seebeck Coefficient

The rather simple ‘Integral method’ of measuring Seebeck coefficient in metallic wires cannot be employed in the case of semiconductors owing to the difficulty of drawing them in wire form. Semiconductors, for the present experiments, are prepared through the mechanical alloying route and are available in pellet form. The experimental arrangement consists of attaching chromel–alumel thermocouples both to the top
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and bottom surfaces of the pellet. The pellet is sandwiched between a cooling platform below and a heating platform above as shown in the photograph in Figure 1.

Figure 1. Pellet pressed between a heating and a cooling platform.

The cooling platform is constructed from a Peltier cooler powered by a 0–15 volt, 0–5 amp power supply. The heating stage is constructed out of a nichrome heater powered by a suitable power supply. This is optional as the top surface would be at a temperature higher than the lower surface which is in direct contact with the Peltier cooler. The main objective of this experimental arrangement is to create a temperature difference varying between 1 °C and 10 °C between the top and bottom surfaces of the pellet. The mean temperature of the sample can be maintained to be close to the ambient temperature. The schematic diagram of the sample with thermocouple probes at either end is shown in Figure 2.

Figure 2. Schematic of the differential technique.

The Seebeck coefficient $S(T)$ of the semiconducting sample at temperature $T$ is given by the relation

$$S_{\text{Chr}} (T) - S(T) = \frac{\Delta V_{13}}{\Delta T},$$

(1)

where $S_{\text{Chr}} (T)$ is the Seebeck coefficient of the chromel probe, and $\Delta V_{13}$ is the voltage developed in chromel–sample thermocouple for a temperature difference of $\Delta T$.

In a similar way,

$$S_{\text{Alu}} (T) - S(T) = \frac{\Delta V_{24}}{\Delta T},$$

(2)

where $S_{\text{Alu}} (T)$ is the Seebeck coefficient of the alumel probe and $\Delta V_{24}$ is the voltage developed in alumel–sample thermocouple for the same temperature difference of $\Delta T$. 

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Hence, $\Delta T$ can be evaluated from the following relation,

$$\Delta T = \frac{(\Delta V_{13} - \Delta V_{24})}{(S_{\text{Chr}} - S_{\text{Alu}})}.$$  \hfill (3)

The temperature difference can thus be more accurately measured as compared to obtaining the difference of two large quantities $T_1$ and $T_2$. 

$(S_{\text{Chr}} - S_{\text{Alu}})$ is the relative Seebeck coefficient of a chromel–alumel thermocouple and is generally represented as $S_{\text{Chr-Alu}}(T)$. Thus the Seebeck coefficient of the sample under study, $S(T)$, is given by the relation

$$S(T) = S_{\text{Chr}}(T) - \left[ \frac{\Delta V_{13}}{(\Delta V_{13} - \Delta V_{24})} \right] S_{\text{Chr-Alu}}(T).$$  \hfill (4)

The mean temperature $T$ of the sample can be evaluated taking the average of the thermoemfs $\Delta V_{12}$ and $\Delta V_{34}$.

$S(T)$, the Seebeck coefficient of the sample, can thus be evaluated by measuring the thermoemf’s $\Delta V_{12}$, $\Delta V_{34}$, $\Delta V_{13}$ and $\Delta V_{24}$. These low level DC signals are measured in the 4-channel DC differential amplifier.

The present experiment has the twin objectives of deciphering the sign of the Seebeck coefficient and its magnitude near room temperature. The experiment is carried out by measuring $\Delta V_{13}$ for different values of $\Delta T$, which is proportional to $(\Delta V_{13} - \Delta V_{24})$. From the slope of $\Delta V_{13}$ against $(\Delta V_{13} - \Delta V_{24})$, $S(T)$ can be evaluated from the known values of $S_{\text{Chr}}(T)$ and $S_{\text{Chr-Alu}}(T)$. The room temperature value of $S_{\text{Chr}}(T)$ is $\approx +22 \mu\text{V/C}$ and of $S_{\text{Chr-Alu}}(T)$ is $\approx 40 \mu\text{V/C}$.

Figure 3 shows a plot of $\Delta V_{13}$ against $(\Delta V_{13} - \Delta V_{24})$ for Sample 1, which is p-type bismuth telluride.

![Figure 3](image_url)

**Figure 3.** Typical data of $\Delta V_{13}$ against $(\Delta V_{13} - \Delta V_{24})$ for Sample 1.

The slope of the linear fit of the data in Figure 3 is $\approx -6.5$ and the absolute Seebeck coefficient of Sample 1 as computed from relation (4) turns out to be around $+280 \mu\text{V/C}$.  

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Figure 4 shows a plot of $V_{13}$ against $(V_{13} - V_{24})$ for sample 2, which is n-type bismuth telluride.

The slope of the linear fit of the data in Figure 4 is $\approx +3.3$ and the absolute Seebeck coefficient of Sample 2 as computed from relation (4) turns out to be $-110 \mu V/C$.

15.4 DC Hall Effect in Semiconductors using an Electromagnet

1. Introduction

When a charge $q$ moves with a velocity $v$ along the X-axis in a magnetic field $B$ along the Z-axis, there is a force $F_B = q(v \times B)$ on the charge. This force $F$ is along the $-Y$-axis if the charge $q$ is positive, and along the $+Y$-axis if the charge $q$ is negative. So the charge moves in the Y-direction, and collects at the edge of the sample along its width.

If the sample is rectangular with a length $l$ along X, width $w$ along Y, and thickness $t$ along Z (Figure 1), the collection of charges along the width produces an electric field $E$ along the $+Y$-direction if $q$ is positive, and $-Y$-direction if $q$ is negative. This electric field will exert a force, $F_E = qE$, on the charge $q$ in a direction opposite to $F_B$, whatever the sign of the charge. The charge accumulation along the width will stop when $F_E$ equals $F_B$ in magnitude. Then
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\[ qE = qBv. \]  \hspace{1cm} (1)

The current in the X-direction \( I \) is given by

\[ I = n (wt) q v. \]  \hspace{1cm} (2)

Here \( n \) is the number density of charge carriers in the conductor.

Substituting for \( v \) in (1) in terms of \( I \), we get

\[ E = \frac{V_H}{w} = \frac{BI}{(nqwt)} \]  \hspace{1cm} (3)

or

\[ V_H = \frac{BI}{(nqwt)} = R_H \left( \frac{BI}{I} \right). \]  \hspace{1cm} (4)

\( V_H \) is the Hall voltage measured along \( Y \), and \( R_H \) is the Hall coefficient

\[ R_H = \frac{1}{nq}. \]  \hspace{1cm} (5)

Thus,

a) The Hall voltage along \( Y \) will be positive if \( q \) is positive and negative if \( q \) is negative;

b) For a given current through the sample, the Hall voltage will be proportional to the magnetic field across the sample;

c) For a given magnetic field, the Hall voltage will be proportional to the current through the sample.

d) The Hall voltage will be inversely proportional to the thickness \( t \) of the sample, but will not depend on its length and width.

e) The Hall voltage will be inversely proportional to the number density of charge carriers.

Let us calculate the Hall voltage for a sample 1 mm (10\(^{-3}\) m) in thickness, for a current of 1 mA, and a magnetic field of 1 kG = 0.1 T. The charge carriers are electrons with a charge \( q \) of \(-1.6 \times 10^{-19}\) coulombs.

If the conductor is a metal, the number of charge carriers is of the order of \( 10^{27}/m^3 \). The Hall voltage \( V_H \) comes out to be

\[ V_H = \frac{(0.1 \times 1 \times 10^{-3})/(10^{27} \times (-1.6 \times 10^{-19}) \times 10^{-3})}{10^{27} \times (-1.6 \times 10^{-19}) \times 10^{-3}} = -6.7 \times 10^{-10} \text{ V}. \]

Thus, in metals the Hall voltage will be in nanovolts.

On the other hand, consider a lightly doped n-type semiconductor with a carrier density of \( 10^{21}/m^3 \).

From equation (4), the Hall voltage will be

\[ V_H = \frac{(0.1 \times 1 \times 10^{-3})}{(10^{21} \times (-1.6 \times 10^{-19}) \times 10^{-3})} = -6.7 \times 10^{-4} \text{ V}. \]

The Hall voltage will be in mV and will be easy to measure.

The sign of the Hall coefficient gives the sign of the majority carrier. It will be positive for p-type semiconductors, and negative for n-type semiconductors.

The resistance \( r \) of the sample can be calculated by measuring the voltage \( V_x \) along the length of the sample (i.e., \( r = V_x/I \)). The conductivity is given by

\[ \sigma = \left( \frac{1}{r} \right) \left( \frac{I}{wl} \right). \]  \hspace{1cm} (6)

The conductivity is related to the carrier concentration and mobility \( \mu \) of the charge carrier by
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\[ \sigma = nq\mu = \frac{\mu}{R_H}, \]  

(7)

One can then find the mobility from a measurement of the conductivity and the Hall coefficient.

In a semiconductor the number density of charge carriers varies as \( \exp(-E_g/2kT) \) for lightly doped silicon or germanium. So the Hall coefficient will show an exponential decrease as the temperature is increased.

2. Sample

In the present experiment, we use a square sample of p- or n-doped germanium, with a length and width of about 5 mm and a thickness of 0.5 mm.

![Figure 2. Schematic diagram of the sample holder.](image)

In Figure 2, the two leads marked \( I \) are the current leads. The Hall voltage is measured at the voltage leads marked \( V_H \).

The resistance of the sample is a few hundred ohms. Red wires are connected to the contacts \( V_H, V_H \) to measure the Hall Voltage along the Y-axis. The contacts \( I, I \), along the X-axis, are connected by green wires to a constant current source. The sample holder is held in a wooden clamp so that the sample can be placed between the pole pieces of an electromagnet, perpendicular to the magnetic field.

3. DC Method of Measuring Hall Coefficient

(A) Calibration of the Electromagnet

First adjust the distance between the pole pieces to be around 1.5 cm.

There are two coils in our electromagnet. Connect the red banana terminal to the red terminal on the power supply, and one of the black banana terminals to the black terminal on the power supply. The black terminals on the magnet are marked as Coil 1 and Coil 2. There is a switch which can be put in position marked coil 1 or in the position marked coil 2. Put this switch in the position marked coil 1. Turn the current knob on the power supply to the extreme left. Switch on the power supply. The current reading is 0.01 A.

Take out the Gauss meter probe. Withdraw the SS cover on the probe, to expose the sensor mounted in a red plastic mount. On the mount, one side is marked to face the north pole. The pole on the right side of our electromagnet is the north Pole and the writing must face this pole. A DPDT switch on the Gauss meter can be put in two positions marked X1 and X10. Put this switch in the position marked X1. Mount the Gauss meter probe on the wooden stand, and push the probe so that it is in the center of the space between the two pole pieces. If the Gauss meter is switched on, it will show a reading of approximately 200 gauss. If the reading is negative, it means that the writing on the sensor mount is facing the south pole. The probe has to be rotated about its axis through 180°.

Adjust the knob on the power supply of the magnet to change the current through coil 1, in steps of approximately 0.04 to 0.05 amps. For each current the Gaussmeter reading is taken. The maximum current will be restricted to 0.36 A, when the field will reach approximately 1 kilogauss.

Repeat the readings by putting the DPDT switch in coil 2 position.
A sample set of readings are shown in Table 1.

**Table 1.** Calibration of the magnet.

<table>
<thead>
<tr>
<th>$I_{\text{mag}}$ (A)</th>
<th>Field (gauss)</th>
<th>$I_{\text{mag}}$ (A)</th>
<th>Field (gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>206</td>
<td>0.01</td>
<td>206</td>
</tr>
<tr>
<td>0.04</td>
<td>263</td>
<td>0.06</td>
<td>319</td>
</tr>
<tr>
<td>0.07</td>
<td>325</td>
<td>0.09</td>
<td>383</td>
</tr>
<tr>
<td>0.1</td>
<td>406</td>
<td>0.14</td>
<td>503</td>
</tr>
<tr>
<td>0.14</td>
<td>506</td>
<td>0.18</td>
<td>621</td>
</tr>
<tr>
<td>0.18</td>
<td>593</td>
<td>0.2</td>
<td>663</td>
</tr>
<tr>
<td>0.2</td>
<td>638</td>
<td>0.23</td>
<td>717</td>
</tr>
<tr>
<td>0.24</td>
<td>716</td>
<td>0.28</td>
<td>844</td>
</tr>
<tr>
<td>0.28</td>
<td>823</td>
<td>0.31</td>
<td>910</td>
</tr>
<tr>
<td>0.32</td>
<td>909</td>
<td>0.33</td>
<td>989</td>
</tr>
<tr>
<td>0.36</td>
<td>1013</td>
<td>0.36</td>
<td>1039</td>
</tr>
</tbody>
</table>

The first two columns refer to data when the current passes through coil 1, and the last two columns refer to the data when the current passes through coil 2. $I_{\text{mag}}$ is the current through the coil in amps, read on the meter in the power supply. The column marked Field is the reading on the digital Gauss meter.

A graph is drawn between the Field and the current. This is shown in Figure 3.

![Figure 3. Calibration graph for the magnet.](image)

From the graph we get the following equations for the field $B$ in terms of the current $I_{\text{mag}}$ for the two coils

- **Coil 1:** \[ B \text{ (in gauss)} = 174.3 + 2310 \times I_{\text{mag}} \text{ (in amps)} \].
- **Coil 2:** \[ B \text{ (in gauss)} = 174.6 + 2408 \times I_{\text{mag}} \text{ (in amps)} \].

One may take the average and write for either coil

\[ B \text{ (in gauss)} = 174.5 + 2359 \times I_{\text{mag}} \text{ (in amps)} \].  \hspace{1cm} (8)
From this equation, \( B \) can be calculated from the value of \( I_{mag} \).

**(B) Measuring Hall Coefficient**

Connect a constant current source to the green wires, taking care to connect the wire marked + to the positive terminal of the CCS, and the wire marked – to the negative terminal of the CCS. The two red wires are connected to a DC microvoltmeter, again taking care to see that the wire marked + is connected to the positive terminal of the microvolt meter, and the wire marked – is connected to the negative terminal.

Mount the Hall probe in the wooden holder and place the sample between the pole pieces of the electromagnet, so that the sample is perpendicular to the magnetic field.

A current \( I \), in the low current mode of the constant current source, of 0.5 mA is set to pass through the sample. The magnetic field is set by adjusting the current \( I_{mag} \) through one of the coils (we may use either one of the coils). The DC microvoltmeter reading is noted. The current \( I_{mag} \) through the coil is increased from 0.01 A to 0.36 A, in steps of 0.04 to 0.05 A. The current \( I \) through the sample is kept constant. For each value of the current \( I_{mag} \) through the magnetic field coil, the reading of the DC microvoltmeter is noted.

Then the current \( I \) through the sample is increased in steps of 0.5 mA and the experiment is repeated. A sample set of data for an n-type germanium sample is given in Table 2.

**Table 2.** Hall effect measurements in n-type germanium sample thickness = 0.5 mm = \( 5 \times 10^{-4} \) m.

<table>
<thead>
<tr>
<th>( I ) (mA)</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{mag} ) (amps)</td>
<td>( B ) (gauss)</td>
<td>( V ) (( \mu )V)</td>
<td>( I_{mag} ) (amps)</td>
</tr>
<tr>
<td>0.01</td>
<td>198</td>
<td>-245</td>
<td>0.01</td>
</tr>
<tr>
<td>0.06</td>
<td>316</td>
<td>-543</td>
<td>0.06</td>
</tr>
<tr>
<td>0.1</td>
<td>410</td>
<td>-769</td>
<td>0.1</td>
</tr>
<tr>
<td>0.14</td>
<td>505</td>
<td>-993</td>
<td>0.14</td>
</tr>
<tr>
<td>0.19</td>
<td>623</td>
<td>-1338</td>
<td>0.19</td>
</tr>
<tr>
<td>0.24</td>
<td>741</td>
<td>-1608</td>
<td>0.24</td>
</tr>
<tr>
<td>0.28</td>
<td>835</td>
<td>-1863</td>
<td>0.28</td>
</tr>
<tr>
<td>0.32</td>
<td>929</td>
<td>-2107</td>
<td>0.32</td>
</tr>
<tr>
<td>0.36</td>
<td>1024</td>
<td>-2355</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( I ) (mA)</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{mag} ) (amps)</td>
<td>( B ) (gauss)</td>
<td>( V ) (( \mu )V)</td>
<td>( I_{mag} ) (amps)</td>
</tr>
<tr>
<td>0.01</td>
<td>198</td>
<td>-676</td>
<td>0.01</td>
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<td>0.06</td>
<td>316</td>
<td>-1744</td>
<td>0.06</td>
</tr>
<tr>
<td>0.1</td>
<td>410</td>
<td>-2368</td>
<td>0.1</td>
</tr>
<tr>
<td>0.14</td>
<td>505</td>
<td>-3122</td>
<td>0.14</td>
</tr>
<tr>
<td>0.19</td>
<td>623</td>
<td>-4262</td>
<td>0.19</td>
</tr>
<tr>
<td>0.24</td>
<td>741</td>
<td>-5283</td>
<td>0.24</td>
</tr>
<tr>
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<td>835</td>
<td>-6091</td>
<td>0.28</td>
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<tr>
<td>0.32</td>
<td>929</td>
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</tr>
<tr>
<td>0.36</td>
<td>1024</td>
<td>-7891</td>
<td>0.36</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( I ) (mA)</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{mag} ) (amps)</td>
<td>( B ) (gauss)</td>
<td>( V ) (( \mu )V)</td>
</tr>
<tr>
<td>0.01</td>
<td>198</td>
<td>-1013</td>
</tr>
<tr>
<td>0.06</td>
<td>316</td>
<td>-2674</td>
</tr>
<tr>
<td>0.1</td>
<td>410</td>
<td>-3968</td>
</tr>
<tr>
<td>0.14</td>
<td>505</td>
<td>-5211</td>
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<tr>
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<td>623</td>
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</tr>
<tr>
<td>0.36</td>
<td>1024</td>
<td>-13050</td>
</tr>
</tbody>
</table>
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The row marked \( I (\text{mA}) \) at the top of the table is the current through the sample. The column \( I_{\text{mag}} \) gives the current through the magnet coil in amps. The column marked \( B \) is the magnetic field in gauss, calculated from \( I_{\text{mag}} \) using equation (8). The column \( V (\mu \text{V}) \) is the reading of the microvoltmeter.

It should be mentioned that the reading on the microvoltmeter will not be zero when the magnetic field is zero. This is because the two contacts to measure the Hall voltage will not be on an equipotential surface. The current enters through point contacts, and so the current lines are not parallel to the length of the specimen at all points.

Note that, in the above table, all Hall voltages are negative because the sample is n-type.

In Figure 4, we plot the Hall voltage as a function of magnetic field for different currents \( I \) through the sample.

![Figure 4](image)

**Figure 4.** Variation of Hall voltage as a function of the magnetic field for different currents through the sample.

For a given current through the sample, the Hall voltage is a linear function of the magnetic field. Table 3 gives values of the slope \( \alpha = \frac{dV_H}{dB} \) for different currents through the sample.

<table>
<thead>
<tr>
<th>( I ) (mA)</th>
<th>Slope ( \frac{dV_H}{dB} = \alpha ) (( \mu \text{V/gauss} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>-2.56</td>
</tr>
<tr>
<td>1</td>
<td>-4.58</td>
</tr>
<tr>
<td>1.5</td>
<td>-6.6</td>
</tr>
<tr>
<td>2</td>
<td>-8.62</td>
</tr>
<tr>
<td>2.5</td>
<td>-10.84</td>
</tr>
<tr>
<td>3</td>
<td>-12.7</td>
</tr>
<tr>
<td>3.5</td>
<td>-14.59</td>
</tr>
<tr>
<td>4</td>
<td>-16.2</td>
</tr>
</tbody>
</table>

The slope \( \alpha \) is in \( \mu \text{V/gauss} \). A plot of \( \alpha \) vs. \( I \) is shown in Figure 5.
The slope $\beta$ of this graph is $-3.95 \times 10^{-3}$ V/tesla.amp = $-39.5$ V/tesla.amp, since 1 tesla = $10^4$ gauss. The thickness of the sample is $0.5$ mm = $5 \times 10^{-4}$ m. The Hall coefficient $R_H$ is given by

$$R_H = \beta t = -39.5 \times 5 \times 10^{-4} = -0.01975 \text{ m}^3.\
\text{coulomb}.$$ 

The number density of charge carriers (electrons) is

$$n_e = \frac{1}{0.01975 \times 1.6 \times 10^{-19}} = 3.16 \times 10^{20} \text{ m}^3.$$ 

Similar measurements were taken on a p-type sample. The data are not given to save space. The Hall voltages are all positive for the p-type sample.

Figure 6 shows a plot of Hall voltage against magnetic field for different currents through the p-type sample.
The slope $\alpha$ for different currents are given in Table 4.

Table 4. Slope $\alpha$ for different currents $I$ from Figure 6.

<table>
<thead>
<tr>
<th>$I$ (mA)</th>
<th>$dV_H/dB$ ($\mu$V/gauss)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.73</td>
</tr>
<tr>
<td>1</td>
<td>1.39</td>
</tr>
<tr>
<td>1.5</td>
<td>2.03</td>
</tr>
<tr>
<td>2</td>
<td>2.65</td>
</tr>
<tr>
<td>2.5</td>
<td>3.43</td>
</tr>
<tr>
<td>3</td>
<td>4.07</td>
</tr>
<tr>
<td>3.5</td>
<td>4.75</td>
</tr>
<tr>
<td>4</td>
<td>5.47</td>
</tr>
</tbody>
</table>

The slope $\alpha$ is plotted against the current $I$ through the sample in Figure 7.

![Figure 7. Slope $\alpha$ vs. $I$ from Table 4.](image)

The slope $\beta$ of this graph is $1.354 \times 10^{-3}$ V/gauss.amp. This is equal to $13.54$ V/tesla.amp. The thickness of the sample is $0.5$ mm $= 5 \times 10^{-4}$ m. The Hall coefficient $R_H$ is

$$R_H = \beta t = 13.54 \times 5 \times 10^{-4} = 0.00675 \text{ m}^3/\text{coulomb}.$$ 

The number density of holes $n_H$ can be calculated from

$$n_H = \frac{1}{(R_H \times 1.6 \times 10^{19})} = \frac{9.26 \times 10^{20}}{\text{m}^3}.$$ 

The geometry is not simple enough to measure the resistivity and hence the mobility.

One may also measure the Hall effect with a solenoid producing a magnetic field of 150 to 200 gauss. The Hall voltage will be small. So we make the magnetic field alternating by passing an AC current through the solenoid and measure the AC Hall voltage with a lock-in amplifier. Such a technique is used for semiconductor samples with low mobility of the carrier.
1. Introduction

The resistance of a metal changes when a magnetic field is applied. Three types of behavior are seen in metals. In non-magnetic metals, the change in resistance is small at low fields (a few kilogauss). In some metals the change becomes large at very high magnetic fields. The behavior depends on the nature of the Fermi surface. In metals like aluminium, sodium, etc., in which the Fermi surface is closed, the magnetoresistance saturates at high fields (of the order of 100 to 1000 kilogauss). But in metals like Bi, which have an equal number of electrons and holes, the resistance does not saturate even at the highest fields. In metals like copper, silver, etc., in which the electrons have open orbits on the Fermi surface in certain crystallographic directions, the magnetoresistance will be large when the applied magnetic field is along those directions.

In ferromagnetic metals, the change in resistance is of the order of 2% at low magnetic fields. But the change in resistance parallel to the magnetic field is different from the change in resistance perpendicular to the magnetic field. This is called anisotropic magnetoresistance. This anisotropy at low fields is shown by permalloy as seen in Figure 1.

![AMR of Permalloy](image)

**Figure 1.** Anisotropic magnetoresistance in permalloy at a magnetic field of 0.1 T.

In this figure $\theta$ is the angle which the direction of resistance measurement makes with the magnetic field. Note that the change in resistance is about 2%, in a magnetic field of 1000 G. Permalloy also has a low saturation field. Permalloy was used for making magnetic reading heads.

Giant magnetoresistance was discovered independently by Albert Fert and Peter Grünberg. They were awarded the Nobel Prize in Physics in 2007. Fert worked on superlattices of (Fe-Cr)$_n$ deposited on GaAs, while Grünberg investigated a trilayer Fe-Cr-Fe deposited on GaAs. Fert observed a change in resistance of 50% at 4.2 K, on the application of a magnetic field, and named the new effect Giant Magnetoresistance.

In a non-magnetic metal, the energy bands for the spin up and spin down electrons are the same. In transition metals like iron, the electrons in the s-band have a low density of states, whereas the electrons in the d-bands have a high density of states. In a metal like copper, the Fermi level is above the d-band. So the s-electrons cannot scatter into the d-band, as all d-band levels will be filled. So copper has a low resistivity. On the other hand in paramagnetic iron, the Fermi level lies also in the d-band. So s-electrons can be scattered into the d-band states. Because the d-band has a high density of states, the s-d scattering is high, and iron has a relatively high resistance. The temperature dependence of resistance is also different.

If iron is in the ferromagnetic state, the density of states in the d-band will be different for spin up and spin down electrons. This is shown in Figure 2. So, spin up electrons will have a different scattering cross
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section, compared to spin down electrons. Hence, the contribution to resistance from spin up and spin down electrons will be different. The two resistances will be in parallel.

![Figure 2](image)

**Figure 2.** (a) A non-magnetic metallic layer sandwiched between two ferromagnetic metallic layers with opposite magnetizations. (b) The bands for spin up and spin down electrons in the three layers. F is the Fermi level.

Consider a trilayer, Fe-Cu-Fe. The two iron layers will be coupled through an exchange interaction, which is oscillatory in character. The exchange interaction between the two layers will depend on the thickness of the intervening non-magnetic layer. As the thickness of this layer (which is in the nanometer scale) increases, the exchange interaction between the two magnetic layers will change from ferromagnetic to antiferromagnetic. In such a case in zero magnetic field, the magnetizations in the two ferromagnetic layers will be antiparallel. In a magnetic field greater than the coercive field, the magnetizations in the two layers will become parallel. The two situations are shown in Figure 3.

![Figure 3](image)

**Figure 3.** A trilayer composed of a FM-NM-FM layer with AFM exchange interaction between the layers.

Electrons, with spin parallel to the magnetization in the material, will have a resistance $R_{\text{para}}$ in one film. Electrons, with spin antiparallel to the magnetization in the material, will have $R_{\text{anti}}$ in the same film. The resistances $R_{\text{para}}$ and $R_{\text{anti}}$ also include the contributions from the scattering at the interface of the FM to NM metal or vice versa. Electrons with spin up in the first material will experience a total resistance of $R_{\text{para}} + R_{\text{anti}}$ in going through the layers when $H = 0$. Electrons with spin down will also experience a total resistance of the same magnitude. The two resistances will be in parallel and so the total resistance of the trilayer will be $(R_{\text{para}} + R_{\text{anti}})/2$. On the other hand, when $H > H_C$, electrons with spin up will experience a resistance $2R_{\text{para}}$, while electrons with spin down will experience a resistance $2R_{\text{anti}}$. So, the total resistance will be $2R_{\text{para}}R_{\text{anti}}/(R_{\text{para}} + R_{\text{anti}})$. If $R_{\text{para}}$ and $R_{\text{anti}}$ are very different, the change in resistance with applied magnetic field will be very large. For example, if $R_{\text{anti}} = 10R_{\text{para}}$, the resistance when $H = 0$ will be $5.5R_{\text{para}}$, while the resistance when $H > H_C$ will be $1.8R_{\text{para}}$. Thus the resistance will decrease on the application of the magnetic field by about 66%. This is a large effect compared to the anisotropic magnetoresistance in permalloy. So Fert named this the Giant Magnetoresistance effect.

2. **Colossal Magnetoresistance**

Colossal magnetoresistance has been discovered in $La_{1-x}M_xMnO_3$, where M is a divalent metal like Ca or Sr. When $x = 0.33$, the resistance on the application of the field decreases by 99.9%. This is shown in Figure 4.

3. Applications

GMR sensors are used for magnetic field sensing, as read heads in hard disk drives, and in magnetic random access memories.

4. Experiment to Measure GMR

The GMR sample used in the present experiment has been provided by Dr. Prasanta of NAL, Bangalore.

The sample is in the form of a chip a few mm square in size and has two leads, which can be connected to an LCR meter for measuring its resistance. Its resistance is of the order of 5 kilo-ohms. The sample is mounted on a wooden mount and can be placed between the pole pieces of an electromagnet, so that the magnetic field is parallel to the surface of the specimen. Different currents can be passed through the electromagnet, and the resistance can be read out directly.

The distance between the pole pieces of the electromagnet is adjusted to be 2 cm. The electromagnet is calibrated using a Gauss meter, and passing different currents through coil 1 or coil 2 of the magnet. The readings for calibration are given in Table 1.

<table>
<thead>
<tr>
<th>CoI 1</th>
<th>B (gauss)</th>
<th>(B_{cal})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>204</td>
<td>195.302</td>
</tr>
<tr>
<td>0.06</td>
<td>307</td>
<td>306.812</td>
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<tr>
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<td>396.02</td>
</tr>
<tr>
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<td>480</td>
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</tr>
<tr>
<td>0.18</td>
<td>564</td>
<td>574.436</td>
</tr>
<tr>
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<td>654</td>
<td>663.644</td>
</tr>
<tr>
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<td>750</td>
<td>752.852</td>
</tr>
<tr>
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<td>829</td>
<td>842.06</td>
</tr>
<tr>
<td>0.34</td>
<td>921</td>
<td>931.268</td>
</tr>
<tr>
<td>0.37</td>
<td>995</td>
<td>998.174</td>
</tr>
<tr>
<td>Coil 2</td>
<td>B (gauss)</td>
<td>(B_{cal})</td>
</tr>
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<td>204</td>
<td>195.302</td>
</tr>
<tr>
<td>0.06</td>
<td>307</td>
<td>306.812</td>
</tr>
<tr>
<td>0.1</td>
<td>389</td>
<td>396.02</td>
</tr>
<tr>
<td>0.14</td>
<td>480</td>
<td>485.228</td>
</tr>
<tr>
<td>0.18</td>
<td>564</td>
<td>574.436</td>
</tr>
<tr>
<td>0.22</td>
<td>654</td>
<td>663.644</td>
</tr>
<tr>
<td>0.26</td>
<td>750</td>
<td>752.852</td>
</tr>
<tr>
<td>0.3</td>
<td>829</td>
<td>842.06</td>
</tr>
<tr>
<td>0.34</td>
<td>921</td>
<td>931.268</td>
</tr>
<tr>
<td>0.37</td>
<td>995</td>
<td>998.174</td>
</tr>
</tbody>
</table>

Coil 1 Interce 174 Slope 2198.4 CoI 2 Interce 172 Slope 2262 Average Interce 173 Slope 2230.2

Figure 5 shows a plot of field against current. The intercepts and slopes are shown in Table 1.
The magnetic field can be calculated from the current using the equation

$$B = 173 + 2230.2I.$$  \hspace{1cm} (1)

The calculated values are shown in column 3 of Table 1.

The Gauss meter probe is now removed, and the GMR sample is placed between the pole pieces of the magnet, so that it lies at the center of the pole pieces. The resistance is directly measured on the LCR meter SM 5118 of Scientific Instruments Co. The current through the magnet is changed in small steps, and the resistance readings are recorded. The magnetic field for a given current is calculated from equation (1). The readings taken in two runs are shown in Table 2.

