Stability and dynamics of fluid flow past deformable solid media

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Outline

- Introduction and motivation.
- Governing equations and stability theory.
- Two new high Reynolds number instabilities.
- Suppression of interfacial instabilities in two-layer flows by a soft solid layer.
- A new instability in viscoelastic flow past a deformable wall.
- Summary and outlook.

Flow past soft deformable solid surfaces

- Biological flows and Bio-technological applications.
- Aerospace and marine propulsion: drag reduction.
- Geophysical phenomena (volcanic tremors).
- Flow in microfluidic devices made of soft elastomers.
- Polymer processing by co-extrusion.

Fluid-dynamic aspects:

- Instabilities and Laminar–Turbulent transition.
- Laminar flow $\Rightarrow$ lower drag, poor mixing.
- Turbulent flow $\Rightarrow$ high mixing & drag.

Salient features of flow past soft solids

- Rigid walls: shear modulus of steel $\sim 10^{10}$ Pa
  Soft materials: shear modulus $\sim 10^4 – 10^6$ Pa $\sim 10^{-5} G_{steel}$.
- Soft interfaces easily deformed by fluid stresses.
- Elasto-hydrodynamic coupling $\Rightarrow$ Interfacial waves.
- Wall dynamics could affect instabilities and transition.
- Anomalous instabilities observed in experiments.
  Rigid tube $Re \simeq 2100$; Deformable tubes: Re much lower.
- **Objective of our work:** To uncover qualitatively new instabilities in flow past deformable solid media.
  (Will not consider tube collapse due to external pressure changes.)

Fluid Governing equations

- Incompressible; both Newtonian and Viscoelastic fluids.

$$\rho(\partial_t + v_j \partial_j) v_i = -\partial_i p + \partial_j \tau_{ij}$$

- **Newtonian fluid:** $\tau_{ij} = \eta(\partial_i v_j + \partial_j v_i)$
- **Viscoelastic fluid:** Upper-Convected Maxwell (UCM) model

  $\lambda[\partial_t \tau_{ij} + v_k \partial_k \tau_{ij} - \partial_k v_i \tau_{kj} - \partial_k v_j \tau_{ki}] + \tau_{ij} = \eta(\partial_i v_j + \partial_j v_i)$
- Material constants: Viscosity η and relaxation time λ.
- Nonzero first normal stress difference under simple shear.
- Applicable for polymer melts; solvent contribution easily added.

Model for deformable solid

- Incompressible linear viscoelastic solid

  u_i: Displacement field in the solid.

  $\Sigma_{ij} = -p_w \delta_{ij} + \sigma_{ij}$

$$\sigma_{ij} = G(\partial_i u_j + \partial_j u_i) + \eta_w \partial_t (\partial_i u_j + \partial_j u_i)$$

  Governing Equations:

$$\partial_i u_i = 0$$

$$\rho \partial_t^2 u_i = \partial_j \Sigma_{ij}$$

- Material constants: Shear modulus G and Viscosity η_w.

- For simplicity, density of solid = density of fluid = ρ

Linear stability theory

- Impose small fluctuations and linearize: $v_i = \overline{v}_i + v_i'$

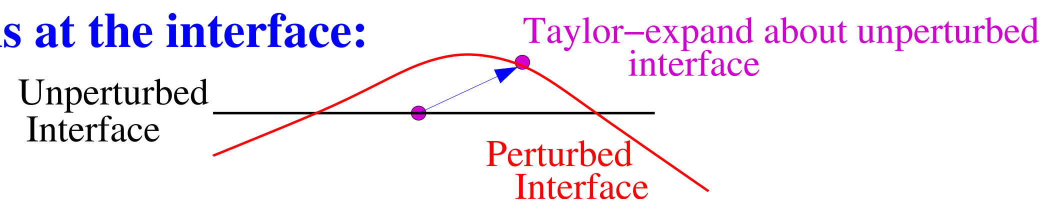
$$v_i'(x,z,t) = \sum \tilde{v}_i(z) \exp[\mathrm{i}kx + \mathrm{i}n\theta] \exp[-\mathrm{i}kct]$$

- k: Wavenumber of fluctuations.

