The role of micro-scale inertia in transport processes

- **Ganesh Subramanian**
(JNCASR, Bangalore)

Micro-scale inertia *(suspensions and emulsions)*

- *Navier-Stokes equations :*

$$\underbrace{Re\,\frac{\partial \boldsymbol{u}}{\partial t} + Re\ \ \boldsymbol{u}\cdot\nabla\,\boldsymbol{u}}_{\text{Inertial acceleration}} = \underbrace{-\nabla\, p}_{\text{Pressure forces}} + \underbrace{\nabla^{2}\boldsymbol{u}}_{\text{Viscous forces}}$$

Reynolds number $Re = \dfrac{Ua\rho}{\mu}$

a : **fibre length**
U : **particle (drop) size**
ν : **fluid kinematic viscosity**

- *Small particles, viscous fluid :* Re **<< 1**

- ***Quasi-steady Stokes equations :***

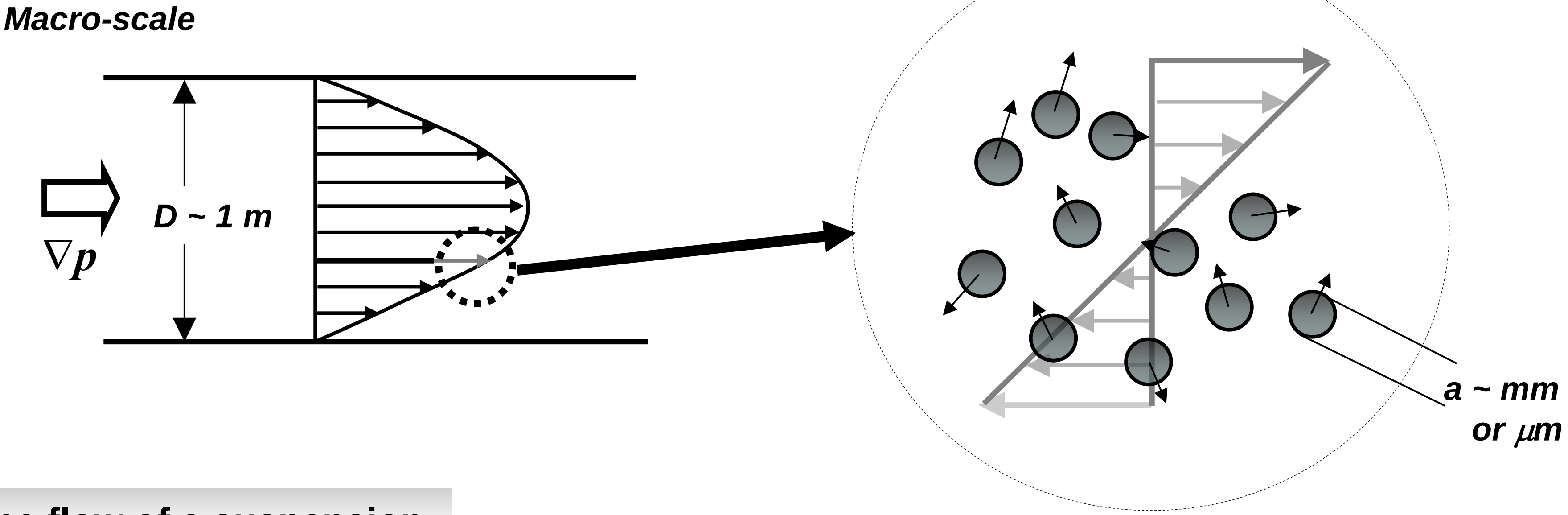
$$-\nabla p + \nabla^{2} u = 0$$

➡ Good approximation for small-scale slow, slowly varying motions

An instantaneous balance between pressure and viscous forces !

Central to the hydrodynamics in suspension rheology

Micro-scale inertia



Pipe flow of a suspension

Micro-structure

a = 10 micron
μ= 1 Pa-sec

Re = O(10^{-9})

Stokes regime

[Polystyrene/PMMA particles in honey or glycerine]

a = 100 micron
μ = 10^{-3} Pa-sec

Re = O(1)

Micro-scale inertia important

[Particles or drops in aqueous media, Cellulose fibres in water – paper and pulp industry]

Applications

Heat (Mass) transfer

- ***Exothermic reactions on catalyst particles***
- ***Suspension polymerization***
- ***Drug delivery using porous polymer particles***
- ***Transport characteristics of immobilized cells/cell-aggregates in bio-reactors***

Momentum transfer

- ***Pyroclastic flows***
- ***Gravity currents (sediment deposition patterns)***
- ***Rheology***

Newtonian : fluid

$$\boldsymbol{Re}\,\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} + \boldsymbol{Re}\;\boldsymbol{u}\cdot\nabla\,\boldsymbol{u} = -\nabla\,\boldsymbol{p} + \nabla^{2}\boldsymbol{u}$$

Averaged equation : for a complex fluid (suspension/emulsion)

$$\underbrace{\boldsymbol{Re}\,\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{t}} + \boldsymbol{Re}\;\boldsymbol{u}\cdot\nabla\,\boldsymbol{u}}_{\text{Macro-scale inertia}} = -\nabla\,\boldsymbol{p} + \nabla\cdot\tau$$

- ***Micro-scale inertia determines the rheology of the fluid***
- τ ***may be regarde as a suitable average over the underlying microstructure***
- ***Velocity fluctuations of the disperse phase may be regarded as contributing to a Reynolds stress***

Principle of reversibility

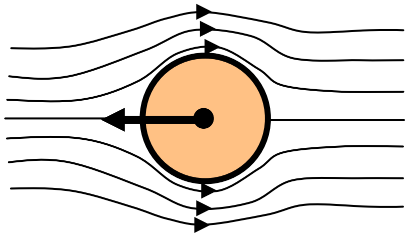
☐ **Mere reversal of boundary conditions should lead to reversal of entire flow field**

' Reversing boundary motion reverses time '