### Table 2. Measurement of GMR.

<table>
<thead>
<tr>
<th>RUN 1</th>
<th>RUN 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R(0)$ (ohms)</td>
<td>$R$ (ohms)</td>
</tr>
<tr>
<td>$I$ (amps)</td>
<td>$B$ (gauss)</td>
</tr>
<tr>
<td>0.01</td>
<td>195</td>
</tr>
<tr>
<td>0.03</td>
<td>240</td>
</tr>
<tr>
<td>0.06</td>
<td>307</td>
</tr>
<tr>
<td>0.09</td>
<td>374</td>
</tr>
<tr>
<td>0.12</td>
<td>441</td>
</tr>
<tr>
<td>0.15</td>
<td>508</td>
</tr>
<tr>
<td>0.18</td>
<td>574</td>
</tr>
<tr>
<td>0.2</td>
<td>619</td>
</tr>
<tr>
<td>0.23</td>
<td>686</td>
</tr>
<tr>
<td>0.27</td>
<td>775</td>
</tr>
<tr>
<td>0.3</td>
<td>842</td>
</tr>
<tr>
<td>0.33</td>
<td>909</td>
</tr>
<tr>
<td>0.36</td>
<td>976</td>
</tr>
<tr>
<td>0.39</td>
<td>1043</td>
</tr>
</tbody>
</table>

$R(0)$ is the resistance when the sample is outside the magnetic field. The % change is defined as

$$\left(\frac{R(B) - R(0)}{R(0)}\right) \times 100.$$  \hspace{1cm} (2)
Figure 6 shows the resistance $R$ as a function of the magnetic field $B$ in the two runs, and Figure 7 shows the % change in resistance as a function of $B$ in the two runs.

We see that the results are highly reproducible. The resistance drops sharply as the field is increased and the drop saturates at a field of around 1000 gauss. The total % change in resistance from 0 to 1000 gauss is 10.7%. Since the magnet has a remnant field of around 170 gauss even when no current passes through the coils, we are unable to take readings in fields below 170 gauss. It is in this region that the resistance drops sharply. One may measure the resistance in this region using a solenoid.

Thus the experiment to measure change in resistance in a magnetic field of a GMR material is relatively simple.
15.6 Direct Measurement of Piezoelectric Constant of PVDF Film

1. Introduction

Only crystals belonging to the 20 point groups which do not have inversion symmetry, can exhibit piezoelectricity. Out of these 20 point groups, crystals belonging to only 10 point groups can exhibit ferroelectricity. This is explained in Section 18.4.

When an electric field and stress are both applied, we may write the coupled equation

\[ D_i = \sum_{j=1}^{3} \epsilon_{ij} E_j + \sum_{j=1}^{6} d_{ij} \sigma_j \]  

(1a)

\[ e_{ij} = \sum_{j=1}^{6} \sigma_{ij} \delta_{ij} + \sum_{k=1}^{3} d_{ik} E_k. \]  

(1b)

\( D \) and \( E \) are vectors with components \( D_i \) and \( E_i \), with \( i \) going from 1 to 3. \([\sigma] \) and \([e]\) are second rank symmetric tensors with components \( \sigma_{ij} \) and \( e_{ij} \), with \( j \) going from 1 to 6. Italic indices \( i, j \) go from 1 to 3, while script indices \( i, j \) go from 1 to 6. (For this notation see Section 18.4).

Since \( D_i \) has the unit of coulomb/meter\(^2\) and \( e_{ij} \) the unit of newton/m\(^2\), \( d_{ij} \) has the unit of coulomb/newton.

For \( a \)-quartz, which belongs to the trigonal system, the only independent piezoelectric constants are \( d_{11} \) and \( d_{14} \). They have the values \( d_{11} = 2.3 \times 10^{-12} \) coulomb/newton and \( d_{14} = 0.67 \times 10^{-12} \) coulomb/newton.

Quartz is a naturally occurring mineral. To produce synthetic quartz crystals, one should go to high temperature, and apply high hydrostatic pressure. These involve relatively expensive techniques.

Polyvinylidene fluoride (PVDF) is a polymer. When a film of this material is subjected to a uniaxial or biaxial stress and a high electric field is applied to it at high temperature, the dipole moments of the fluorine atoms align themselves parallel. The dipole moment per polymer unit becomes large. This is called poling. When the specimen returns to room temperature, and the electric field and stress are removed, its piezoelectric constants become high. PVDF films have become commercially available and have found many applications.

In the direct method of measurement, one has to apply a stress, and measure the charge created by the stress. In ceramic materials, the elastic compliance coefficients \( s_{ij} \) are in the range of \( 10^{-11} \) m\(^2\)/N, and the dimensions of the specimens are in mm. Large forces need to be applied to get an appreciable charge separation.

PVDF has a larger elastic compliance coefficient (in the range \( 5 \times 10^{-10} \) m\(^2\)/N), and it can be prepared as a film with thickness of the order of a few tens of microns. Even a small load creates a large stress. Charges collect on the metalized surfaces of the film. This area is large (of the order of 3 cm \( \times \) 1 cm). So, appreciable charge separation can be observed even when we apply forces of the order of a few newtons to stretch the film. A direct measurement of \( d_{31} \) is feasible in such films.

2. Experimental Set Up

To stretch the film by applying a load, the setup shown in Figure 1 has been designed.
A is a rectangular steel base which can be clamped to a table. B is a steel upright fixed to A at one end. There is a horizontal steel plate C fitted to the upright B. The film F is fixed, on either side, to two Perspex clamps D and E. Each clamp is made of two identical pieces, and the film is clamped between the pieces with screws. Before clamping the film, the Perspex surface is made rough with emery paper, and the film is stuck with superglue. The Perspex clamp D is fixed to the plate C. The leads to the metalized faces of the film come out through the Perspex block D. The other Perspex block E has a wire tied to its center. This passes over an aluminium pulley P fixed to a steel upright at the end of A. The wire carries a weight hanger with a hook. The film can be stretched by placing slotted weights on the weight hanger. This arrangement avoids the bending or twisting of the film when the weights are added to the hanger.

3. Charge Amplifier

When the film is loaded or unloaded, equal charges of opposite sign are developed on the two faces. To measure the amount of charge developed, we have built a charge amplifier. The essential element of the charge amplifier is the circuit shown in Figure 2.

The film F is represented by a capacitor and a resistor in parallel within a dotted rectangle. The film is connected to the op-amp through leads. The stray capacitance of the leads is $C_S$. When the film gets charged, a current $dq/dt$ flows through $R$ and charges the capacitor $C_1$. If $V(t)$ is the voltage across $C_1$, then

$$\frac{dq}{dt} = C_1 \frac{dV}{dt}.$$  \hfill (2)

Integrating we get

$$q = C_1 V.$$  \hfill (3)

The capacitance of the film and the stray capacitance do not play any significant role. The resistor $R_1$ across $C_1$ allows the capacitor to discharge before the next measurement is made. The time constant $C_1 R_1$ should be of the order of seconds. In our case, $C_1$ is 110 nF and $R_1$ is 50 megohms.

The voltage developed across $C_1$ is further amplified by a factor 4.7, and then goes through a peak detector to charge a 10 $\mu$F capacitor. This will be charged to the peak voltage, 4.7 V. Then the capacitor will discharge slowly. So we note the peak voltage.

If $d_{31}$ is positive, a tensile stress in the direction 1 (along the length of the film) will cause a positive charge to accumulate on the face normal to direction 3 (along the thickness). If the tensile stress is reduced, the charge will decrease. Note that the charge on the film will be discharged within a time of the order of 0.1 sec or less due to the ions present in air, and the leakage resistance of the PVDF film. So, when we load again with a 0.5 kg load, the same charge will collect on the film, and the DMM, which measures the output of the peak detector, will measure the same voltage. While loading we may accidentally disturb the film, and so the DMM reading will differ from one loading to another.
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If we remove the load, then the face of the film normal to the direction 3 will get a negative charge. In the peak detector we use a diode which will allow the voltage to go through only if it is in the forward direction. So, we reverse the input voltage by putting a reversing switch. Then, the output voltage of the diode will be positive. But, we have to note it with a negative sign. The voltage read on the DMM, after unloading, is written with a negative sign. There is a shorting switch across the output capacitor. This allows us to short the capacitor and bring the voltage across it to zero. Then we open the switch and get the next reading.

The front panel of the charge amplifier looks as shown in Figure 3.

Figure 3. Front panel of charge amplifier.

M is the mains switch. Power is on when this switch is pressed down. The switch S₁ is the shorting switch for the output capacitor. When taking a reading, it is put in position O (OPEN). After taking the reading, it is pressed to position S (SHORT) to short the capacitor, and then put back in position OPEN to take the next reading. The switch S₂ has two positions, Forward (FOR) and Reverse (REV). When readings are taken while increasing the load on the film, it is put in forward position. When readings are taken with decreasing load, it is put in the reverse position. The two leads from the film are connected to the BNC connector marked Input. A DMM in the DC 20 V range is connected to two banana plugs marked O/P. The pot at the bottom right helps to adjust the offset at the output to zero, before a reading is taken.

4. Procedure for Measurement

Put a dead weight of 0.5 kg so that the film is taut.

Connect the leads from the film to the BNC connector marked Input. Connect a DMM in the DC 20 V range to the banana terminals marked O/P. Put S₂ in position FOR. Press S₁ to position SHORT. The DMM should read zero. Then put S₁ in position OPEN. If the DMM does not show zero, turn the Offset pot till the DMM reads zero.
Now slowly put a 0.5 kg slotted weight on the weight hanger, without disturbing the weight hanger much. The DMM will show a maximum reading \(V_1\), which is to be noted. Press switch \(S_1\) to SHORT, and then put back to OPEN. Adjust the Offset pot so that the DMM shows zero. Now add another 0.5 kg slotted weight to the weight hanger, taking care to see that the weight hanger suffers the least disturbance. Note the reading \(V_2\). \(V_1\) is proportional to the charge \(q_1\) when the first weight is loaded. \(V_2\) is proportional to the charge \(q_2\) when the second weight is loaded. So, the total charge \((q_1 + q_2)\) for the two weights together is proportional to \((V_1 + V_2)\). Repeat the procedure till the six weights, each of 0.5 kg, are loaded.

For taking readings during unloading, put switch \(S_2\) in position REV. Then, unload the weights one by one without disturbing, and note the DMM reading. Each time, after noting the DMM reading, put switch \(S_1\) in position SHORT for a minute; then put it back in position OPEN, and adjust the pot so that the DMM reads zero, before removing the next weight.

One cycle consists in loading the six weights one by one, and then unloading them one by one. Repeat this operation three times.

A sample set of readings are shown in Table 1.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Load (newtons)</th>
<th>Run1 V O/P (volts)</th>
<th>(V_2) O/P (volts)</th>
<th>(V_3) O/P (volts)</th>
<th>(V_4) ave (volts)</th>
<th>(V_5) total (volts)</th>
<th>Charge (coulombs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.9</td>
<td>2.98</td>
<td>3.41</td>
<td>3.38</td>
<td>3.26</td>
<td>3.26</td>
<td>6.36 x 10^{-08}</td>
</tr>
<tr>
<td>2</td>
<td>9.8</td>
<td>3.63</td>
<td>3.43</td>
<td>3.67</td>
<td>3.58</td>
<td>6.84</td>
<td>1.40 x 10^{-07}</td>
</tr>
<tr>
<td>3</td>
<td>14.7</td>
<td>3.45</td>
<td>3.80</td>
<td>3.48</td>
<td>3.58</td>
<td>10.42</td>
<td>2.16 x 10^{-07}</td>
</tr>
<tr>
<td>4</td>
<td>19.6</td>
<td>3.46</td>
<td>3.20</td>
<td>3.28</td>
<td>3.31</td>
<td>13.73</td>
<td>2.86 x 10^{-07}</td>
</tr>
<tr>
<td>5</td>
<td>24.5</td>
<td>3.30</td>
<td>3.19</td>
<td>3.10</td>
<td>3.20</td>
<td>16.93</td>
<td>3.54 x 10^{-07}</td>
</tr>
<tr>
<td>6</td>
<td>29.4</td>
<td>2.88</td>
<td>2.93</td>
<td>2.96</td>
<td>2.92</td>
<td>19.85</td>
<td>4.16 x 10^{-07}</td>
</tr>
</tbody>
</table>

The first column gives the serial number of the 0.5 kg weight added (while loading), or removed (while unloading). The second column gives the force in newtons. While unloading, this is negative because the force decreases by this amount. The third, fourth and fifth columns give the observed DMM voltage, in the three runs, each time a slotted weight of 0.5 kg is added/removed. Because we put the switch \(S_2\) in the reverse position, the observed voltages are written with a negative sign when unloading the weights. The sixth column is the average of the voltages in columns 3, 4 and 5. Column 7 gives the sum of the average voltages up to the weight added. The charge is calculated from the relation

\[
q = \frac{CV}{4.7}. \tag{4}
\]

Here \(C\) is 110 nF, 4.7 is the amplification factor of the amplifying circuit, and \(V\) is \(V_{\text{total}}\), the sum of the average voltages up to the weight added.

A plot of \(q\) as a function of force \(F\), while loading, is shown in Figure 4(a).
A plot of charge vs. force, while unloading, is shown in Figure 4(b).

We find that the slope, while loading, is $1.45 \times 10^{-8}$ C/N. While unloading, it is $1.53 \times 10^{-8}$ C/N. The average slope is $1.49 \times 10^{-8}$ C/N to the second place of decimals.

The metalized area $A_C$, is $12 \times 30$ mm$^2$. The charge is collected on this area. The load acts on an area $A_L = 28 \, \mu m \times 16$ mm, where $28 \, \mu m$ is the thickness of the film, and $16$ mm is its width.

Now $d_{31}$ is the charge collected per unit area divided by the applied stress. So,

$$d_{31} = \text{slope} \times \left( \frac{A_L}{A_C} \right)$$

$$= 18.5 \times 10^{-12} \frac{C}{N}$$  \hspace{1cm} (5)
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The manufacturer gives a typical value of \(23 \times 10^{-12}\) C/N, with a variation of \(\pm 20\%\). The observed value agrees within these limits.

15.7 Elastic and Piezoelectric Properties of PZT

1. Introduction

The piezoelectric effect is discussed in Section 18.4. When a stress is applied to a piezoelectric material, charge separation occurs. Conversely, when an electric field is applied to the material, its dimensions change. This effect, called the piezoelectric property of the crystal, is described by the constants \(d_{ij}\) of the material (here the index \(i\) in Calibri goes from 1 to 3 and the index \(j\) in script goes from 1 to 6).

An example of such a piezoelectric material is \(\alpha\)-quartz. It has two piezoelectric constants \(d_{11}\) and \(d_{14}\) which have the respective values \(2.3 \times 10^{-12}\) and \(0.67 \times 10^{-12}\) coulomb/newton. Here the index 1 is along the unique axis of quartz.

Quartz is a naturally occurring mineral. To produce synthetic quartz crystals involves high temperatures, and relatively expensive techniques.

2. Ceramic Piezoelectric Materials

After World War II there was intense research in ferroelectric barium titanate which had a piezoelectric coefficient larger than quartz. This led to lead zirconium titanate–PZT. These are called piezoelectric ceramic materials. Other materials in this category are lithium niobate, lithium tantalate and sodium tungstate.

Barium titanate crystallizes in the perovskite structure \(\text{ABO}_3\), where \(A\) is the heavy metal ion like Ba, \(B\) is the small metal ion like titanium. The crystal structure is shown in Figure 1.

![Figure 1. Perovskite structure \(\text{ABO}_3\). Heavy atom \(A\) is at the center of the cube. Atoms \(B\) are at the corners and black atoms are oxygen.](image)

We see that the \(B\) atom is coordinated octahedrally with the oxygen atoms. One may substitute at the \(A\) site, other alkaline earth metals or rare earths, and on the \(B\) site trivalent or pentavalent ions of radii matching that of titanium ion. In Pb(Zr,Ti)O\(_3\), Pb is the \(A\) ion, and Zr and Ti occupy the \(B\) sites randomly. Such substitutions lead to a distortion of the perovskite structure, and provide a handle to vary the piezoelectric properties to suit different applications. The piezoelectric coefficients are large – nearly two orders of magnitude larger than in quartz.

These are ceramic materials prepared by a solid state reaction. The chemicals in the right proportion are mixed, ball milled, dried and then heated to 1200 °C for solid state reaction to take place. The material is then pulverized to the desired particle size, formed, and sintered at 1300 °C. They are ground and lapped to the
desired shape and size, and metalized on two sides. They are then subjected to high electric field at elevated temperatures in oil. This is called poling, and this is what gives rise to the large piezoelectric coefficients of the material. These materials are relatively cheap, and can be prepared with specific properties to match a variety of applications.

3. Applications

The following list is collected from the ‘General information’ provided by Sparkler Ceramics in Pune. These materials are available in a variety of shapes and sizes such as disks, plates, bars, rings, rods, tubes, etc. Some of the typical applications are: (i) High Voltage Generators for gas lighters; (ii) Fuses for explosives; (iii) Ultrasonic cleaners; (iv) Ultrasonic welders; (v) Ultrasonic atomizers; (vi) Nebulizers; (vii) Strain and excitation gauges; (viii) Accelerometers; (ix) NDT transducers; (x) Flowmeters; (xi) Dynamic force and pressure measurers; (xii) SONAR; (xiii) Deepwater hydrophones; (xiv) Piezoelectric actuators/ translators.


In the following paragraphs, we shall describe an experiment to measure the elastic and piezoelectric properties of PZT. Much of the material in this section is taken from Sparkler Ceramics write-up on ‘Measuring Properties of Piezoelectric Ceramics’ [1]. This is available on the website of Sparkler Ceramics www.sparklerceramics.com.

The PZT specimens are poled. The PZT sample contains many grains oriented at random. During poling an electric field is applied at high temperature, which aligns the spontaneous electric polarization in the grains when the specimen is cooled. This means that the poling direction is a unique axis. This implies that the properties along the poling direction are different from the properties in a plane perpendicular to the poling direction. We label the axis in the poling direction as 3, and the two orthogonal axes, perpendicular to the poling direction, as 1 and 2. For this symmetry, we have only two independent piezoelectric coefficients $d_{33}$ and $d_{31}$. $d_{33}$ is related to the charge produced along the poling direction, when a tensile stress $\sigma_3$ is applied along the poling direction. $d_{31}$ refers to the charge developed along the poling direction, when a tensile stress $\sigma_1$ is applied along the axis 1.

Before we describe the experiment proper, let us explain the terminology. Table 1, taken from [1], gives the definition of the various symbols used.

<table>
<thead>
<tr>
<th>Sl. No.</th>
<th>Term</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$k_{33}, k_{31}, k_p$</td>
<td>Piezoelectric coupling factors</td>
</tr>
<tr>
<td>2</td>
<td>$K_T^3$</td>
<td>Dielectric constant</td>
</tr>
<tr>
<td>3</td>
<td>$D$ or tan $\delta$</td>
<td>Dissipation factor</td>
</tr>
<tr>
<td>4</td>
<td>$s_{33}^D, s_{11}^D, s_{33}^E, s_{11}^E$</td>
<td>Elastic constants</td>
</tr>
<tr>
<td>5</td>
<td>$d_{33}, d_{31}$</td>
<td>Piezoelectric charge coefficients</td>
</tr>
<tr>
<td>6</td>
<td>$g_{33}, g_{31}$</td>
<td>Piezoelectric voltage constants</td>
</tr>
<tr>
<td>7</td>
<td>$Q_m$</td>
<td>Quality factor</td>
</tr>
<tr>
<td>8</td>
<td>$f_m$</td>
<td>Frequency of minimum impedance or resonance frequency</td>
</tr>
<tr>
<td>9</td>
<td>$f_n$</td>
<td>Frequency of maximum impedance or antiresonance frequency</td>
</tr>
</tbody>
</table>

The principle of the experiment is the following. When an AC voltage of frequency $f$ is applied to a PZT specimen, across the conducting contacts perpendicular to the direction of poling, the impedance $Z$ of the specimen varies with frequency as shown in Figure 2.

The impedance is a minimum at the resonance frequency $f_m$, and is a maximum at the anti-resonance frequency $f_n$. We find the resonance and anti-resonance frequencies $f_m$ and $f_n$ by experiment. From their values, we calculate the elastic and the piezoelectric constants, and coupling factors of the piezoelectric specimen.
For carrying out such experiments, we have to use different types of specimens. The types used are shown in Figure 3 (figure taken from [1]). Figure (a) shows a specimen which is poled along the length. The width and thickness in the case of rectangular parallelepiped specimens, or the diameter in the case of cylindrical specimens, should be small compared to the length. Figures 3(b) and 3(c) show specimens poled along the thickness. The metalized areas are perpendicular to the thickness. The thickness should be small compared to the length, in the case of rectangular parallelepipeds as shown in Figure (b), or should be small compared to the diameter for cylindrical specimens as shown in Figure (c).

<table>
<thead>
<tr>
<th>Dimensional Requirements</th>
<th>1 ≥ 2.5 w, t, d</th>
<th>1 ≥ 3.5 w, t,</th>
<th>d ≥ 10 t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suggested Dimensions</td>
<td>l = 15 mm</td>
<td>l = 40 mm</td>
<td>d = 40 mm</td>
</tr>
<tr>
<td></td>
<td>d = 5 mm</td>
<td>w = 10 mm</td>
<td>t = 3 mm</td>
</tr>
<tr>
<td></td>
<td>t = 3 mm</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Suitable for the</td>
<td>k_{33}</td>
<td>k_{33}</td>
<td>k_{p}</td>
</tr>
<tr>
<td>determination of</td>
<td>K_{33}</td>
<td>K_{33}</td>
<td>K_{33}</td>
</tr>
<tr>
<td></td>
<td>D</td>
<td>D</td>
<td>D</td>
</tr>
<tr>
<td></td>
<td>S_{33}</td>
<td>S_{33}</td>
<td>S_{33}</td>
</tr>
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<td></td>
<td>S_{33}</td>
<td>S_{33}</td>
<td>S_{33}</td>
</tr>
<tr>
<td></td>
<td>d_{33}</td>
<td>d_{33}</td>
<td>d_{33}</td>
</tr>
<tr>
<td></td>
<td>E_{33}</td>
<td>E_{33}</td>
<td>E_{33}</td>
</tr>
<tr>
<td></td>
<td>Q_{m}</td>
<td>Q_{m}</td>
<td>Q_{m}</td>
</tr>
<tr>
<td></td>
<td>density</td>
<td>density</td>
<td>density</td>
</tr>
</tbody>
</table>

Notations:
l = length.
t = thickness.
w = width.
d = diameter.

Figure 3. Configurations of samples used in measurement.

5. Experimental Procedure

We procured ten specimens each, in rectangular parallelepiped shape, poled along the length as shown in Figure 3(a), and poled along the thickness as shown in Figure 3(b). The dimensions of the specimens
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were length \( l = 25 \text{ mm} \), width \( w = 5 \text{ mm} \) and thickness \( t = 3 \text{ mm} \). The metalized faces on the specimens are perpendicular to the poling direction.

We attached 0.3 mm insulated wires to the metalized faces using silver epoxy. The specimens were mounted in a metal box with a plastic top. An AC voltage of about 1 volt RMS is applied to the BNC connector, BNC1. There are two specimens, PZT1 and PZT2. PZT1 is poled along its thickness. In this specimen, the metalized faces, shown shaded, are along its length and width. PZT2 is poled along its length. In this specimen, the metalized faces, shown shaded, are parallel to its width and thickness. The central pin of BNC1 is connected to one of the metalized faces (called the top face) of PZT1 and PZT2. The bottom face of PZT1 is connected to a resistance R1, which is about 2.2 k\( \Omega \). The bottom face of PZT2 is connected to a resistance R2 of 1 megohm. The free ends of the resistors are connected to the ground pin of the BNC1 connector. So, the AC voltage will drive a current through both PZTs. The current through PZT1 will be large because the current flows along its thickness, while the current through PZT2 will be small because the current flows along its length. The voltage across R1 or R2 is connected to the connector BNC2 through the DPDT switch SW2. The connections are shown in Figure 4.

![Figure 4. Connections to specimens PZT1 and PZT2 in the sample box.](image)

The ground pin of the BNC1 and BNC2 connectors are connected to the metallic box to shield the contents from stray voltages.

Figure 5 shows the connections in the piezoelectric measurement box.

![Figure 5. Piezoelectric measurement box.](image)

A signal generator, in the frequency range 20 to 100 kHz, is connected to the BNC connector 1 in Figure 5. The output from the signal generator is applied to a 1 k pot \( P_1 \). At the variable point in the pot, we may set the desired voltage. This voltage goes to two terminals of a DPDT switch SW1, as well as the terminals of the BNC connector 2 (Figure 5). If the BNC connector 2 (Figure 5) is connected to the BNC connector 1 on the sample box (Figure 4), the AC output from the pot will drive currents through the PZT samples. The voltage across the resistor R1 or R2 connected to the PZT samples in the sample box, and appearing at BNC2 on the sample box, is connected to BNC terminal 3 on the measurement box. This voltage is connected to the other two terminals of the DPDT switch SW1 (Figure 5). The central terminals of SW1 are connected to the BNC connector 4. A true RMS voltmeter connected to BNC4 will measure either the
input voltage from the pot P1, or the voltage across the resistor connected to the PZT sample in the sample box. An ordinary DMM will not measure the true voltage at frequencies used in this experiment.

The switch SW1 on the measurement box is put in position ‘I’ to measure the input voltage, i.e., the voltage coming out of the variable terminal of pot P1. To start with, the signal generator frequency is kept at 40 kHz, and its output is adjusted so that the voltage read on the true RMS voltmeter is about 0.7 volts. The switch SW2 on the sample box is set so that it measures the voltage across R1 connected to PZT1 sample.

The frequency is increased in steps. The input voltage and output voltages are measured at each frequency when the switch SW1 is put in position I and II respectively.

Table 2 shows a set of sample readings for piezo rod PZT1 (poled along the thickness).

| Density ρ  | 7650 kg/m³ |
| length l   | 0.025 m    |
| Resistance R1 | 2200 ohms |
| K₃   | 1752       |

<table>
<thead>
<tr>
<th>f (kHz)</th>
<th>V_in (mV)</th>
<th>V_R1 (mV)</th>
<th>I (μA)</th>
<th>f (kHz)</th>
<th>V_in (mV)</th>
<th>V_R1 (mV)</th>
<th>I (μA)</th>
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</thead>
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The first and fifth columns give the frequency in kHz. The second and sixth columns give the RMS value of the input voltage. This is kept sensibly constant over the entire frequency range. The third and seventh columns give the voltage across 2.2 kΩ resistance R. The fourth and eighth columns give the current, obtained by dividing the voltage in the third and seventh columns by 2.2 kΩ.

Figure 6 shows a plot of the current as a function of frequency.

![Figure 6](image-url)
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At the resonance frequency, the current is a maximum, and at the anti-resonance frequency, the current is a minimum. The values of \( f_m \) and \( f_n \) are 54.6 kHz and 57.9 kHz respectively.

The ratio \( (f_n - f_m)/f_m \) is calculated, and found to be 0.060. The variation of the coupling parameter \( k_{31} \) with this ratio is given in Figure 7, adapted from ref. [1]. From this figure, the value of \( k_{31} \) comes out to be 0.35. Note that this measurement will not give the sign of \( k_{31} \). For that, one will have to measure \( d_{31} \) directly, by measuring the charge created for a given stress. From the sign of the charge, the sign of \( d_{31} \), and hence, the sign of \( k_{31} \) can be found. The value of \( k_{31} \) given by Sparkler Company for this specimen is –0.31. The magnitude agrees with the value measured.

From \( f_m \) one can calculate the elastic compliance coefficient \( s_{11}^E \) using the relation

\[
s_{11}^E = \frac{1}{4\pi f_m^2}.
\]

Here \( \rho \) is the density of the material. The value for \( s_{11}^E \) comes out to be \( 1.75 \times 10^{-11} \) m²/N. The data from Sparkler Company is \( 1.90 \times 10^{-11} \) m²/N.

Using the value of \( k_{31} \) we can calculate the piezoelectric charge coefficient \( d_{31} \) from the relation

\[
d_{31} = k_{31} \cdot (\epsilon_0 K_{E3}^T s_{11}^E)^{0.5}.
\]

Using our values of \( k_{31} \) (with a minus sign), and \( s_{11}^E \), and \( K_{E3}^T \), we get a value of \( -1.83 \times 10^{-10} \) C/N. The value given by Sparkler is \( -1.70 \times 10^{-10} \) C/N. Here \( \epsilon_0 \) is \( 8.85 \times 10^{-12} \) F/m.

The piezoelectric voltage constant \( g_{31} \) can be obtained from the relation

\[
g_{31} = \frac{d_{31}}{(\epsilon_0 K_{E3}^T)}.
\]
From our data, \( g_{31} \) comes out to be \(-1.18 \times 10^{-2} \) Vm/N. Sparkler’s value is \(-1.10 \times 10^{-12} \) Vm/N.

For the PZT2 sample (specimen poled along its length), the DPDT switch SW2 on the sample box is set so that it measures the voltage across R2. Since the impedance of this specimen is very high, we have to use a higher input voltage. The same measurements as before are made in the range of frequencies 40 to 80 kHz. Data are shown in Table 3.

**Table 3.** Sample PZT2 poled along its length.

<table>
<thead>
<tr>
<th>( f ) (kHz)</th>
<th>( V ) (mV)</th>
<th>( V_{R2} ) (mV)</th>
<th>( I ) (( \mu )A)</th>
<th>( f ) (kHz)</th>
<th>( V ) (mV)</th>
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<td>2000</td>
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</table>

We plot the results for the current as a function of frequency from 40 kHz to 58 kHz to find \( f_m \). This plot is shown in Figure 8(a).

![Figure 8(a). Current vs frequency for length poled specimen.](image-url)
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From the graph the resonant frequency $f_m$ can be measured and is found to be 55.0 kHz. To get the anti-resonance frequency $f_n$, we plot the results in the range 58 to 80 kHz in Figure 8(b). The current is small and we get a broad minimum in the current. The frequency $f_n$ is around 70 kHz.

Figure 8(b). Current vs frequency for length poled specimen.

From these data, the ratio $(f_n - f_m)/f_m$ is found to be 0.270. Figure 9 shows a plot of $k_{33}$ vs. this ratio (adapted from Ref. [1]). From this figure, the value of $k_{33}$ is read off at $(f_n - f_m)/f_m = 0.27$. The value comes out to be 0.67. The actual data from Sparkler is 0.7.

Figure 9. (adapted from Ref. [1]).
From the measured anti-resonance frequency, we can calculate the elastic compliance coefficient $s_{33}^D$ from the relation

$$s_{33}^D = \frac{1}{4\rho l^2 f_n^2}. \quad (4)$$

This comes out to be $1.06 \times 10^{-11} \text{ m}^2/\text{N}$. The value given by Sparkler is $0.82 \times 10^{-11} \text{ m}^2/\text{N}$. The piezoelectric charge coefficient $d_{33}$ can be calculated from the expression

$$d_{33} = k_{33}^* \left( \varepsilon_0 k_T^3 s_{33}^D \right)^{0.5}. \quad (5)$$

From our data $d_{33}$ comes out to be $2.72 \times 10^{-10} \text{ C/N}$. The piezoelectric voltage coefficient $g_{33}$ can be calculated from

$$g_{33} = \frac{d_{33}}{\left( \varepsilon_0 K_T^3 \right)}. \quad (6)$$

From our data $g_{33}$ comes out to be $1.75 \times 10^{-2} \text{ Vm/N}$.

