Light, Space and Time[*]

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Einstein's work in 1905 on Special Relativity and on the Quantum Nature of Light - two of his three monumental achievements in that year - have had profound consequences for the further development of physics. His bringing space and time together more intimately than before was a major first step on the way to his later General Relativity, and was accepted quite quickly. The idea of the Photon, the Particle of Light, however went through a more complicated history, and waited for almost two decades before widespread acceptance. This talk will cover these two aspects of Einstein's work in his Year of Miracles.

I will attempt to convey to you the background and the meanings of two of Albert Einstein's three path breaking discoveries of 1905 - the first one of March on the nature of light, and the second of June on the natures of space and time, his Special Theory of Relativity. In each case a little bit of history is involved. The titles and publication details of the two papers are:

'On a heuristic point of view concerning the production and transformation of light', Annalen der Physik <u>17</u>, 132 (1905), received March 18.

'On the electrodynamics of moving bodies', Annalen der Physik <u>17</u>, 891 (1905), received June 30.

The light quantum - christened 'photon' in 1926 by the chemist G.N.Lewis - is the second elementary particle in the history of physics, the first being the electron discovered by J.J.Thomson in 1897. However unlike the latter, the photon was not the result of an experimental discovery but of an 'inner revolution', a reinterpretation of existing theoretical ideas. It all began in 1859 when Gustav Kirchoff showed that if radiation is in equilibrium with material bodies at a common temperature T - called 'temperature radiation' - its energy density per unit frequency interval is a <u>universal</u> function $U(\nu, T)$ of frequency ν and temperature T. He then posed the problem - measure $U(\nu, T)$ experimentally and derive it from theory.

In 1893 Wilhelm Wien proved a theorem about the form of this function:

$$U(\nu, T) = \nu^3 \ f(\nu/T), \tag{1}$$

and in 1896 he proposed a specific form leading to Wien's Law:

$$U(\nu, T) = a \ \nu^3 \ e^{-b\nu/T}. \tag{2}$$

This was obeyed well for high frequencies. In 1899 Max Planck found a link between $U(\nu, T)$ and the average energy $\epsilon(\nu, T)$ of a charged material oscillator of natural frequency ν at temperature T, continously emitting and absorbing radiation according to Maxwell's electromagnetic equations:

$$U(\nu, T) = \frac{8\pi\nu^2}{c^3} \ \epsilon(\nu, T) \ , \ \epsilon(\nu, T) = \frac{c^3}{8\pi} \cdot \nu \ f(\nu/T). \tag{3}$$

In mid 1900 Lord Rayleigh (followed later by James Jeans) showed that if one applied the classical equipartition theorem to radiation, there is a definite

prediction:

$$U(\nu, T) = \frac{8\pi\nu^2}{c^3} \cdot k\,T \ , \ f(\nu/T) = \frac{8\pi k}{c^3} \cdot T/\nu. \qquad (4)$$

Planck himself could have obtained this result if he had applied the equipartition theorem to the material oscillator.

Later, in October 1900, Planck was told by his experimental colleague Rubens that while Wien's Law held at high frequencies, the Rayleigh-Jeans law was valid at low frequencies. He then immediately set to work to find a mathematical interpolation between the two. For this he worked with the entropy $S(\epsilon)$ of the material oscillator written in terms of its energy. From thermodynamics we have the relations

$$\frac{dS(\epsilon)}{d\epsilon} = \frac{1}{T} \ , \ \frac{d^2 S(\epsilon)}{d\epsilon^2} = -\frac{1}{T^2}\frac{dT}{d\epsilon} \ , \qquad (5)$$

so by simple substitutions the validity of the Wien and Rayleigh Jeans laws gave:

$$\left(\frac{d^2 S(\epsilon)}{d\epsilon^2}\right)^{-1} = \left(\begin{array}{l} -b\nu\epsilon \text{ in Wien, small } \epsilon, \text{ limit (non classical)} \\ -\epsilon^2/k \text{ in Rayleigh Jeans, large } \epsilon, \text{ limit (classical)} \end{array}\right) \qquad (6)$$