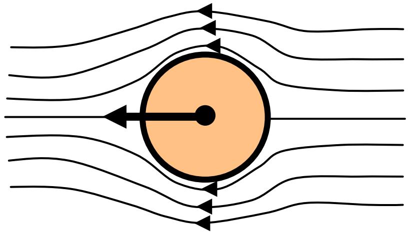
Different from well-known mechanical reversibility in classical mechanics

• Restrictions imposed by reversibility

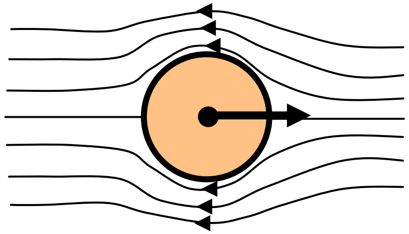
☐ **Fore-aft symmetric body - Velocity field must be fore-aft symmetric**



1 : Fore-aft asymmetric velocity field



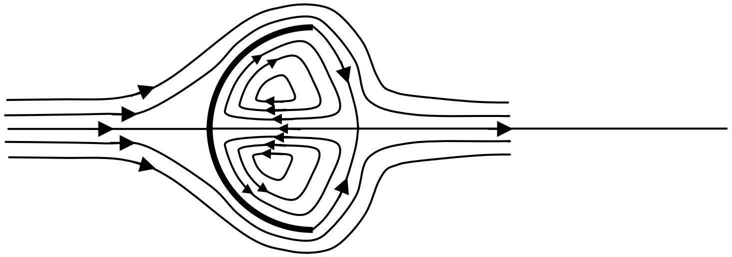
2 : Reverse time



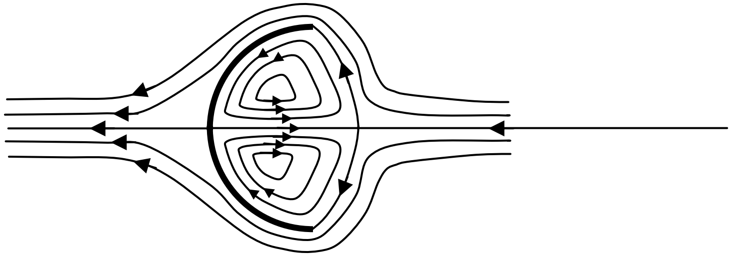
3 : Reverse boundary motion

→ ***2 and 3 are different streamline patterns !***

☐ **Both fore and aft-separation possible for fore-aft asymmetric body**



1. Aft-separation

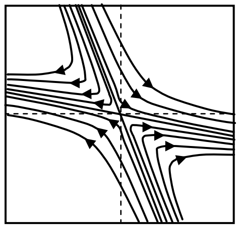


2. Reverse time: fore-separation !

Cross-section of a spherical cap

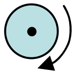
❑ Ubiquitousness of closed streamlines in Stokes flows

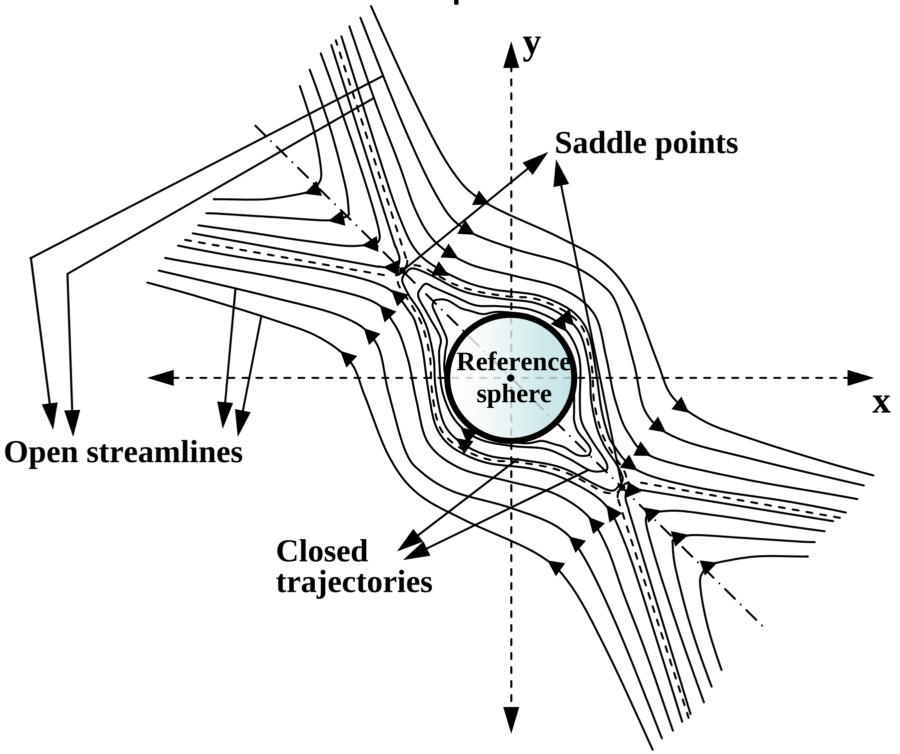
- Planar vortical Stokes flow + rotating particle = closed streamlines around particle

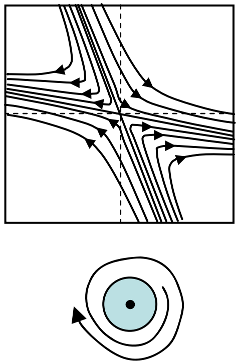


Hyperbolic linear flow

+

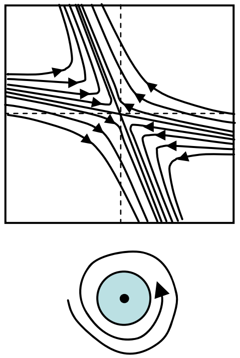






Assume near-field
Streamlines spiral out

Reverse flow



Near-field streamlines
must spiral in

- *Spiralling streamlines are excluded by reversibility*

- ***Reversibility leads to closed streamlines and closed particle pathlines***
- ***SMALL amount of (micro-scale) inertia has BIG effects***
- ***Note that <u>dynamic reversibility</u> does not imply <u>kinematic reversibility</u>***
- ***Kinematic irreversibility (chaos) will only become relevant for complex configurations and on sufficiently long time scales in the dilute limit***