References


3. Piezoelectric Ceramics APC International

Chapter 16

Experiments in Phase Transitions
16.1 Introduction to Phase Transitions

1. Introduction

When we heat water, it starts to boil at a temperature of 100 °C under atmospheric pressure. Above 100 °C, it exists as steam. Similarly, if we cool water, it freezes at 0 °C under atmospheric pressure. This is not a chemical change. Water consists of molecules, each molecule having two atoms of hydrogen and one atom of oxygen. The chemical composition of water does not change when it transforms from liquid to vapor, or liquid to solid. However, the physical properties of steam, liquid water and ice are very different. We say that water undergoes a phase transition at 100 °C when it changes from liquid to vapor. Similarly, at 0 °C water undergoes a phase transformation from liquid to solid.

A phase of a material is a macroscopic state of the material which is homogeneous, and for which we may define quantities like density, internal energy, entropy, specific heat, etc. At a certain value of external parameters like temperature, pressure, magnetic field, etc., the same chemical material changes from one phase to another in which the thermodynamic properties are different.

The liquid to vapor phase transition was the first to be investigated very thoroughly. It was found that there is a critical temperature for this phase transition. The liquid to vapor phase transition can be brought about only if the temperature of the substance is below the critical temperature. If the temperature is below the critical temperature, then the phase transition occurs when the pressure is changed. Below a certain pressure $P$, at a temperature $T$ (less than the critical temperature), the material exists in the vapor phase. When the pressure is more than this value $P$, the material exists in the liquid phase. At this pressure $P$ and temperature $T$, the material coexists in both phases in equilibrium. A plot of this pressure $P$ versus temperature $T$ leads to a curve separating the liquid and vapor phases. This curve is called the boiling curve. At points on the boiling curve, the liquid and vapor phases coexist in equilibrium.

One may draw a similar curve for the solid to liquid transition. This is called the melting curve. On the melting curve, the material coexists in the solid and liquid phases in equilibrium. One may also draw a similar curve when the material directly goes from solid to vapor phase. This is called the sublimation curve. These curves mark the regions in pressure–temperature space where one or the other phase is the stable phase.

Figure 1 shows such a phase diagram.

![Phase Diagram](image)

Figure 1. Phase diagram for a vapor-liquid-solid transition.

In the phase diagram (Figure 1), the part of the curve from A to the triple point marks the sublimation curve where the solid directly goes into the vapor phase on heating. From the triple point to the critical point, the curve separates the vapor and liquid phases. The liquid phase exists only if the temperature is below the critical temperature and the pressure is high. If the temperature is above the critical temperature, there is no phase separation between the liquid and vapor phases. The part of the curve between the triple point and B is the melting curve. To the left of the melting curve the material exists in the solid phase. There is no critical
Experiments in Phase Transitions

temperature for the solid-liquid transition, i.e., the melting curve from the triple point to B will go up and up endlessly as the pressure is increased.

2. Critical and Triple Points

The critical temperature and pressure are important parameters for the material. Below the critical temperature, an application of pressure will cause the vapor to liquefy. If any gas has to be liquefied by compression, its temperature must be below the critical temperature of the material. While the critical temperatures of gases like ammonia are above room temperature, the critical temperatures of gases like nitrogen, oxygen, hydrogen and helium are below room temperature. In the domestic refrigerator one uses a gas, the critical temperature of which is above room temperature. The gas liquefies when compressed, and evaporates when the pressure is reduced. But to liquefy gases like nitrogen, the gas must first be cooled below its critical temperature.

The existence of a critical temperature is related to the symmetry of the material in the liquid and vapor phases. In both these phases, the material is isotropic, i.e., its properties are the same in all directions. A phase transformation involving phases which have the same symmetry will have a critical temperature.

A solid has a lower symmetry than a liquid. A solid–liquid phase transition will not have a critical point.

The triple point is one at which all three phases coexist in equilibrium. It has specific values for the temperature and pressure for a given material. The triple point of pure water can therefore be used as a standard for calibration of thermometers.

3. Thermodynamics of Phase Transitions

A material in a given phase has the following thermodynamic properties at a given temperature and pressure: $U$ the internal energy, $S$ the entropy, and $V$ the volume for unit mass of the material. From these, we may define other thermodynamic functions. The Gibbs free energy $G$ is one such function defined by

$$G = U - TS + pV.$$  

(1)

This function is of importance in discussing the theory of phase transitions. Usually, phase transformations are achieved by changing temperature $T$ or pressure $p$. The change in temperature and pressure changes the thermodynamic properties $U$, $S$ and $V$. The total change in the Gibbs free energy $G$ due to a change $dT$ in temperature and $dp$ in pressure is

$$dG = (dU - TdS + pdV) - SdT + V dp.$$  

The term in the brackets is zero due to the first law of thermodynamics. So,

$$dG = -SdT + V dp.$$  

(2)

Then,

$$\left( \frac{\partial G}{\partial T} \right)_p = -S,$$  

(3a)

and

$$\left( \frac{\partial G}{\partial p} \right)_T = V.$$  

(3b)

$G$ is a function only of $T$ and $p$ and its derivatives are related to the entropy and volume.

The same quantity of material in different phases will have different values of $G$. At any given temperature and pressure, the phase with the lowest Gibbs free energy will be stable. When two phases coexist on a boiling or melting curve, the Gibbs free energies of the two phases are equal. Figure 2 shows the Gibbs free energy for phases 1 and 2 as a function of temperature at constant pressure.
Phase 1 could be the vapor phase and phase 2 the liquid phase. Note that $G$ decreases as temperature is increased because $(\partial G/\partial T)_p$ is negative. Phase 1 which has a larger negative slope, and so a larger entropy, can be identified with the vapor phase. The two curves intersect at a temperature $T_0$. Above this temperature, phase 1 (vapor phase) has the lower Gibbs free energy, and therefore, is the stable phase. Below this temperature, phase 2 (liquid phase) has the lower Gibbs free energy, and so is stable. At $T_0$, both phases have the same Gibbs free energies per unit mass. So, both phases are stable and can coexist. We may have $m_1$ grams of phase 1 and $m_2$ grams of phase 2 at $T_0$ and they will coexist. Keeping the total mass fixed, we may convert $\delta m$ grams of phase 2 (liquid) to phase 1 (vapor), by giving some heat. The Gibbs free energy of the total system will remain the same.

While the Gibbs free energies per unit mass are the same for the two phases at $T_0$, the slopes of the two curves at $T_0$ are different, i.e.,

$$
\left( \frac{\partial G_1}{\partial T} \right)_p = -S_1 \neq \left( \frac{\partial G_2}{\partial T} \right)_p = -S_2.
$$

The entropy per unit mass of phase 1 at $T_0$ is larger than the entropy of unit mass of phase 2 at $T_0$ at the given pressure. So, if we want to convert unit mass of the material from phase 2 to phase 1 at $T_0$, we need to give a quantity of heat $L$

$$
L = T_0 \left( S_1 - S_2 \right).
$$

This is called the latent heat of the material.

Similarly,

$$
\left( \frac{\partial G_1}{\partial p} \right)_{T_0} = V_1 \neq \left( \frac{\partial G_2}{\partial p} \right)_{T_0} = V_2.
$$

This implies that the specific volume $V_1$ (volume per unit mass in phase 1) is different from the specific volume $V_2$. For example, the specific volume of steam at 100 °C and 1 atmosphere pressure is several hundred times larger than the specific volume of water at the same temperature of boiling.

This type of transition is called a first order transition. In such transitions, the following will hold.

1. The Gibbs free energy for unit mass is the same for the two phases at the transition point.
2. The first derivatives of $G$ with respect to $T$ and $p$ are discontinuous for the two phases at the transition point.
3. Such a transition will be characterized by a latent heat at the transition point.

First order transitions usually show the phenomenon of super-cooling or super- heating. If you take a quantity of water in a clean tube and heat it, the water may be heated beyond the boiling point without transforming to the vapor phase. Similarly, steam can be cooled substantially below the boiling point before transforming to water. These phenomena are called super-heating and super-cooling, respectively.

We may also have a phase transformation in which the Gibbs free energy curves do not intersect but touch at the transformation point. This is shown in Figure 3.
Experiments in Phase Transitions

![Figure 3](image-url)

**Figure 3.** Specific Gibbs free energy as a function of temperature for a second order phase transition.

In such a phase transition, not only is the Gibbs free energy per unit mass the same for the two phases at the transition point, but the first derivatives of $G$ with $T$ and $p$ are also the same. The two phases have the same specific entropies and specific volumes at the transition point. However, the second derivatives of $G$ with $p$ and $T$ are different for the two phases at the transition point.

\[
\left(\frac{\partial^2 G_1}{\partial T^2}\right)_p = -\left(\frac{\partial S_1}{\partial T}\right)_p \neq \left(\frac{\partial^2 G_2}{\partial T^2}\right)_p = -\left(\frac{\partial S_2}{\partial T}\right)_p \tag{7a}
\]

\[
\left(\frac{\partial^2 G_1}{\partial p^2}\right)_{T_0} = \left(\frac{\partial V_1}{\partial p}\right)_{T_0} \neq \left(\frac{\partial^2 G_2}{\partial p^2}\right)_{T_0} = \left(\frac{\partial V_2}{\partial p}\right)_{T_0} \tag{7b}
\]

and

\[
\left(\frac{\partial^2 G_1}{\partial T \partial p}\right)_{p,T_0} = -\left(\frac{\partial S_1}{\partial p}\right)_{p,T_0} \neq \left(\frac{\partial^2 G_2}{\partial T \partial p}\right)_{p,T_0} = -\left(\frac{\partial S_2}{\partial p}\right)_{p,T_0} \tag{7c}
\]

Since $S_1 = S_2$, there is no latent heat of transition at the transition point. But equation (7a) implies that the specific heat will show a discontinuous jump at the transition. Similarly, $V_1 = V_2$ at the transition point. But equation (7b) implies that the compressibility (and hence the bulk modulus) of the material will vary discontinuously at the transition point. Such a transition is called a second order transition.

A classic example of a second order transition is from liquid helium I (LHeI) to liquid helium II (LHeII), when liquid helium is cooled below 2.13 K. The specific heat jump in this case is shown in Figure 4.

![Figure 4](image-url)

**Figure 4.** Specific heat jump in liquid helium at the $\lambda$ transition.

Examples of second order transitions are the following:

1. Paramagnetic to ferromagnetic transitions.
2. Metal insulator transition in strontium-doped lanthanum manganites.
3. Superconducting transition in the absence of an applied magnetic field.
4. Landau’s Theory of Phase Transitions

Consider a paramagnetic to ferromagnetic transition. In the paramagnetic phase, the magnetic moments are oriented in a disordered fashion, and there is no net magnetic moment in the absence of a magnetic field. But below a certain temperature $T_C$, called the Curie temperature, the material develops a spontaneous magnetization, which grows from zero at $T_C$ to a value which approaches a limit as $T$ tends to absolute zero. This is shown in Figure 5.

![Saturation magnetization of nickel plotted against temperature](image)

**Figure 5.** Magnetization as a function of temperature below the transition temperature in ferromagnetic nickel.

The paramagnetic state is a disordered state. On the other hand, the ferromagnetic state at absolute zero is a perfectly ordered state. We may define a parameter $m = M(T)/M(0)$, where $M(T)$ is the magnetization at temperature $T$ and $M(0)$ is the magnetization at absolute zero. This parameter quantifies the order in the ferromagnetic material. This order parameter grows from 0 at $T_C$ to unity in the perfectly ordered state.

Landau assumed that for any phase transition, we may define such an order parameter $m$ which is zero in the high temperature phase, and non-zero in the low temperature phase. In a first order phase transition, the order parameter jumps discontinuously from 0 to a value less than unity at $T_C$, and grows to unity as the temperature is reduced below $T_C$. In a second order transition, the order parameter does not show a discontinuity at $T_C$. It grows from zero at $T_C$ to unity as the temperature is lowered. First order transitions are also called discontinuous transitions, and second order transitions continuous transitions.

Landau assumed that, in a continuous transition near the transition temperature, the order parameter $m$ will be small, and one may expand the Helmholtz free energy per unit mass of the material, $F$, in a power series of the order parameter.

$$ F = F_0 + am^2 + \beta m^4 + \ldots $$

He also assumed that the coefficient $a$ is a function of temperature and varies as

$$ a = a_0 (T-T_C), $$

and $\beta$ is independent of temperature. $a_0$ and $\beta$ are positive constants. So, when $T$ is more than $T_C$, both $a$ and $\beta$ are positive, while when $T$ is less than $T_C$, $a$ is negative while $\beta$ is positive. At a maximum or minimum value of $F$ as a function of the order parameter $m$, we have

$$ \frac{dF}{dm} = 0 = 2m (a + 2\beta m^2). $$

Above $T_C$, $(a + 2\beta m^2)$ is a positive number for all values of $m$. $F$ has only one minimum value at $m = 0$. But at $T < T_C$, $F$ has three extremum values at

$$ m = 0 $$

and at

$$ m = \pm \sqrt{\left( \frac{-a}{2\beta} \right)} = \pm \left( \frac{a_0}{2\beta} \right)^{1/2} (T_C - T)^{1/2}. $$
At $m = 0$, $F$ is a maximum, whereas at the two other values of $m$, $F$ has equal minimum values. Above $T_C$, the disordered state is stable, while below $T_C$, either of the two ordered phases is equally probable, because the two phases have the same minimum free energy. Figure 6 illustrates this situation.

**Experiments in Phase Transitions**

Landau’s theory predicts that the order parameter will grow as $(T_C - T)^{1/2}$ below $T_C$ and close to $T_C$. If we apply a driving field, then we have to add a term $-mF$ in the free energy expression, where $F$ is the driving force. The driving field for paramagnetic to ferromagnetic transition will be the magnetic field $H$. Then above $T_C$ in the disordered phase, the minimum of free energy will occur at

$$2ma + 4bm^3 - F = 0. \quad (12)$$

Above $T_C$, both $a$ and $b$ are positive, and $m$ is small. We may neglect the term in $m^3$ in comparison to the term in $m$. Then, the minimum free energy occurs at a value of $m$ given by

$$m = \frac{F}{2a_0(T - T_C)}. \quad (13)$$

If we are dealing with a magnetic material undergoing paramagnetic to ferromagnetic transition, the order parameter $m = M(T)/M(0)$ and the driving field $F$ is $H$, the applied magnetic field. The magnetic susceptibility $\chi$ is defined by

$$\chi = \left(\frac{\partial M}{\partial H}\right)_T. \quad (14)$$

From equation (13), $\chi$ will be proportional to $1/(T - T_C)$. This is the Curie–Weiss law.

Experiments close to the critical temperature indicate deviations from Landau theory. Near the critical temperature, the magnetization varies as $(T_C - T)^\beta$, the reciprocal of the magnetic susceptibility in the paramagnetic region, $\chi^{-1}$, varies as $(T - T_C)^\gamma$, and the specific heat $C_H$ at constant magnetic field varies as $(T - T_C)^{\gamma + \delta}$. The susceptibility and specific heat are termed as response functions. The exponents $\alpha'$, $\beta$ and $\gamma'$ are called critical exponents. Exponents with a prime attached to them refer to the behavior as one approaches $T_C$ by lowering the temperature, while exponents without the prime, refer to the case when $T_C$ is approached by raising the temperature. These critical exponents are the same for all materials undergoing para- to ferro-magnetic transitions, irrespective of the differences in the microscopic interactions responsible for the magnetic behavior of these materials. Not only so; different phase transitions obey similar critical behavior with the same values of the critical exponents. For example, a magnetic material with a single component magnetic moment (i.e., $m = +1$ or $-1$ along a given direction) has the same behavior as a vapor-liquid transition near its critical point. The critical exponents for analogous response functions for a superconducting material and an XY Heisenberg ferromagnet are the same. We can divide all phase transitions into a certain number of universality classes. For systems belonging to a given universality class, the critical exponents for analogous response functions are the same.

These factors do not come out of the Landau formalism. The Landau theory is said to be a mean field approximation and is valid only at temperatures far away from the critical temperature.
5. Fluctuations and Coherence Length

Let us consider a paramagnetic material near the Curie temperature. We may define a magnetization \( m(\mathbf{r}) \). Suppose you take two different points at positions \( \mathbf{r}_1 \) and \( \mathbf{r}_2 \). The magnetizations at these two points may be different and they may fluctuate with time. We define a correlation function between the magnetizations by finding the value of the scalar product of \( m(\mathbf{r}_1) \) and \( m(\mathbf{r}_2) \), averaged over a long period of time. If the values of the magnetizations vary completely randomly, independent of each other, this time average will be zero. But if the variations are not completely random, then this time average will be non-zero. The magnetizations at the two points are then said to be correlated. This correlation function will only depend on the distance between the two points. If the distance is small, the correlation function may be expected to be large, whereas, if the distance is large, the correlation function will tend to zero. The correlation function will vary with distance \( r \) between two points as \( \exp(-r/\xi) \). \( \xi \) is called the correlation length. It is a function of temperature. As one approaches the critical temperature, the correlation length increases. For a first order or discontinuous transition, the correlation length becomes finite at the transition temperature. For a continuous transition (second order), the correlation length approaches infinity as the transition temperature is approached.

Landau theory is valid when the correlation length is small. Near the transition temperature it breaks down, as the correlation length becomes large.

6. Scaling and Critical Exponents

The fact that the response functions diverge at the critical point shows that the thermodynamic potential has a part which is singular at the critical temperature. This singular function is homogeneous. The thermodynamic potential, which is a singular homogeneous function, satisfies the relation

\[ G(\lambda^{a_x}x, \lambda^{a_y}y) = \lambda G(x, y), \]  

(15)

where \( \lambda \) is any positive number, and \( a_x \) and \( a_y \) are the scaling exponents for the variables \( x \) and \( y \) respectively. This is called the scaling law. If we approach the critical point by changing \( x \) and keeping \( y \) constant, \( a_x \) is called the scaling power for the path along \( x \).

To explain this law by a physical example, let us consider a large ferromagnet. Its thermodynamic potential is a function of temperature and magnetic field. If we break it into two equal pieces keeping the temperature and magnetic field the same, the thermodynamic potential of each piece will be the same function of \( T \) and \( H \) as the original large piece, but multiplied by the factor \( 1/2 \). The magnetic moment (magnetization into volume) of each piece will be half the magnetic moment of the large piece.

A thermodynamic function is a derivative of the thermodynamic potential. For example, the magnetization is

\[ M = -\left( \frac{\partial G}{\partial H} \right)_T. \]  

(16)

The magnetic susceptibility is

\[ \chi = \left( \frac{\partial M}{\partial H} \right)_T = -\left( \frac{\partial^2 G}{\partial H^2} \right)_T. \]  

(17)

The specific heat at constant magnetic field, \( C_H \), is given by

\[ C_H = T \left( \frac{\partial S}{\partial T} \right)_H = -T \left( \frac{\partial^2 G}{\partial T^2} \right)_H. \]  

(18)

\( \chi \) and \( C_H \) are called response functions.

The exponent for the thermodynamic function is the exponent for \( G \) (which is 1), minus the exponent for the path multiplied by the order of the derivative. Thus the exponents in the scaling function for \( M, \chi \) and \( C_H \) are:

\[ M : 1-a_H, \]  

(19a)

\[ \chi : 1-2a_H, \]  

(19b)

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The critical exponent for a thermodynamic function as a function of \((T - T_C)\) or \((T_C - T)\) near the critical temperature is the ratio of the exponent of the thermodynamic function to the exponent of the path. Thus,

\[
M \propto (T_C - T)^\beta, \quad \beta = \frac{(1-a_H)}{a_T}, \quad \text{(20a)}
\]

\[
\chi \propto (T - T_C)^{-\gamma'}, \quad -\gamma' = \frac{(1-2a_H)}{a_T}, \quad \text{(20b)}
\]

\[
C_H \propto (T - T_C)^{-a'}, \quad -a' = \frac{(1-2a_T)}{a_T}, \quad \text{(20c)}
\]

The denominator is \(a_T\), because we are finding the variation as a function of temperature. The three critical exponents depend on the two path exponents \(a_H\) and \(a_T\). Therefore, one can get a relation between the three critical exponents by eliminating the two path exponents. This relation is

\[
a' + 2 \beta + \gamma' = 2. \quad \text{(21)}
\]

This is called a scaling law.

The second implication of scaling is the following. Take equation (15) with \(x\) as the magnetic field \(H\), and \(y\) as \(\varepsilon = (T-T_C)/T_C\), differentiate it with respect to \(H\) and use equation (16). We get

\[
M(\lambda^{a_H} H, \lambda^{a_T} \varepsilon) = \lambda^{1-a_H} M(H, \varepsilon). \quad \text{(22)}
\]

If we choose \(\lambda = H^{-1/a_H}\), then

\[
\frac{M(H, \varepsilon)}{H^{(1-a_H)/a_H}} = M(1, \frac{\varepsilon}{H^{a_T/a_H}}). \quad \text{(23)}
\]

Thus, a plot of \(M/H^{(1-a_H)/a_H}\) vs. \(\varepsilon/H^{a_T/a_H}\) must show the magnetizations of a material measured at different values of \(H\) falling on a single curve. This curve must be the same for different magnetic materials. \(M/H^{(1-a_H)/a_H}\) is called the scaled magnetization and \(\varepsilon/H^{a_T/a_H}\) the scaled temperature.


16.2 Martensite to Austenite Phase Transition in Shape Memory Alloy Nitinol

1. Introduction

As an example of a first order phase transition we discuss the phase transition in the shape memory alloy Nitinol.

The shape memory alloy NITINOL (Nickel Titanium Naval Ordnance Laboratory) was discovered in 1959 by William J Buehler of the U.S. Naval Ordnance Laboratory. This alloy exhibits two remarkable effects, viz. ‘shape memory’, wherein the alloy can ‘memorize’ a predetermined shape and return to this shape under certain temperature conditions, and ‘pseudoelasticity’, where large strains of the order of 8-10% can be recovered in applications. These unique features have made Nitinol a remarkable engineering material with applications in such diverse fields as cardiovascular surgery, orthodontics, solid-state heat engines, aerospace and toy industry.

The serendipitous discovery of ‘memory metal’ took place at the U.S. Naval Ordnance Laboratory when Buehler noticed a remarkable acoustic damping change of a Ni-Ti ingot with temperature change, near room temperature. This unusual event unfolded when an assistant of Buehler was transporting several melted Ni-Ti bars from an arc furnace to a table. One of the bars which had cooled to near room temperature
accidentally fell on the concrete floor, and made a ‘dull thud’ sound while a bar at a higher temperature made a characteristic ‘metallic sound’ on being dropped to the floor. This quick test of Buehler to determine the damping capacity of the Ni-Ti alloy has now come to be known as ‘pseudoelasticity’. Intuitively Buehler reasoned that the startling acoustic damping must be related to a major atomic structural change, related only to minor temperature variation. Further metallurgical studies like microhardness and microstructure led Buehler and his group to a significant conclusion that in this alloy, major atomic movements occur in a rather low temperature regime near room temperature.

The revelation of the unique ‘shape memory effect’ in Nitinol came a little later. In the early 1960s, Buehler prepared a long thin Nitinol strip for use in demonstrations of the material’s unique damping properties. The strip was bent into short folds longitudinally, forming a sort of metallic accordion. The strip could be repeatedly compressed and stretched (as an accordion) at room temperature. In a review meeting this strip was passed around the conference meeting and everyone flexed the strip repeatedly. A technical director, who was a pipe smoker, accidentally heated the compressed strip. The strip, to everyone’s amazement, transformed at once to the original longitudinal strip. The mechanical memory discovery, while not made in Buehler’s metallurgical laboratory, was the missing piece of the puzzle of the earlier mentioned acoustic damping and other unique changes during temperature variation. This serendipitous discovery became the ultimate payoff for Nitinol. It is now well recognized that the low temperature martensite to high temperature austenite phase change occurring near room temperature is responsible for the remarkable shape memory and pseudoelastic behavior of Nitinol.

2. Martensite to Austenite Transformation

The martensite-austenite transformation involves cooperative movement of atoms, unlike nucleation and growth mechanism commonly encountered in first-order phase transitions.

Figure 1 depicts the structural relationship between the austenitic and martensitic phases that leads to the shape memory effect.

![Shape Memory Alloys](image)

**What are Shape Memory Alloys?**

- **Shape Memory Alloys (SMAs)** are metallic alloys that undergo a solid-to-solid phase transformation which can exhibit large recoverable strains. Example: Nitinol

**Austenite**
- High temperature phase
- Cubic Crystal Structure

**Martensite**
- Low temperature phase
- Monoclinic Crystal Structure

Twinned Martensite Detwinned Martensite

**Figure 1.** Structural changes involved in martensite-austenite phase transition. Note that a cooperative movement of 0.8 Å brings about this phase change. The process of detwinning leads to large recoverable strains.

It may be noted that a cooperative movement of atoms through a shear transformation can bring about the cubic (high temperature austenite phase) to monoclinic (low temperature martensite phase) structural transition. This type of diffusion-less transformation is also known as ‘military transformation’, and is characterized by a low enthalpy change. Further, the martensitic phase is ‘twinned’ so that the overall shape is retained. The volume change accompanying the martensite-austenite first-order phase transition is less...
than 1% and emphasizes the subtle nature of this phase transformation. The process of ‘detwinning’ on application of stress in the martensitic phase accounts for the large recoverable strain in this system.

Another unique feature of this transition is that the phase change occurs over a temperature range, so that during the heating cycle one can define the austenite start \((A_S)\) and austenite finish \((A_F)\) temperatures. Martensitic start \((M_S)\) and martensitic finish \((M_F)\) temperatures characterize the reverse transition during the cooling cycle. Figure 2 shows these characteristic temperatures in the thermally induced phase transition in Nitinol.

3. Resistivity Studies Across the Martensite-Austenite Transition in Nitinol

Electrical resistivity measurements provide a convenient probe to track the ‘first order’ martensitic phase transformation in a shape memory alloy. The experimental arrangement essentially consists of a Nitinol wire with the conventional four probes spot-welded as indicated in Figure 3. A chromel–alumel thermocouple is also spot-welded at the center for measuring the temperature of the sample.

Figure 3. Sample holder arrangement for four probe resistivity measurement as a function of temperature.

This sample holder is sandwiched between two nichrome heaters which are connected in parallel. A 10 volt, 1.5 amp power supply is adequate to heat the sample to around 150 °C. A constant current of 10 milliamperes is passed through the leads 1 and 4 from DC constant current source. The voltage developed
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across the leads 2 and 3, which is proportional to the resistance of the sample, is amplified in a high quality DC amplifier (typically with a gain of 100). Its output is read in a digital panel meter. The thermo-emf from the chromel–alumel thermocouple, after cold-junction compensation, is again amplified in a DC amplifier and its output read in a digital panel meter. NBS table for Type-K thermocouple is used to convert the thermo-emf to the appropriate temperature value. A typical plot of the resistivity vs. temperature data is given in Figure 4.

![Figure 4. Martensite–austenite phase transition in Nitinol.](image)

The salient features of this plot are (1) the drop in the resistivity accompanying the phase transition, (2) near-ideal reversibility in the resistivity value after a thermal cycle, (3) hysteresis of around 20°C which is typical of a first-order transition. It may also be noted that both the forward and reverse transformations occur over a small temperature range as indicated in Figure 2. The characteristic temperatures \(A_S, A_F, M_S\) and \(M_F\) can be estimated using the data in this plot.

16.3 Paraelectric to Ferroelectric Transition in Modified Barium Titanate

1. Introduction

Ferroelectricity is a property of certain non-conducting crystals, or dielectrics, that exhibit spontaneous electric polarization (separation of the center of positive and negative electric charge, making one side of the crystal positive and the opposite side negative) that can be reversed in direction by the application of an appropriate electric field. Ferroelectricity is named by analogy with ferromagnetism, which occurs in materials such as iron. Iron atoms, being tiny magnets, spontaneously align themselves in clusters called ferromagnetic domains, which in turn can be oriented predominantly in a given direction by the application of an external magnetic field.

Ferroelectric materials – for example, barium titanate (\(\text{BaTiO}_3\)) and Rochelle salt – are composed of crystals in which the structural units are tiny electrical dipoles. In each unit the centers of positive charge and negative charge are slightly separated. In some crystals these electric dipoles spontaneously line up in clusters called domains, and in ferroelectric crystals the domains can be oriented predominantly in one direction by a strong external electric field. Reversing the external field reverses the predominant orientation of the ferroelectric domains, though the switching to a new direction lags somewhat behind the change in the external electric field. This lag of electric polarization behind the applied electric field is called ferroelectric hysteresis, named by analogy with ferromagnetic hysteresis. Hysteresis loop characterizes the ferroelectric phase in a material (Figure 1).
Ferroelectricity ceases in a given material above a characteristic temperature, called its Curie temperature, because the heat agitates the dipoles sufficiently to overcome the forces that spontaneously align them. Near the Curie point or transition temperature, thermodynamic properties including dielectric, elastic, optical, and thermal constants show an anomalous behavior. This is due to a distortion in the crystal as the phase structure changes.

Figure 2 illustrates the changes in a ferroelectric material which transforms from a ferroelectric tetragonal phase to paraelectric cubic phase in barium titanate. The permittivity curve represents data measured on a BaTiO$_3$ ceramic. The arrows show possible directions of the spontaneous polarization (in two dimensions). The unit cell is represented by a square (of size $a_C \times a_C$) in the cubic phase and a rectangle (of size $c_T \times a_T$) in the tetragonal phase.

As ferroelectricity ceases above the Curie temperature, materials lose their intrinsic polarization in the paraelectric phase. The electrical susceptibility $\chi$ in the paraelectric phase follows the well-known Curie–Weiss law, viz.,

$$\chi = \frac{A}{T-T_C},$$

where $A$ is a constant. This is the same form of expression as the Curie–Weiss law of magnetic susceptibility.

A convenient method of determining electrical susceptibility is to measure the capacitance of a parallel plate capacitor containing the ferroelectric substance as a dielectric. The capacitance $C$ is related to susceptibility through the expression

$$C = C_0 (1 + \chi).$$

Here, $C_0$ is the capacitance without a dielectric and is determined from the relation.
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\[ C_0 = \varepsilon_0 A/d, \]  

where \( \varepsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1} \), \( A \) is the area of the specimen and \( d \) is its thickness.

![Figure 3](image-url)

**Figure 3.** Ferroelectric to paraelectric transition in a sample of PZT.

Typical data on PZT (Lead Zirconate Titanate) ceramic are given in Figure 3. Electrical susceptibility is a measure of the ease of polarization of a material. Figure 4 gives the data of \( 1/\chi \) in the paraelectric region.

![Figure 4](image-url)

**Figure 4.** Plot of \( 1/\chi \) versus \((T-T_c)\) in the paraelectric region.

2. Experimental Arrangement

To measure the transition temperature of a ferroelectric sample under investigation, the sample is inserted in a planar heater arrangement similar to the one used for electrical resistivity studies. The nichrome heater is powered by a 0-30 V, 0-3A DC power supply. The capacitance of the specimen is measured through an LCR meter operating at 1 kHz. A chromel-alumel thermocouple is placed in close proximity to the sample for temperature measurement. An electronic unit incorporating linearization circuitry for type-K thermocouple...
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and cold-junction compensation facilitates direct measurement of the temperature of the sample with a resolution of 0.1 °C.