Planck then guessed that for all ϵ one just adds the two:

$$\left(\frac{d^2 S(\epsilon)}{d\epsilon^2}\right)^{-1} = -(b\nu\epsilon + \epsilon^2/k) \qquad (7)$$

from which by elementary integration and appeal to the $R - J$ limit he obtained his famous Law of Temperature Radiation:

$$\begin{aligned} \frac{dS(\epsilon)}{d\epsilon} &= \frac{1}{T} = \frac{1}{b\nu}\ln(1 + kb\nu/\epsilon) \Rightarrow \epsilon(\nu, T) = h\nu/(e^{h\nu/kT} - 1), \ h = bk \\ &\Rightarrow \ U(\nu, T) = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{h\nu/kT} - 1} \end{aligned} \qquad (8)$$

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Two months later in December 1900 he proposed the physical postulate of the quantisation of the possible energies of the material oscillator, which would lead to this expression for its average energy $\epsilon(\nu, T)$.

Now we come to Einstein's work in 1905. He considered radiation on its own, not the link to matter, in the non-classical Wien limit of high frequencies and contained in a spatial volume V and in a frequency range $\Delta \nu$. This radiation has energy

$$
\begin{aligned}
E \simeq \frac{8\pi\nu^2 \Delta\nu}{c^3} \cdot V \cdot h\nu \; e^{-h\nu/kT} \;\; &= \;\; \mathcal{N} \cdot V \cdot e^{-h\nu/kT}, \; \mathcal{N} = 8\pi h\nu^3 \Delta\nu/c^3 \\
&= \;\; E(V,T).
\end{aligned}
\tag{9}
$$

Using again the thermodynamic relation (5) for temperature, he calculated the entropy of this radiation in terms of volume V and energy E:

$$
\begin{aligned}
\frac{dS}{dE} \;\; &= \;\; \frac{1}{T} = \frac{k}{h\nu}(\ln V - \ln E/\mathcal{N}) \Rightarrow \\
& \;\; S(E,V) = \frac{k}{h\nu}(E \ln V + E - E \ln E/\mathcal{N}).
\end{aligned}
\tag{10}
$$

Now let v_0 be the total volume and v a part of it; then we get

$$
S(E,v) - S(E,v_0) = \frac{kE}{h\nu} \ln\left(\frac{v}{v_0}\right).
\tag{11}
$$

From the Boltzmann relation $S = k \ln W$ linking entropy to statistical probability we get

$$
W = e^{(S(E,v)-S(E,v_0))/k} = \left(\frac{v}{v_0}\right)^{E/h\nu},
\tag{12}
$$

which Einstein interpreted in these words:

"If monochromatic radiation of frequency ν and energy E is enclosed.... in the volume v_0, the probability that at a randomly chosen instant the total radiation energy will be found in the portion v of the volume v_0 is

$$W = \left(\frac{v}{v_0}\right)^{E/h\nu}.$$

From this we further conclude that monochromatic radiation of low density (within the range of validity of Wien's radiation formula) behaves thermodynamically as if it consisted of mutually independent energy quanta of magnitude $h\nu$".

This is how the concept of light quanta was born. Einstein applied it immediately to explain three experimentally known phenomena: Stokes' rule in photo luminiscence, the photoelectric effect, and the ionization of gases by ultraviolet light. Of these the most famous is the photoelectric equation

$$E = h\nu - P \tag{13}$$

connecting electron energy E, light frequency ν and work function P.

R.A.Millikan did extensive experiments over the decade 1905-1915 to check this equation for visible light. Maurice de Broglie checked it for X-rays, Jean Thibaud and Ellis for gamma rays. But at the end Millikan said:

"I spent ten years of my life testing that 1905 equation of Einstein's, and, contrary to all my expectations, I was compelled in 1915 to assert its unambiguous verification in spite of its unreasonableness, since it seemed to violate everything we knew about the interference of light".