Heat (Mass) transfer :

Neutrally buoyant particles or drops in shearing flows

Collaborator: Don Koch,
Cornell University

Governing equation for heat transfer - Convection-diffusion equation

$$\underbrace{Pe\,(u\cdot\nabla T)}_{\text{Convection by fluid velocity field}} = \underbrace{\nabla^2 T}_{\text{Conduction}}$$

Peclet number, $Pe = \dfrac{aU}{D}$

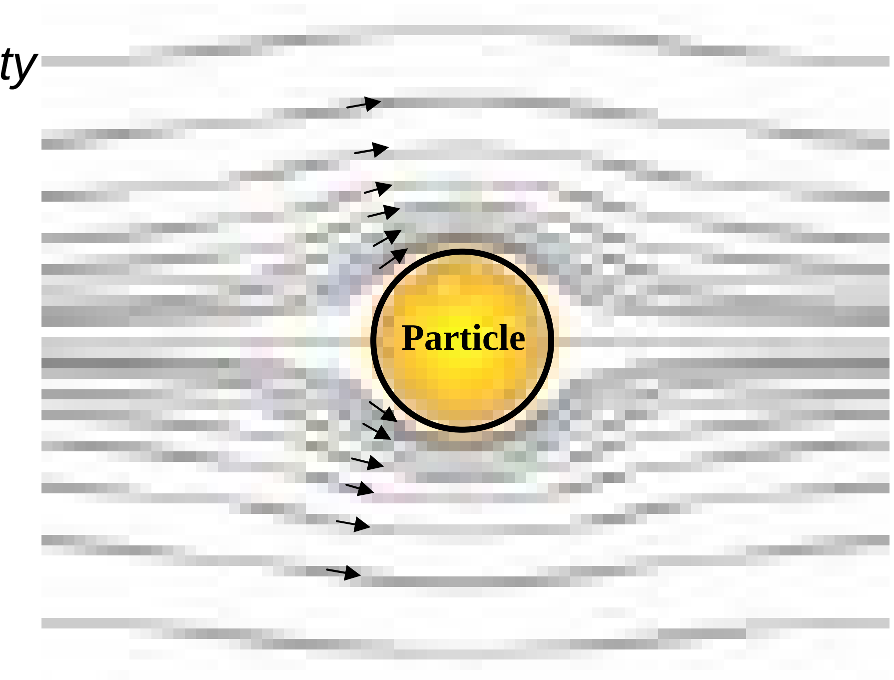
$$\begin{cases} T = T_0 \longrightarrow \text{on particle surface} \\ T = T_\infty \longrightarrow \text{ambient fluid} \end{cases}$$

- ***Physical properties assumed to be constant***
- u ***obtained as a solution of the Stokes equations (Re = 0) or with weak inertia***
- ***High*** Pe ***- transfer of heat or mass across a thin boundary layer***

Heat transfer from particle : open streamlines

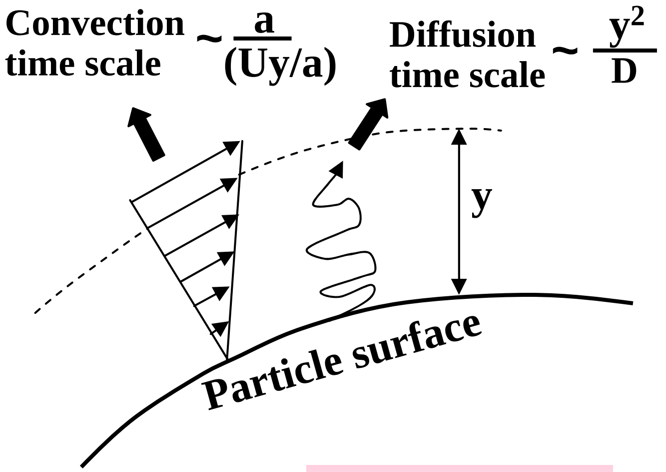
$$u = \boxed{U} + \Gamma \cdot x + h.o.t$$

Slip velocity

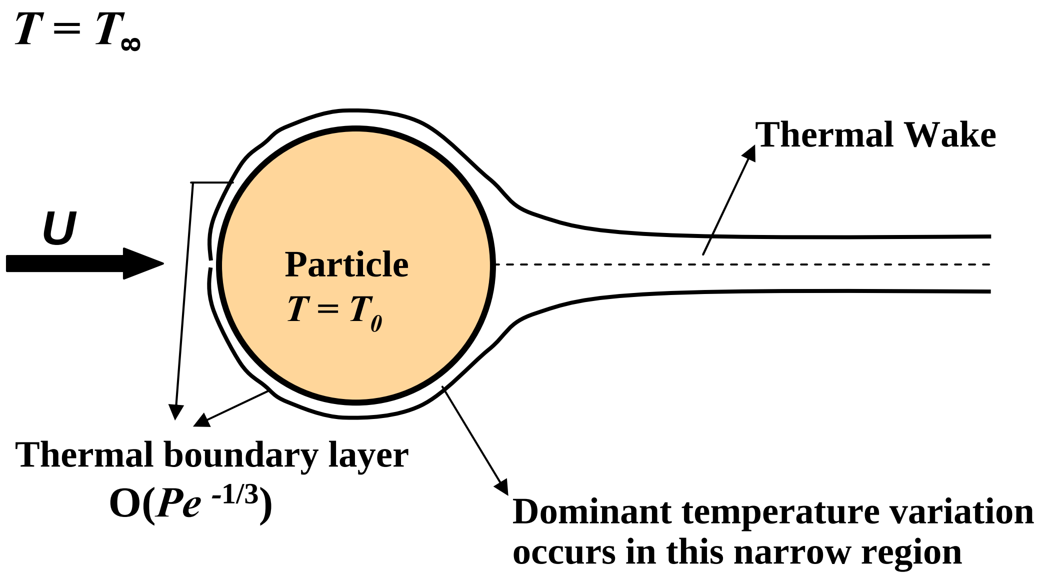


Open streamlines for relative translation of fluid and particle lead to a *thin boundary layer* and the familiar scaling of heat transfer rate with Peclet number