3. Ferro-Para Transition in Modified Barium Titanate

Using the experimental arrangement described above, the phase transition or Curie temperature $T_C$ of a ferroelectric ceramic, modified barium titanate (BaTiO$_3$), can be directly determined. This polycrystalline form of BaTiO$_3$ is widely used as a dielectric in high-permittivity commercial capacitors. In its pure single-crystal form, BaTiO$_3$ has $T_C \approx 120$ °C. Above $T_C$, the paraelectric phase has a cubic structure. Below $T_C$ it is ferroelectric with a tetragonal structure. The Curie temperature of BaTiO$_3$ decreases by the substitution of small amounts of ions such as Ca$^{2+}$, Sr$^{2+}$ for the Ba$^{2+}$ ion.

Table 1 gives the capacitance as a function of temperature. Figure 5 shows typical data across the phase transition in a modified barium titanate system. The Curie temperature is around 80 °C for this material.

Table 1. Data for modified BaTiO$_3$.

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Figure 5. Ferro-para transition in a modified barium titanate system.

The data in Table 1 are plotted in Figure 5. The transition temperature is around 79 °C.
16.4 Tracking of the Ferromagnetic-Paramagnetic Transition in Nickel Through Electrical Resistivity

1. Introduction

Nickel is a 3d transition metal and has been extensively studied for its magnetic and electronic transport properties. It is now well recognized that the partially filled 3d and 4s orbitals make up the conduction band and influence both its ferro-magnetic and electronic transport properties. Electrons carry both charge and spin. However, spin is normally ignored in traditional electronics, which is primarily concerned with the manipulation of the charges by electric fields in different materials. Mott developed the well known s-d scattering model for explaining the anomalies observed in the electrical resistivity behavior across the ferromagnetic-paramagnetic transition in metals like nickel. Historically, the effect of spin on the mobility of electrons was explored for the first time in this work and led to the concept of spin polarized currents in ferromagnetic metals. Albert Fert explored this model both experimentally and theoretically in great detail, which culminated in the discovery of giant magnetoresistance in magnetic multilayers in the year 1988.

The present experiment has twin objectives, viz., (1) to understand the anomalous behavior of the electrical resistivity across the phase transition through the concept of spin polarized current, and (2) to show that the temperature derivative of the electrical resistivity near the Curie temperature behaves like the magnetic specific heat.

2. Experimental Set up

The measurement system is based on the well-known four-probe method for resistivity measurement along with a thermocouple probe in good thermal contact with the sample for direct temperature measurement. The precision resistivity set up developed for such measurements is shown in Figure 1.

![Figure 1. Experimental set up to measure resistivity of nickel.](image)

The sample holder arrangement used in such studies is depicted in Figure 2. The nickel sample used in the present studies is a commercially pure (99%) wire and the current, voltage and thermocouple leads are spot welded as indicated in Figure 2.

![Figure 2. Schematic of nickel sample holder configured for four-probe resistivity measurements as a function of temperature.](image)
An AC current source with high voltage compliance, operating at 400 Hz, drives an AC current of 20 mA through the sample. A reference signal is also derived from this source. The AC signal developed across the sample is measured using an analog lock-in technique. The synchronously demodulated signal is further amplified and read through a 4½ digit DPM.

For accurate temperature measurement, a linearizing circuit was designed for the chromel–alumel thermocouple. This proprietary circuit employs a second degree polynomial curve fitting technique for correcting the nonlinear thermo-emf versus temperature characteristic of the chromel–alumel thermocouple. The thermo-emf of the chromel–alumel thermocouple, which is in good thermal contact with the sample under study, is amplified with a gain of 200 in a high quality front end instrumentation amplifier, configured around INA 101 (Burr-Brown make). Since a single junction thermocouple is employed, Cold Junction Compensation (CJC) is effected through the usage of an IC temperature transducer like AD 590. The linearized voltage gives the temperature (1 mV corresponds to 1 °C). It is read through a 4½ digit DPM with a resolution of 0.1 °C.

A compact furnace with a nominal resistance value of 6 Ω designed for reaching temperatures up to 600 °C is used. Kanthal wire is wound around an alumina tube and forms the heating element of the furnace. A screw terminal board is provided at the top of the furnace for easy connection of the leads coming from the sample holder. The furnace is powered through a 30 volt, 5 amp DC power supply. The sample holder wherein the six leads are threaded through a 6-hole ceramic tube is inserted in the furnace such that the sample is located in the constant temperature zone of the furnace.

3. **Ferromagnetic-Paramagnetic Transition in Nickel**

Experimental data for the resistivity of nickel over the temperature range from 300 to 425 °C, collected using the system described above, is given in Figure 3.

![Figure 3](image-url)  
*Figure 3. Ferromagnetic-paramagnetic transition in nickel tracked through resistivity measurements.*

The main feature of this plot is the anomalous variation of resistance near 360 °C. There is a marked change in the slope around the Curie temperature, which is about 357 °C for nickel. This can be seen more clearly in the plot of dR/dT versus temperature (Figure 4). This curve bears a close resemblance to the \( \lambda \)-type specific heat anomaly observed near a second order phase transition.
4. Energy Band Structure and Magnetic Transition in Nickel

The high electrical resistivity of transition metals like nickel as compared to Cu, Ag and Au was first explained by N F Mott. It may be noted that the conduction band states in metals like Ni, Co, Fe (which belong to the transition metal series) have wave functions derived mainly from the s orbitals just as in Cu, Ag and Au, and that the effective number of conduction electrons is not much less than in the noble metals. However, the mean free path in transition metals is much smaller due to the scattering of electrons from s band states to vacant states in the d band. The partial occupancy of the narrow d band is responsible for the ferromagnetism in transition metals and there is a direct connection between the magnetic properties and their electrical conductivity.

The band structure of ferromagnetic nickel can be represented rather schematically as in Figure 5. The spin-down electrons (spin parallel to the direction of magnetization) occupy a 4s band accommodating about 0.3 electrons/atom and a partially filled 3d band containing 0.6 holes/atom. The 3d band has a very high density of states at the Fermi energy $E_F$ and this is a typical feature of a transition metal. The spin-up electrons also occupy a 4s band containing about 0.3 electrons/atom. However, the Fermi level lies above the spin-down 3d band and hence there are no d holes in this band. The overlap of the 3d and 4s bands in Nickel is responsible both for the ferromagnetism and its high electrical resistivity.

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The variation of the resistivity of Ni as a function of temperature in the range 300 to 425 °C is shown in Figure 3. The important feature of this curve is that the resistivity of Ni varies nonlinearity with temperature in the ferromagnetic phase with the temperature coefficient of resistivity increasing with temperature, and reaching a maximum near the Curie temperature which is around 357 °C. The ferromagnetic-paramagnetic phase transition manifests as a marked decrease in the temperature coefficient of resistivity near $T_c$ followed by a linear increase of resistivity with temperature. A qualitative explanation of the anomalous behavior of resistivity with temperature across the magnetic phase transition can be given based on the band structure diagram in Figure 5. If there is no spin flip during scattering, then, for temperatures lower than the Curie

"Figure 4. Data for $dR/dT$ against temperature for nickel."
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temperature we would expect only half of the conduction electrons to make transitions to the empty states of the 3d band. On the other hand, above $T_c$, when the spin-split 3d bands coalesce, holes with both spin directions will be present, and all the conduction electrons can make transitions to the empty states in the 3d band. Thus, the destruction of ferromagnetism leads to an increase in the electrical resistivity.

![Figure 5](image-url)

**Figure 5.** Band structure of ferromagnetic nickel (schematic). a) Spin-up electrons. b) Spin-down electrons.

It is clear that the spin-up and spin-down electrons of the 4s band carry currents in parallel but with different conductivities in the ferromagnetic region, leading to values of resistivity lower than that expected from an extrapolation of the data from the paramagnetic region. This two-current model played a significant role leading to the discovery of Giant Magneto-resistance in magnetic multilayers.

### 16.5 Ferromagnetic to Paramagnetic Transition Studied using AC Susceptibility Technique

#### 1. Introduction

The magnetic susceptibility of a material can be measured using AC techniques. In a ferromagnetic material, the susceptibility is large (of the order of a thousand), while in the paramagnetic phase, the susceptibility is less by about six orders of magnitude. So, AC susceptibility can be used to trace the FM to PM transition in small amounts of a ferromagnetic material. This is a contactless technique, and samples of about 50 mg will provide a sufficiently large signal.

In La$_{1-x}$Sr$_x$MnO$_3$ compounds, the paramagnetic to ferro-magnetic transition temperature $T_M$ decreases as the Sr concentration decreases. This change in transition temperature, with Sr concentration, is tracked with the AC susceptibility setup. The samples were prepared by Dr. V Ganesan of the UGC-DAE CSR in Indore.

Ni-Zn ferrite beads are used in RF applications. They are available commercially. The FM to PM transition in the bead of Ni-Zn Ferrite is also studied using this set up.

#### 2. Primary and Secondary Coils

In the AC susceptibility measurement, we use a primary coil with two secondary coils wound over it. This is shown in Figure 1.
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Figure 1. Primary and secondary coil windings.

On a plastic former, about 10 cm in length and about 6 cm in diameter, wind a copper coil containing about 2000 turns. This is the primary coil PC. On the primary coil, symmetrically about the center, wind two secondary coils of SWG 36 to 40 enamel coated copper wire. Each secondary coil contains the same number of turns (about 500). The secondary coil SC2 is wound in the opposite sense to the secondary coil SC1. The leads of PC are brought to banana terminals P1 and P2. The leads of the secondary coil SC1 are brought to two banana terminals S1 and S1'. The leads of SC2 are brought to the terminals S2 and S2'.

If a sinusoidal AC voltage from a signal generator, at 1.5 kHz frequency and peak-to-peak amplitude 3 V, is applied to PC, secondary voltages will appear between the terminals S1, S1' and the terminals S2, S2'. Let the RMS voltages measured be $V_1$ and $V_2$. Connect S1' to S2, and measure the RMS voltage across S1 and S2'. If the secondary coils are wound in opposition, this will be the voltage $|V_1 - V_2|$. On the other hand, if the coils have been wound in the same direction, this voltage will be $|V_1 + V_2|$. If we get the latter voltage connect S1' to S2', and measure the voltage across S1 and S2. It will now be $|V_1 - V_2|$. We must connect the terminals appropriately to get an output which is the difference between the secondary voltages. If the coils are identical in every respect, this difference will be zero. But it is not possible to wind the coils exactly identical, and so the difference will be a few millivolts.

3. Block Diagram of AC Susceptibility Circuit

Figure 2 shows the block diagram of the circuit.

T is a 1:1 isolation transformer. The input terminals of the transformer, $T_1$ and $T_2$, are connected to the output terminal of a signal generator. The primary coil terminals are connected to the terminals $B_1$ and $B_2$.
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on the box. (This is a BNC connector and is marked B on the box). Then the primary coil is connected in series with a pot $P_1$, of 1 kΩ, across the output terminals of the transformer $T$. $P_1$ is the amplitude-adjust pot for balance. The terminals $D_1$, COM and $D_2$ on the box are connected to the terminals $S_1$, $S_1'$ and $S_2'$ (or $S_2$ if the secondary coils are not wound in opposition). The output voltages of the two secondary coils, relative to ground, are connected to a DPDT switch DP1. The central terminals of DP1 are connected to the AC differential amplifier 1. If the two secondary coils are in opposition, the voltage across the terminals $S_1$ and $S_2'$ relative to ground will be $V_1$ and $V_2$.

![Figure 2. Block diagram of the AC susceptibility circuit.](image)

The output of differential amplifier 1 will be $V_1 - V_2$. It will be a few millivolts AC, and will come out at F.

We have to balance this output to zero. This is done as follows. The variable point on the pot $P_1$ is connected to a phase shifter. There is a pot $P_2$ in the phase shifter. By turning the pot, the phase of the output voltage of the phase shifter can be changed relative to the phase of the input voltage. The voltage from $P_1$, after phase shifting, comes out at G. This phase shifted voltage $V_3$ at G, and the output at F of differential amplifier 1 (Diff.Amp.1) are fed to a second differential amplifier 2 (Diff.Amp.2). By adjusting the pot $P_1$ and the pot $P_2$, we can make the output of differential amplifier 2 very small, a few mV. This is shown in Figure 3(a). The output of Diff.Amp.2 comes out at H. This output H is then fed to a phase sensitive detector AD630. The reference signal to the AD630 comes from the phase shifted output at G of the phase shifter. AD 630 will give a DC output at J, which is further amplified at the two-stage output amplifier. The amplified DC output comes out at K. This output K is fed to one pair of terminals of the DPDT switch DP2. $P_3$ is an offset adjust pot. The AC output H from Diff.Amp.2 is fed to the other pair of terminals of DP2. The middle terminals of DP2 are connected to the terminals marked OP. In one position of DP2 (up), we measure the AC output voltage at H of Diff.Amp.2, if the multimeter connected to the terminals marked OP is put in AC 2 V range. Thus, we can adjust the pots $P_1$ and $P_2$ to get the AC balance at the output of Diff.Amp.2, when there is no sample within the coil SC1. In the other position of DP2, we measure the DC output of the final
Experiments in Phase Transitions

output amplifier. This is done by putting the multimeter in the DC 20 V range. The DC offset of the amplifier can be made zero by adjusting the offset pot $P_3$.

If the sample is suspended to hang in the coil SC1, the multimeter will show a large DC output which will vary with temperature.

What is the purpose of DP1? When we wind the two secondary coils, sometimes the coil SC1 may have a higher output than the coil SC2, and sometimes the reverse will be the case. Then the output of Diff.Amp.1 will change phase by 180 degrees depending on voltage of SC1 being more than the voltage of SC2, or vice versa. This is shown in Figures 3(a) and 3(b). Note the curves marked $V_1-V_2$. In one case we may be able to balance the output voltage of Diff.Amp.1, within the range of variation of $P_2$ (Figure 3(a)). In the other case, this will not be possible. In the latter case, we reverse the voltage ($V_1-V_2$) by putting the switch DP1 up. Now, we will be able to balance by adjusting $P_1$ and $P_2$ (Figure 3(c)). $V_3$ is the voltage coming out of the phase shifter.

![Figure 3(a).](image)

![Figure 3(b).](image)
Figure 3(c). Illustrating the need for DP1.

4. Front Panel of the Circuit Box

The front panel of the AC susceptibility circuit box is shown in Figure 4.

Figure 4. Front panel of the circuit box.
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Mains switches the power on. The BNC socket T should be connected to the signal generator. The BNC socket B should be connected to the banana terminals PC of the primary coil (Figure 1). The terminals D1, COM, D2 should be connected to terminals S1, S1’ and S2 of the secondary coils (Figure 1). The pot P₁, marked AMP is the amplitude balancing pot, and the pot P₂, marked PHASE is the phase balancing pot. DP1 and DP2 are the two DPDT switches. O/P are either a pair of banana terminals or a BNC Connector to be connected to a digital multimeter. The pot P₃ marked OFFSET is the DC offset adjusting pot.

5. Insert with Sample

The insert is shown in Figure 5.

![Sample insert tube](image)

Figure 5. Sample insert tube.

C is a circular Hylam disc with a central hole. A Pyrex glass tube G, of 3 cm diameter, closed at the bottom end, is fixed with Araldite to this hole. On the tube G, a heater of 5 to 10 ohm resistance is wound. The two leads of the heater come out through the circular disc C to the terminals marked HT. When the tube is inserted in the primary coil, the circular disc rests on the top of the former, on which the primary coil is wound. The heater is at the center of the secondary coil SC1.

A second circular plate B of Hylam has a Hylam rod R fixed to a hole in its center. At the end of the rod, a small length of about 1 cm is cut, so that we have a plane face 1 cm × 5 mm. On the circular side of this cut, we fix a Pt100 thermometer with Araldite. The leads of the thermometer come to the RCA socket TH on B. On the flat side, we put the sample S, which has a length less than 1 cm and a width of the order of 5 mm, and fix it to the rod with Teflon tape. The thickness of the sample is in mm.

When the plate B is placed on the circular disc C, the sample and thermometer hang at the center of the heater HH, which is at the center of the secondary coil SC1.

The sample is in the ferromagnetic phase at room temperature. So, it has a large magnetic susceptibility. This causes the output voltage \( V₁ \) to become larger than the output voltage \( V₂ \) by a few mV. This difference in voltage will depend on the frequency and the amplitude of the AC signal.

6. Procedure

First, place the circular disc C on the primary coil holder. The insert B with the sample is not put in the glass tube. Then connect the signal generator to the BNC connector marked T on the circuit box. Connect the terminals of the primary coil to the BNC connector marked B on the circuit box. The terminals, S1 and S2’, of the secondary coils, should be connected to the terminals, D1 and D2, on the
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circuit box. The terminals S1' and S2 are connected with a wire. S1' is connected to the terminal marked COM on the circuit box. A DMM is connected to the output terminals, marked O/P on the circuit box. Put the switch DP1 down, and the switch DP2 in position marked AC. Put the DMM in AC 200 mV range.

Switch on the signal generator, and set the frequency to 1.5 kHz. Check that the SG output is set to sine mode, and adjust the peak-to-peak amplitude of the signal generator to 3 V. Note the DMM AC voltage, and adjust the pots P₁ and P₂ alternately till you get a minimum voltage on the DMM. Now, put the switch DP2 in the position DC, and put the DMM in DC 20 V range. Adjust the offset pot P₃ to set the DC voltage on the DMM to zero. When you first switch on the instrument, the reading on the DMM will keep changing slowly because the offset changes slowly. Wait for half an hour till it becomes steady, and again adjust the offset to zero reading on the DMM.

Now, place sample 1 (LSMO with Sr content of 0.25) on the flat portion of the rod R and fix it with Teflon tape. Put the sample insert in the glass tube. Adjust the AC output of the signal generator, so that the DC reading on the DMM is between 1 and 2 V DC. Remove the insert with the sample from the tube, and check if the DMM reading goes to zero. Otherwise, adjust the offset pot to make it zero. Put the sample back in G.

Connect a DC power supply to the heater terminals on the circular disc C. Connect the RCA socket TH of the Pt100 thermometer to a temperature indicator. Set a voltage of about 5 V on the DC power source. The temperature will slowly increase. Adjust the voltage till the temperature increases at the rate of 1 °C per minute. Note the temperature and DMM DC output, initially for every 5° rise in temperature, till you reach about 60 °C. Between 60 °C and 80 °C, note the DMM readings for every degree rise in temperature. Continue taking the readings till the DMM reading goes to nearly zero, and remains reasonably constant.

The experiment is repeated with the LSMO(Sr = 0.3) and a Ni-Zn ferrite bead. A set of readings for the samples Laₙ₋₁SrₓMnO₃ (x = 0.25 and x = 0.3) and for Ni-Zn ferrite is given in Table 1.

Table 1. Sample readings for LSMO (x = 0.3, 0.25) and Ni-Zn ferrite bead. Frequency: 1.5 kHz.

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<th>Temp (°C)</th>
<th>V_DC (volts)</th>
<th>Temp (°C)</th>
<th>V_DC (volts)</th>
<th>Temp (°C)</th>
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Figures 6, 7 and 8 show plots of the output V_DC against temperature for the three samples. Figures 9, 10 and 11 show the temperature derivatives of V_DC against temperature. The temperature of the peak is T_MP, the temperature of the midpoint of the transition.
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Figure 6. FM to PM transition in LSMO (Sr = 0.25).

Figure 7. FM to PM transition in LSMO (Sr = 0.3).

Figure 8. FM to PM transition in a Ni-Zn ferrite bead.
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**Figure 9.** Derivative plot of Figure 6 for LSMO (Sr = 0.25).

**Figure 10.** Derivative plot of Figure 7 for LSMO (Sr = 0.3).

**Figure 11.** Derivative plot of Figure 8 for Ni-Zn ferrite.

In Table 2, the midpoint temperatures of the transitions are collected for the three samples.

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We see that as the Sr concentration $x$ is reduced from 0.3 to 0.25, the transition temperature $T_{MP}$ comes down by about 14°C.

### 16.6 Metal-Insulator Transition in a Pellet of Strontium-Doped Lanthanum Manganite

#### 1. Introduction

Lanthanum manganite has the perovskite structure $\text{ABO}_3$ discussed earlier (Figure 1, Section 15.7). In a stoichiometric compound, all lanthanum atoms are in the 3+ state. Manganese atoms are also in the 3+ state, and oxygen atoms are in the 2− state.

In the lanthanum manganite system, we have manganese atoms, each surrounded by six oxygen atoms at the corners of an octahedron. Two octahedra share a corner. In such an octahedral system, a distortion may occur in such a way that the lengths of two Mn-O bonds along the Z-axis become different from the lengths of the four MnO bonds perpendicular to the Z-axis. This distortion in symmetry results in the removal of the degeneracy of electronic levels, and a consequent lowering of energy. This is the Jahn–Teller distortion. This distortion occurs below a certain temperature, and leads to a change in the band structure of the material. Above the transition temperature, there is a band gap, leading to a semiconductor-like decrease in resistance as the temperature is increased. Below the transition temperature, the band gap is closed. This leads to a metallic-type increase of resistance with increasing temperature. Pure $\text{LaMnO}_3$ is insulating. When some of the La$^{3+}$ ions are replaced with divalent ions like Sr$^{2+}$, an equal number of Mn$^{3+}$ ions become Mn$^{4+}$. The Mn$^{3+}$ and Mn$^{4+}$ ions are distributed randomly. In this disordered system, below a certain temperature, Jahn–Teller distortion sets in, leading to an insulator-metal transition as the temperature is lowered. This transition is also accompanied by a para- to ferro- magnetic transition. The paramagnetic to ferromagnetic transition in two LSMO samples with different doping levels of strontium was studied in Section 16.5. The transition temperature increases as the Sr concentration increases. Here we study the metal to insulator transition in an LSMO sample with a Sr concentration close to 0.15. The transition temperature is much below room temperature. This sample was prepared and supplied by Dr. V Ganesan of UGC-DAE CSR in Indore.

#### 2. Experimental Setup

The normal boiling point of liquid nitrogen is 77 K. The transition temperature of the sample is around 110 K. In Section 18.3 we discuss how low temperature experiments can be carried out in a bath cryostat or with a closed cycle refrigerator (CCR). This experiment could be done using a CCR. However, for doing simple resistivity measurements down to 90 K, a CCR is not necessary. If liquid nitrogen is available on location, a simple dipstick cryostat can be used for the measurement of resistivity. It is easy to fabricate, and is very cost effective. Such a dipstick cryostat is described in Figure 1.

The cryostat consists of an external Pyrex glass flask to which an aluminium flange is attached at the top end. From the top brass flange a stainless steel (SS) tube of about 2 to 3 mm diameter hangs inside the vessel. The top flange is fixed to the bottom flange by brass bolts and nuts with a neoprene O ring in between the top and bottom flanges. At the bottom end of the SS tube a copper rod is brazed. The sample in the form of a rectangular pellet is fixed to the bottom end of the copper rod, which is flattened. A heater of a few watts is fixed to the copper rod at its top end. A platinum resistance thermometer (PRT) is also fixed on the other side of the bottom portion of the copper rod carrying the sample. A number of copper radiation shields are brazed to the SS tube. Electrical leads from the PRT, current and voltage leads to the sample, and leads to the heater are brought out through the electrical feedthrough. A rotary vacuum pump is connected to evacuate the glass flask.
The sample has a resistance of the order of several kilo-ohms. So it is enough to use a two-probe measurement for the resistance. Copper leads are attached, using silver paint, to the sample on a thin insulating substrate. The substrate is fixed to the copper block with superglue. Thin copper leads are taken from the silver painted contacts to the electrical feedthrough.

3. Experimental Procedure

Connect the regulated DC power supply to the heater leads. Connect the leads of the Pt 100 thermometer to a temperature indicator. Connect a DMM in the 200 kΩ range to the electrical leads from the sample.

First switch on the roughing pump. Within a few minutes the vacuum inside the glass tube will reach a value of 0.02 millibar. Then slowly insert the glass tube into the neck of a 20 liter liquid nitrogen storage dewar. The temperature indicator connected to the Pt 100 resistor shows the temperature of the copper block. This temperature will start falling. Slowly increase the depth of immersion of the tube till the flange of the glass tube rests on the neck of the Dewar. The sample will cool to a temperature of −170 °C (about 103 K) in one to one and a half hours.

Insert thermocol blocks between the flange of the glass tube and the neck of the dewar to raise the cryostat above the level of liquid nitrogen. Switch on the power supply and set a voltage of 2.5 V. The temperature of the copper block will start increasing. Keep the rate of increase between 1 and 2 degrees centigrade per minute. Note the resistance of the sample, on the DMM, for every five degree rise in temperature. If the rate of rise of temperature decreases, slightly raise the voltage on the power supply. Never raise the DC power supply voltage beyond 4 V. It will be possible to come up to +20 °C by raising the level of the flange above the neck of the dewar by inserting thermcol pads and also by raising the power supply voltage.

A sample set of data is given in Table 1.

A plot of variation of resistance with absolute temperature is shown in Figure 2. A sharp transition is seen around 169 K. Above this temperature, the resistance decreases as the temperature increases. This is the behavior of semiconductors. Below this temperature, the resistance increases as the temperature increases. This is the behavior seen in metals.
### Table 1. Metal-insulator transition in a LSMO pellet.

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<th>$T$ (°C)</th>
<th>$T$ (K)</th>
<th>$R$ (kΩ)</th>
<th>$T$ (°C)</th>
<th>$T$ (K)</th>
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</table>

**Figure 2.** Metal-insulator transition in a sample of LSMO.

### 16.7 Superconducting Transition in YBCO

1. **Introduction**

A brief introduction to superconductivity is given below.

Superconductivity was discovered by Kammerling Onnes in 1911 when he found that the resistance of a frozen capillary column of mercury went abruptly to zero when cooled to the boiling point of liquid helium. Figure 1 shows schematically the variation in resistance as a function of temperature in a superconductor.
Experiments in Phase Transitions

Figure 1. Resistance vs. temperature for a superconductor.

If a current is induced in a metal in the superconducting state, it will persist for a long time as the material has no resistance. Using NMR techniques, File and Mills (PRL 10, 93, (1963)) in their experiment, estimated that the decay time of the persistent current was longer than 100,000 years. A theoretical estimate (Kittel: *Introduction to Solid State Physics*, Seventh Edition, Wiley India, page 359) states that the persistent current will remain for a period longer than the age of the universe.

Associated with the vanishing of resistance, the superconductor becomes a perfect diamagnet in the superconducting state. Consider the following experiment:

A sphere of the material, which will become superconducting below a temperature $T_C$, is kept above this temperature in an external magnetic field $B$. Then, some magnetic flux lines penetrate the material. If now the material is cooled below $T_C$, these magnetic flux lines are expelled, so that $B$ inside the sphere is zero. This means that

$$B = \mu_0(H + M) = 0 \quad \text{or} \quad M = -H.$$  \hspace{1cm} (1)

One may also take the material in the normal state, cool it below $T_c$ and then apply an external magnetic field $B$. Even then, the flux lines do not penetrate the sphere. Thus, a superconductor is characterized by

1. a vanishing resistance and
2. perfect diamagnetism.

Superconductivity is a phase transition from the normal state. Below $T_C$, the superconducting phase has a lower free energy than the normal phase, and so is the stable phase. If the superconductor is placed in a magnetic field, the free energy will increase, since the free energy due to the magnetic field is $-\int \mathbf{M} \cdot d\mathbf{H}$. In a diamagnetic material, $\mathbf{M}$ and $\mathbf{H}$ are in opposite directions. In the normal phase, the material is paramagnetic, but the susceptibility is very small. In the superconducting phase, the diamagnetic susceptibility is very large. As the magnetic field is increased, the free energy of the superconducting phase will be equal to the free energy of the normal phase at a critical field $H_C$, and will exceed the free energy of the normal phase when $H$ exceeds $H_C$. The material will revert to the normal state at $H_C$, and the resistance will have a non-zero value and the magnetic field will penetrate the material.

The phase transition from a normal to a superconducting phase in the absence of an external applied magnetic field, is a second order phase transition. It will be accompanied by a discontinuous jump in specific heat. This is shown for aluminium which has a $T_C$ of 1.163 K. The data are taken from Norman E Phillips, *Physical Review* 114, 676, (1959). Above a magnetic field $H$ of 100 oersteds, aluminium becomes normal. So, even when it is cooled below 1.163 K in a field of 150 oersteds, the specific heat follows the curve for the normal state (Figure 2).
The existence of a critical field implies that there is a limit to the current that a superconductor can carry. If this current produces a magnetic field more than $H_c$, then the material will turn normal.

There are two types of superconductors. Figure 3 shows a plot of $-M$ vs. $H$ in the two cases. In the first one, the material reverts abruptly to the fully normal state when the magnetic field exceeds a critical value $H_c$, as shown in the Figure 3. In the second type, the material breaks into normal and superconducting regions at the field $H_{c1}$. Some magnetic flux starts penetrating the normal regions, which coexist with superconducting regions. As the field is increased further, the normal regions grow in volume, and the superconducting regions shrink, till at a second higher value $H_{c2}$, the entire material becomes normal. Now magnetic flux fully penetrates the material.

**Figure 2.** Specific heat jump in aluminium in zero magnetic field at the superconducting transition temperature.

**Figure 3.** $-M$ vs. $H$ curves for type I and type II superconductors.

Between $H_{c1}$ and $H_{c2}$, the material will still have zero resistance in type II superconductors. In type I materials, $H_c$ is small. Such materials in the superconducting state cannot be used to generate large magnetic fields. In type II materials, $H_{c2}$ can be very large. So, type II materials can be used to produce high magnetic fields (up to $H_{c2}$) in the superconducting state. The critical field also depends on temperature. This is shown in Figure 4.
Type II superconductors can carry very large currents. The transport $J_c$ (current per unit area of cross section) is the maximum current density at which the material reverts to the normal resistive state. $J_c$ in NbTi and Nb$_3$Sn wires at 4.2 K is shown as a function of the magnetic field in Figure 5. These are the two materials which are used commercially to produce large magnetic fields.

The major application of superconducting magnets is in particle accelerators to bend high energy beams and in MRI machines. Superconducting magnets can also be used for separating magnetic materials present in tiny quantities in non-magnetic ores.

Figure 4. Temperature dependence of $B_{c2}$ for some type II superconductors.

Figure 5. Transport $J_c$ at 4.2 K as a function of magnetic field in NbTi and Nb$_3$Sn wires. (from sust421171f34_.online.jpg).
2. High Temperature Superconductors

Though superconductivity was discovered in 1911, the highest transition temperature of 22.3 K was reached in Nb$_3$Ge after intense efforts for more than seventy years following the discovery of superconductivity.

Bardeen, Cooper and Schrieffer (BCS) provided a theoretical explanation for superconductivity in 1957. They showed that the electron-phonon interaction in metals leads to the pairing of electrons, one with spin up and the other with spin down. Such pairing occurs below a transition temperature characteristic of the metal. This results in the formation of Cooper pairs of the electrons below the Fermi level. These bound electrons are separated from excited states by a small gap in energy. To raise a bound electron pair, energy must be supplied to break the Cooper pair. All the Cooper pairs move in unison and cannot be scattered. This leads to vanishing electrical resistance in the superconductor.