- Complex wave speed $c = c_r + \mathrm{i}\, c_i$

$c_i > 0 \Rightarrow$ Unstable $\quad c_i < 0 \Rightarrow$ Stable $\quad c_i = 0 \Rightarrow$ Neutrally stable.

Conditions at the interface:



- Non-trivial couplings between the base flow and height fluctuations in the interface conditions.

High Reynolds number inviscid modes

- $Re \gg 1$, Fluid inertial stresses $\sim$ Wall elastic stresses $\boxed{\rho V^2 \sim G}$

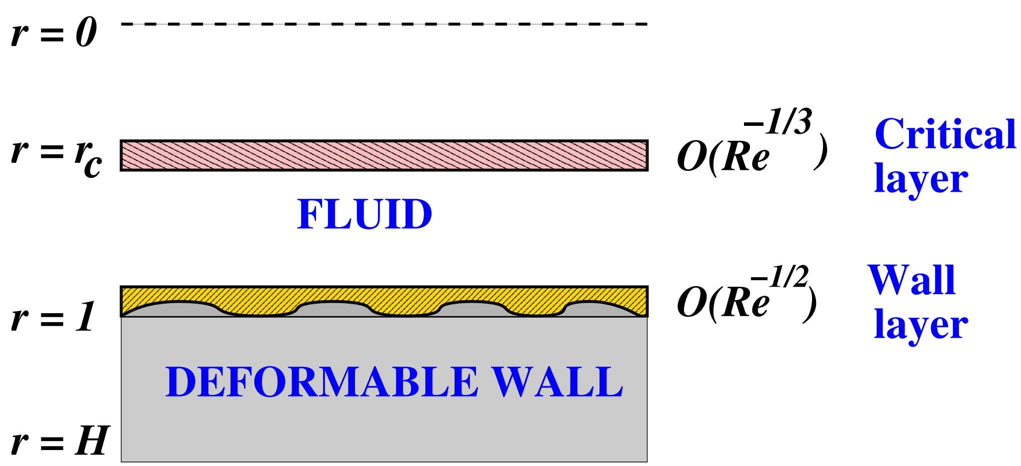
$$\Rightarrow \boxed{V \sim (G/\rho)^{1/2}}$$

- Inviscid fluid governing equations.
- Classical theorems of stability for flow in a deformable tube $\Rightarrow$ Can a given flow be unstable in the inviscid limit ?
- Developing flow in the entrance region, flow in a converging tube, fully-developed flow subjected to non-axisymmetric perturbations.
- Singular perturbation in the inviscid limit: Viscous effects important in two thin layers even at high Re.