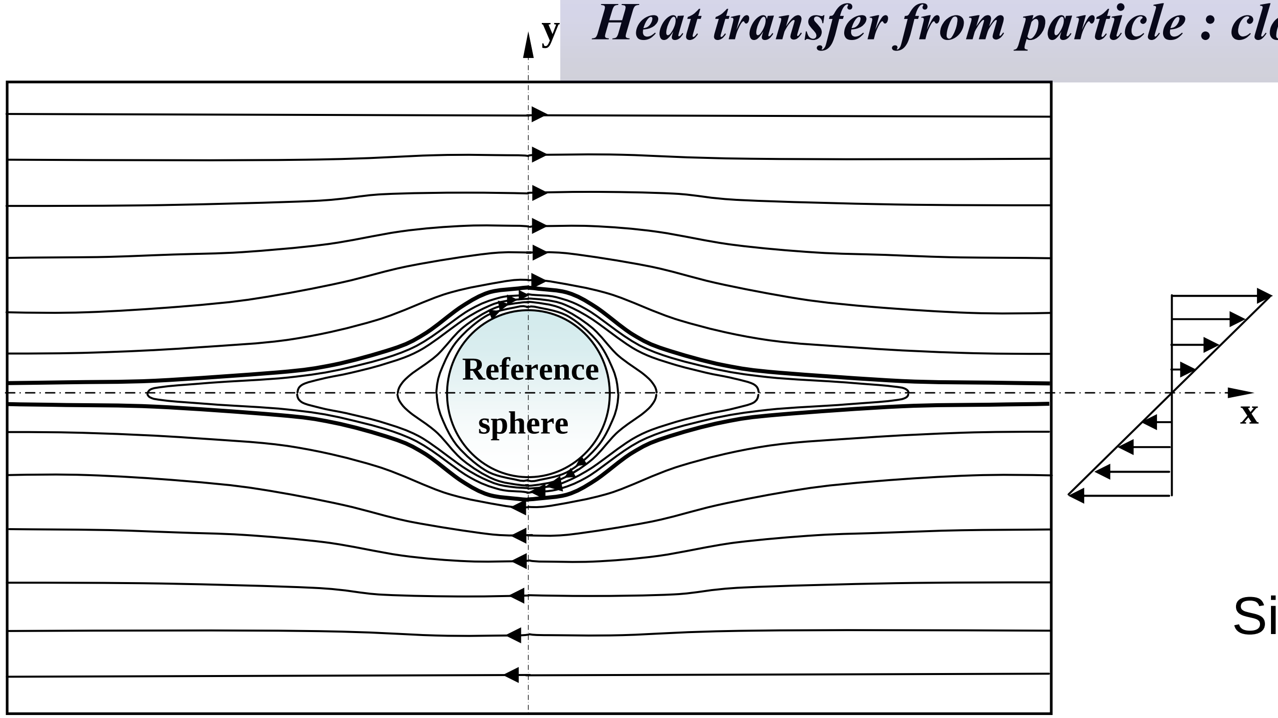
$Nu = \# \, Pe^{1/3}$ for $Pe >> 1$



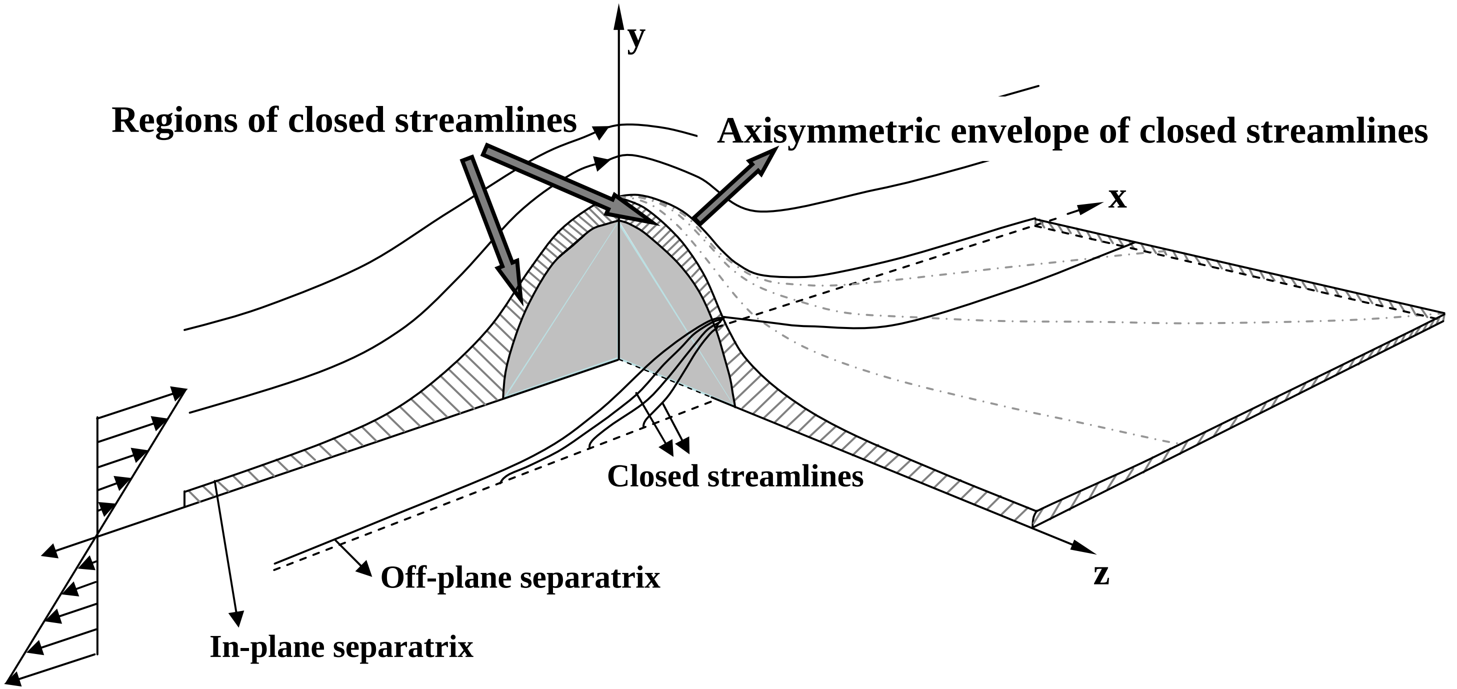
$$\frac{a}{(Uy/a)} \sim \frac{y^2}{D}$$



Heat transfer from particle : closed streamlines (Re= 0)