The BCS theory was very successful in explaining the properties in many typical superconductors. The weakness of the electron-phonon interaction responsible for the pairing mechanism leads one to believe that one cannot obtain superconductivity at high temperatures.

In 1986 Bednorz and Müller observed the resistance versus temperature behavior in Ba-La-Cu-O system and suggested that one may obtain superconductivity in this system with a transition temperature of 30 K. Tanaka obtained a transition temperature of 30 K in barium-doped La$_2$CuO$_4$ by the end of 1986. Early in 1987 Chu and Wu reported a superconducting transition temperature of 93 K in YBa$_2$Cu$_3$O$_7$. Soon, many scientists were involved in research on high temperature superconductors. The highest temperature reported so far is 135 K in Hg Ba$_2$Ca$_2$Cu$_3$O$_8$.

The structure of YBa$_2$Cu$_3$O$_7$ is shown in Figure 6. This is a structure of three distorted perovskite (ABO$_3$) cubes placed one above the other with some lattice positions vacant. The perovskite structure ABO$_3$ is cubic, where A is a heavy atom and B is a lighter atom. The YBCO structure can be deficient in oxygen. Oxygen deficiency reduces the superconducting transition temperature. When there are 6.41 atoms of oxygen, the material is not superconducting.

![Figure 6. Structure of YBa$_2$Cu$_3$O$_{7-x}$](image)
The structure is orthorhombic with $a$ and $b$ lattice parameters slightly different, with the difference depending on oxygen deficiency. The $c$ lattice parameter is nearly 3 times the lattice parameters $a$ and $b$. When the oxygen deficiency is 1, there are no O1 atoms along the $b$-axis and the lattice parameters $a$ and $b$ become equal. As the oxygen deficiency decreases, O1 lattice sites are occupied more and more. The lattice parameter $b$ becomes larger than $a$. The structure becomes orthorhombic. Superconductivity occurs when the oxygen deficiency becomes less than 0.5. The transition temperature is maximum when the deficiency is 0.07.

There are copper–oxygen planes parallel to the $ab$-plane. Here, copper is surrounded by four oxygens. There are copper–oxygen chains along the $b$-axis. In these chains, copper is surrounded by two oxygens. The copper–oxygen planes are thought to be responsible for superconductivity. The copper–oxygen chains are the charge reservoirs. The electron–phonon mechanism postulated by BCS is certainly not the mechanism responsible for the pairing of the electrons. The exact mechanism of pairing is still not understood though many theoretical models have been made.

When the high-temperature superconductors were discovered, there was the expectation that a variety of applications, which were possible with conventional superconductors cooled to 4.2 K, would be realized with these materials using liquid nitrogen as the coolant. Unfortunately, these materials have inherent technological disadvantages. So, though some demonstrations have been made, such large-scale applications have not resulted so far.

### 3. Experiment to Measure the Superconducting Transition Temperature in a Sample of YBCO

A sample of YBCO, prepared by Sri Venkateswarulu, at UGC-DAE CSR, Indore, through the kind courtesy of Dr V Ganesan was used in this experiment. The sample, prepared by solid state reaction, and well annealed in oxygen, had a semicircular shape with a diameter of about 10 mm and a thickness of about 1 mm. Four contacts were made in a line with silver paste and the sample was fixed to a copper plate of about 1 mm thickness with superglue. The superglue insulated electrically the pellet from its copper base. Because the film of superglue was thin, the thermal contact was not reduced substantially.

A Closed Cycle Refrigerator (CCR), developed at RRCAT, Indore, was given on long-term loan to the Indian Academy of Sciences, Bengaluru, through the kind courtesy of the Director of RRCAT. This CCR was used for this experiment. The working of a CCR is discussed in Section 18.3.

The copper plate on which the YBCO sample was mounted, was screwed to the cold head of the second stage of the CCR. The two extreme contacts were the current lead contacts, and the two middle contacts were the voltage lead contacts. The current lead contacts were soldered to two pins of a feedthrough, and the voltage lead contacts to two other pins.

After mounting the specimen and soldering the leads to the pins of the feedthrough, the copper radiation shield was screwed on to the first stage of the cryocooler. The radiation shield will reach temperature 90 K on cooling and will reduce the radiation from room temperature reaching the sample. There was a vacuum shroud put around this radiation shield.

The chilled water system providing cooling water circulation to the compressor of the CCR was switched on.

Then, a valve connecting the CCR to a rotary-cum-diffusion pump setup was opened. The rotary pump was started to get a rough vacuum of about 0.01 mbar and then the diffusion pump was switched on. When the vacuum reached a value of about $10^{-4}$ mbar, and after checking that the chilled water had reached its minimum temperature of 12 °C, the switch on the cryocooler panel was put on to start the cryocooler. The temperature of the second stage was measured with a Pt1000 resistance thermometer. It has a resistance of 1000 ohms at 0 °C. The resistance of Pt1000 started to fall as the sample was cooled.

The leads of a heater on the second stage cold head, with a resistance of about 35 ohms were connected to a DC 30 V 2A power supply. The current leads on the sample were connected to a constant current source in the low current mode. The voltage leads were connected to channel 1 of a DC differential amplifier. The terminals of channel 2 of the differential amplifier were shorted, the selector was put to read channel 2, and the voltage reading on the amplifier panel meter was reduced to zero by pressing the push switch. This corrected for the offset in the DC amplifier. The channel switch was then put to read the voltage across the sample. The constant current source was switched on, and the current was set to 20 mA. The reading on the panel meter on the differential amplifier was about 0.080 mV or 80 μV when the sample temperature
was near room temperature. We did not take readings as the sample was cooling because it was difficult to adjust the cooling rate. When the PRT resistance reading on a DMM was 257 ohms, corresponding to a temperature of about 90 K, the reading on the panel meter of the differential amplifier went rapidly to zero and remained at zero as the resistance of the PRT decreased further. When the resistance of the PRT reached 200 ohms, corresponding to a sample temperature of about 77 K, the cryocooler was switched off.

To check that the YBCO pellet had become superconducting, the current was increased in steps to 40, 60 and 80 mA. The reading on the differential amplifier remained zero. If the YBCO had a resistance, this four-fold increase in current should have caused the panel meter of the differential amplifier to show a reading increasing with current.

The power supply connected to the heater was switched on and the voltage adjusted till the resistance of the Pt1000 thermometer increased by 4 to 5 ohms per minute. The PRT reading and the panel meter reading on the differential amplifier were taken every minute. The differential amplifier reading remained zero till the PRT resistance reached about 248 ohms. Then, the panel meter reading started to go up fast. The heating voltage was reduced slightly to reduce the heating rate and readings of the amplifier panel meter and the PRT resistance were taken, as the resistance changed by 0.5 ohms every time. This was done till the differential amplifier reached a voltage of about 26 μV. Now, the reading on the differential amplifier started to change slowly. The heater voltage was readjusted to maintain a rise of 4 to 6 ohms per minute in the resistance of PRT, and the reading of the resistance was noted every time the reading of the amplifier panel meter increased by 1 μV. This was continued till the Pt resistance reached 510 ohms (about 150 K).

The heater, the constant current source and the differential amplifier were switched off. The valve connecting the cryocooler with the diffusion pump setup closed and the diffusion pump switched off. After waiting for 30 minutes and checking that the diffusion pump became cold, the valves on the pump setup were closed and the rotary pump switched off.

A sample set of readings are given in Table 1 for two runs on the same sample with slightly different currents through the sample. The first column gives the PTR value in ohms. The temperature is estimated from the resistance using the interpolation formula

$$T \text{ (in Kelvin)} = 30.6811 + 0.2309R + 8.76 \times 10^{-6}R^2,$$

where $R$ is the resistance of PRT in ohms. This is given in column 2. The third column gives the differential amplifier voltage in microvolts.

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<th>Pt 1000 Res. (Ω)</th>
<th>Temp (K)</th>
<th>$V$ (μV)</th>
<th>Pt 1000 Res. (Ω)</th>
<th>Temp (K)</th>
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Figure 7 shows a plot of the voltage across the sample versus temperature in Kelvin.

The figure shows an abrupt transition starting at a temperature of 90.2 K and ending at a temperature of 88.5 K. The midpoint of the transition is at 89.4 K.

In the normal region, the readings for the second run are slightly lower in voltage because the current through the sample is slightly lower.

If the sample was left in the CCR for a few days, the water in the atmosphere reacted with Ba in the sample to form a thin semiconducting layer. If the experiment was repeated, the resistance did not go to zero.
below the transition temperature. This was because we now had a semiconductor layer in series with the superconducting region, and the two resistances were in series. We still saw a sudden fall in resistance at the transition temperature. The fractional change in amplifier voltage, calculated from \( \frac{V_{\text{above}} - V_{\text{below}}}{V_{\text{above}}} \), decreased, the longer the sample was exposed to the atmosphere in the CCR. In the above, \( V_{\text{above}} \) and \( V_{\text{below}} \) are the voltages measured above and below the transition temperature. Also, the temperature at the midpoint of the transition comes down slowly by a few degrees.

16.8 Structural and Melting Transition in KNO\(_3\) using Differential Thermal Analyzer

1. General Description

Differential Thermal Analysis (DTA) is a technique to record a thermal event in the sample by comparing it with a reference material which is devoid of any physical or chemical changes in the temperature range of interest. In the conventional experimental arrangement, a single block with symmetrical cavities for the sample and the reference is heated uniformly in the constant temperature zone of a tubular furnace. The block acts as a heat sink, ensuring that the sample and reference are heated at the same rate so that there is no temperature difference between them in the absence of a thermal event. Further, the block also exchanges heat with the sample so that the sample temperature re-equilibrates with the furnace temperature after a thermal event in the sample. However, the temperature of the sample must not re-equilibrate too rapidly because the DTA signal (the differential temperature) will then be very small. It is thus recommended to have a sample container of low thermal conductivity so as to get an adequate DTA signal. It follows that the temperature change accompanying a transition is directly proportional to the enthalpy change of the transition. An absolute quantitative measurement of the heat of transition requires a mathematical treatment of the heat flow between the sample and the block. A practical method of evaluating the change of enthalpy is to calibrate the DTA instrument with a material whose change in enthalpy is known from other studies and compare the peak areas.

In the conventional DTA technique, the thermocouples are housed in the sample and reference cavities in a block made out of aluminium for the temperature range 25–500 °C and in a stainless steel block for the temperature range up to 800 °C. This sample block is located in the constant temperature zone of a tubular furnace. For the temperature range up to 800 °C, the conventional type K (chromel–alumel) thermocouple is employed.

The block diagram of the DTA system is given in Figure 1. It consists of the following subsystems, viz.,

a) Precision temperature linearizer & \( \Delta T \) amplifier;

b) Programmable power supply (PPS);

c) Furnace with the sample holder;

d) PC based data acquisition system (PC DAS).

2. Precision Temperature Linearizer

The thermocouple input from the chromel–alumel thermocouple in thermal contact with the reference material (alumina) is routed through a differential input filter followed by a high quality instrumentation amplifier configured for a gain of 200. Since temperature measurements are carried out using a single junction thermocouple, reference point compensation is required and an IC temperature transducer provides the necessary cold junction compensation (CJC). The amplified thermocouple voltage with CJC is linearized using a propriety circuit that employs a fourth order polynomial curve fitting for the thermo-emf versus temperature data. The fitting accuracy is \( \pm 0.5 \) °C for the temperature range 25–800 °C.

The linearized temperature is read through a 4 \( \frac{1}{2} \) digit DPM with a resolution of 0.1 °C. The analog output of the linearized temperature (1 mV corresponding to 1 °C) is available at the rear panel of the unit as TEMP O/P.
The temperature differential between the reference and the sample, $\Delta T$, is the central quantity of interest. This signal is fed to the terminals marked $\Delta T$ INPUT at the rear panel of the unit. The signal is initially filtered using a special differential filter followed by a high stability, high gain differential Instrumentation amplifier, viz., INA 101. The gain of the amplifier can be set from the front panel of the instrument to 200, 500 or 1000. The DC offset is less than 1 $\mu$V referred to the input stage even when a high gain of 1000 is used. The amplified voltage corresponding to $\Delta T$ is brought out at the rear panel of the unit and is marked as $\Delta T$ OUTPUT for further processing in the PC based data acquisition system.

3. **Power Amplifier (Programmable Power Supply)**

This unit powers the furnace designed for high temperature DTA studies on materials. The output rating of the programmable power supply is 30 V and 5 amps. The USB socket in the rear panel provides the required interface to the computer. The maximum power output of 100 watts is optimum
Experiments in Phase Transitions

for attaining temperatures in the range of 600 °C and also for achieving linear rates of heating up to 20 °C/minute.

4. Furnace

The compact furnace has been built using non-magnetic materials with provision to house the DTA sample holder. Nichrome wire of 32 SWG is wound around an alumina tube of dimensions 27mm OD and 23mm ID and of length 200mm. The number of turns/inch of nichrome wire wound on the alumina tube is 18 turns/inch. The resistance of the heating coil thus wound is 13 Ω. The nominal current carrying capacity of this wire is 1 ampere. Brackets made out of Syndanio material, which is a good thermal insulator, are fitted at either end of the alumina tube to thermally isolate it from an outer cover made out of powder-coated mild steel enclosure. The nichrome wire heater is covered with K-wool along the entire length for thermal insulation. The efficiency of this heater arrangement is around 10 °C/watt. The furnace is powered from the 100 watt programmable power supply.

5. Data Acquisition System

The DAS is a dual-channel 16 bit A/D converter with an appropriate preamplifier. The gain of the preamplifier is optimized prior to A/D conversion to enhance the accuracy of measurement. This card is interfaced to the computer through an USB port. The analog outputs from the electronic system, viz., \( T \) and \( \Delta T \) form the inputs to the DAS.

6. DTA Software

This menu-driven software dta.exe has been developed in LabView environment for acquiring and graphic display in real time of a DTA run. Significant features of this software are:

a) Software selectable rate of heating from 1 °C/minute to 20 °C/minute;

b) Real time digital filtering of the data for improved signal-to-noise ratio;

c) Real time graphic display of \( \Delta T \) versus \( T \) signal for online observations of thermal events occurring in the sample

d) Auto scaling of the data along both \( X \)-and \( Y \)-axes.

The data from the run is also stored in a separate file for further analysis. Software has also been developed for numerical computation of the peak areas for estimating enthalpy changes.

7. Operating Instructions for Running the DTA Experiment

a) Fill the sample (in the form of powder or thin filings) in the bore provided in the sample holder.

b) Fill the alumina powder which is the reference material in the other bore marked with a + sign on the outer side.

c) Insert the two chromel–alumel thermocouples into the bores containing the sample and the reference. Color code for the two thermocouples are:

Reference : Chromel – Red color sleeve
Alumel – Black color sleeve

Sample : Chromel – White color sleeve
Alumel – Grey color sleeve

d) Make the connections of the thermocouple to the terminal strip provided at the top of the furnace as given below.
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Terminal 1. Chromel (Reference – Red)
Terminal 2. Alumel (Reference – Black) joined to
    Alumel (Sample – Grey)
Terminal 3. Chromel (Sample – White)

e) Connect the shielded wire provided with Red End connector to the TC INPUT terminals at the rear panel of the Differential thermal analysis unit. The other end of the wire is connected to Terminals 1 and 2 (Red wire to Terminal 1 and Black wire to Terminal 2) on the top of the furnace.

f) Connect the shielded wire provided with Black End connector to the ΔT INPUT terminals at the rear panel of the DTA unit. The other end of the wire is connected to Terminals 3 and 1 at the top of the furnace.

g) Connect the TEMP output terminals from the DTA unit to Channel 0 in the Channel Selector box using the black-shielded wire.

h) Connect the ΔT Output at the rear panel of the DTA unit to Channel 1 in the Channel Selector box using the yellow-shielded wire.

i) Connect the D/A output from the Channel Selector box to the PID OUTPUT terminals at the rear panel of the POWER AMPLIFIER unit.

  Purple wire in the end connector should go to the RED terminal of the PID OUTPUT.

j) Switch on the computer and select the dta.exe file.

  The menu-driven software will prompt you to enter the necessary details for running the DTA experiment.

  The photograph of the complete system is given in Figure 2.

![Figure 2. PC based DTA system.](image)

The performance of the system is demonstrated by some typical data on KNO$_3$ and CuSO$_4$·5H$_2$O. The PID parameters can be easily altered in the software to achieve the optimum linear rate of heating. It is worth mentioning that to achieve this linear rate of heating, sufficient care has to be taken in positioning the sample holder inside the furnace.

Figure 3 gives typical data on a DTA run carried out on KNO$_3$. The DTA signals observed near 129°C and 340°C corresponding to a structural transition and melting transition respectively, conform well with the published literature.
Figure 3. DTA of KNO₃.

Figure 4 gives the results of a DTA run carried out on CuSO₄·5H₂O in an open container. The first three peaks observed in the DTA plot are due to the following reactions:

\[ \text{CuSO}_4 \cdot 5\text{H}_2\text{O} \rightarrow \text{CuSO}_4 \cdot 3\text{H}_2\text{O} + 2\text{H}_2\text{O} (L) \]  \hspace{1cm} (a)

\[ 2\text{H}_2\text{O} (L) \rightarrow 2\text{H}_2\text{O} (g) \]  \hspace{1cm} (b)

\[ \text{CuSO}_4 \cdot 3\text{H}_2\text{O} \rightarrow \text{CuSO}_4 \cdot \text{H}_2\text{O} + 2\text{H}_2\text{O} (g) \]  \hspace{1cm} (c)

Figure 4. DTA of CuSO₄·5H₂O.
Chapter 17

Experiments in Percolation
17.1 Percolation Threshold for Resistance in Composites

1. Introduction

One can make a composite of an insulator and a conducting material, both of which are granular. One can mix fine-grained powders of these materials in different proportions by volume, and make pellets out of them in a hydraulic press. Whether the resultant material is insulating or conducting depends on the fractional volume \( p \) occupied by the conducting material. When this fractional volume is below a threshold \( p_c \), the material is insulating; when it is above this threshold, the material is conducting. Near, and above, the threshold concentration, the conductivity will vary as \( (p-p_c)^t \), where \( t \) is a critical exponent.

2. Percolation

One can make a lattice model for percolation. Consider a square lattice of points (Figure 1).

![Figure 1](image1.png)

**Figure 1.** A two-dimensional lattice with two adjacent points connected randomly by four conducting links.

Suppose we connect two adjacent lattice points *randomly* with a conducting link (shown by a line). There are 4 links \( (n) \) in the above figure and 36 lattice points \( (N) \). These conducting links will not make the lattice conduct from left to right. We increase the number of links. Above a critical number of links, one may have a conducting path from left to right. This is shown in Figure 2.

![Figure 2](image2.png)

**Figure 2.** A conducting path from left to right.
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The ratio of this critical number \( n_c \) to the total number of lattice points \( N \), in the limit \( N \) tending to infinity, is called the percolation threshold \( p_c \). If we have a one-dimensional lattice, then \( p_c \) must be 1, since any break in the chain will make the chain insulating. In higher dimensions, \( p_c \) will be less than 1.

In a continuum percolation model, we replace \( p = n/N \) by volume fraction \( \Phi \).

If \( R_N \) is defined as the probability of spanning a system of size \( N \), it will vary with \( p \) (or \( \Phi \)) as shown in Figure 3.

![Figure 3. Variation of \( R_N \) as \( p = n/N \) is varied.](image)

In the lattice model, the value of \( p_c \) will depend on (1) the number of dimensions, (2) the type of lattice, and (3) the coordination number for a given site.

Table 1 gives the percolation threshold for different types of lattices in different dimensions. From this table, we see that \( p_c \) decreases as the number of dimensions increases, and as the coordination number of a given site increases. The percolation threshold \( p_c \) also depends on the shape of overlapping objects.

<table>
<thead>
<tr>
<th>Lattice</th>
<th>Dimension ( D )</th>
<th>Coordination No. ( z )</th>
<th>( p_c ) or ( \Phi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Honeycomb</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Square</td>
<td>2</td>
<td>3</td>
<td>0.697</td>
</tr>
<tr>
<td>Triangular</td>
<td>2</td>
<td>6</td>
<td>0.657</td>
</tr>
<tr>
<td>Overlapping discs</td>
<td>2</td>
<td>–</td>
<td>0.676</td>
</tr>
<tr>
<td>Simple cubic</td>
<td>3</td>
<td>6</td>
<td>0.3116</td>
</tr>
<tr>
<td>BCC</td>
<td>3</td>
<td>8</td>
<td>0.2464</td>
</tr>
<tr>
<td>FCC</td>
<td>3</td>
<td>12</td>
<td>0.198</td>
</tr>
<tr>
<td>Overlapping spheres</td>
<td>3</td>
<td>–</td>
<td>0.2895</td>
</tr>
</tbody>
</table>

Table 1. Percolation thresholds for different types of lattices. (from Torquato et al. Ref. [1].)

We may define a quantity \( p_L \), which is the expected size of the largest cluster divided by \( N \). When \( p < p_c \), the value of \( p_L \) tends to zero as \( N \) tends to infinity. When \( p \) tends to 1, \( p_L \) will tend to \( p \). So, \( p_L \) behaves like an order parameter in a continuous phase transition. Near the critical threshold, \( p_L \) will vary as

\[
p_L \propto (p - p_c)^\beta.
\]

\( \beta \) is a critical exponent believed to be universal.

Suppose we take two lattice points at \( \mathbf{r} \) and \( \mathbf{r}' \), NOT in the largest cluster. The probability that the two points are connected will vary as

\[
\exp \left[-\frac{|\mathbf{r} - \mathbf{r}'|}{\xi}\right].
\]

\( \xi \) is called the correlation length. Near \( p_c \) the correlation length will vary as

\[
\xi \propto (p - p_c)^{-v}.
\]

As \( p_c \) is approached, the correlation length becomes large.
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The overall electrical conductivity of the material will vary with \( p \) or \( \Phi \) as shown in Figure 4.

\[ \langle \sigma \rangle \sim (p - p_c)^\nu. \]

(4)

We see that the conductivity starts increasing from \( p_c \). Near and above \( p_c \), the conductivity will vary as

\[ \langle \sigma \rangle \sim (p - p_c)^\nu. \]

The values of the critical exponents for two and three dimensions are given in Table 2.

<table>
<thead>
<tr>
<th>Exponent</th>
<th>Dimension</th>
<th>Dimension</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta )</td>
<td>5/36</td>
<td>0.41</td>
</tr>
<tr>
<td>( \nu )</td>
<td>1.33</td>
<td>0.88</td>
</tr>
<tr>
<td>( \tau )</td>
<td>0.33</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2. Critical exponents for percolation.

3. Experimental Procedure

Six pellets of mixtures of graphite and white cement, in different proportions by volume, were prepared: Sample denoted by \( n \) has \( n\% \) graphite and \((100-n)\%\) white cement. Samples with \( n = 5, 6, 7, 8, 9, \) and \( 10 \) were prepared by Prof. M S R Rao of the Department of Physics, IIT, Madras. The samples were mounted on either side of a rectangular copper strip of 2.5 cm width, and 5 cm length. There are three samples on each side.

The resistance of sample 5 was in excess of 50 megohms. As the volume fraction of graphite increased from sample 5 to sample 8, the resistance decreased to a value of 2.2 kilohms. So a two-probe constant voltage method was used to measure the resistance of these samples. Each sample was connected with a resistance \( r_j \) in series. The samples, with their series resistances, were connected in parallel to a 9 V battery. Let the voltage across the resistance \( r_j \) for the sample \( j \) be \( V_j \). The total applied voltage is \( V_0 \) (close to 9 V). Then the resistance \( R_j \) of the sample is

\[ R_j = \frac{(V_0 - V_j)}{V_j}. \]

(5)

The resistances \( r_j \) connected in series with the sample \( j \) were

\[ r_5 = 470 \, k\Omega, \quad r_6 = 47 \, k\Omega, \quad r_7 = 10 \, k\Omega, \quad \text{and} \quad r_8 = 220 \, \Omega. \]

Samples 9 and 10 had resistances in the ohms range. A four-probe method was used. A constant current source was connected to these samples in series so that the same current flowed through the samples. Two voltage leads from each sample were taken out to measure the voltage across the sample on a DMM in the DC 200 mV range. From the current \( I \) and the voltage \( V \), the resistance \( R \) is measured as

\[ R = \frac{V}{I}. \]

(6)

Sample readings are given in Table 3 for samples 5 to 8, and in Table 4 for samples 9 and 10.
Table 3. Resistance at room temperature of composites of white cement and graphite. Samples 5 to 8 \( V_0 = 8600 \text{ mV} \).

<table>
<thead>
<tr>
<th>Sample No.</th>
<th>Graphite %</th>
<th>( r_j (k\Omega) )</th>
<th>( V_j (\text{mV}) )</th>
<th>( R_j (\Omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5</td>
<td>470</td>
<td>28</td>
<td>( 1.44 \times 10^8 )</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>47</td>
<td>1202</td>
<td>( 2.89 \times 10^5 )</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>10</td>
<td>1085</td>
<td>( 6.93 \times 10^4 )</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>0.22</td>
<td>453</td>
<td>( 3.96 \times 10^3 )</td>
</tr>
</tbody>
</table>

Table 4. Resistance at room temperature of composites of white cement and graphite. Samples 9 and 10 Current \( I = 10.7 \text{ mA} \).

<table>
<thead>
<tr>
<th>Sample ( j )</th>
<th>( V (\text{mV}) )</th>
<th>( R (\Omega) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>112.5</td>
<td>10.5</td>
</tr>
<tr>
<td>10</td>
<td>49.9</td>
<td>4.7</td>
</tr>
</tbody>
</table>

4. Analysis of Data

In Table 5 the resistance values at 27.5 °C are collected for some values of the concentration \( p \) (5 to 10% for the samples 5 to 10).

<table>
<thead>
<tr>
<th>( p )</th>
<th>( R(\Omega) )</th>
<th>( 1/R )</th>
<th>( (p - p_c) )</th>
<th>( (p - p_c)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1.44E+08</td>
<td>6.94E-09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.92E+05</td>
<td>3.42E-06</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>6.93E+04</td>
<td>1.44E-05</td>
<td>0.7</td>
<td>0.49</td>
</tr>
<tr>
<td>8</td>
<td>3.99E+03</td>
<td>2.51E-04</td>
<td>1.7</td>
<td>2.89</td>
</tr>
<tr>
<td>9</td>
<td>1.05E+01</td>
<td>9.51E-02</td>
<td>2.7</td>
<td>7.29</td>
</tr>
<tr>
<td>10</td>
<td>4.67E+00</td>
<td>2.14E-01</td>
<td>3.7</td>
<td>13.69</td>
</tr>
</tbody>
</table>

Figure 5 shows a plot of \( 1/R \) against \( p \). This is fitted to a second degree polynomial.

\[
\frac{1}{R} = A + Bp + Cp^2.
\]

![Figure 5](image.png)

**Figure 5.** A plot of \( 1/R \) against concentration \( p \). Fit is to a second degree polynomial.

The values of the coefficients, with errors, are given in Table 6.
Experiments in Percolation

Table 6

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.689</td>
<td>0.189</td>
</tr>
<tr>
<td>B</td>
<td>-0.222</td>
<td>0.053</td>
</tr>
<tr>
<td>C</td>
<td>0.017</td>
<td>0.003</td>
</tr>
</tbody>
</table>

According to theory, above \( p_c \), \( 1/R \) should be proportional to \( (p - p_c)^2 \), i.e.,

\[
\frac{1}{R} = G (p - p_c)^2 = Gp_c^2 - 2Gp_c p + Gp^2.
\]

So

\[
A = Gp_c^2, \\
B = -2Gp_c.
\]

Therefore, the critical concentration \( p_c = (-2A/B) \). Using the values of \( A \) and \( B \) from Table 6, we get

\[
p_c = \frac{-2 \times 0.689}{-0.222} = 6.3.
\]

For \( p = 7 \) to 10, the values of \( (p - p_c) \) and \( (p - p_c)^2 \) are given in columns 4 and 5 respectively of Table 5. A plot of \( 1/R \) against \( (p - p_c)^2 \) is shown in Figure 6, for the concentrations 7 to 10. We get a reasonable fit to a linear relation between \( 1/R \) and \( (p - p_c)^2 \) as required by theory.

Thus, for this composite of white cement and graphite, the percolation threshold is about 6.3 volume % of graphite. Above this concentration, the conductivity varies roughly as \( (p - p_c)^2 \).

For further information, the following references may be consulted

17.2 Variable Range Hopping in a Composite

1. Introduction

In a metal, the conduction electrons move freely. When an electric field $E$ is applied, they develop a drift velocity $v_D$, in the direction of $E$, given by

$$v_D = \mu E,$$

where $\mu$ is called the mobility of the charge carriers. Electrons are scattered by the vibrating atoms. So the mobility decreases as the temperature increases.

The electrical conductivity in a metal or a doped semiconductor is given by

$$\sigma = nq\mu.$$

Here $n$ is the number of charge carriers per unit volume, and $q$ is the charge on the carriers. In metals, $n$ is very large ($\approx 10^{27}/m^3$) and independent of temperature. Since the mobility decreases as the metal is heated, the electrical conductivity $\sigma$ of a metal will decrease as the temperature increases. The resistance, which is inversely proportional to the conductivity, will increase in a metal as the temperature is increased. In a semiconductor, the number density $n$ of carriers increases exponentially with temperature. For lightly doped semiconductors,

$$n = n_0 \exp\left(\frac{-E_g}{2kT}\right).$$

Here $E_g$ is the energy band gap of the material, $k$ is the Boltzmann’s constant and $T$ is the absolute temperature. This exponential increase in the number of charge carriers more than compensates for the decrease in the mobility as the temperature of a semiconductor increases. So, the electrical conductivity of a semiconductor increases rapidly as its temperature is increased. Consequently, the resistance of a semiconductor decreases rapidly with temperature.

But this simple model will be inapplicable in the composite, where we have conducting grains of graphite interspersed with insulating grains of white cement. In such a case, the electron can move freely in graphite, but will find an energy barrier at the interface between graphite and white cement grains. The hopping model is appropriate for such a situation.

2. Activated Conduction Model

In this model, the electron hops from one site to another. Figure 1 shows schematically the potential barrier in a simple lattice model.

![Figure 1](image)

Figure 1. One-dimensional model for hopping conduction.

Let us assume that we have two sites A and A' on either side of the origin O at equal distance from O. An electron has to cross a potential barrier in hopping from O to A, or O to A'. The sites will represent graphite grains, and the potential barrier the white cement grains. The potential barrier on either side is shown in the figure, and for simplicity is assumed to be equal in height. This barrier has a height $E_g$. At a temperature $T$ (in Kelvin) the probability for an electron to cross the barrier and go from O to either A or A' is proportional to $\exp\left(-\frac{E_g}{kT}\right)$. The number of electrons which will jump in unit time from O to A will be equal to the number which will jump in the reverse direction O to A'. The net current will be zero.

