Design and Optimisation Problems in Wireless Sensor Networks

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Outline of Talk

1. Wireless Sensor Networks (WSNs): Quick Overview
2. Optimum Event Detection
3. Other Algorithm Design Work
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1. Wireless Sensor Networks (WSNs): Quick Overview

2. Optimum Event Detection

3. Other Algorithm Design Work
Wireless Networks of Multifunction Smart Sensors (WSNs)

- A smart sensor node is popularly called a *mote*

**Sensing:** temperature, chemicals, light, infrared, biosensors, strain, sound, vibration (often using MEMS technology)

**Processing:** e.g., 16-bit, 8 Mhz, 48KB flash, 10 KB RAM
  - with a simple OS

**Digital radio:** e.g., ISM band; a few Kbps

**Battery:** e.g., 100mAh ("button" batteries) to 2000mAh (2 AA batteries)
Motes are deployed and self-organise into a mesh network
Nodes contain pieces of a distributed signal processing algorithm
Computations are performed on the measurements
Partial results flow over the links
Downstream nodes combine their inputs and forward the results
The MAC schedules the transmissions in the network

Embedded distributed smart instrumentation
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Potential Applications of Wireless Sensor Networks

- Environment monitoring
- Self-monitoring structures
- Industrial sensor networks
  - Chemical factories, refineries, power distribution yards
  - Energy and environment management networks in large buildings
  - Emerging ISA 100 standard (IEEE 802.15.4 PHY, TDMA MAC)
- Ecological monitoring and wild life management
- Monitoring mobile patients in hospitals and homes
- Locating people in large buildings or public transportation systems
- Detecting intruders in border areas (smart fields, electronic “trip wires”)
Design and Optimisation Problems in WSNs

- **Node placement**
  - Sensing coverage, and radio connectivity

- **Topology design**
  - Discover a topology that has good throughput and delay performance
  - Related to node placement

- **Node localisation**: GPS might not work in certain domains

- **Packet forwarding**: Nodes sleep-wake cycle; table driven routing not practical
  - Discover routes on the fly

- **Time synchronisation across the network**

- **Distributed signal processing**
  - Event detection and classification
  - Object tracking
Outline of Talk

1. Wireless Sensor Networks (WSNs): Quick Overview

2. Optimum Event Detection

3. Other Algorithm Design Work
The Classical Change Detection Problem (CDP)

- **The Setting:** Discrete time, $k \geq 0$. The change occurs at a random time $T \geq 0$.
  - Need to detect the change
  - We have a prior on $T$, and the observations $X_k^{(i)}$, $1 \leq i \leq N$, $k \geq 1$
  - Conditionally i.i.d. in time and space

- **The Problem:** Determine a stopping time $\tau$, such that

$$
\min \ E \left[ (\tau - T)^+ \right]
$$

s.t. $P(\tau < T) \leq \alpha$

- **The Solution:** The optimal $\tau$ is given by

[Shiryaev '63]

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\tau = \inf \{ k : \Pi_k \geq \Gamma_\alpha \}
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where $\Pi_k = P(T \leq k \mid X_{[1:k]})$
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Limitations of the Classical CDP

- **Recent motivation:** event detection in wireless sensor networks
  - The classical model does not account for many practical aspects
- **Sleep-wake scheduling of sensors (cost for making observations)**
  - Optimal detection with sleep-wake scheduling: [Infocom 2008]
- **Transient change:** The event occurs and then disappears in a finite time
  - Quickest transient change detection: [IWAP 2010]
- **Networked sensors**
  - Random access packet network between the sensors and the fusion centre [SECON 2007, ACM TOSN 2010]
- **Large extent network:** spatial variation in the way the change (event) affects the sensors
  - Distributed detection and localisation in large wireless sensor networks: [Allerton 2009]
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Nodes make periodic observations (e.g., temperature; gas concentration (e.g., LPG))

Measurements (or local decisions) are sent to the “fusion centre”...

... over a random access network
The Sequential Detection Problem with Network Delays

- The sequencer at the fusion centre releases complete batches of samples to the detection algorithm

- Minimise detection delay s.t. a false alarm probability bound, i.e.,

$$\min E (\tilde{U} - T) I\{\tilde{T} \geq T\}$$

subject to

$$Pr (\tilde{T} < T) \leq \alpha$$
Theorem

*If the sampling is periodic at rate $r$ and the batch sojourn time process $D_b$, $b \geq 1$, is stationary with mean $d(r)$, then*

$$\min_{\Pi_\alpha} E\left((\tilde{U} - T) I_{\{\tilde{T} \geq T\}}\right)$$

$$= l(r) (1 - \alpha) + d(r) (1 - \alpha) + \frac{1}{r} \min_{\Pi_\alpha} E\left(\tilde{K} - K\right)^+$$

*where $l(r)$ is the delay due to (coarse) sampling, and $K$ is the index of the first sample after the change.*

- A cross-layer result: combines network performance & statistical performance for the first time
- The problem $\min_{\Pi_\alpha} E\left(\tilde{K} - K\right)^+$ is the classical one
- The threshold will depend on the sampling rate $r$
Collocated Nodes: Change Detection over a Network
Decision and Network Delays Decouple

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If the sampling is periodic at rate $r$ and the batch sojourn time process $D_b, b \geq 1$, is stationary with mean $d(r)$, then