Simple shear flow (α = 0)
Re = 0



Heat transfer from particle : closed streamlines (Re = 0)

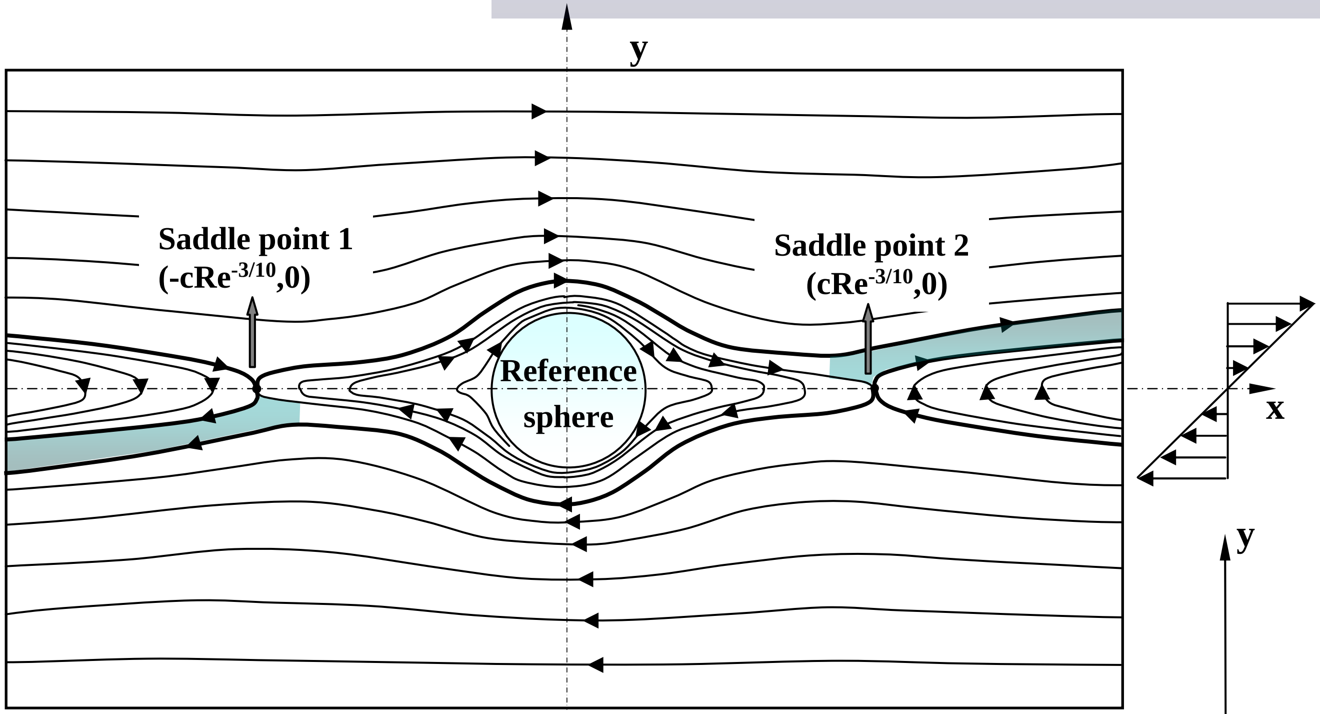
Closed streamlines around the sphere give diffusion limited heat transfer even at large *Pe*

⬍

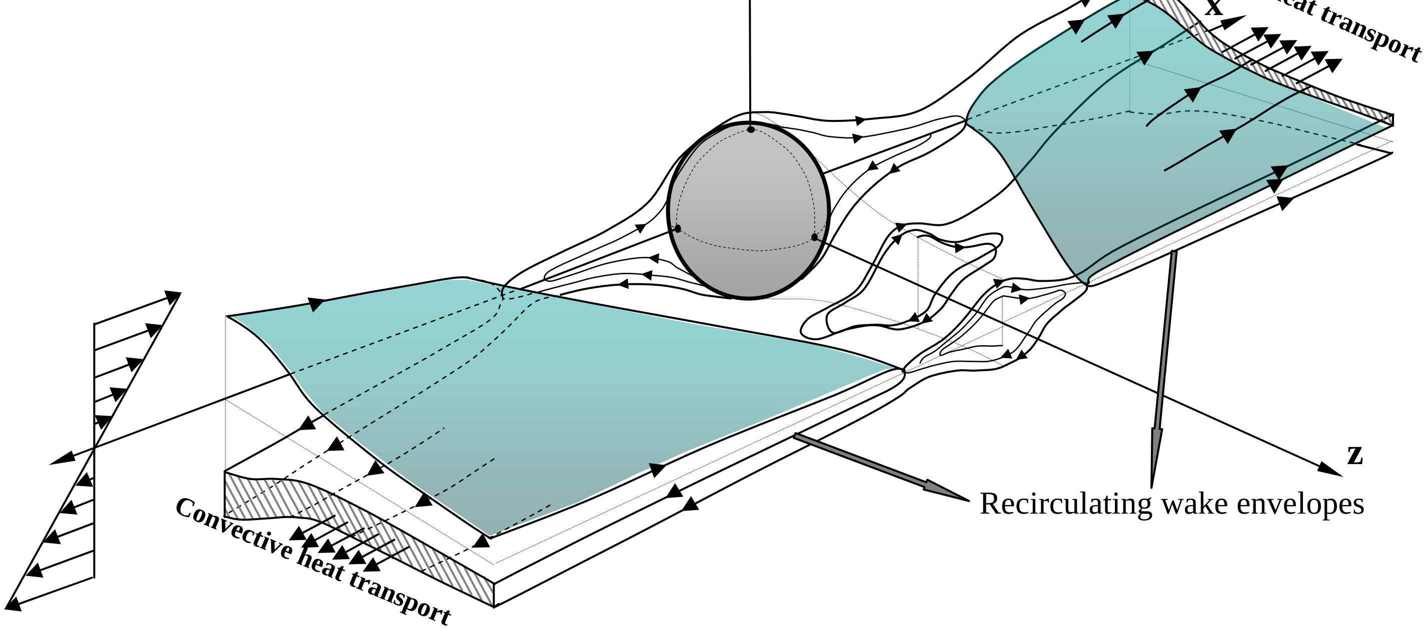
$$Nu = 4.5 \quad \text{for } Pe >> 1 \ \text{ and } \ Re = 0, \alpha = 0$$