If an electrical field $E$ is applied parallel to the line, the potential barrier to the left will be lowered by $eV$ and the barrier on the right will be raised by $eV$, since the potential at A' will be $+V$ and the potential at
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A will be \(-V\), relative to the potential at the origin. Remember that the charge on the electron is \(-e\). So the probability for an electron to jump from O to A will now be proportional to

\[
\exp \left( -\frac{(E_a - eV)}{kT} \right),
\]

while the probability of hopping from O to A will now be proportional to

\[
\exp \left( -\frac{(E_a + eV)}{kT} \right).
\]

If \(eV\) is small compared to \(E_a\), i.e., for small electric fields, the difference in probability will be proportional to

\[
\left( \frac{eV}{kT} \right) \exp \left( -\frac{E_a}{kT} \right).
\]

This results in a current, in the direction of the field, and proportional to the field. So, the electrical conductivity \(\sigma\) in such a material will be proportional to \((1/T) \exp (-E_a/kT)\), and the resistance of the material will vary as

\[
R \propto T \exp \left( \frac{E_a}{kT} \right).
\]

A plot of \([\ln (R) - \ln (T)]\) versus \(1/T\) will give a straight line with slope \(E_a/k\).

3. Mott’s Variable Range Hopping Model

The above model is too simple for the composite of conducting and insulating materials. In a disordered material, the grain sizes of graphite and white cement may not be uniform, and the distance between two graphite grains will have a distribution. The potential barrier will depend on the distance between two centers, between which the electron hops. The electron is localized at center O, because it is trapped at the site for some time. Its wave function will decrease as \(\exp (-ar)\), where \(a\) is the inverse of the localization length. The probability that the electron will hop a distance \(r\) will be proportional to the square of the wave function, i.e., to \(\exp (-2ar)\). In addition, the probability will depend on \(\exp (-E_a(r)/kT)\), where \(E_a(r)\) is the barrier height between two grains at a distance \(r\). So the probability that an electron hops a distance \(r\) is proportional to

\[
\exp \left( -\frac{2ar + E_a(r)}{kT} \right).
\]

Mott used the following argument to get the dependence of \(E_a(r)\) on \(r\). Consider a sphere of radius \(r\). Let the number density of states (i.e., the number of energy levels per unit energy difference per unit volume) be \(N_0\). The volume of the sphere is \(\frac{4\pi r^3}{3}\). The number of energy levels per unit energy interval in this volume is

\[
n = N_0 \left( \frac{4\pi r^3}{3} \right).
\]

The average spacing of the energy levels is

\[
\Delta E = \frac{1}{n} = \frac{3}{(4\pi N_0 r^3)} = \frac{\beta_0}{r^3},
\]

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where \( \beta_0 \equiv 3/(4\pi N_0) \). \( \Delta E \) takes the place of \( E_a(r) \) in the equations (6–8) for electric conduction. We see that the localization of the electron will favor short hops, while \( \Delta E \) will favor long hops. So, there is a value of \( r \) for which the hopping probability becomes a maximum.

This value of \( r \) can be obtained by maximizing

\[
\exp \left( - \left( 2ar + \left( \frac{\beta_0}{kTr^2} \right) \right) \right).
\]

The maximum occurs when

\[
r^4 = \left[ \frac{3\beta_0}{2akT} \right].
\]

Putting this value of \( r \) in the expression for the probability, the maximum probability is

\[
\exp\left( -\left( \frac{T_0}{T} \right)^{1/4} \right),
\]

where

\[
T_0 = \left( \frac{8}{3} \right)^4 \left( \frac{3\alpha^3 \beta}{2k} \right).
\]

So, conductivity will increase with temperature as

\[
\left( \frac{1}{T} \right) \exp\left( -\left( \frac{T_0}{T} \right)^{1/4} \right),
\]

and resistance \( R \) will vary as

\[
R \propto T \exp\left( \frac{T_0}{T} \right)^{1/4}.
\]

\( T_0 \) will be a constant for a given specimen.

Mott’s expression for the dependence of \( E_a \) on \( r \) is not derived from rigorous arguments. But more rigorous theoretical calculations support Mott’s hypothesis.

Unless one makes a high precision resistivity measurement over a wide temperature range, it will be difficult to decide whether the temperature variation of resistance follows a simple activation model, or the variable range hopping model.

4. Experimental Procedure

Samples 9 and 10 of composites of white cement with graphite, mentioned in the previous section, were mounted on a copper plate with superglue. A Pt100 thermometer was also attached to the copper block. The current and voltage leads for the two samples, as well as the leads of the platinum thermometer, were brought to a terminal block. The samples hang from the terminal block inside the furnace. A constant current was sent through both the samples in series and the voltages across the samples were measured with a DMM. The temperature indicator connected to the platinum resistance thermometer gave the sample temperature in \(^\circ\)C. A DC power supply was connected to the furnace, and the voltage of the power supply was adjusted to give a rate of rise of temperature of approximately 1 \(^\circ\)C per minute.

Sample readings are given in Tables 1 and 2.
Experiments in Percolation

Table 1. Current $I = 10.7$ mA. Sample 9.

<table>
<thead>
<tr>
<th>$T$ (°C)</th>
<th>$T$ (K)</th>
<th>$1/T$ (K)</th>
<th>$V$ (mV)</th>
<th>$R$ (Ω)</th>
<th>ln$R$-ln$T$</th>
<th>$(1/T)^{0.25}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>27.5</td>
<td>300.5</td>
<td>0.003328</td>
<td>112.5</td>
<td>10.514</td>
<td>-3.352</td>
<td>0.2401</td>
</tr>
<tr>
<td>37.5</td>
<td>310.5</td>
<td>0.003221</td>
<td>111.4</td>
<td>10.411</td>
<td>-3.395</td>
<td>0.2383</td>
</tr>
<tr>
<td>42.5</td>
<td>315.5</td>
<td>0.00317</td>
<td>110.9</td>
<td>10.364</td>
<td>-3.415</td>
<td>0.2376</td>
</tr>
<tr>
<td>47.5</td>
<td>320.5</td>
<td>0.00312</td>
<td>110.2</td>
<td>10.299</td>
<td>-3.437</td>
<td>0.2364</td>
</tr>
<tr>
<td>52.5</td>
<td>325.5</td>
<td>0.003072</td>
<td>109.4</td>
<td>10.224</td>
<td>-3.460</td>
<td>0.2352</td>
</tr>
<tr>
<td>57.5</td>
<td>330.5</td>
<td>0.003026</td>
<td>108.5</td>
<td>10.140</td>
<td>-3.484</td>
<td>0.2347</td>
</tr>
<tr>
<td>62.5</td>
<td>335.5</td>
<td>0.002981</td>
<td>107.3</td>
<td>10.028</td>
<td>-3.510</td>
<td>0.2339</td>
</tr>
<tr>
<td>67.5</td>
<td>340.5</td>
<td>0.002937</td>
<td>105.9</td>
<td>9.897</td>
<td>-3.538</td>
<td>0.2323</td>
</tr>
<tr>
<td>77.5</td>
<td>350.5</td>
<td>0.002853</td>
<td>103.8</td>
<td>9.700</td>
<td>-3.587</td>
<td>0.2315</td>
</tr>
<tr>
<td>82.5</td>
<td>355.5</td>
<td>0.002813</td>
<td>102.3</td>
<td>9.560</td>
<td>-3.615</td>
<td>0.2302</td>
</tr>
<tr>
<td>87.5</td>
<td>360.5</td>
<td>0.002774</td>
<td>100.6</td>
<td>9.401</td>
<td>-3.646</td>
<td>0.2297</td>
</tr>
<tr>
<td>92.5</td>
<td>365.5</td>
<td>0.002736</td>
<td>99.1</td>
<td>9.261</td>
<td>-3.675</td>
<td>0.2287</td>
</tr>
<tr>
<td>97.5</td>
<td>370.5</td>
<td>0.002699</td>
<td>97.7</td>
<td>9.130</td>
<td>-3.703</td>
<td>0.2279</td>
</tr>
<tr>
<td>102.5</td>
<td>375.5</td>
<td>0.002663</td>
<td>95.8</td>
<td>8.953</td>
<td>-3.736</td>
<td>0.2271</td>
</tr>
<tr>
<td>107.5</td>
<td>380.5</td>
<td>0.002628</td>
<td>94.4</td>
<td>8.868</td>
<td>-3.764</td>
<td>0.2264</td>
</tr>
<tr>
<td>112.5</td>
<td>385.5</td>
<td>0.002594</td>
<td>92.1</td>
<td>8.607</td>
<td>-3.801</td>
<td>0.2256</td>
</tr>
<tr>
<td>117.5</td>
<td>390.5</td>
<td>0.002561</td>
<td>90.1</td>
<td>8.420</td>
<td>-3.836</td>
<td>0.2249</td>
</tr>
<tr>
<td>122.5</td>
<td>395.5</td>
<td>0.002528</td>
<td>88.4</td>
<td>8.261</td>
<td>-3.868</td>
<td>0.2242</td>
</tr>
<tr>
<td>127.5</td>
<td>400.5</td>
<td>0.002497</td>
<td>87.1</td>
<td>8.140</td>
<td>-3.895</td>
<td>0.2235</td>
</tr>
</tbody>
</table>

In Tables 1 and 2, the first column gives the temperature in degrees Celsius, and the second in degrees Kelvin. The third column gives the value of $1/T$ ($T$ in K). The fourth column gives the measured voltage in millivolts. The fifth column gives the resistance $R$ in ohms. The sixth column gives ln$R$-ln$T$, where ln is logarithm to base e. The last column gives $(1/T)^{0.25}$.

Figure 2 shows a plot of ln$R$-ln$T$ against $1/T^{1/4}$ for samples 9 and 10.
The fit to the VRH model is reasonably good. The slopes of the lines are 33 for sample 9 and 28.5 for sample 10.

Thus, the temperature variation of resistance in the conducting samples of the composite of white cement and graphite obey the Mott’s variable range formula reasonably well.
Chapter 18

Appendix
18.1 Sensors

1. Introduction

Before we talk about sensors, we need to understand how a measurement is made electronically. The first step in measuring any variable, like temperature or resistance, is to convert the variable into an electrical signal through a sensor. For example, using a piezoelectric sensor we convert force into a voltage signal. This signal is often very small, and is beset with noise. We have to condition the signal by amplifying it, and eliminating unwanted noise using suitable filters. Then, the processed signal is sent to a measuring instrument like an oscilloscope or a voltmeter, and is recorded. This is how a measurement is made electronically.

![Figure 1](image)

**Figure 1.** (a) Analog signal as a function of time. (b) Same signal in digitized form.

Signals can be of two types: (i) analog or (ii) digital. An analog signal varies continuously with time, as shown in Figure 1(a). The same signal in a digitized form is shown in Figure 1(b). The digitized signal can read to two digits. It changes discretely in steps of one in the last digit. Digital signals can be processed with microprocessors and computers. To convert from analog to digital signals, one uses an analog to digital (A to D) converter circuit and vice-versa. Digital signal processing is advantageous, but costlier, compared to analog signal processing. In all the experiments described in this manual, we use only analog signals, and analog processing with operational amplifiers. The final output is read on a digital Multimeter.

In this section, we will deal with sensors which we have used in the experiments. Temperature sensors are used to measure temperature. We have used the other sensors to show how they generate a signal, when they are subjected to a given stimulus. We will discuss these sensors. We will not deal with signal processing. We give references to a few books which deal with operational amplifiers and their applications.

2. Sensors and Actuators

There are many physical variables which need to be measured accurately, if we want to understand the behavior of physical systems. Examples of physical variables are displacement, velocity, temperature, pressure, electric and magnetic fields, and light intensity. We use sensors to measure the values of the physical variables. Some property of a sensor changes when the physical variable changes. A measurement of this property is used to measure the value of the variable.

As an example, consider this situation. A liquid is heated, and we want to know how its temperature changes with time. The sensor used could be a thermocouple, one junction of which is immersed in the liquid, and the other junction is kept in an ice bath. The thermocouple generates a thermo-emf, when the two junctions are at different temperatures. The thermo-emf increases as the temperature difference increases.
Appendix

This thermo-emf can be amplified, and measured with a millivoltmeter. Thus, the thermocouple is a sensor to measure temperature.

Consider another example. A magnetic field is to be measured. A lightly doped semiconductor is used. It is placed in a magnetic field, directed along its thickness. A current is passed along the length of the semiconductor. Then a voltage appears along the width of the semiconductor, which is transverse both to the direction of the current, and the direction of the magnetic field. This is called the Hall voltage, and a measurement of the Hall voltage, at a given value of the current, can be used to sense, and measure, the magnetic field.

An actuator is used to control a physical variable, like the position of a tip or the temperature of a bath. In scanning probe microscopes, we need to position the tip of a lever exactly on a sample, and to move the tip by specific amounts in a given direction. We use a piezoelectric tube or rod for this purpose. The dimensions of a piezoelectric material change by precise amounts when a voltage is applied to the piezoelectric material. The tip is attached to one end of a piezoelectric rod. The other end of the rod is fixed. We can change, and fix, the position of the tip by controlling the voltage applied to the piezoelectric rod. This is an example of an actuator.

In this section we will discuss only sensors.

3. Active and Passive Sensors

Consider the two sensors described in the previous subsection. The thermocouple generates a thermo-emf on its own, when one junction of the thermocouple is heated. It does not need an external power supply or current. Such a sensor, that does not need any external excitation to enable the measurement of a physical variable, is called a passive sensor.

The Hall sensor, used to measure a magnetic field, requires an external excitation, namely a current through it, if it has to function. Such a sensor is called an active sensor.

4. Factors Important in the Selection of Sensors

Different sensors can be used to measure a physical variable. For example, temperature can be measured using a mercury-in-glass thermometer, a resistance thermometer or a thermocouple. Magnetic field can be measured using a search coil, a Hall probe, a magnetoresistive material, or a Superconducting Quantum Interference Device (SQUID). Which sensor we choose will depend on various factors.

The factors of importance in selecting a sensor are:

(i) Sensitivity
(ii) Linearity
(iii) Dynamic response
(iv) Range
(v) Precision, Accuracy and reproducibility
(vi) Ease of measurement
(vii) Cost

We will briefly discuss each factor, taking the example of sensors used to measure temperature.

(i) Sensitivity

This can be defined as the change in the required property of the sensor, for unit change in the quantity of the physical variable.

Let us consider the platinum resistance thermometer. In a platinum resistance thermometer, the resistance $R$ varies with temperature. If we take a platinum resistance thermometer with a resistance of 100 ohms at ice point (called Pt100), the resistance changes by 0.39 ohms with every degree rise in temperature. This is the sensitivity of the Pt 100 resistance thermometer. If we use a chromel–alumel thermocouple, the thermo-emf changes by 40 $\mu$V for one degree Centigrade change in temperature. This is the sensitivity of the thermocouple.
(ii) **Linearity**

The value of the property, used to measure the variable, may vary linearly with the variable, or non-linearly with the variable. Figure 2(a) shows the variation of resistance of a platinum resistance thermometer from 0 to 300 °C. In Figure 2(b) is shown the thermo-emf of a copper–constantan thermocouple over the same temperature range, when the reference junction is maintained at 0 °C.

![Figure 2](image)

**Figure 2.** (a) Variation of resistance of Pt 100 with temperature. (b) Variation of thermo-emf of Cu–constantan thermocouple with temperature.

We see from Figure 2 that the variation of resistance of a Pt 100 thermometer is linear with temperature in the range 0 to 300 °C, while the variation of the thermo-emf of a copper–constantan thermocouple is nonlinear. The slope of the graph increases as the temperature increases.

All sensors need calibration. Calibration implies determining the values of the property of a sensor at a few well-defined standard values of the variable. For temperature measurement, the standard points are internationally accepted triple, melting, or boiling points of a few materials. If the response of the sensor is linear, we need to calibrate the sensor only at two standard points. For a sensor with nonlinear characteristics, the calibration has to be done at many standard points. A linear response also implies an easier interpolation to measure the variable.

A sensor which gives a linear response is to be preferred over one which has a nonlinear response.

(iii) **Dynamic response**

Often one measures a variable varying with time. The response of the sensor to such a time-varying variable is called the dynamic response. If the change in the variable $X(t)$ is a unit step function at time $t = 0$, as shown in Figure 3, the response of the sensor may follow one of the characteristics discussed below:

![Figure 3](image)

**Figure 3.** Change in the variable $X(t)$. 

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(a) First order response

The differential equation satisfied by the response \( R(t) \) to a unit step function of the variable is

\[
\frac{dR(t)}{dt} + \frac{R(t)}{\tau} = SX(t).
\]  

(1)

Here, \( S \) is the sensitivity of the sensor and \( \tau \) is called the relaxation time.

The response of the system is

\[ R(t) = R_{\text{max}} \left( 1 - e^{-t/\tau} \right). \]  

(2)

This response is shown in Figure 4.

![Figure 4. Dynamic response of the first order sensor.](image)

The response is delayed, and reaches its maximum value after infinite time. Characteristically, if \( t/\tau > 5 \), the response reaches a value more than 0.993 \( R_{\text{max}} \). The relaxation time \( \tau \) is dependent on the properties of the sensor, and how it is mounted. A sensor showing such a response is called a first order sensor.

One may take the Laplace transform of the response \( R(t) \). The Laplace transform of a function \( F(t) \) is defined as

\[
F(s) = \int_0^\infty F(t) e^{-st} \, dt.
\]  

(3)

For a first order system, the response function \( R(s) \) is

\[ R(s) = \frac{R_{\text{max}}}{s + (1/\tau)}. \]  

(4)

If the signal varies in a sinusoidal fashion with time, with a frequency \( f \), then the response of the sensor at \( \omega = 2\pi f \) is given by

\[ R(\omega) = \left\{ \frac{R(0)}{1 + j\omega\tau} \right\} \exp(j\omega t), \]  

where \( R(0) \) is the response at zero frequency and \( j = \sqrt{-1} \). The amplitude of response is \( R(0)/[1 + (\omega\tau)^2]^{1/2} \), and the phase lag between the variable and the response is \( \phi = \tan^{-1}(\omega\tau) \). The amplitude of response will be flat if \( \omega\tau \ll 1 \). If \( \omega\tau \gg 1 \), the response will decrease as \( 1/\omega \). So, such a sensor will be useful for measuring a sinusoidal signal if the frequency of the signal is much smaller than \( 1/2\pi \).
Appendix

An example of a first order system is the thermocouple and resistance thermometer. The value of \( \tau \) is directly proportional to the thermal capacity of the sensor and inversely proportional to a constant related to the heat loss to its surroundings.

(b) Second order response

The response of a second order sensor will satisfy the following differential equation, when a unit step variable \( X(t) \) is applied to it.

\[
\frac{d^2 R}{dt^2} + 2\gamma \omega_0 \left( \frac{dR}{dt} \right) + \omega_0^2 R = SX(t).
\]

Here, the parameter \( \gamma \) describes the damping, and the parameter \( \omega_0 \) represents the natural frequency. For example, consider a strain sensor bonded to a beam to measure its strain. The beam, on which the sensor is mounted, will have a natural frequency and a damping.

Such a sensor will show three different types of responses, namely (i) under-damped, (ii) critically damped, and (iii) over-damped. These are shown in Figures 5(a), 5(b) and 5(c). \( \gamma < 1 \) represents an under-damped sensor, \( \gamma = 1 \) a critically damped sensor, and \( \gamma > 1 \) an over-damped sensor.

![Figure 5(a)](image)

**Figure 5(a).** Dynamic response function of an under-damped second order sensor for \( R_0 = 1 \).

![Figure 5(b)](image)

**Figure 5(b).** Dynamic response function of a critically damped second order sensor for \( R_0 = 1 \).
Appendix

$$R = R_0[1-\frac{1}{(b-a)}(b e^{-\omega t} - a e^{-bt})]$$
$$a = [\Gamma-(\Gamma^2-1)^{1/2}]\omega_0; \quad b = [\Gamma+(\Gamma^2-1)^{1/2}]\omega_0$$

Figure 5(c). Dynamic response function of an over-damped second order sensor for $R_0 = 1$.

When the variable changes from one value to another, we see that the response of the sensor oscillates about the steady value $R_0$ for some time, before it settles down in the under-damped case (Figure 5(a)). For the critically damped case (Figure 5(b), the response of the sensor approaches the steady value without oscillations in a shorter time compared to the over-damped case (Figure 5(c)).

If the variable varies in a sinusoidal fashion with time, with an amplitude $X_0$, we replace $X(t)$ in equation (6) by $X_0 e^{j\omega t}$. In this case, the response $R(\omega)$ is complex and is given by

$$R(\omega) = \left\{ \frac{S X_0}{(\omega_0^2 - \omega^2) + j2\Gamma\omega_0\omega} \right\} e^{j\omega t}. \quad (7)$$

The response will show a resonant behaviour at $\omega = \omega_0$, and the response will differ in phase from the variable. When the applied frequency $\omega \ll \omega_0$, the response will be reasonably flat. This is the range of frequencies in which the sensor can be used for measurement.

(iv) Range

This is the range of the variable over which the sensor could be used. Let us take the example of a platinum resistance thermometer and a germanium resistance thermometer. The platinum resistance thermometer can be used from 20 K to 700 K. Below 20 K the sensitivity of the thermometer becomes very low. On the other hand, the germanium resistance thermometer is very sensitive below 10 K, and loses its sensitivity above this temperature. So its range is from 4 K to 10 K, while the range of the platinum resistance thermometer is from 20 K to 700 K.

(v) Precision, accuracy and reproducibility

If we measure the same variable, say melting temperature of a solid, twice with a given sensor, the two values will not be the same. If the two values agree to within a certain limit, which is small (say a millidegree Celsius), we say that the sensor is precise to within this limit.

There is a difference between precision and accuracy. Accuracy is the difference between the measured value and the true value of the variable. Precision is the difference in the value of the same variable measured with the same sensor repeatedly. A sensor may be precise to within a millidegree Celsius, but may be accurate only to 0.01 °C. One should not confuse precision with accuracy.

Often we use a sensor over a long period of time, say several years. If we measure a standard variable, say the melting point of pure silver, with the sensor over a long period of time, we may find the value changing continuously with time in one direction. We then say that the sensor reading is not reproducible over this period of time. Ageing may change the calibration of a sensor. To make accurate measurements one needs to calibrate the sensor periodically.
Appendix

A sensor is used often with associated instruments to carry out the measurement. The precision of the measurement will depend on the errors involved with all the associated instruments. For example, let us say that we use a microvoltmeter to measure the thermo-emf of a thermocouple, and the microvoltmeter can read only to a precision of ±1 μV. A temperature difference of 1 °C corresponds to a thermo-emf of about 40 μV. This can be measured only to within ±1 μV with the microvoltmeter. The temperature difference of 1 °C can only be measured with a precision of 2.5%. If we want to measure the change in temperature of 1 °C with a precision of ±0.5%, we need to amplify the thermo-emf by a factor of 5.

(vi) Ease of measurement

Some properties can be measured more easily than others. Precision resistance measurement is relatively easier than precision measurement of other properties. So, if a sensor measures a variable using its resistance change, and if the sensitivity of resistance change is high, one would prefer to use a resistive sensor.

(vii) Cost

Cost is a major factor if funds available are limited. One will have to balance the cost of a sensor against its other advantages like better sensitivity, linearity and faster response time.

4. Sensors for Different Variables

Table 1 gives the sensors used to measure different variables. This table is not exhaustive. We will discuss only some of the sensors listed in the table. The choice is dictated by the sensors used in the experiments described in earlier Chapters.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sensor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement</td>
<td>Potentiometer</td>
</tr>
<tr>
<td></td>
<td>Differential transformer</td>
</tr>
<tr>
<td>Stress, Strain</td>
<td>Strain gauge</td>
</tr>
<tr>
<td></td>
<td>Piezoelectric sensors</td>
</tr>
<tr>
<td>Pressure</td>
<td>Diaphragm pressure gauge</td>
</tr>
<tr>
<td></td>
<td>Capacitive pressure gauge</td>
</tr>
<tr>
<td></td>
<td>Piezoelectric sensors</td>
</tr>
<tr>
<td>Liquid flow</td>
<td>Orifice meter</td>
</tr>
<tr>
<td></td>
<td>Turbine flow meter</td>
</tr>
<tr>
<td>Temperature</td>
<td>Thermocouples</td>
</tr>
<tr>
<td></td>
<td>Resistance thermometers</td>
</tr>
<tr>
<td></td>
<td>Radiation pyrometers</td>
</tr>
<tr>
<td>Magnetic field</td>
<td>Search coil</td>
</tr>
<tr>
<td></td>
<td>Hall Effect sensors</td>
</tr>
<tr>
<td></td>
<td>GMR sensors</td>
</tr>
<tr>
<td></td>
<td>Squid sensors</td>
</tr>
<tr>
<td>Light intensity</td>
<td>Photodiodes</td>
</tr>
<tr>
<td></td>
<td>Phototransistors</td>
</tr>
</tbody>
</table>
Appendix

In the discussion below, the sensors are grouped according to the property which changes with the variable.

SENSORS DEPENDENT ON RESISTANCE

In these sensors, the resistance of the sensor changes with the variable. The resistance change can be measured using a Wheatstone bridge shown in Figure 6.

![Wheatstone Bridge](image)

Figure 6. Wheatstone bridge for a resistive sensor.

In this bridge the two arms marked \( R \) have equal precision resistances. The arm \( R_3 \) has a variable resistance and \( S \) is the resistive sensor. When the variable is at a reference value (say the temperature is at ice point), the precision resistor \( R_3 \) is adjusted to balance the bridge. Then \( R_3 \) is equal to \( S_0 \), the value of the resistance of the sensor at the reference value of the variable. The resistance of the sensor changes to \( S_0 + \Delta S \) when the variable changes from the reference value. This will give an out-of-balance signal \( V_0 \), which can be related to the change in resistance \( \Delta S \) of the sensor by the following equation.

\[
V_0 = V_S \left\{ \frac{(S_0 + \Delta S)}{(R + S_0 + \Delta S)} - \frac{R_3}{(R + R_3)} \right\}.
\]

Here \( R_3 = S_0 \) when the bridge is initially balanced, and \( V_S \) is the source voltage applied to the bridge. If \( \Delta S \) is small compared to \( S_0 \), then

\[
V_0 = V_S \Delta S \left[ \frac{R}{(R + S_0)^2} \right].
\]

For small values of the change \( \Delta S \) compared to \( S_0 \), the out-of-balance voltage varies linearly with \( \Delta S \).

If \( V_S \) is an applied DC voltage, then \( V_0 \) will be positive if \( \Delta S \) is positive, and \( V_0 \) will be negative if \( \Delta S \) is negative. The output voltage \( V_0 \) can be amplified and measured.

If \( V_S \) is an applied AC voltage, then \( V_0 \) will also be an AC voltage. But the phase of \( V_0 \) will be the same as the phase of \( V_S \) if \( \Delta S \) is positive. \( V_0 \) will differ in phase by 180° with \( V_S \) if \( \Delta S \) is negative. Using phase sensitive detection discussed in Section 11.1, the sign and magnitude of \( \Delta S \) can be determined.

(i) Resistive position sensor

A convenient sensor for determining the linear or angular position and displacement is based on the principle of the potentiometer.

In a potentiometer, a wire is wound either on a linear former, or on a former which is in the form of a torus. The total resistance of the potentiometer is \( R_T \). There is a variable sliding contact which moves along the length of the winding. In the linear potentiometer (Figure 7(a)) the sliding contact moves along a line, and in the circular potentiometer (Figure 7(b)) it moves along a circle. The variable
Figure 7. (a) a potentiometer for measuring linear displacement. (b) a potentiometer for measuring angular displacement.

Point divides the total resistance in two parts, part 1 having a resistance $R_1$ and part 2 having a resistance $R_2$. If a potential $V_0$ is applied across the total resistance, the voltage at the variable point has a value

$$V = \frac{V_0 R_2}{R_T}.$$  \hfill (10)

If the windings are uniform, the displacement of the variable point (linear in Figure 7(a) and angular in Figure 7(b)) will be proportional to $V$.

(ii) Resistive temperature sensor

One may have two classes of resistive temperature sensors, namely (a) those having a positive temperature coefficient of resistance (PTR), and (b) those having a negative temperature coefficient of resistance (NTR).

Among the PTR resistive temperature sensors, the most useful sensor is the platinum resistor. Platinum can be obtained in high purity, can be drawn in the form of thin wires, and is not corroded in the atmosphere. Platinum resistors have the added advantage that the resistance varies linearly with temperature over a very large range of temperature, and the sensitivity is relatively high. For Pt 100 and Pt 1000 sensors, the resistance as a function of temperature is given by the formula

$$R(T) = R_0 \left[ 1 + AT + BT^2 + C (T - 100) T^3 \right].$$  \hfill (11)

Here, $R_0 = 100$ ohms for Pt 100, (or 1000 $\Omega$ for Pt 1000) at 0 $^\circ$C,

$$A = 3.9083 \times 10^{-3} / ^\circ C,$$

$$B = -5.7550 \times 10^{-7} / ^\circ C^2,$$

$$C = -4.183 \times 10^{-12} / ^\circ C^4 \text{ for } T < 0$$

$$= 0 \text{ for } T > 0.$$  \hfill (11)

Here $T$ is the temperature in Celsius scale.

Platinum resistance thermometers in a glass or ceramic enclosure usually weigh around a few 100 mg and have dimensions 2 mm dia and about a few cm length. One can also get platinum resistance thermometers in a foil of about 2 mm $\times$ 2 mm size. The Pt thermometers can be used from 14 to 873 K. Below 14 K the thermal capacity is large, and such Pt resistance thermometers are not suitable for measuring rapidly varying temperatures.

The thermistor is an example of an NTR thermometer. Figure 8 shows the temperature variation of resistance of a typical thermistor.
Figure 8. Temperature variation of the resistance of a thermistor.

The resistance of a thermistor increases rapidly as the temperature decreases. The sensitivity of a thermistor, which is the slope of the curve in Figure 8, increases as the temperature decreases. Every thermistor needs to be calibrated individually. The calibration is not stable over long periods of time. The thermistor also has a mass comparable to the large Pt resistance thermometer and thus is not suitable for recording temperatures varying rapidly.

In the experiments described in earlier chapters, the Pt 100 resistance thermometer is extensively used to measure temperature. It is relatively inexpensive.

(iii) Strain gauge

This is a resistive sensor used for measuring strain. The bonded strain gauge is shown in Figure 9.

Figure 9. Bonded strain gauge to measure strain in a cantilever when bent.

A patterned metallic film, between protecting foils, forms the sensor element. It is bonded to the structure undergoing strain, for example, a cantilever. The resistance of the film changes when the cantilever is bent.
Appendix

This change in resistance is measured using a Wheatstone bridge. The fractional change in resistance is

\[ \frac{\Delta R}{R} = K \varepsilon. \]  \hspace{1cm} (12)

Here, \( \varepsilon \) is the strain causing the change in resistance, and \( K \) is called the gauge factor. This linear relation is valid for small strains. For larger strains, the relation between \( \Delta R/R \) and \( \varepsilon \) is nonlinear. A strain gauge needs calibration.

The unbonded strain gauge is shown in Figure 10.

![Figure 10](image)

**Figure 10.** Unbonded strain gauge for measuring force.

In this case, a movable platform will slide on a fixed platform when a force is applied. Resistance wires are stretched between pegs on the fixed platform and pegs on the movable platform. When a force is applied to the movable platform, the wires get stretched and the resistances of the wires change. Unbonded strain gauges can be used to measure displacement or force.

(iv) **Photoconductive element to measure light intensity**

A semiconducting film such as CdS will change its resistance when light falls on it. This is used to measure intensity of light in a light flux meter.