In fact, Millikan was not alone in his reluctance to accept the light quantum idea, so were Planck and Niels Bohr. In fact in 1913 Planck wrote about Einstein:

"That he may sometimes have missed the target in his speculations, as for example, in his theory of light quanta, cannot really be held against him".

In spite of such opposition, over the years Einstein sharpened his ideas about radiation quanta. In 1909 he went back to the complete Planck Law, not just the Wien limit, and calculated the fluctuation of energy of radiation using statistical thermodynamics. This was really a reinterpretation of Planck's interpolation of 1900. If we consider temperature radiation in a unit spatial volume and in a frequency range $\Delta\nu$, and we bring in Planck's oscillator energy ϵ, we have for the radiation energy E:

$$
\begin{aligned}
E &= U(\nu, T)\Delta\nu = \alpha\,\epsilon,\ \alpha = 8\pi\nu^2\Delta\nu/c^3: \\
(\Delta E)^2 &= k\,T^2\,\frac{dE}{dT} = k\,T^2\alpha\,\frac{d\epsilon}{dT} = -\alpha\,k\left(\frac{d^2 S(\epsilon)}{d\epsilon^2}\right)^{-1} \\
&= \alpha\,k\left(\frac{\epsilon^2}{k} + b\,\nu\,\epsilon\right)\ \text{by Planck's interpolation} \\
&= \alpha\,\epsilon^2 + \alpha\,h\,\nu\,\epsilon
\end{aligned}
$$

$$
= \underset{\underset{\substack{\text{classical RJ} \\ \text{wave term}}}{\downarrow}}{\frac{c^3}{8\pi\nu^2\Delta\nu}E^2} \quad + \quad \underset{\underset{\substack{\text{quantum Wien} \\ \text{photon term}}}{\downarrow}}{h\nu\,E} \tag{14}
$$

So he interpreted electromagnetic radiation as having simultaneously wave like and particle like properties - the first instance of such duality -

"It is my opinion that the next phase in the development of theoretical physics will bring us a theory of light that can be interpreted as a kind of fusion of the wave and the emission theory".

At this time he also pictured radiation as made up of "independently moving point like quanta with energy $h\nu$". In 1916 he showed that apart from possessing energy $h\nu$, the light quantum carries momentum $h\nu/c$ as well. And then he wrote to his close friend Michele Besso:

"I do not doubt any more the reality of radiation quanta, although I still stand quite alone in this conviction".

And in spite of Planck's and Bohr's views to the contrary, he was convinced of the need to apply quantum mechanics to the radiation field:

"The properties of elementary processes....make it seem almost inevitable to formulate a truly quantized theory of radiation".

The photon was finally accepted in 1923-24 after the discovery of the Compton effect, while Dirac quantized the electromagnetic field in 1927.

Today the photon is a century old, and is accepted as one of the elementary particles of nature, the carrier of electromagnetic forces. Its cousins the W and Z bosons and the gluons are the carriers of the weak and the strong nuclear forces. Photons are used to probe atomic and molecular processes, with the help of lasers, to 'watch what happens' in chemical reactions at incredibly short time scales like 10^{-12} or 10^{-15} seconds. They have been used in crucial experiments testing the foundations of quantum mechanics - Alain Aspect's experiments to check violations of Bell's inequalities, and Leonard

Mandel's beautiful experiments to display the quantum interference of light. Dirac once said that the photon interferes only with itself, and when a photon meets a beam splitter, it is in a Hamlet - like dilemma - whether to be transmitted or be reflected - the most fundamental probabilistic behaviour in quantum mechanics. Quantum teleportation too was first achieved using photons.