- Nu – dimensionless heat transfer in conduction units
- $Nu = 1$ for pure conduction ($Pe = 0$)

Heat transfer from particle : finite Re



Simple shear flow
Re finite



Heat transfer from particle : finite Re

- At small but nonzero *Re* the flow topology changes.

- Streamlines approach the sphere along the vorticity axis and spiral outward in the flow-gradient plane eventually exiting through the channels indicated in green.

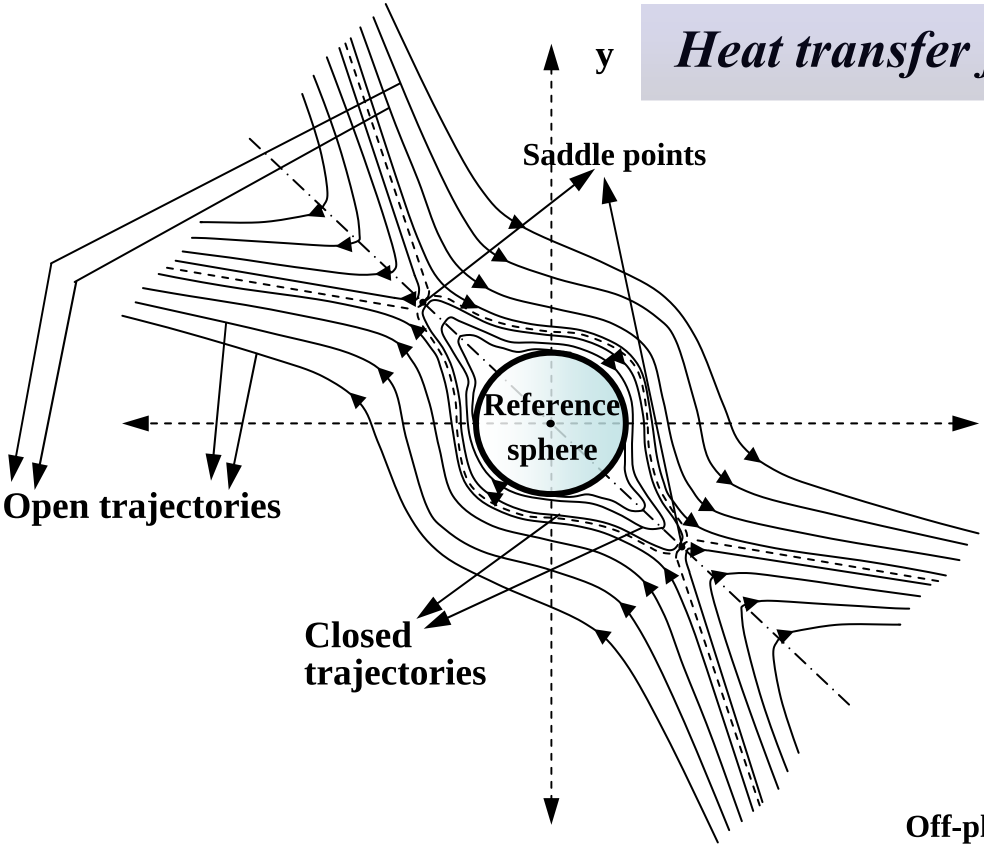
- The open streamlines result in enhanced heat transfer.

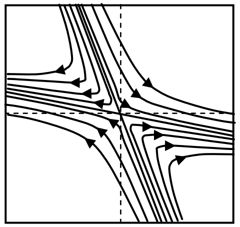
↕

$$Nu = 0.33\,(Re\;Pe)^{1/3} \quad \text{for} \quad Pe \gg 1 \text{ and } \quad Re \ll 1$$

(Subramanian & Koch, PRL 2006)

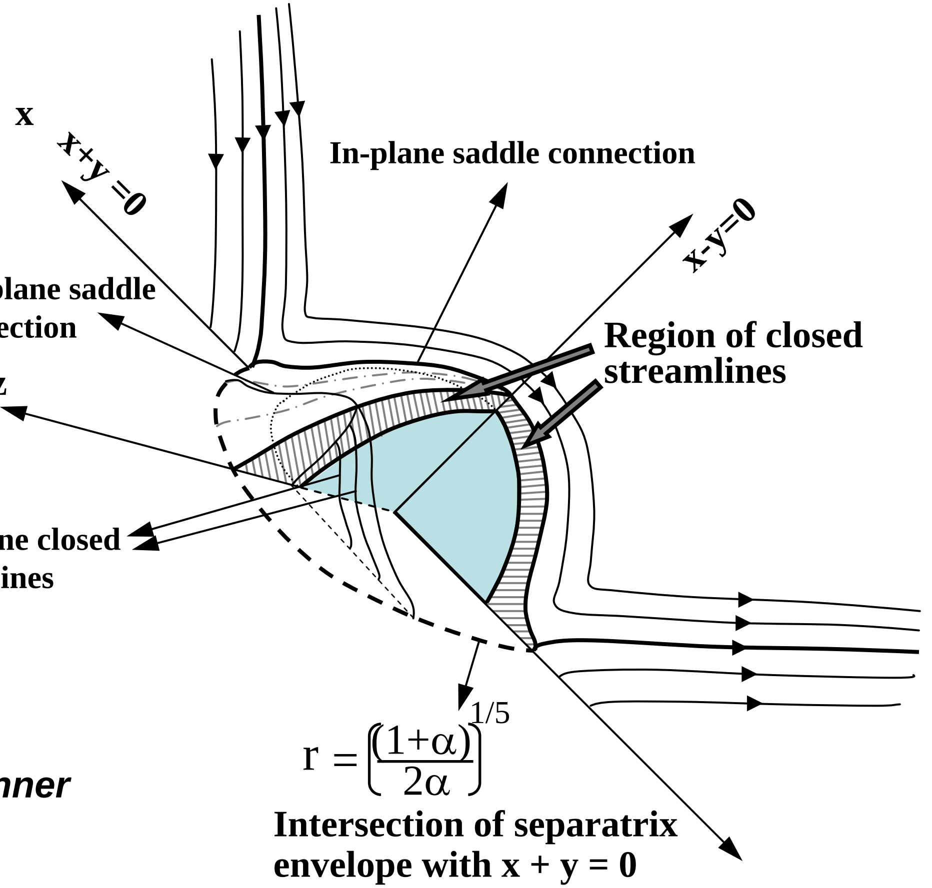
Heat transfer from particle : closed streamlines (Re=0)