Figure 11(a) shows such a light sensing film and Figure 11(b), plotted on a log-log scale, shows how the resistance of the film changes with the intensity of the light beam.

![Figure 11](image)

**Figure 11.** (a) Photoresistive film on a substrate. (b) Variation of resistance with light flux in lumens.
Appendix

(v) Magnetic field sensors using resistance change

The giant magnetoresistance (GMR) effect was discovered in 1980 by Albert Fert in France, and Peter Grünberg in Germany. In ferromagnetic materials, the energy bands of down spin and up spin electrons are split. Down spin electrons have a spin magnetic moment in the direction of the remnant magnetization in the ferromagnetic metal. Up spin electrons have a magnetic moment in a direction opposite to the direction of the remnant magnetization in the material. So the density of states at the Fermi Level is larger for up spin electrons, the energy band of which is raised relative to the Fermi Level. The density of states at the Fermi level for down spin electrons, the energy band for which is lowered relative to the Fermi Level, has a smaller value. The scattering rate for up spin electrons is more than the scattering rate for down spin electrons.

The total resistivity can be thought of as a parallel combination of the resistivity contributed by up spin and down spin electrons.

Now consider a three layer film in which two ferromagnetic layers of different materials are separated by a layer of a non-magnetic material. This is shown in Figure 12.

![Figure 12. A three layer GMR Film.](image)

The initial directions of remnant magnetization in the two films are parallel. The magnetization in the low coercivity film can be reversed by the application of a low magnetic field.

Consider measuring the resistance normal to the layers. In the absence of the field, the direction of the spin of an electron, with reference to the direction of magnetizations in the film, does not change in going from film 1 to 2. An up spin electron in film 1 remains an up spin electron in film 2. So also for the down spin electron. But if the magnetizations in the two films are anti-parallel, an up spin electron in film 1 becomes a down spin electron in film 2. Similar is the case for a down spin electron. Since the resistivity for up spin and down spin electrons are appreciably different in the two films, a reversal of magnetization, by the application of a suitable magnetic field, will cause a large change in the resistance of the films as shown in Figure 13.

![Figure 13. Resistance change in a GMR trilayer when the magnetizations change from anti-parallel to parallel configurations.](image)

This change in resistance can be appreciable and can be used to measure magnetic field.

An experiment is described in Section 15.5 to measure the change in resistance of a GMR sensor in a magnetic field.

(vi) Anisotropic magneto resistance (AMR) to measure the direction of the magnetic field

A permalloy (Ni–Fe) film is grown on a substrate in a high magnetic field, parallel to its length. The initial magnetization $M$ will be parallel to its length. The resistance of the film depends on the angle between the direction of the current $I$ and the direction of magnetization $M$. The direction of $M$ changes when the magnetic field direction is changed from parallel to the length of the film to parallel to its width. So, the angle that $M$ makes with the current changes, and the resistance of the film changes.
Shorting bars are laid out on the permalloy film at an angle of 45° to the length as shown in Figure 14. The current flows through the permalloy film perpendicular to the shorting bars. There are four strips of permalloy. In two of these strips, the current flows at +45° to the easy axis of magnetization, which is along the length. In the other two strips, the current flows at −45° to the direction of easy magnetization. The four strips form the four arms of a Wheatstone bridge. When a magnetic field is applied along the width, the direction of magnetization in the film changes from the easy direction and makes an angle different from ±45° with the current. So, the resistances in the four arms change. An out-of-balance voltage arises in the Wheatstone bridge. This is amplified and plotted as a function of the transverse magnetic field strength. This is shown in Figure 15.

One can keep the magnitude of the magnetic field constant, but change its orientation, from along the width to along the length. The change in fractional resistance with orientation is shown in Figure 16.
Thus, AMR allows the measurement of the magnetic field as well as its direction. The response of the film to change in direction of the magnetic field is very fast.

Sensors Producing an Output Voltage

(i) Thermocouple

A thermocouple is a passive sensor. Two wires of different metals, say copper (red) and constantan (blue), are fused together at the two ends, and the two junctions are maintained in two baths at different temperatures $T_2$ and $T_1$.

![Figure 17. Thermocouple.](image)

A thermo-emf is generated which can be read on a millivoltmeter. The thermo-emf generated is given by

$$V = \int_{T_1}^{T_2} S \, dT.$$  \hfill (13)

$S$ is called the thermoelectric power of the thermocouple. $S$ depends on temperature $T$ and also on the thermocouple materials.

For a given reference temperature $T_1$ (which is usually 0 °C), the thermo-emf varies with temperature $T_2$ in a nonlinear fashion. Table 2 gives the different types of thermocouples and the range of temperatures over which they are used.

<table>
<thead>
<tr>
<th>Type</th>
<th>Thermocouple materials</th>
<th>Range of use</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>Chromel–Constantan</td>
<td>From −50 to 740 °C</td>
</tr>
<tr>
<td>J</td>
<td>Iron–Constantan</td>
<td>From −40 to 750 °C</td>
</tr>
<tr>
<td>K</td>
<td>Chromel–Alumel</td>
<td>From −200 to 1350 °C</td>
</tr>
<tr>
<td>T</td>
<td>Copper–Constantan</td>
<td>From −200 to 350 °C</td>
</tr>
<tr>
<td>R, S</td>
<td>Platinum–Rhodium alloys</td>
<td>From 0 to 1600 °C</td>
</tr>
<tr>
<td></td>
<td>Tungsten–Rhenium alloy</td>
<td>From 0 to 2350 °C</td>
</tr>
</tbody>
</table>

Figure 18 shows how the thermo-emf varies with temperature for some of these thermocouples. Chromel–Alumel thermocouple is commonly used because of its reasonably large sensitivity and range, and its very nearly linear characteristic.

Many experiments described in this manual use the copper–constantan thermocouple for measuring temperature differences. Section 15.2 describes an integral method for measuring the thermo-emf of a copper–constantan thermocouple as a function of temperature.

A thermocouple can also be used to measure pressure in an evacuated chamber. A constant source of heat causes one junction of the thermocouple, kept inside the vacuum chamber, to become hot. For a given heat power, the final temperature reached depends on the rate at which the junction loses heat by convection in the environment.
surrounding air. The lower the pressure, the lower is the loss of heat, and the higher the temperature reached by the junction. This is the principle of the thermocouple gauge used in vacuum systems. Thermocouple gauges can be used to measure pressures from 100 millibar to about 0.01 mbar. A rotary pump evacuates a vessel from atmospheric pressure to a pressure in the range 0.1 to 0.01 mbar.

(ii) Piezoelectric sensor to measure load

When a load is applied to a piezoelectric material, charges are developed on the material. The charge can be measured with a charge amplifier.

The equivalent electric circuit for a piezoelectric sensor is shown in Figure 19.

Here $R$ is the leakage resistance of the piezoelectric material and $C$ its capacitance. The time constant, $CR$, is of the order of a few milliseconds. So the charge collected on the crystal will leak away in a short time. A charge amplifier is used to measure the charge. This is shown in Figure 20.

---

**Figure 18.** Thermo-emf against temperature. Reference junction at 0 °C.

**Figure 19.** Equivalent circuit of a piezoelectric sensor far below its natural frequency.

**Figure 20.** Charge amplifier circuit.
Appendix

Besides measuring static force, this arrangement can also be used for forces changing with time. The dynamic response of a piezoelectric sensor extends to several kHz. It can be used to measure rapidly changing forces.

A piezoelectric system for measuring force is shown schematically in Figure 21.

![Figure 21](image1)

Figure 21. An arrangement for measuring force using a piezoelectric sensor.

The hydrostatic pressure applied to the membrane loads the piezoelectric crystal through the piston. The charge developed by the crystal is proportional to the load on the crystal.

In Sections 15.6 and 15.7, we deal with experiments on the measurement of the piezoelectric constant of PVDF film and the frequency response of PZT bars.

(iii) Hall effect sensor to measure magnetic field

Figure 22 shows how a Hall effect sensor works.

![Figure 22](image2)

Figure 22. Hall effect measurement.

A doped semiconductor is used in the form of a rectangular slab of thickness $t$. By connecting a constant current source (CCS) to two contacts along the length of the slab, a current $I$ (of a few milliamps) is sent through the semiconductor along its length. The slab is placed in a magnetic field $B$ applied along its thickness. Then a voltage appears at two opposite points along the width. This voltage $V_H$ is called the Hall voltage. For a given current $I$, the Hall voltage is

$$V_H = R_H I \frac{B}{t}.$$  \hspace{1cm} (14)

The coefficient $R_H$ is given by

$$R_H = \frac{1}{nq}.$$  \hspace{1cm} (15)
where \( n \) is the number density of charge carriers, and \( q \) has the magnitude of electronic charge and sign of the charge carrier. \( R_H \) is positive in p-type semiconductors, and negative in n-type semiconductors. For lightly doped semiconductors with \( n \) of the order of \( 10^{21} \) per cubic meter, the Hall voltage is of the order of several hundred microvolts at \( B = 1 \) kG and a current of 1 mA.

Hall effect magnetic field sensors with inbuilt amplifiers are commercially available for measuring magnetic fields of several kilogauss.

An experiment on Hall effect is described in Section 15.4.

**Sensors Depending On Variable Coupling**

**Differential Transformer**

In a differential transformer there is a primary coil and two secondary coils as shown in Figure 23. When an AC current passes through the primary coil, voltages \( V_A \) and \( V_B \) will be induced in the two secondary coils. The voltages are balanced. When a ferromagnetic rod \( R \) is placed between the primary coil and the secondary coils, the two secondary voltages become unbalanced, and a differential voltage appears across the terminals CD. This differential voltage depends on the position of the ferromagnetic rod between the secondary coils. Thus, the out-of-balance voltage changes as the position of the ferromagnetic rod changes between the secondary coils.

![Figure 23. Differential transformer to measure displacement.](image)

Linear voltage differential transformers can be used to measure small changes of length. They are useful in measuring the thermal expansion of materials in the form of rods.

The AC susceptibility experiment described in Section 16.5 is based on this variable coupling.

**Sensors Dependent on Change of Capacitance**

(i) **Capacitance pressure gauge**

Figure 24 shows a schematic representation of a capacitance pressure gauge.

![Figure 24. Capacitive pressure gauge (from www.Instrumentation today.com).](image)
Appendix

In this gauge, the diaphragm, which is metallic, is separated from a fixed metallic plate by a dielectric, which may be vacuum or air. The deformation of the metallic diaphragm, on application of pressure, will change the capacitance between the diaphragm and the fixed metallic plate. The change in capacitance may be measured by using an AC Wheatstone bridge. The change in capacitance can be calibrated against pressure.

(ii) Capacitance level sensor for a liquid

A cylindrical capacitance consists of two concentric cylinders of different radii mounted coaxially, one inside the other, as shown in Figure 25.

![Figure 25. Cylindrical capacitance for liquid level sensing.](image)

The total length of the capacitor is \( l \). A liquid, of dielectric constant \( \varepsilon_r \), fills the capacitor to a length \( l' \). The rest of the length of the capacitor is filled with air, for which one may take the dielectric constant to be 1. The radius of the inner tube is \( r_1 \), and the radius of the outer tube is \( r_2 \). The capacitance of such a condenser is

\[
C (l') = \frac{2\pi \varepsilon_0}{\ln (r_2/r_1)} \left[ \left( l - l' \right) + \varepsilon_l l' \right].
\]

So the capacitance changes linearly with the height \( l' \) to which the liquid is filled. A measurement of the capacitance, using one of a number of AC techniques, will enable one to find the level to which the liquid is filled.

This is a cursory introduction to some sensors which we use. To get more information the following references may be consulted.


18.2 Evaporation and Sputtering Techniques for Thin Film Deposition

1. Introduction

Many materials are prepared in the form of thin films. Different techniques are used to prepare thin films of materials. In this section, we provide an introduction to two of the commonly used techniques, namely, evaporation and sputtering.
Appendix

2. Deposition of Thin Films by Evaporation

Figure 1 shows a schematic of an evaporation deposition system.

The material to be deposited (say aluminium) is in a crucible, which is heated to above the melting temperature of the material through a resistive heater (Figure 1(a)). The substrate on which the film is to be deposited, and the crucible containing the material are in a chamber that is evacuated to a vacuum of $10^{-6}$ mbar pressure or better, with a diffusion or turbomolecular pump, backed by a rotary pump. The evaporating atoms of the material will effuse out of the crucible. Because the pressure is very low, the mean free path of the atoms in the vapor is large. Most of the atoms travel in a straight line and fall on the substrate, and stick to it. The substrate can be heated, if required. This method of depositing films has the advantage that the deposition rate is high.

If the melting point of the material is high, one may use a focused beam of electrons to produce the heating (Figure 1(b)). This is called electron beam evaporation. In this case the heat, deposited by the electrons in the crucible, is high, and so there is a need to water cool the crucible.

The rate of production of the vapor atoms, at a vapor pressure $p$ and temperature $T$ (in Kelvin), is given by the Langmuir formula

$$ R = 0.0583 \left( \frac{M}{T} \right)^{1/2} p. $$

Here $R$ is the rate of evaporation in g/cm$^2$, and $M$ is the atomic weight of the element.

The types of sources used to evaporate, or sublime, the material are shown in Figure 2.

![Figure 1](image1.png)

Figure 1. Deposition of thin film by evaporation.

![Figure 2](image2.png)

Figure 2. Different types of evaporation sources (from Ref. [3]).
Appendix

A table of temperatures and support source materials used in evaporation of elements is given in Reference 2.

Distribution of Film Thickness on Substrate

If the source is a point, and the perpendicular distance from the source to the substrate is $h$, then the deposition rate at different distances $l$ from the center of the substrate will be different. For a source of radius $r$, one will have to sum over the deposition rates from different points in the source.

The film thickness $d$ will vary as one moves away from the point on the substrate, where the thickness of the deposited film has its maximum value $d_0$. The variation of the thickness $d/d_0$ is shown as a function of $(l/h)$ in Figure 3, taken from Ref. [3]. The graphs are drawn for different radii $r$ of the source.

![Figure 3](image)

Figure 3. Deposited thickness, as a fraction $d/d_0$ of maximum thickness as a function of $l/h$ for sources of different $r/h$ values. P is a point source and S is a small area source.

Thus, for a film of uniform thickness we have to use a source of large radius compared to the distance $h$ of the source from the substrate. To improve the uniformity of film thickness, one can rotate the source. For very large area coatings, one uses many sources distributed on a ring.

When depositing compounds, one should be aware that the different elements in the compound may deposit at different rates on the substrate. Thus, the deposit may differ considerably in stoichiometry from the compound in the target. Controlling the substrate temperature can improve the stoichiometry.

For example, in evaporating oxides, one needs high temperatures of the order of 1500 °C. At these temperatures, the oxides may dissociate. The released oxygen may react with the material of the evaporation boat, or with residual hydrocarbons in the vacuum chamber. This will affect the oxygen stoichiometry of the deposited film. This can be overcome by using laser beam evaporation, e-beam evaporation or flash evaporation. The hot spot is local, and the evaporation source metal is not hot. The evaporation is fast, and there is less time for chemical decomposition to take place.

Growth of Film

The atoms arrive singly on the substrate, and they migrate on the substrate due to their kinetic energy. Some of them may leave by re-evaporation. When two atoms collide on the substrate, they merge. In this way nuclei are produced, which grow as separate islands. Eventually the islands merge, and a continuous film of the material is obtained on the substrate. It is obvious that this growth process will depend on the temperature of the substrate. Controlling the temperature of the substrate at an optimal value helps in the growth of films.

3. Thin Film Deposition by Sputtering

A second process for the deposition of thin films is by sputtering. Figure 4 illustrates this process.
Appendix

A high voltage applied between the anode and the cathode initiates a discharge in the argon gas. The positively charged argon ions are attracted to the target, which is the cathode, and impinge on the target. The argon ions will lose energy in the target by elastic and inelastic processes. In elastic collisions, part of the kinetic energy of the argon ions is transmitted to the target atoms. Those target atoms, which acquire a kinetic energy larger than the binding energy of the target atom, will escape from the target. These are called the sputtered atoms. Some of the sputtered atoms will be negatively charged, some positively charged, and some neutral. The positively charged target atoms will fall back on the target. The negatively charged target atoms will be attracted by the substrate which is the anode and which is at ground potential. The neutral and negatively charged target atoms will suffer collisions with the argon ions, and will be scattered. After many such collisions, some of the target atoms will deposit on the substrate.

Secondary electrons, produced by the inelastic collisions of the argon ions with the target, help to maintain the discharge by ionizing the neutral argon atoms.

4. Quantities which Determine the Rate of Sputtering

Only a fraction of the argon ions, incident on the target, will create target atoms for sputtering. The ratio of the number of target atoms that get sputtered to the number of ions incident on the target is called the sputtering yield \( Y \). This sputtering yield is shown in Figure 5, as a function of energy of the incident ion for different target materials with argon as the rare gas.

Figure 4. Schematic of sputter deposition.

Figure 5. Sputtering yield as a function of ion energy for different targets, with argon gas (from Ref. [4]).
Appendix

This yield will depend on (1) the energy $E$ of the incident ion, (2) the surface binding energy $U$ of the target atom, (3) the ratio of the mass of the target atom to the mass of the rare gas ion, $M_t/M_i$, and (4) the nuclear energy loss function $S_n(E)$. There is a minimum energy of the ion, called the threshold energy $E_{TH}$, for sputtering target atoms. This threshold energy depends on the ratio of the mass $M_t$ of the target atom to the mass $M_i$ of the rare gas ion, which is causing the sputtering. The variation of $E_{TH}$ with the mass ratio $M_t/M_i$ is shown in Figure 6.

Figure 6. Ratio of threshold energy $E_{TH}$ to the surface binding energy $U$ of the target atom plotted as a function of $M_t/M_i$ (from Ref. [4]).

If $J$ is the number of ions falling on the target in one second, the power in the ion beam is $P = JE$, where $E$ is the energy of the incident ions. The $J$ ions, falling on the target in one second, produce $JY(E)$ sputtered target atoms in one second, and the sputtering uses up an energy $JY(E)U$ to tear these atoms from the surface of the target. So, the energy efficiency of the sputtering process is

$$\eta = \frac{JY(E)U}{JE} = \frac{Y(E)U}{E}. \quad (2)$$

The energy efficiency for sputtering is a maximum $\eta_{max}$ at an energy $E = 7E_{TH}$. In the range $3 < E/E_{TH} < 10$, the energy efficiency $\eta > 0.8\eta_{max}$. For copper, $U = 4$ eV, and $E_{TH} = 30$ eV. So efficiency is maximum at an energy $E$ of 210 eV. Between 90 eV and 300 eV for the incident energy of the ions, the efficiency is more than $0.8\eta_{max}$.

Schematic of DC Sputter Deposition System

A schematic of a typical DC diode sputter deposition system is shown in Figure 7. The rare gas is fed to the vacuum chamber in a continuous stream to maintain a pressure of about 50 to 80 millibar. The target is insulated from the vacuum chamber which is earthed. The target is maintained at a few kilovolts negative voltage relative to the chamber. Surrounding the target, but connected to the chamber, is a dark shield. The argon ions are accelerated in the space between the shield and the target. The substrate, on which the sputtered film is obtained, is connected to a negative DC bias with respect to the chamber. The substrate can be heated to maintain it at a high temperature. The target is water cooled.

In a diode sputtering system, one needs to maintain a high argon pressure to have an appreciable flux of argon ions. This makes the mean free path of the target atoms low and so the film deposition rate is very low.

One can reduce the argon ion pressure and still maintain a satisfactory flux of ions by using a triode sputtering system. Here we have a filament which produces electrons. The accelerated electrons produce the argon ions. The number of atoms which are ionized will depend on the filament current, which produces the stream of electrons. This decouples ion production from the energy with which the ions hit the target. The flux of argon ions and the energy with which the ions hit the target can be independently controlled. Since the pressure of argon gas in a triode sputtering system is very much lower than the pressure in the diode
Appendix

sputtering system, the mean free path of the target atoms is longer in the triode sputtering system. This leads to a larger deposition rate of the target atoms on the substrate.

![Diode sputter deposition system](image1)

**Figure 7.** DC sputtering system (from Ref. [4]).

The ionization efficiency of the electrons can be increased by making their path a spiral. This is done by a magnetic field perpendicular to the electric field accelerating the electrons. This is called the EXB configuration. This spiral path increases the ionization efficiency of the electrons, thus increasing the ion flux and reducing the potential applied to the target. This technique is called Magnetron Sputtering. A schematic of this is shown in Figure 8.

![Magnetron sputter deposition system](image2)

**Figure 8.** Magnetron sputtering system (from Ref. [4]).
Appendix

The EXB (crossed electric and magnetic field) configuration can be achieved in many different ways. Interested readers may consult the literature.

The target atoms may travel, without collisions, in a straight line to the substrate, or may undergo collisions with other atoms and diffuse to the substrate. Some of the sputtered atoms on the substrate may be reflected back. The total deposition rate is mainly determined by the ballistic and diffusive components.

DC sputtering has the disadvantage that it cannot be used with insulating targets. As the ions impinge on the insulating target, the target becomes positively charged, repelling a further influx of ions. For insulating targets, one uses an RF voltage between the target and the shield.

RF sputtering has the following advantages. It can be used with metal, insulator and composite targets. Charging effects on insulating targets are avoided. It operates at a pressure of 1 to 15 millibar. However, the deposition rates are low for some materials. Stray magnetic fields will disturb the sputtering process. Efficient heat removal from the target is necessary.

5. Monitoring the Thickness of the Deposited Film

The thickness of the film, during deposition, can be monitored using a quartz crystal monitor. A thin quartz crystal, with gold plated electrodes, is used for this purpose. The crystal has a natural frequency \( f \) of vibration. This frequency changes as the mass of the film deposited varies. The thickness of the film is monitored by following this frequency change. Since the mass deposited, for the same thickness of the film, depends on the density of the material of the film, the calibration of the quartz crystal monitor depends on the density of the target material.

The following references may be consulted for further study:
3. Vacuum Evaporation of Thin Films by Dr R.E. Hurley, School of Electrical and Electronics Engineering, Queen’s University, Belfast. slideplayer.com/?slide/5876720.
4. Lecture 3: Fundamentals of Sputter Deposition. tam.northwestern.edu/summerinstitute/_links/_courses/../L03.pdf.

18.3 Low Temperature and Vacuum Techniques

1. Introduction

Experiments at low temperature are important for the following reasons:

a) Atoms vibrate with smaller and smaller amplitude as the temperature of a solid is reduced. Electron scattering effects becomes less. X-ray diffraction spots become sharper as the temperature is reduced.

b) Many phase transitions involve very small differences in the free energy of the two phases. Therefore, they occur at low temperatures. Notable examples are the magnetic transitions from a disordered paramagnetic phase to an ordered magnetic phase.

c) Some novel phenomena are observed at low temperatures.

(i) In a two-dimensional metal, one observes the quantum Hall effect. In this effect, the Hall conductivity is quantized in units of \( e^2/h \) as the magnetic field is changed.

(ii) Some metals become superconducting at low temperatures. In the superconducting phase, the resistance of the metal is zero.

(iii) Liquid helium (\( ^4 \)He), cooled to a temperature below 2.17 K, flows through narrow channels without viscosity. This is called superfluidity.
Appendix

(iv) Bose–Einstein condensation is seen in a gas of alkali atoms, which have a total spin quantum number zero, at temperatures of the order of a few tens of a nanokelvin.

2. Attainment of Temperatures Below 0 °C using Liquid Cryogens

Temperatures below the ice point can be achieved by liquefying permanent gases. Gases can be liquefied by applying pressure, only if the temperature of the gas is below its critical temperature $T_c$. The so-called permanent gases have a critical temperature below 0 °C. Permanent gases will liquefy only when they are cooled below 0 °C. To liquefy the gas, it must be first compressed. The compressed gas is then expanded, either (a) adiabatically, in an expansion engine or (b) isentropically, through a narrow orifice, to cool the gas progressively till it reaches a temperature below its critical temperature. Then the gas liquefies when this process proceeds further.

The normal boiling points of different gases are given in Table 1. The normal boiling point is the boiling temperature when the pressure on the gas is 1 atmosphere.

<table>
<thead>
<tr>
<th>Gas</th>
<th>Normal Boiling Point in Kelvin</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oxygen</td>
<td>90.2</td>
</tr>
<tr>
<td>Argon</td>
<td>87.3</td>
</tr>
<tr>
<td>Nitrogen</td>
<td>77.4</td>
</tr>
<tr>
<td>Neon</td>
<td>27.1</td>
</tr>
<tr>
<td>Hydrogen</td>
<td>20.3</td>
</tr>
<tr>
<td>Helium $^4$He</td>
<td>4.21</td>
</tr>
</tbody>
</table>

Helium has two isotopes, one with a mass number 4 and another with mass number 3. $^4$He occurs with an abundance of 99.7%.

Of the gases listed in Table 1, liquid nitrogen and liquid helium are commonly used in low temperature laboratories because (1) nitrogen is non-combustible and boils at 77 K; it is easy to use, and (2) helium has the lowest boiling point of 4.2 K.

If the pressure on the liquid is reduced, its boiling temperature decreases. The triple point of nitrogen is 63.15 K. Below this temperature, nitrogen will become a solid. Helium has no solid-liquid-vapor triple point. $^4$He will remain a liquid even when it is cooled to absolute zero, as long as the pressure on it is below 25 atmospheres. This unique behavior of liquid $^4$He is because of the large zero-point energy in the solid state, due to the vibrations of its atoms.

One may do low temperature experiments by immersing a sample in the liquid cryogen. With liquid nitrogen, one may be able to do measurements from 77 K to room temperature with a suitably designed cryostat. If we are able to reduce the pressure on liquid nitrogen with a vacuum pump, the measurements may be extended theoretically down to its triple point. With liquid $^4$He, one may carry out measurements from 4.2 K to room temperatures. By lowering the pressure on liquid helium, the temperature of measurement may be extended down to 1.8 K. To reach temperatures below 1.8 K with $^4$He, one will have to use very large vacuum pumps. This is not economically viable.

Temperatures down to 0.3 K can be achieved by pumping on liquid $^3$He, which has a normal boiling point of 3 K. But liquid $^3$He is difficult to obtain.

Temperatures down to a few degrees millikelvin can be attained by using a dilution refrigerator, in which one uses a mixture of liquid $^3$He in liquid $^4$He. One can lower the temperature to a few tens of a microkelvin by adiabatically demagnetizing the nuclear spins in copper, which is already cooled to a few degrees millikelvin with a dilution refrigerator. This is about the lowest temperature that can be attained in a macroscopic material, with a mass of about a few milligrams.

In a cloud of an alkali vapor, at very low pressure (10$^{-8}$ mbar or lower) containing a billion atoms, one can achieve a temperature of the order of ten degrees nanokelvin using a technique called laser cooling, followed by evaporative cooling.

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This is as close to absolute zero of temperature as we can go with currently available techniques.

3. Storage of Liquid Cryogens

Liquid cryogens have to be stored in double-walled vessels of stainless steel. The space between the walls is evacuated to a pressure of $10^{-6}$ mbar or lower, and the outer wall of the inner vessel is surrounded by a multilayer insulation of aluminized Mylar. The heat leak from the outer wall, at a temperature much above the boiling point of the liquid, to the cryogenic liquid at low temperatures is reduced by the vacuum and the multilayer insulation. Using such vessels, an evaporation rate of 1% per day can be achieved. For liquid helium, lightweight containers of aluminium, with fiberglass neck, are available.

If one wants to use liquid cryogens for cooling, one needs a machine to liquefy nitrogen or helium. These machines are sophisticated, and need trained operators to run them efficiently, and to maintain them. In small laboratories, it may be difficult to have such expensive machines.

In major cities in the country, it is now possible to buy liquid nitrogen, and even liquid helium, from a supplier. The cost of liquid nitrogen will be around twenty to thirty rupees per liter, and the cost of liquid helium around Rs one thousand per liter. If one wants to cool a large mass of material, then one cannot avoid the use of liquid cryogens. But if one wants to perform experiments with a few grams of a material, closed circuit refrigerators are currently available and are found in many laboratories in the country.

4. Closed Cycle Refrigerators

Closed cycle refrigerators produce low temperatures with a refrigerator, in which helium gas undergoes a closed cycle of operations. Closed cycle refrigerators are of two types: (a) those operating at high frequencies, of the order of a few hundred or thousand cycles per second, and (b) those operating at low frequencies, of the order of a few cycles per second.

The machines operating on the reverse Stirling cycle are of the first kind. These were developed first by Kohler. They have become very popular in producing liquid nitrogen.

The second kind of machines, operating at a few hertz, is the one commonly used in closed cycle refrigerators for low temperature measurements, normally down to 10 K. These are based on the Gifford–McMahon cycle which is explained below.

A schematic diagram of the Gifford–McMahon refrigerator is shown in Figure 1.

![Schematic diagram of a G-M refrigerator](from Ref. [1]).
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The cycle of operations is the following. The compressor compresses helium gas to a pressure of about 120 psi. The compressor is similar to the one used in a domestic refrigerator. The heat of compression is removed by injecting oil during compression. The helium discharged at high pressure, after removal of heat, contains oil vapor. If this oil is not removed completely, it will freeze on cooling, and will clog the lines.

Oil removal (OR) is done by centrifugation. The oil collects in drops and returns to the compressor through a low pressure line. The high pressure gas then passes through a micropore filter F which removes the final traces of oil. After the oil is completely removed, the gas enters a buffer vessel.

The thermodynamic cycle of operations consists of the following stages:

Stage 1: The displacer D in the expansion cylinder is at the bottom of its stroke. Valve V\textsubscript{}2 is closed and V\textsubscript{}1 is opened. The pressure below the piston rises from a low value \(P_o\) to the value \(P_2\) in the buffer vessel. The warm space is above the displacer piston and its temperature is at 300 K.

(NOTE: We are describing the operation when the machine has run for some time and has cooled the regenerator. In every cycle the gas cools in stage 3 due to adiabatic expansion. This is described below. When this gas passes through the regenerator in stage 4, it deposits its cold in the regenerator and picks up heat from the regenerator. Thus the regenerator becomes cold. Now read on.)

Stage 2: Valve V\textsubscript{}1 remains open. The displacer moves up. The gas in the warm space moves through the regenerator (REG) to the cold space below the piston. The heat in the warm gas is deposited in the regenerator, as it cools. This process takes place at constant pressure. As the gas cools, more gas is drawn from the buffer vessel.

Stage 3: When the displacer reaches the top position, valve V\textsubscript{}1 closes and valve V\textsubscript{}2 opens. The pressure drops because of the additional volume available to the gas. The temperature of the gas falls slightly due to expansion.

Stage 4: The displacer moves down pushing the cold gas through the regenerator to the compressor. This gas picks up the heat stored in the regenerator, and enters the compressor at room temperature.

The advantages of the G–M refrigerator are:

1. The compressor and expander are separate, thus reducing the vibration of the cold parts.
2. The expander operates at low frequencies of a few hertz.
3. The pressure difference across the displacer is very small. So the clearance between the displacer piston and the cylinder need not be very small.
4. It is simple to multistage the G–M refrigerator.

G–M refrigerators can be of two or three stages. The first stage operates between, say, 300 K and 90 K, the second between 90 K to 10 K, and the third between 10 K to a still lower temperature.

In a two-stage refrigerator, one may get about 20 watts refrigeration at 90 K, and 1 watt refrigeration at 10 K.