In 1905 Einstein had written to his friend Konrad Habicht about his several papers that year, and he had used the phrase 'truly revolutionary' only in connection with the light quantum concept. Here is what he said near the end of his life, in a letter of December 1951 to Besso:

"All the fifty years of conscious brooding have brought me no closer to the answer to the question, 'What are light quanta?'. Of course today every rascal thinks he knows the answer, but he is deluding himself".

No better way than this to conclude this part of my talk!

Now I turn to my second topic, the Special Theory of Relativity, where the history goes further back in time. In 1685 Newton in his Principia had expressed definite views on the natures of space and time. Space was intrinsically homogeneous, isotropic and Euclidean, and was the same for all observers; time flowed uniformly and was also common to all observers. Thus each was absolute on its own. Then, based on Galilean-Newtonian mechanics, there was a Principle of Relativity: there exists in Nature a special class of so-called inertial frames or observers, moving with respect to one another with uniform velocities, for whom the laws of mechanics are the same and

take their simplest form. It was important for the further development of physics that Newton expressed these views in clear and unambiguous terms.

Then Maxwell discovered his field equations of electromagnetism around 1865, and found that they predicted the existence of electromagnetic waves with a certain speed calculable from theory. He identified light with such waves, thus unifying optics, electricity and magnetism. He evidently accepted the Newtonian view of space and time, which implied a definite law for addition of velocities. This led to the conclusion: the Maxwell equations could not be valid in all inertial frames of mechanics, but only in a subset of them at rest relative to one another. In these alone the speed of light would have the calculated value of 300,000 km/sec. In all other inertial frames of mechanics the speed would be both frame and direction dependent. In brief - the principle of relativity of mechanics could not hold for electromagnetism, absolute rest would be meaningful, and departures from it could be detected by electromagnetic measurements.

This was the background for the exquisitely precise experiments of Michelson and Morley, and as you all know they showed no effects of the anticipated kind. The explanation offered for this was the rather adhoc Lorentz-Fitzgerald contraction hypothesis: moving material objects shrink in their direction of motion.

Meanwhile Einstein seems to have approached the problem entirely on his own in a strikingly original fashion. From around age 16 he asked himself such questions as: what does a light wave look like if I travel alongside

it with light speed; what happens to my image in a mirror if I and the mirror both move at that speed? After years of puzzling over these and related questions, some time in 1905 over a marathon discussion with Besso it suddenly struck him: time was the culprit, it was not absolute and the same for all observers! Simultaneity is affected by motion! It appears that the writings of the philosopher David Hume on the differences of psychological time from person to person also inspired him to this conclusion. To Besso he wrote: "Thanks to you, I have completely solved my problem". From this inspired insight to completion of the paper was a mere 5 to 6 weeks. And in 1924 he recalled:

"After seven years of reflection in vain (1898-1905), the solution came to me suddenly with the thought that our concepts and laws of space and time can only claim validity in so far as they stand in a clear relation to our experiences; and that experience could very well lead to the alteration of these concepts and laws. By a revision of the concept of simultaneity into a more malleable form, I thus arrived at the special theory of relativity".

Einstein's guiding idea was that a common principle of relativity had to hold for both mechanics and electromagnetism. He elevated this to a postulate, and added as another postulate the constancy of the speed of light in all inertial frames. From these, assuming homogeneity and isotropy of space and uniformity of time, he derived the new space time transformation equations known as the Lorentz transformations (in the simplest form) to go

from one inertial frame to another:

$$x' = (x - vt)/\sqrt{1 - v^2/c^2}, \ t' = (t - v\,x/c^2)/\sqrt{1 - v^2/c^2},$$
$$y' = y, \qquad z' = z. \tag{15}$$

Here x, y, z, t and x', y', z', t' are the space and time coordinates assigned to one and the same event in two inertial frames with a relative velocity v in the x direction.

These are to be contrasted with the earlier Galilean Newtonian equations

$$x' = x - vt \ , \quad t' = t \ , \quad y' = y \ , \quad z' = z \tag{16}$$

He also established the invariance of Maxwell's equations under the new transformations, and dispensed with the ether as the carrier of electromagnetic waves. In accord with his postulated relativity principle he went on to modify Newton's mechanics to fall in line with electromagnetism. His world famous formula

$$E = m\,c^2 \tag{17}$$

was a fallout of this effort.