Planar linear flow

Re = 0



$$r = \left(\frac{1+\alpha}{2\alpha}\right)^{1/5}$$

Intersection of separatrix envelope with x + y = 0

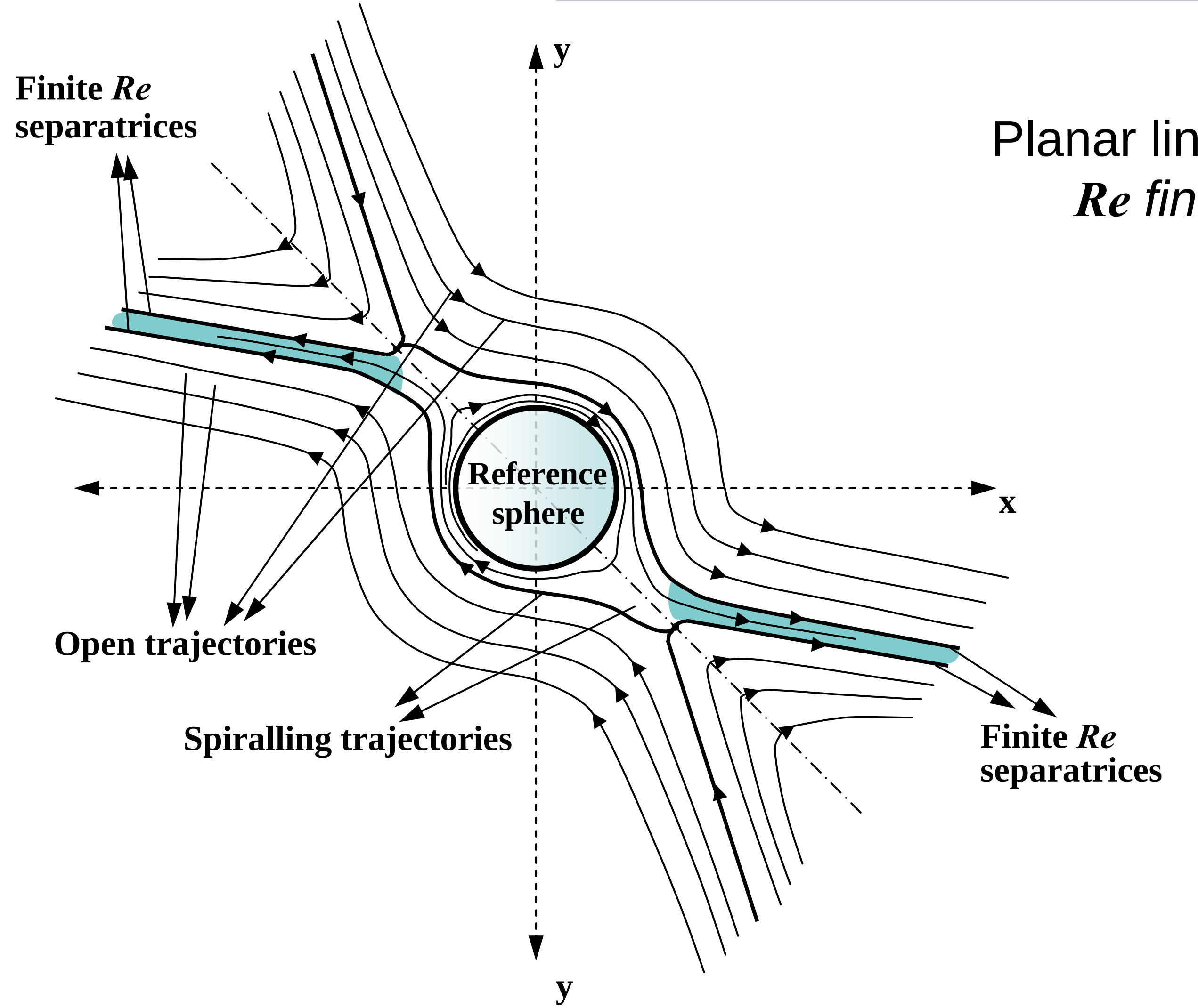
$\boldsymbol{Nu}\ \alpha\ (1-\alpha)^{-1}$ for $\boldsymbol{Pe} >> \mathbf{1}$ and $\boldsymbol{Re = 0}$