J. Fluid. Mech, **395**, 211 (1999); **407**, 291 (2000).

Inviscid instability

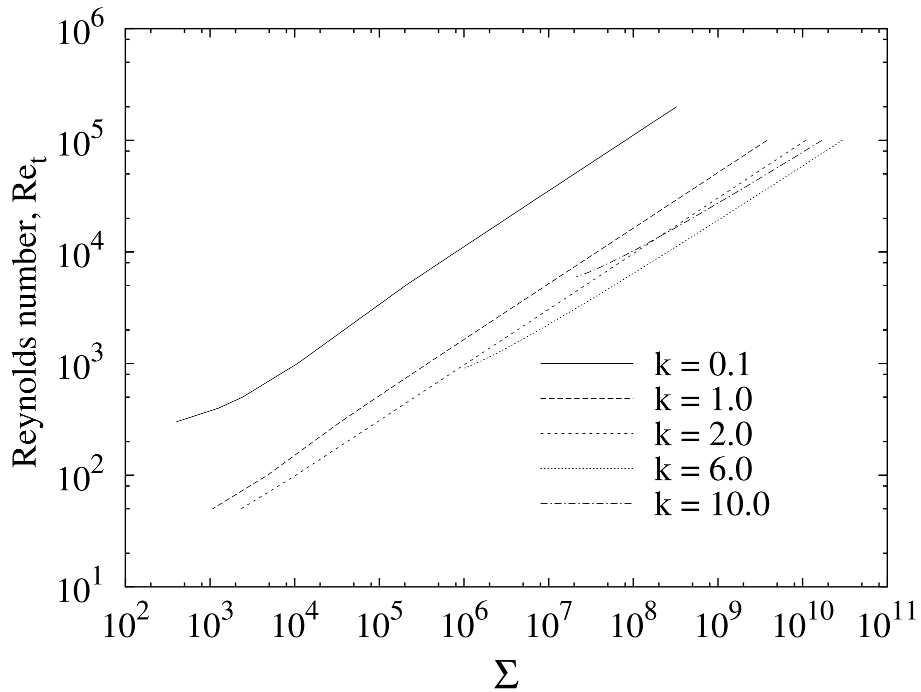
- Inviscid limit: Logarithmic singularity at $r = r_c$ where $U(r_c) = c_r$
- Critical and wall layers:



- Multiple solutions at leading order for $c^{(0)}$.
- Flow unstable in the inviscid limit when $\rho V^2/G$ increases.
- Inviscid results corroborated by numerical solution at large Re.

Reynolds number vs. Nondimensional elasticity Σ

- Flow in entrance region: $X = 0.050, H = 2$; $\Sigma = \rho G R^2/\eta^2$



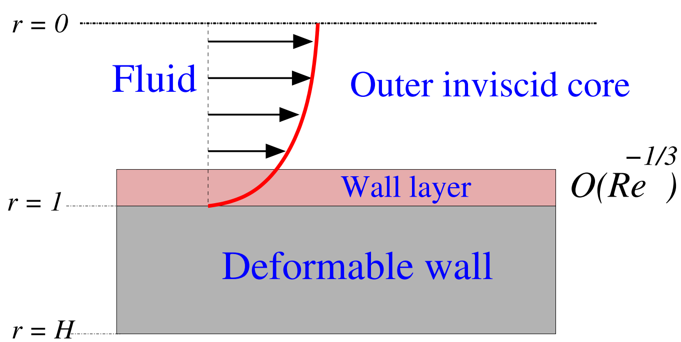
- Instability extends to moderate Reynolds number $Re \sim \Sigma^{1/2}$

Summary of Inviscid instability

- Instability requires the presence of the critical layer singularity.
- Absent in fully developed flow in a deformable tube subjected to axisymmetric disturbances and in plane Couette flow in a channel.
- Instability predicted for developing flow in a deformable tube, flow in a converging deformable tube, and for fully-developed flow subjected to axisymmetric disturbances.
- Absent in rigid tubes.
- Instability driven by convective transport of energy in the critical layer.
- Critical Reynolds number $Re \propto \Sigma^{1/2}$; continues to moderate Re.

High-Re Wall mode instability

- $Re \gg 1$; wavespeed of fluctuations $O(Re^{-1/3})$ smaller than the maximum velocity of the base flow.
- Viscous stresses confined to a layer of thickness $O(Re^{-1/3})$ smaller than the tube radius *near the fluid-solid boundary*.
- Viscous shear stresses in the wall layer $\sim$ Elastic stresses in the solid.
- $\frac{\eta V}{R Re^{-1/3}} \sim G \Rightarrow \boxed{Re \propto \Sigma^{3/4}}$; $\boxed{V \gg (G/\rho)^{1/2}}$



Euro Phys J B, **19**, 607 (2001); *Phys Fluids*, **14**, 2324 (2002)

