The Journey from Maxwell to Faraday
(From Fields to Strings)

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- Matured and became highly successful in the 1970’s-80’s.
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- Mathematically similar to statistical mechanics and hence its universality and power.
String Theory

- A very nontrivial generalisation of QFT - introduced to study gravitational interactions.
- In many special cases, it appears to give a miraculous reformulation of QFTs. How?
- Can be used to do calculations in strongly interacting QFT using classical gravity! Why?
- My work: To try and elucidate how string theory naturally emerges from QFT.
Maxwell vs. Faraday

- Two pictures of Classical Electromagnetism:

- Localised pointlike fields vs. extended lines of flux.
Maxwell vs. Faraday

- Maxwell’s (classical field) picture is good for electrodynamics.
- Since the strength of interactions is small.
- QED is a successful theory of quantum fields.
- Faraday’s picture is not quantitatively useful.
- What are the equations governing the diffuse lines of flux?
Maxwell vs. Faraday

- But this is not always the case.
- Sometimes the flux lines are bunched up into a string-like tube (e.g. QCD).

- Can now try to write equations for the dynamics of these strings.
- This is a more natural & useful reformulation.
“We may assume [that] when we pass to the quantum theory the lines of force become all discrete and separate from each other.... We now have a model in which the basic entity is the line of force, a thing like a string, instead of a particle. The strings will move about and interact with one another according to quantum laws” -- Paul Dirac (1956)
From Fields to Strings

- *In Field Theory, a sum over paths:*

\[ \sum_{\text{paths}} e^{iS} = \int [D X(\tau)] e^{iS[X(\tau)]} \]

- *Point particle point of view.*

- *Captured by Feynman diagrams of QFT.*
From Fields to Strings

$$\sum_{\text{surfaces}} e^{iS} = \int [DX(\sigma, \tau)] e^{iS[X(\sigma, \tau)]]}$$

- How can a sum over paths be also a sum over surfaces?

- $$\sum_{\text{paths}} = \sum_{\text{surfaces}} ??$$
From Fields to Strings

- Remarkably, there exists a general way to reorganise the Feynman Diagrams for point particles into a sum over surfaces for strings.
- The graphs essentially give a triangulation of the moduli space of Riemann surfaces.
- Schwinger proper times of QFT translate into parameters of the worldsheets of flux tubes.
- Basically thickened graphs get glued into surfaces.
QFT amplitudes can be recast into String theory amplitudes!

Simple examples seem to give a very explicit picture of this connection.

Indications of an extra holographic dimension where the Faraday flux lives.

But need to developed further to obtain explicit Faraday equations for strings.
To Conclude...To Continue

- Recasting QFT in the language of strings is a passage from Maxwell to Faraday.
- Can see how this can happen on very general grounds.
- Potentially very powerful in trying to understand dynamics of strongly interacting QFTs in many contexts.
- Might also shed light on puzzles of quantum gravity.