The Lorentz transformation equations contain all the key features of relativistic spacetime in an uncluttered and transparent manner: dependence of the concept of simultaneity on the frame of reference; the earlier postulated Lorentz-Fitzgerald contraction hypothesis; the causal structure of space time; and so on. Unlike the photon story, special relativity was accepted very

quickly by the physics community. In a 2004 talk, C.N.Yang has very perceptively contrasted the positions of Lorentz, Poincarè and Einstein on these issues. Poincarè had the philosophy of relativity clearly in mind already in 1904 when he said:

"The principle of relativity, according to which the laws of physical phenomena should be the same, whether for an observer fixed, or for an observer carried along in a uniform movement of translation; so that we have not and could not have any means of discerning whether or not we are carried along in such a motion".

However, as Yang says, the idea of relativity of simultaneity eluded him. Lorentz had his transformation equations already in 1904, but he did not understand the physical meaning of the transformed time variable t'. In fact he said in 1915:

"The chief cause of my failure was my clinging to the idea that only the variable t can be considered as the true time and that my local time t' must be regarded as no more than an auxiliary mathematical quantity".

All in all this table captures the situation succinctly:

	Mathematics	Physics	Philosophy
Poincaré	$\checkmark$	$\times$	$\checkmark$
Lorentz	$\checkmark$	$\times$	?
Einstein	$\checkmark$	$\checkmark$	$\checkmark$

Three years after Einstein's work, the mathematician Hermann Minkowski showed how to handle relativistic space time in a truly elegant mathematical formalism which proved crucial for later developments. It is in the nature of

special relativity that it acts as a restrictive principle, not as a model of any specific phenomena. Einstein himself expressed this in 1911:

"The Principle of Relativity is a principle that narrows the possibilities; it is not a model, just as the Second Law of Thermodynamics is not a model".

In general terms Special Relativity rules out action at a distance and propagation speeds greater than light speed. Here are some examples of Special Relativity 'in action' as a guiding and limiting principle: de Broglie's 1923 idea of waves associated with matter; the Dirac electron equation found in 1928; the discovery of the positron and the general concept of antimatter; the Pauli spin statistics theorem of 1940, of which there have been many later variants; and the modern theories of the electroweak interactions of elementary particles, and of quantum chromodynamics. In fact Pauli ended his 1940 paper by saying:

"In conclusion we wish to state, that according to our opinion the connection between spin and statistics is one of the most important applications of the special relativity theory".
And Dirac on many occasions said the idea of antimatter was a straightforward consequence of special relativity.

Today the working of particle accelerators, nuclear energy production, cosmic rays are all constant reminders of the validity of special relativity.

It seems that Dirac once asked Yang what he thought was Einstein's greatest contribution to physics. Yang replied that he thought it was his General Relativity, the supreme classical theory of gravitation. To which

Dirac replied: No, his greatest achievement was to show that time was to be treated on par with space, the two had to be transformed together. Mathematically the Lorentz transformation is much more beautiful than the Galilean transformation.

It seems Einstein once said "..... the presentation of science to the general public must be as simple as possible, but not more so".

I am aware you are not the general public, being much better informed. Yet I have given this talk in that spirit - and I hope I have been faithful to him and intelligible to you.

Thank you.

Suggestions for further reading

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2. A Pais, 'Subtle is the Lord...' The Science and the Life of Albert Einstein, Oxford University Press, Oxford 1982.

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5. C.N.Yang, 'Einstein's Impact on Theoretical Physics in the 21st century', reprinted in Association of Asia Pacific Physical Societies Bulletin Vol.15,

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12. N.Mukunda, 'Einstein's Life and Legacy', Resonance Vol.10, No.5, p.75-85 (2005).