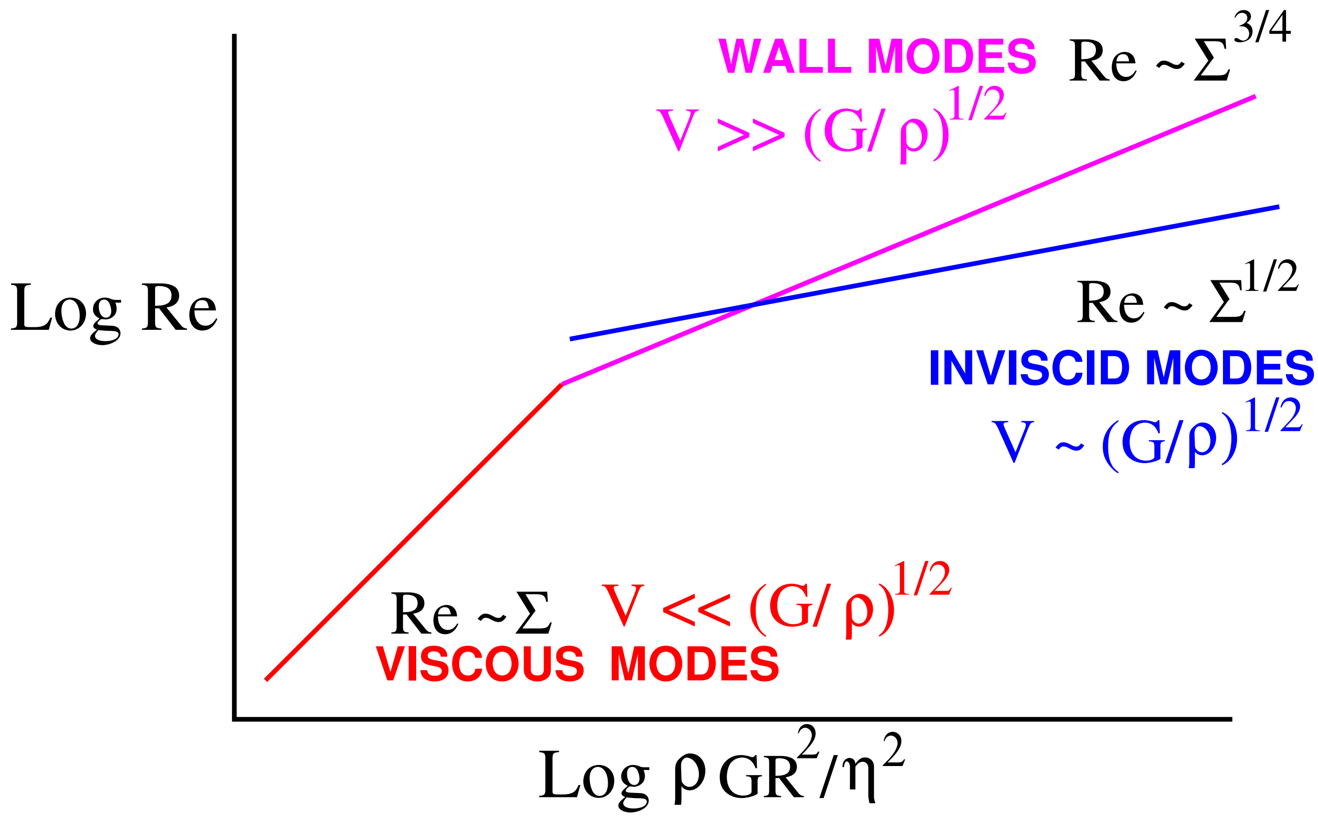
Wall mode instability in flow past deformable solids

- Rigid tubes: Wall modes are stable (Gill, 1965)
- Present asymptotic analysis: $Re \gg 1$, $Re \propto \Sigma^{3/4}$

  Leading order: Fluid shear stresses in the wall layer subdominant; c real, with multiple solutions, $c^* \propto (G/\rho)^{1/2}$.