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Theorem

If the sampling is periodic at rate $r$ and the batch sojourn time process $D_b$, $b \geq 1$, is stationary with mean $d(r)$, then

$$\min_{\Pi} \mathbb{E}\left((\tilde{U} - T)I\{\tilde{T} \geq T\}\right)$$

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- A cross-layer result: combines network performance & statistical performance for the first time
- The problem $\min_{\Pi} \mathbb{E}\left(\tilde{K} - K\right)^+$ is the classical one
  - The threshold will depend on the sampling rate $r$
Example: \( \rho = 0, p = 0.0005, \alpha = 0.01, \sigma = 0.3636, N = 10 \)

- Stability requirement \( N \times r < \sigma \), the success rate
- Mean detection delay vs. sampling rate \( r \)

The optimal sampling rate is \( r = 0.029 \) (sampling interval = 34 slots)
Detect an event at a random time, $T$

- $n$ sensors in the observation field
- Sensor $i$ makes the measurements $X^{(i)}_k$, $k \geq 1$
- How many sensors should we turn on?
- Tradeoffs: detection delay, false alarm rate, battery life
Detect an event at a random time, $T$

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Collocated Nodes: Optimal Node Energy Management

- Detect an event at a random time, $T$
  - $n$ sensors in the observation field
- Sensor $i$ makes the measurements $X^{(i)}_k, k \geq 1$
- How many sensors should we turn on?
- Tradeoffs: detection delay, false alarm rate, battery life
fixed number of observations

$M_k = m$

$M_{k+1} = m$

$M_{k+2} = m$

Decisions:
stop and declare change or continue sampling

number of observations is dynamically controlled

$M_k$

$M_{k+1}$

$M_{k+2}$

Decisions:
stop and declare change or determine how many sensors to activate
Collocated Nodes: Optimal Node Energy Management

The Decision Rule

- At sample time $k$, the observation vector is $X^{(M_k)}_k$
  - the vector of observations from $M_k$ sensors
- Compute $\Pi_k = P\{T \leq k \mid X^{(M_1)}_1, \ldots, X^{(M_k)}_k\}$
  - $\Pi_k =$ posterior probability of the event having occurred at or before $k$

An example

- $n = 10$ sensors
- False alarm cost = 100.0
- Sensor activation cost = 0.5
- Delay cost = 1 per sample delay
- $f_0 \sim \mathcal{N}(0, 1), f_1 \sim \mathcal{N}(1, 1)$
- Optimal algorithm
  - Stop if $\Pi_k \geq 0.9$
  - Else use $M_k$ as in the plot
Collocated Nodes: Optimal Node Energy Management

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- At sample time $k$, the observation vector is $X^{(M_k)}_k$
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  compute $\Pi_k = \Pr \{ T \leq k \mid X^{(M_1)}_1, \ldots, X^{(M_k)}_k \}$
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The Decision Rule

- At sample time $k$, the observation vector is $X_k^{(M_k)}$
- the vector of observations from $M_k$ sensors
- Compute $\Pi_k = P \left\{ T \leq k \left| X_1^{(M_1)}, \ldots, X_k^{(M_k)} \right. \right\}$
- $\Pi_k =$ posterior probability of the event having occurred at or before $k$

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Collocated Nodes: Optimal Node Energy Management
Comparison with a Fixed Number of Active Sensors

- The optimum rule initially keeps just one sensor active
- Rapidly increases the number of active sensors when the event occurs
- Plots on the right show the average cost comparison with various fixed number of sensors
Problem: An event happens at an unknown time $T$ and at an unknown location $\ell_e$. Detect and locate the event.

Objective: Raise an alarm with a small detection delay subject to Time to False Alarm (TFA) $\geq \gamma$ and Time to False Isolation (TFI) $\geq \gamma$. 

Before the Event

$X_k^{(i)} \sim f_0.$

After the Event

$X_k^{(i)} \sim \begin{cases} f_1, & \text{if } i \in N_e(r_s), \\ f_0, & \text{if } i \notin N_e(r_s). \end{cases}$

$\text{Event}$
### Before the Event

\[ X_k^{(i)} \sim f_0. \]

### After the Event

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**Problem:** An event happens at an unknown time \( T \) and at an unknown location \( \ell_e \). **Detect** and **locate** the event.

**Objective:** Raise an alarm with a small detection delay subject to **Time to False Alarm (TFA) \( \geq \gamma \)** and **Time to False Isolation (TFI) \( \geq \gamma \).**
We deployed 10 nodes in the ROI in a deterministic manner such that we get $N = 9$ regions and

$N_1 = \{1, 2, 4\}$, $N_2 = \{2, 4, 5\}$, $N_3 = \{2, 3, 5\}$, $N_4 = \{4, 6, 7\}$, $N_5 = \{4, 5, 7\}$, $N_6 = \{5, 7, 8\}$, $N_7 = \{6, 7, 9\}$, $N_8 = \{7, 9, 10\}$, and $N_9 = \{7, 8, 10\}$.

- Nikiforov (1995) provides the optimal centralised algorithm
- Distributed algorithms are based on fusion of local CUSUM statistics
- We also provide an approach for a power-law signal decay model
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Other WSN Algorithm Design Work

- **WSN node placement**
  - Minimum cost
  - Performance objectives (delay and network reliability)

- **Node localisation without GPS**
  - Hop count based localisation
  - Theory based on random graph models

- **Geographical forwarding of alarm packets in WSN**
  - Optimal forwarding when nodes sleep-wake cycle
  - Algorithms for “hole” avoidance