It is now possible to get three-stage G–M cryocoolers to cool superconducting magnets. This is called a dry magnet system, as it does not use liquid helium for cooling the magnet.

5. Bath Type of Cryostats

A schematic of a bath type of cryostat for work from 4.2 K to room temperature, using a glass dewar to contain the cryogens, is shown in Figure 2. The glass dewar must be made of well-annealed Pyrex glass.
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$G_1$ is a Pyrex glass dewar to which a metal flange $F'$ is fixed with Araldite. A top flange $F$ has five holes to which the tubes $T_1$ to $T_4$ are brazed. $F$ is fitted to $F'$ with bolts and nuts. An O-ring seal shown as a black circle in the figure makes this joint leak tight.

The tubes $T_1$, $T_2$, $T_3$ and $T_4$ are thin-walled stainless steel tubes. The tube $T_5$ allows the space in $G_1$ to be pumped out to get rid of the air before filling liquid helium in $G_1$.

![Figure 2. Schematic of a liquid helium bath cryostat.](image)

At the other ends of $T_1$ and $T_2$, a brass flange $F_1$ is brazed. $S$ is a sample chamber of copper. It has a brass flange $F'_1$ which can be fitted to $F_1$ with screws. An indium O-ring seal between the flanges makes it vacuum tight. The sample chamber can be evacuated through tube $T_1$ and filled with helium gas at a pressure of $10^{-2}$ to $10^{-3}$ mbar. This exchange gas serves to carry away the heat from the sample in the sample chamber to the liquid helium bath in $G_1$. The sample, mounted inside $S$, will then cool slowly to a temperature slightly more than 4.2 K, the boiling point of liquid helium in $G_1$. Once the sample is cooled to this temperature, the exchange gas can be pumped out through $T_1$ to isolate the sample thermally from the liquid helium bath. A heater attached to the sample can be switched on to raise its temperature. Thus, one can measure the properties of the sample from a temperature close to 4.2 K to a higher temperature. The leads attached to the heater, the thermometer, and other wires for measuring the properties, come out through the vacuum-tight feedthrough at the free end of the tube $T_2$.

A helium transfer tube is inserted in tube $T_3$ through a vacuum-tight joint, and it is used to fill liquid helium in $G_1$ from a storage dewar. The tube $T_4$ is connected to a gas bag to recover the evaporating helium gas from $G_1$. Before filling $G_1$ with liquid helium, the glass dewar must be evacuated through the tube $T_5$ to remove all air in this space. The valve $V$ is then closed, and the glass Dewar $G_1$ filled with liquid helium.

Note: The cryostat described so far can also be used with liquid nitrogen to make measurements on the sample from 77 K to room temperature. In this case, the tube $T_4$ is not connected to a gas bag, but is open to the atmosphere. We do not have to recover the evaporating nitrogen gas. It can be let into the atmosphere. Also the second heat shielding dewar, described below, is not required if we are using liquid nitrogen as the cryogen.

The glass dewar $G_1$ is surrounded by a second glass dewar $G_2$ containing liquid nitrogen. Thus, the outer glass wall of $G_1$ will be maintained at a temperature of 77 K. The heat leaking from the outer wall of $G_2$, which is at room temperature, to the liquid helium bath will be reduced enormously by this arrangement. Using thin-walled stainless steel tubes $T_1$ to $T_4$ reduces the heat leak by thermal conduction from the room.
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temperature flange F to the liquid helium bath, because of the low thermal conductivity of stainless steel, and the low cross-sectional area and the long length of the tubes. Thin fins RS of copper attached to the tubes \( T_1, T_2, \) and \( T_3 \) will reduce this heat more. The evaporating helium will take away the heat coming down the tubes, through heat exchange with the copper fins.

The block B of fiber glass supports the dewar \( G_1 \), inside the dewar \( G_2 \).

Though Pyrex has a low thermal expansion coefficient and can withstand thermal shocks, there is always the danger of the glass dewar imploding some time. To protect the user in such an event, the outer dewar \( G_2 \) is enclosed within a thick cylindrical transparent plastic shield.

One may replace the outer and inner glass dewars by suitably designed stainless steel containers, making a metal cryostat. This is preferable from the point of view of safety. The glass dewar allows one to see inside, and to note the level of liquid helium in \( G_1 \). If we are using a metal cryostat, this will not be possible. We have to use level sensors in the liquid helium space.

Cryostats can be constructed for different types of studies, and will differ in construction. In making cryostats, one should understand how the mechanical, thermal and electrical properties vary, as the temperature of the different materials is lowered to 4.2 K. This understanding is essential to choose proper materials and proper design for the construction of cryostats. Reference [1] gives a clear account of the temperature dependence of the properties of materials, and the principles involved in cryostat design.

A simple dipstick type of cryostat in which resistivity measurements can be done from 90 K to room temperature is described in Section 16.6.

6. Sample Mounting in a Closed Cycle Refrigerator

If we are using closed cycle refrigerators, the sample will be mounted on a copper plate which will be attached to the 10 K stage by screws. A copper radiation shield enclosing the 10 K stage will be screwed to the 90 K stage. The entire arrangement will be in a vacuum enclosure. The leads will be brought out through a vacuum-tight feedthrough.

7. Temperature Sensors for Low Temperature Experiments

Temperature sensors depend on the variation of some property of a material with temperature. There are different types of temperature sensors (see Section 18.1). However, we have to choose one standard thermometer against which all other sensors can be calibrated. This ensures that the temperature of a material, measured by any of the temperature sensors, will yield the same value within certain acceptable uncertainty.

The standard thermometer is the constant volume gas thermometer. If the gas used is at a low pressure, then the change in pressure of a given volume of gas is proportional to the change in temperature. Experiments have shown that the temperature calculated on the basis of the ideal gas law

\[
p V = RT
\]

is independent of the gas used as long as the pressure of the gas is very low. This temperature reading varies very nearly in a linear fashion with pressure over a wide temperature range. The filling gas used in such a thermometer is helium, which has the lowest boiling point of all gases, and can be obtained in high purity. Such thermometers are maintained in National Standards Laboratories all over the world, and are used to calibrate secondary thermometers.

The constant volume gas thermometer is cumbersome to use in laboratories. One prefers to use other thermometers such as resistance thermometers (see Section 18.1), or capacitance thermometers. The capacitance thermometers depend on the variation of the dielectric constant of a material with temperature.

These secondary thermometers have the advantages of (a) small size and (b) low cost.

In choosing a thermometer, the important criteria are (a) the sensitivity, (b) the linearity of the calibration, (c) the speed of response and (d) the reproducibility over long periods of time.

Platinum resistance thermometers (Section 18.1) are very commonly used above 20 K.
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Negative TCR thermometers are made of semiconducting materials, like doped germanium, silicon or gallium arsenide. The resistance of these thermometers increases exponentially below 20K. They are very sensitive below 20 K. But they cannot be used above 20 K.

Both positive and negative TCR thermometers have an appreciable thermal capacity, which makes their response to changing temperature sluggish.

Thermocouples have the advantage that their thermal capacity is small, and so they can follow rapid changes in temperature. But the thermo-emf of thermocouples is a highly nonlinear function of temperature, and so these thermometers need calibration at many standard temperatures to get an accurate polynomial fit to the temperature variation of their thermo-emf. Also, the thermo-power of the commonly used copper–constantan and chromel–alumel thermocouples becomes too small below 20 K. The only thermocouple which can be used down to 4.2 K is the gold(iron)–chromel thermocouple. The gold wire contains a few atomic per cent of iron. Because of the Kondo effect, this thermocouple has a thermo-power of about 16μV/K even at 4.2 K.

Both resistance thermometers and thermocouples are sensitive to the magnetic field. If one would like to measure temperature accurately in the presence of a high magnetic field, one uses amorphous carbon glass thermometers. The resistance of such a thermometer is insensitive to the magnetic field.

Other thermometers such as capacitance thermometers are not very extensively used and will not be discussed here.

8. Vacuum Techniques

All low temperature measurements are made in vacuum. So, some knowledge of vacuum techniques is essential to do low temperature research.

Before we discuss the pumps used, we should know how pressure is measured. The unit for pressure is the bar. The bar is exactly 100,000 newton/m². The atmospheric pressure is nearly 1 bar. In vacuum practice, the unit for measuring pressure is a millibar. If the pressure in a chamber is in the range 10⁻² to 10⁻³ millibar, the vacuum is said to be low. If the pressure is in the range 10⁻⁴ to 10⁻⁶ mbar, the vacuum is said to be high. If the pressure is below 10⁻⁷ mbar, the vacuum is said to be ultrahigh.

To produce a vacuum, one will have to pump out air from the chamber. The common vacuum pump used for this purpose is the rotary vacuum pump. In this pump, there is a vane with a spring mounted on a rotor which rotates eccentrically within a static chamber. The spring presses the vane against the wall of the stator. As the rotor rotates, the spring-mounted vane traps a certain volume of the gas from the chamber in each rotation. This gas trapped between the rotor and the stator is compressed and pushed out to the atmosphere. The pump will gradually evacuate the chamber to which it is connected. Such a rotary pump will produce a limiting vacuum of the order of 10⁻³ mbar. As the pressure decreases, the density decreases. So the mass of gas pushed out in each rotation decreases as the pressure falls. The pumping speed of a pump is specified in liters per hour. A pump with a higher speed will evacuate a chamber of a given volume in a shorter time. The rotor and the stator are housed in an oil sealed box. So some amount of oil vapor will contaminate the vacuum chamber.

To obtain a vacuum in the range 10⁻³ to 10⁻⁶ mbar, one should connect an oil diffusion pump between the rotary pump and the chamber. Once the rotary pump reduces the pressure in the chamber to below 10⁻² mbar, a heater is switched on to vaporize the oil in the diffusion pump. The hot oil molecules will rise. They are cooled with water so that they will condense and drop back into the oil bath at the bottom of the diffusion pump. During this circulation of the oil vapor, the hot oil molecules will collide with the air molecules near the mouth of the chamber, and push them down towards the exit at the bottom of the diffusion pump chamber. The rotary pump will push this gas out.

The vapor pressure of the oil used in the diffusion pump must be much below the ultimate vacuum one wants to reach. Silicone oils with very low vapor pressure are used in the diffusion pump. This oil, when it is hot, will get oxidized if it comes into contact with air. So the heater in the diffusion pump must not be switched on till the vacuum in the chamber has been reduced to below 10⁻² mbar by the rotary pump. While stopping the pumps at the conclusion of the experiment, one must switch off the heater and wait long enough for the diffusion pump to cool before turning off the rotary pump and admitting air in the chamber. Since the diffusion pump uses hot oil to pump the chamber, some of the oil molecules will find their way into the chamber and contaminate the vacuum chamber. To prevent this contamination, one may use a trap with liquid nitrogen between the diffusion pump and the vacuum chamber.
Appendix

Where one needs a clean vacuum free of contamination with oil vapor, nowadays one uses a turbomolecular pump instead of a diffusion pump. In this pump we have a rotor turbine with many vanes, and the turbine rotates at a high speed. The molecules of air in the chamber collide with the vanes and are impelled to move towards the exit of the pump which is connected to a backing rotary pump. The turbopump should be started only after the rotary pump has evacuated the chamber to a low vacuum.

The speed of evacuation not only depends on the speed \( S_p \) of the pump in liters/second, it also depends on the conductance \( S_C \) of the connection between the pump and the chamber. The effective speed \( S \) will be given by

\[
\frac{1}{S} = \frac{1}{S_p} + \frac{1}{S_C}.
\]

The conductance \( S_C \) will be more, the larger the area of cross section and the smaller the length of the connection. So, the vacuum connection must be designed to have a large area of cross section and a small length.

Pressure from 1 bar to \( 10^{-3} \) mbar is measured by a Pirani or thermocouple gauge (see Section 18.1). To measure a pressure below \( 10^{-3} \) mbar, one uses an ionization gauge. Electrons from a hot filament are accelerated towards a grid and ionize the gas molecules by collision. The positively charged gas molecules move towards an electrode maintained at a negative potential with respect to the grid. The ionization current is a measure of vacuum. It will be more, the higher the pressure in the gauge. The ionization gauge must be switched on only after the pressure, as measured by the Pirani or thermocouple gauge, reaches a value in the \( 10^{-3} \) mbar range.

The above gives a very brief account of vacuum techniques.

For a more detailed account of low temperature techniques for measuring different properties, the following references may be consulted.


For vacuum techniques, the following reference may be consulted.

1. Fundamentals of Vacuum technology Compiled by Dr. Walter Umrah, Oerlikon-Leybold Vacuum (free pdf download).

18. 4 Tensor Properties of Crystals

1. Introduction

In this section, a brief introduction is given of second, third and fourth rank tensor properties of crystals, and the effect of the symmetry of the crystal in reducing the number of independent constants.

A single crystal of any material has many symmetry operations. When an operation, like rotation about a point, is performed on an object, and the resulting object is indistinguishable from the object before the operation in all respects, the operation is said to be a symmetry operation. For example, if a perfect square is rotated through an angle \( \pi/2 \) about an axis passing through its center, and perpendicular to its plane, the resulting square is identical with the square before rotation. The square after the operation can be exactly superposed on the square before the operation. We say that rotation through \( \pi/2 \), about an axis perpendicular to the plane of the square and passing through its center, is a symmetry operation of the square.

The types of symmetry operations are rotations, reflections, inversion, roto-reflections and roto-inversions. If there are many symmetry operations of an object passing through a given point, these operations leave the point unchanged. Such a group of symmetry operations is called a point group.
Appendix

In a crystal, the unit cell is repeated periodically along three non-coplanar directions. If you move the crystal through a repeat distance along any of the three directions, the arrangement of atoms in the resulting figure exactly coincides with the arrangement before translation. So a crystal has a group of translations as symmetry operations.

Combining fractional translations with rotations and reflections results in new symmetry operations, called screw axes and glide planes.

The rotational symmetry operations in a crystal must be consistent with the translational symmetry operations. This restricts rotational operations to an angle $2\pi/n$, where $n$ can only be 1,2,3,4 and 6. The collection of these symmetry operations along with mirror planes, inversion, roto-reflection and roto-inversion, which will leave a point unchanged, will be 32 in number. So any crystal must belong to one of the 32 point groups. The point group symmetry has an important role to play in determining what properties a crystal may have, and what symmetry the properties will exhibit.

All properties of the crystal are relations between two fields, an applied field and the resulting field. For example, if we apply an electric field $E$, the crystal will develop an electrical displacement field $D$. If we apply a stress field, the crystal will develop a strain field. The two fields will be related linearly through a property of the crystal. A linear vector relation means that each component of one vector is dependent linearly on each component of the other vector. In the cited example we may write this relation as

$$D = \varepsilon_0 \left[ \varepsilon_r \right] E.$$  \hspace{1cm} (1)

Written in full, equation (1) consists of three equations shown in equation 2(a, b, c).

$$D_1 = \varepsilon_0 \left( \varepsilon_{r,11} E_1 + \varepsilon_{r,12} E_2 + \varepsilon_{r,13} E_3 \right),$$ \hspace{1cm} (2a)

$$D_2 = \varepsilon_0 \left( \varepsilon_{r,21} E_1 + \varepsilon_{r,22} E_2 + \varepsilon_{r,23} E_3 \right),$$ \hspace{1cm} (2b)

$$D_3 = \varepsilon_0 \left( \varepsilon_{r,31} E_1 + \varepsilon_{r,32} E_2 + \varepsilon_{r,33} E_3 \right).$$ \hspace{1cm} (2c)

Note that the two vectors $E$ and $D$ need not be parallel in a crystal. So $\varepsilon_r$, which is called the dielectric constant, is not a scalar. A scalar property will not depend on the direction of the applied field. That is why $\varepsilon_r$ in equation (1) is put in square brackets. We see that $\left[ \varepsilon_r \right]$ actually consists of nine quantities, $\varepsilon_{r,ij}$ which relate the $i^{th}$ component of $D$ to the $j^{th}$ component of $E$. These nine quantities are said to constitute the relative dielectric constant tensor. In this case, we may show from energy considerations that $\varepsilon_{r,ij} = \varepsilon_{r,ji}$. Thus, the number of independent parameters of the dielectric constant tensor reduces to six. The point group symmetry of the crystal may introduce further relations between the parameters.

Consider a crystal having a fourfold axis of rotation along the $c$ axis. This crystal belongs to the tetragonal system, with the lattice parameters $a = b \neq c$. Consider the dielectric constant tensor. Suppose we apply an electric field $E$ along the $c$-axis, which is taken to be the $Z$-axis. Referred to $X$, $Y$, $Z$-axes the electric field has components $E_x = E_y = 0$ and $E_z = E$. The equations 2(a, b, c) reduce to

$$D_x = \varepsilon_{r,13} E,$$ \hspace{1cm} (3a)

$$D_y = \varepsilon_{r,23} E,$$ \hspace{1cm} (3b)

$$D_z = \varepsilon_{r,33} E.$$ \hspace{1cm} (3c)

Now rotate the coordinate system through $\pi/2$ about the $Z$-axis. The new coordinate axis $X'$ will be along the old $Y$-axis; the new $Y'$-axis will be along the old $-X$-axis; and the new $Z'$-axis along the old Z-axis. So, in the new coordinate system, the applied electric field is still $(0,0,E)$. In the new coordinate system, the equations 2(a,b,c) now become

$$D_{X'} = \varepsilon'_{r,13} E = D_y = \varepsilon_{r,23} E.$$

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\[ D_Y' = \varepsilon'_{r,23} E = -D_X = -\varepsilon_{r,13} E. \] (4b)

\[ D_Z' = \varepsilon'_{r,33} E = D_Z = \varepsilon_{r,33} E. \] (4c)

Here \([\varepsilon_r]\) is the relative dielectric constant tensor in the \(X', Y', Z'\) coordinate system. But this rotation is a symmetry operation. So the properties of the crystal must be the same as the properties before rotation. So, \(\varepsilon'_{r,13} = \varepsilon_{r,13}\), \(\varepsilon'_{r,23} = \varepsilon_{r,23}\), and \(\varepsilon'_{r,33} = \varepsilon_{r,33}\). Therefore, for a tetragonal system,

\[ \varepsilon_{r,13} = \varepsilon_{r,23} = -\varepsilon_{r,23}. \] (5)

This implies that both \(\varepsilon_{r,13}\) and \(\varepsilon_{r,23}\) must be zero. By applying the electric field along the \(X\)-axis, and following a similar argument, we can show that

\[ \varepsilon_{r,11} = \varepsilon_{r,22}, \] (6)

and

\[ \varepsilon_{r,21} = 0. \] (7)

So, for this system, the only non-zero independent constants in the relative dielectric constant tensor are \(\varepsilon_{r,11} = \varepsilon_{r,22}\) and \(\varepsilon_{r,33}\). We can measure \(\varepsilon_{r,11}\) by applying the electric field along the \(a\)-axis of the crystal, and measuring the electric displacement along the same direction. Similarly, \(\varepsilon_{r,33}\) can be measured by applying the electric field along the \(c\)-axis, and measuring the displacement in the same direction. Two independent measurements are required to specify the dielectric constant of a crystal belonging to a tetragonal system.

For example, in a crystal belonging to the cubic system, \(\varepsilon_{r,11} = \varepsilon_{r,22} = \varepsilon_{r,33}\), and all the cross constants \(\varepsilon_{ij}\) (\(i \neq j\)) are 0. If we substitute this condition in equations 2(a,b,c), we will see that, for any direction of \(E\)

\[ \mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}. \] (8)

Here \(\varepsilon = \varepsilon_{r,11} = \varepsilon_{r,22} = \varepsilon_{r,33}\) is the single constant representing the tensor \([\varepsilon_r]\).

Examples of second rank tensors are linear thermal expansion coefficient \([\alpha]\), thermal conductivity \([K]\), electrical conductivity \([\sigma]\), dielectric constant \([\varepsilon_r]\), and magnetic permeability \([\mu_r]\).

2. Elastic Property

The fields can be scalar, vector or a tensor. The temperature in any material is a scalar field. The electric field \(\mathbf{E}\) is a vector field. We will now deal with tensor fields. Stress is the force divided by the area on which the force acts. Area is represented by a vector \(\mathbf{A}\) along the normal to the area. Force is a vector \(\mathbf{F}\). So

\[ \mathbf{F} = [\sigma] \mathbf{A}. \] (9)

\([\sigma]\) is a second rank tensor, like the dielectric constant tensor, and is called the stress tensor. From the theory of elasticity we can show that

\[ \sigma_{ij} = \sigma_{ji}. \] (10)

The stress field has six independent components, just as a vector field \(\mathbf{E}\) has three independent components. Note that stress is not a material property. It is a description of an applied field. So, crystal symmetry has nothing to do with the stress tensor. Normally, one uses a simpler notation to denote the six constants, \(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{12}, \sigma_{23}, \) and \(\sigma_{31}\) by \(\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5,\) and \(\sigma_6\).
Appendix

Similarly, strain connects the displacement \( \mathbf{u} \) of a point with its position vector \( \mathbf{r} \) by the relation

\[
\mathbf{u} = [e] \mathbf{r}.
\]

(11)

The strain tensor \([e]\) is again a symmetric second rank tensor having six independent constants, \(e_1, e_2, e_3, e_4, e_5\) and \(e_6\).

Hooke’s law states that stress \([\sigma]\) is proportional to strain \([e]\), that is,

\[
\sigma_i = \Sigma_j C_{ij} e_j.
\]

(12a)

In equation (12a), the index \(i\) runs from 1 to 6; so also the index \(j\). We will use italic indices, \(i, j\), to go from 1 to 3, and script indices, \(i', j'\), to run from 1 to 6.

Conversely,

\[
e_i = \Sigma_j s_{ij} \sigma_j.
\]

(12b)

\(C_{i,j}\) and \(s_{i,j}\) are fourth rank tensors. They are properties of the material. The total number of components in a fourth rank tensor is \(3^4 = 81\). But the symmetry of the stress and strain tensors reduces this number to 36. We may further show that \(C_{i,j} = C_{j,i}\), and \(s_{i,j} = s_{j,i}\). \(C_{i,j}\) are called the elastic constants, and \(s_{i,j}\) are called the elastic compliance coefficients. They are 21 in number. Because they are material properties, crystal symmetry will reduce the number further. Thus, for cubic crystalls, the number of independent components are \(C_{11} = C_{22} = C_{33}, C_{12} = C_{23} = C_{31}\) and \(C_{44} = C_{55} = C_{66}\). The remaining constants are all zero.

3. Piezoelectricity

In some materials, for example crystalline quartz, the application of stress produces an electric displacement field \(\mathbf{D}\). The electric displacement \(\mathbf{D}\) is associated with the production of positive and negative charges on the two faces of the crystal. So we may also say that piezoelectricity is the production of charge on the application of stress.

Conversely the application of an electric field produces a strain in the material. The relation connecting \(\mathbf{D}\) and the stress \([\sigma]\) is given by

\[
\mathbf{D}_j = \Sigma_i d_{ij,k} \sigma_k.
\]

(13)

In equation (13) we have used the full notation for the second rank stress tensor. The coefficients \(d_{ij,k}\) are called the piezoelectric coefficients of the material. In the compressed notation, \(\sigma_j\), for the stress tensor, we may write the piezoelectric constants as \(d_{i,j}\). In general, there are 18 piezoelectric coefficients. Crystal symmetry will reduce this number.

Out of the 32 point groups, 11 have a center of inversion symmetry. In addition, there is the non-centrosymmetric cubic point group 432. For crystals belonging to these 12 point groups, all piezoelectric coefficients \(d_{i,j}\) will be zero. Piezoelectricity will NOT be exhibited by crystals belonging to these 12 point groups. Piezoelectricity is exhibited by crystals belonging to the remaining 20 point groups.

In a piezoelectric material, an electric polarization develops only when a stress is applied. In a ferroelectric material, a spontaneous polarization is present even in the absence of stress. Only 10 point groups, out of the above 20, will show the ferroelectric property. This means that all ferroelectric crystals are piezoelectric, but all piezoelectric crystals are not ferroelectric.

For further study of the effect of symmetry on tensor properties of crystals, the following references may be consulted:


Appendix

Appendix
18.5

NBS Table for Thermo-EMF Chromel–Alumel Thermocouple

Table 1. Thermoelectric EMF in mV Chromel–Alumel Thermocouple Reference Junction at 0 ◦ C. Temperature is in
increments of 1 ◦ C.

Temp◦ C
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
190
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
360
370
380
390
400
410
420
430
440

0

1

2

3

4

5

6

7

8

9

0.000
0.397
0.798
1.203
1.612
2.023
2.436
2.851
3.267
3.682
4.096
4.509
4.920
5.328
5.735
6.138
6.540
6.941
7.340
7.739
8.138
8.539
8.940
9.343
9.747
10.153
10.561
10.971
11.382
11.795
12.209
12.624
13.040
13.457
13.874
14.295
14.713
15.135
15.554
15.975
16.397
16.820
17.243
17.667
18.091

0.039
0.437
0.838
1.244
1.653
2.064
2.478
2.893
3.308
3.723
4.138
4.550
4.961
5.369
5.775
6.179
6.580
6.981
7.380
7.779
8.178
8.579
8.980
9.383
9.788
10.194
10.602
11.012
11.423
11.836
12.250
12.665
13.081
13.498
13.916
14.335
14.755
15.175
15.596
16.017
16.439
16.862
17.285
17.709
18.134

0.079
0.477
0.879
1.285
1.694
2.106
2.519
2.934
3.350
3.765
4.179
4.591
5.002
5.410
5.815
6.219
6.620
7.021
7.420
7.819
8.218
8.619
9.020
9.423
9.828
10.235
10.643
11.053
11.465
11.877
12.291
12.707
13.123
13.640
13.958
14.377
14.797
15.217
15.638
16.059
16.482
16.904
17.328
17.752
18.176

0.119
0.517
0.919
1.326
1.735
2.147
2.561
2.976
3.391
3.806
4.220
4.633
5.043
5.450
5.856
6.259
6.660
7.060
7.460
7.859
8.258
8.659
9.061
9.464
9.869
10.276
10.684
11.094
11.506
11.915
12.332
12.748
13.165
13.582
14.000
14.419
14.839
15.259
15.680
16.102
16.524
16.947
17.370
17.794
18.218

0.158
0.557
0.960
1.366
1.776
2.188
2.602
3.017
3.433
3.848
4.262
4.674
5.084
5.491
5.896
6.299
6.701
7.100
7.500
7.899
8.298
8.699
9.101
9.504
9.909
10.316
10.725
11.135
11.547
11.960
12.374
12.790
13.206
13.624
14.042
14.461
14.881
15.301
15.722
16.144
16.566
16.989
17.413
17.837
18.261

0.198
0.597
1.000
1.407
1.817
2.230
2.644
3.059
3.474
3.889
4.303
4.715
5.124
5.532
5.937
6.339
6.741
7.140
7.540
7.939
8.338
8.739
9.141
9.545
9.950
10.357
10.766
11.176
11.588
12.001
12.416
12.831
13.248
13.665
14.084
14.503
14.923
15.343
15.764
16.186
16.608
17.031
17.455
17.879
18.303

0.238
0.637
1.041
1.448
1.858
2.271
2.685
3.100
3.516
3.931
4.344
4.756
5.165
5.572
5.977
6.380
6.781
7.180
7.579
7.979
8.378
8.779
9.181
9.585
9.991
10.398
10.807
11.217
11.630
12.043
12.457
12.873
13.290
13.707
14.126
14.545
14.965
15.385
15.806
16.228
16.651
17.074
17.597
17.921
18.346

0.277
0.677
1.081
1.489
1.899
2.312
2.727
3.142
3.557
3.972
4.385
4.797
5.206
5.613
6.017
6.420
6.821
t.220
7.619
8.019
8.418
8.819
9.222
9.626
10.031
10.439
10.848
11.259
11.671
12.084
12.599
12.915
13.331
13.749
14.167
14.587
15.007
15.427
15.849
16.270
16.693
17.116
17.540
17.964
18.388

0.317
0.718
1.122
1.530
1.941
2.354
2.768
3.184
3.599
4.013
4.427
4.838
5.247
5.653
6.058
6.460
6.861
7.260
7.659
8.059
8.458
8.860
9.262
9.666
10.072
10.480
10.889
11.300
11.712
12.126
12.540
12.956
13.373
13.791
14.209
14.629
15.049
15.469
15.891
16.313
16.735
17.158
17.582
18.006
18.431

0.357
0.758
1.163
1.571
1.982
2.395
2.810
3.225
3.640
4.055
4.468
4.879
5.288
5.694
6.098
6.500
6.901
7.300
7.699
8.099
8.499
8.900
9.302
9.707
10.113
10.520
10.930
11.341
11.753
12.167
12.582
12.998
13.415
13.833
14.251
14.671
15.091
15.511
15.933
16.355
16.778
17.201
17.624
18.049
18.473

401


Appendix

18.6 Circuits and Mechanical Items and Address of the Company

The following is the complete list of the circuits and mechanical items supplied/manufactured by Ajay Sensors and Instruments.

**Electronic Instruments**

1. Regulated DC Power Supply
2. Temperature indicator
3. Constant current source
4. DC differential amplifier
5. Signal generator
6. Power amplifier
7. High resistance circuit
8. A.C. Bridges circuit
9. Capacitance measuring circuit
10. Thermal relaxation circuit
11. Integrator
12. Lock-in amplifier
13. Feigenbaum circuit
14. Chua circuit
15. Charge amplifier
16. True RMS voltmeter
17. AC Susceptibility circuit
18. AC transformer power supply, 10V, 20 V and 40 V

**Mechanical items**

1. Rectangular box furnace
2. Rigidity modulus setup
3. Young’s modulus by flexural vibrations setup
4. Insert for calibration of a Si diode and copper–constantan thermocouple
5. Stefan’s constant box
6. Thermal conductivity of copper box
7. Thermal conductivity of a poor conductor box
8. Insert for TCR of copper and energy band gap of silicon
9. Thermal diffusivity of brass box
10. R-L-C box used in AC experiment
Appendix

11. Capacitance box
12. Cylindrical capacitance jar
13. Passive filter box
14. Variable voltage box
15. Capacitance cell
16. Insert with ceramic and polymer capacitor
17. Setup for piezoelectric constant for PVDF
18. B–H curve setup for a hard magnetic material
19. B–H curve setup for a soft ferrite
20. Mutual inductance coil
21. Low resistance box
22. Primary and secondary coils with insert with Ni-Zn soft ferrite bead

ADDRESS OF THE COMPANY

AJAY SENSORS & INSTRUMENTS
45/17, Gubbanna Industrial Garden,
12th A Cross, 6th Block,
Rajaji Nagar,
Bangalore 560 010
E-mail: ajaysensors @ yahoo.com

18.7 Books for Study

1. Condensed Matter Physics
   (i) Introduction to Solid State Physics, C Kittel (John Wiley & Sons Inc.)

2. Materials Science
   (i) Callister’s Materials Science and Engineering, W D Callister, (adapted by R. Balasubramanian),
       (Wiley India (P) Ltd. 2007).

3. Phase Transitions
Appendix

4. Nonlinear Dynamics

(i) Nonlinear Dynamics: Integrability, Chaos and Patterns, M Lakshmanan and S Rajasekhar, (Springer India Pvt Ltd. 2003).

5. Introduction to Magnetism and Magnetic Materials


6. NanoScience


7. Introduction to Electronics

(ii) Principles of Electronics, V K Mehta (free download).
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