  First correction: Fluid tangential velocity and stresses in the wall layer dominant; Flow unstable when $Re^{-1/3}(\rho V^2/G)^{1/2} \sim O(1)$.
- Instability continues to low Re.
- Importance of tangential wall motion in this instability.
- Independent of the details of the velocity profile: a generic feature in flow past deformable solid surfaces at $Re \gg 1$.

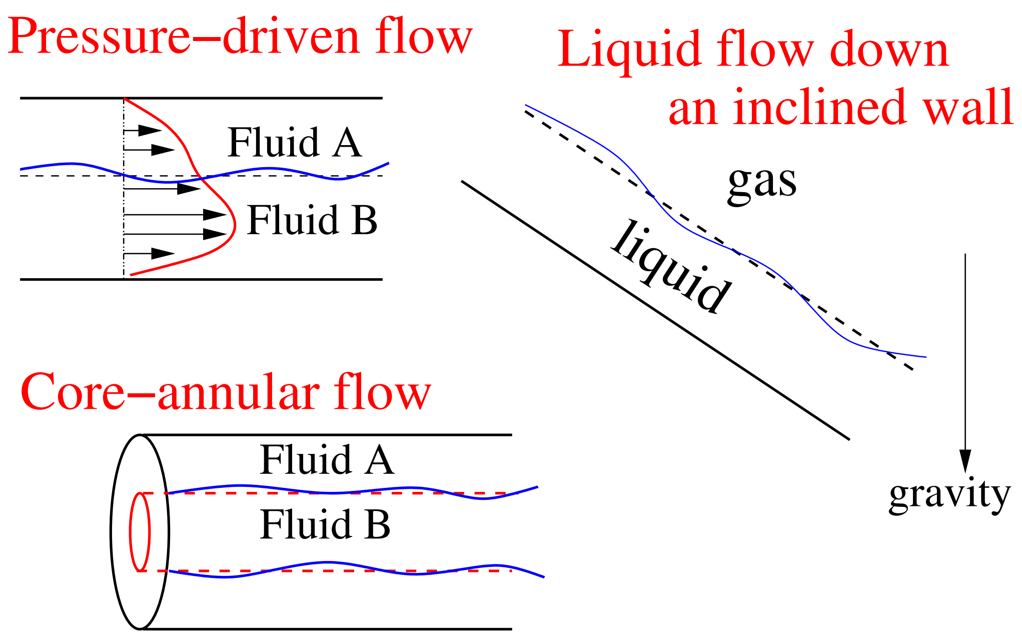
Instabilities in Newtonian flow past a deformable solid



- Instabilities absent in flow through rigid tubes/channels.

Suppression of flow-induced interfacial instabilities in two-layer flows by a soft solid layer

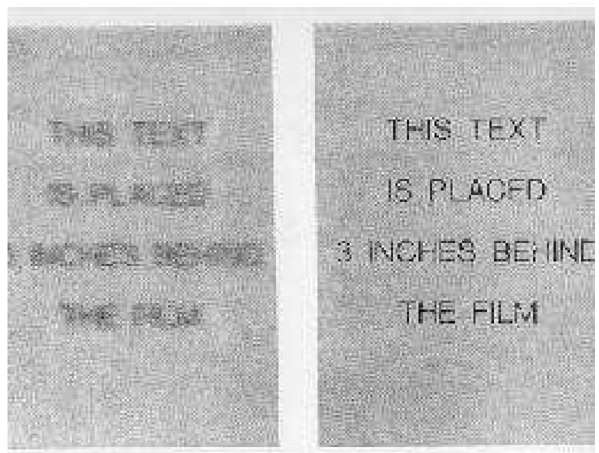
- Coextrusion of polymer melts; lubricated pipelining in oil recovery.



- Interfacial instabilities due to viscosity and/or elasticity stratification.

Example of an interfacial instability

- Effect on the optical quality of a three-layer co-extruded film.



(Mavridis & Shroff, *Polym. Engg and Science*, 1994)

Can deformable solid layers suppress interfacial instabilities ?

Two-layer flow past a soft solid

Rigid surfaces: Long-wave asymptotic analysis.

$$c = c^{(0)} + kc^{(1)} + \cdots$$

$c^{(0)} \to$ real; system neutrally stable to leading order.

$c^{(1)} \to$ imaginary; system unstable under certain configurations.

• Driven by discontinuity of shear rate/normal stress difference at the interface.

Soft solid surface:

At fluid-solid interface: $\tilde{v}_z = -\mathrm{i}kc\tilde{u}_z$; If $\tilde{v}_z \sim \tilde{u}_z \sim O(1)$ in the bulk, $\Rightarrow$ $\tilde{v}_z \sim O(k)$ at the interface. Similarly $\tilde{v}_x \sim O(k)$

$\Rightarrow$ $\boxed{\tilde{v}_z = \tilde{v}_x = 0}$ to leading order at the interface.

$c^{(0)}$ unaffected by solid deformability; same as in rigid surfaces.

Phys Fluids, **16**, 4426 (2004); *J Non-Newt Fluid Mech*, **117**, 163 (2004); **125**, 143 (2005).

Two-layer flow past a soft solid

• Fluid stresses at leading order cause deformation in the solid layer.

⇒ Solid deformation affects the $O(k)$ velocity field in the fluid, through velocity continuity conditions.

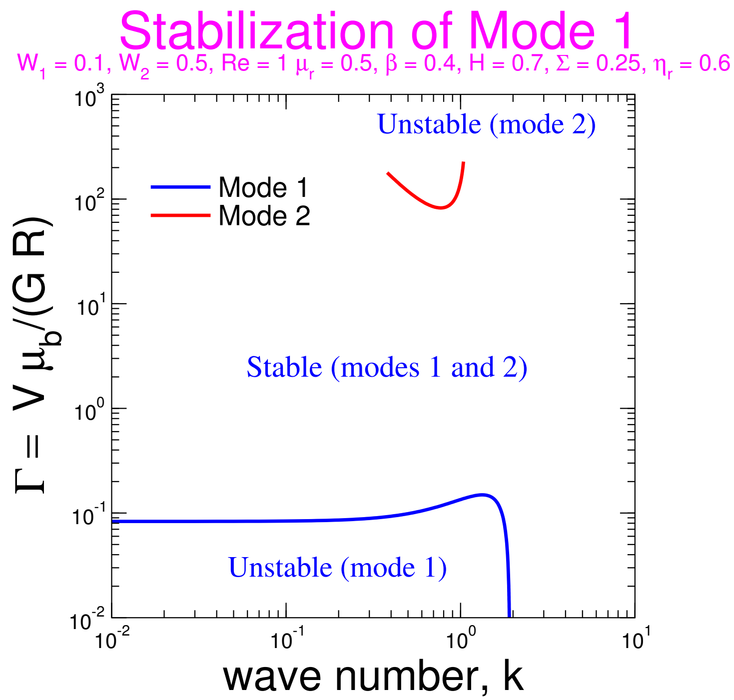
$$c = c^{(0)} + \mathrm{i}k \left[Re f_1(\mu_r, \frac{d_1}{d_2}) + f_2(\frac{\lambda_1}{\lambda_2}, \frac{d_1}{d_2}) + \frac{V\eta H}{GR} f_3(H, \mu_r, \frac{d_1}{d_2}) \right]$$

Contribution from soft solid layer: $f_3(H, \mu_r, d_1/d_2)$ can be stabilizing or destabilizing depending on $\mu_r, H, d_1/d_2$.

• Stabilizing mechanism works for different flow-induced interfacial instabilities.

• Does this continue to finite k ?

Results for two-layer viscoelastic Couette flow



- Short-wave fluctuations in the two-fluid interface unaffected by the solid layer; stabilized only by nonzero interfacial tension.
- A wide window in shear modulus where both modes are stable.

High-frequency modes in viscoelastic Couette flow

• Are there qualitatively new instabilities in viscoelastic flows that are absent in Newtonian fluids ?

Viscous diffusion time $(R^2\rho/\eta) \ll \lambda$ Relaxation time of the fluid.

$\Rightarrow Re \ll W = \lambda V/R.$

• Frequency of fluctuations $\gg$ shear rate of base flow.

$\Rightarrow$ Probe only the 'elastic part' of the UCM fluid.

• Free shear waves in a viscoelastic fluid: wavespeed $c^* \propto \sqrt{\frac{G}{\rho}} \propto \sqrt{\frac{\eta}{\lambda\rho}}$

$\Rightarrow c^*/V \propto Re^{-1/2}W^{-1/2} \propto Re^{-1/2}$ for $W \sim O(1)$.

Asymptotic analysis in the small parameter $Re^{1/2}$, for $Re \ll 1$.

J Non-Newt Fluid Mech, **125**, 121 (2005)

High-frequency modes in viscoelastic flow

Rigid channel

$$c = Re^{-1/2} c^{(0)} + c^{(1)} + \cdots$$

$c^{(0)} \rightarrow$ multiple real solutions (both upstream and downstream waves)

$c^{(1)} \rightarrow$ negative imaginary part; fluid viscous effects stabilizing.

Deformable wall

Flow appears even at leading order in $c^{(0)}$.

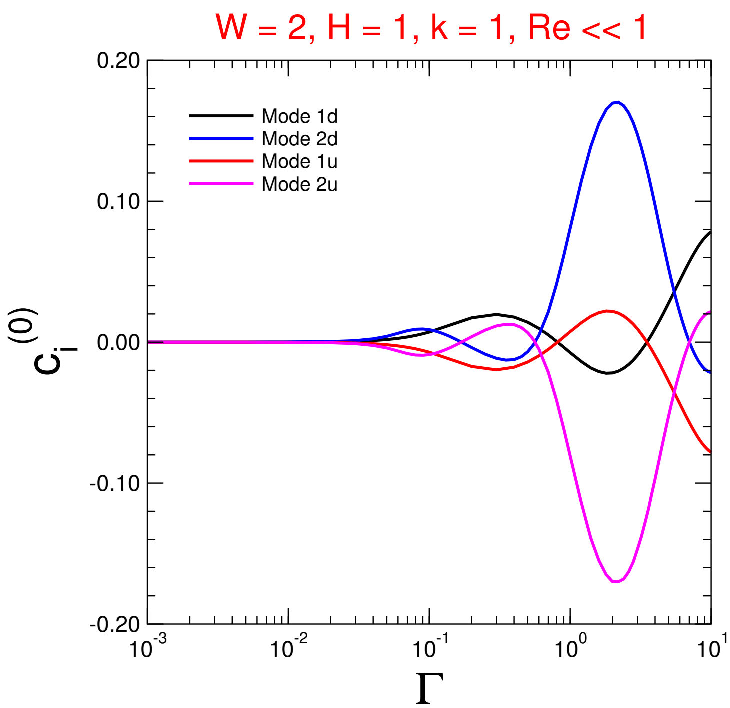
Flow destabilizing for downstream waves, with $c_i^{(0)} \propto \Gamma^2 = (V\eta/(GR))^2$ for small Γ.

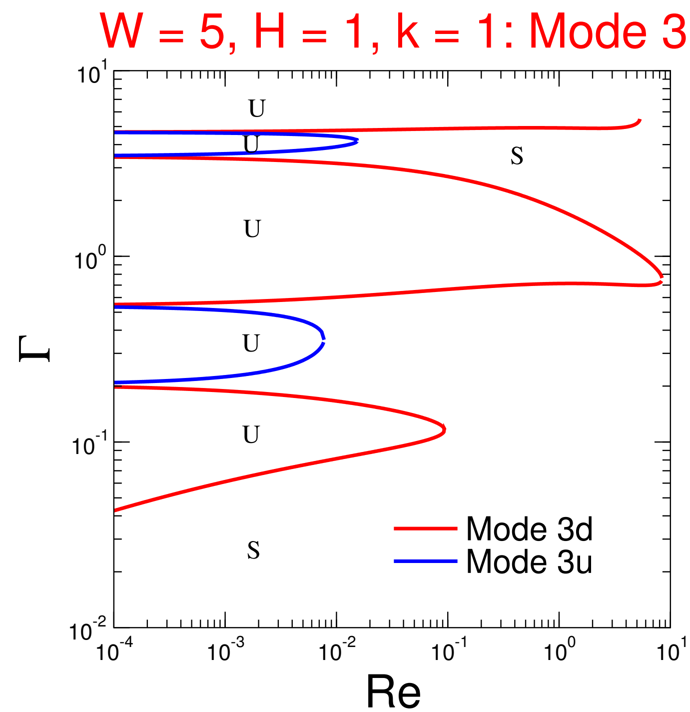
First correction $c^{(1)}$: Stabilizing due to viscous effects

$\Rightarrow$ For small Re and $W \sim O(1)$, $\boxed{\Gamma \propto Re^{1/4}}$ or $\boxed{Re \propto \Sigma^{4/3}}$

Results for Growth rate and Neutral stability

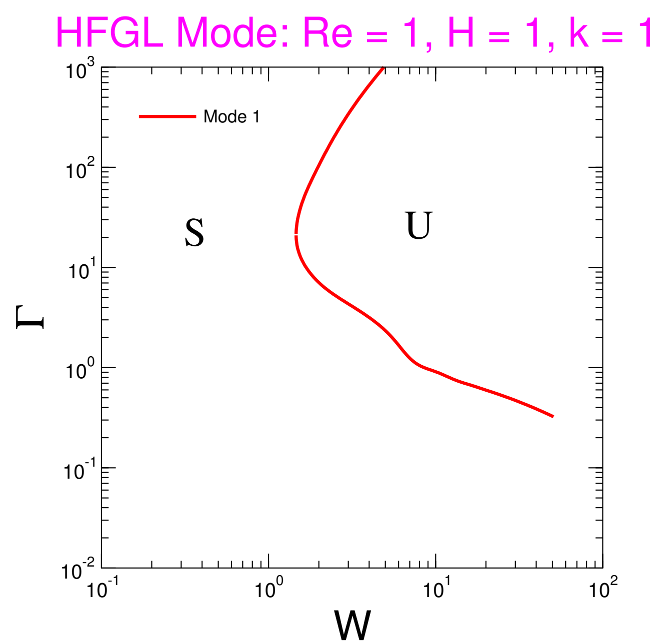
- Oscillatory variation of $c_i^{(0)}$ with Γ for $\Gamma \sim O(1)$.





- Instability exists only upto $Re \sim 1$.

Results: Γ–W Neutral curve



- Instability absent in Newtonian fluids.
- Solvent viscosity has a stabilizing effect on this instability $\Rightarrow$ Instability expected in concentrated polymer solutions and melts at low Re.

Summary and outlook

- Qualitatively new instabilities at high Re in Newtonian fluid flow past deformable solid surfaces.
- Soft, deformable solid layer coatings can suppress two-fluid interfacial instabilities in viscoelastic and Newtonian fluids.
- New class of unstable modes in viscoelastic flow past a deformable wall (absent in Newtonian fluids).

Open questions

- Flow in the nonlinear regime ?
- Both computations and experiments can throw more light.

Acknowledgements

Prof. V Kumaran, IISc Bangalore (Newtonian fluid instabilities)

A Sameer Kumar, IIT Kanpur (Viscoelastic instabilities)

Lalit Kumar, IIT Kanpur (Two-layer flows)