The closed streamline region gets thinner
Higher temperature gradients

Heat transfer from particle : finite Re

Planar linear flow

***Re** finite*

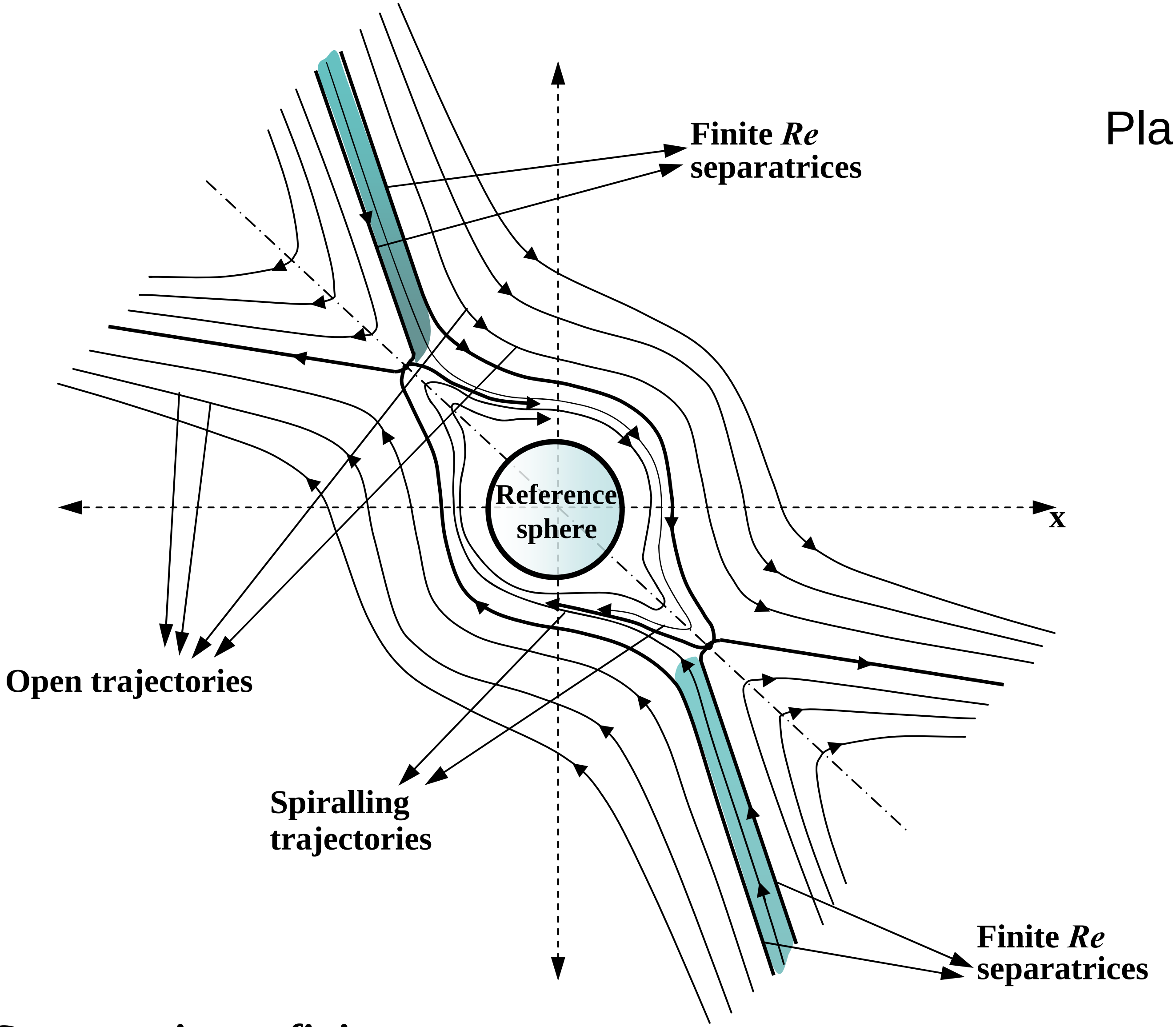


Plane of symmetry (z = 0)

Heat transfer from particle : finite Re

Planar linear flow

***Re** finite*

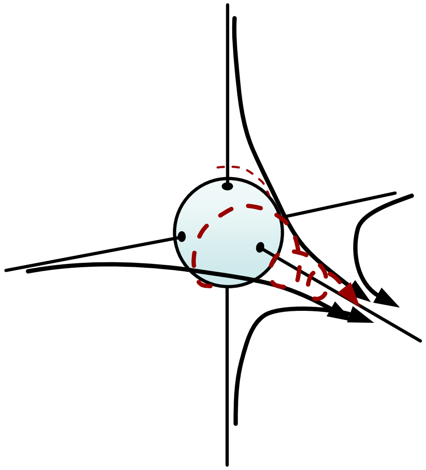


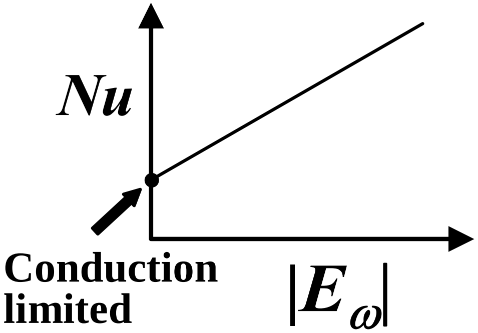
Cross-section at finite z

(Subramanian & Koch, Physics of fluids, 2006)

Heat transfer to small particles in a turbulent flow

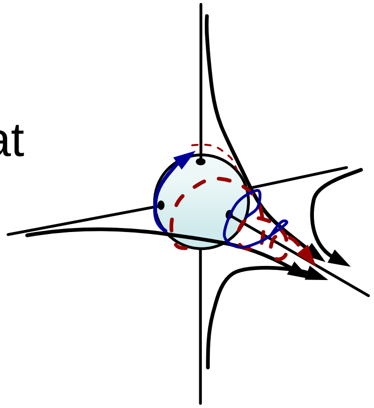
If the particle ***Re*** is zero (particle much smaller than the Kolmogorov scale), Batchelor showed that the heat transfer depends on the extension rate parallel to the vorticity axis

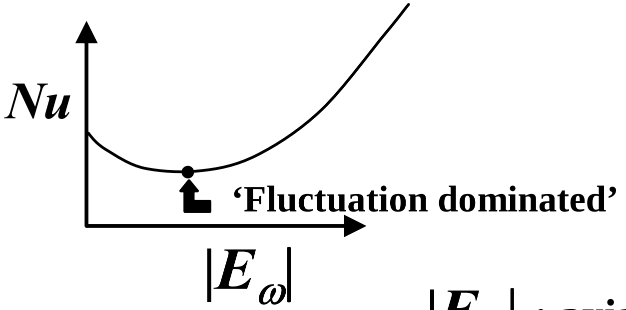




Re = 0

At finite ***Re***, microscale Inertia induces a flow that opposes the imposed axial extension reducing the heat transfer





Finite Re

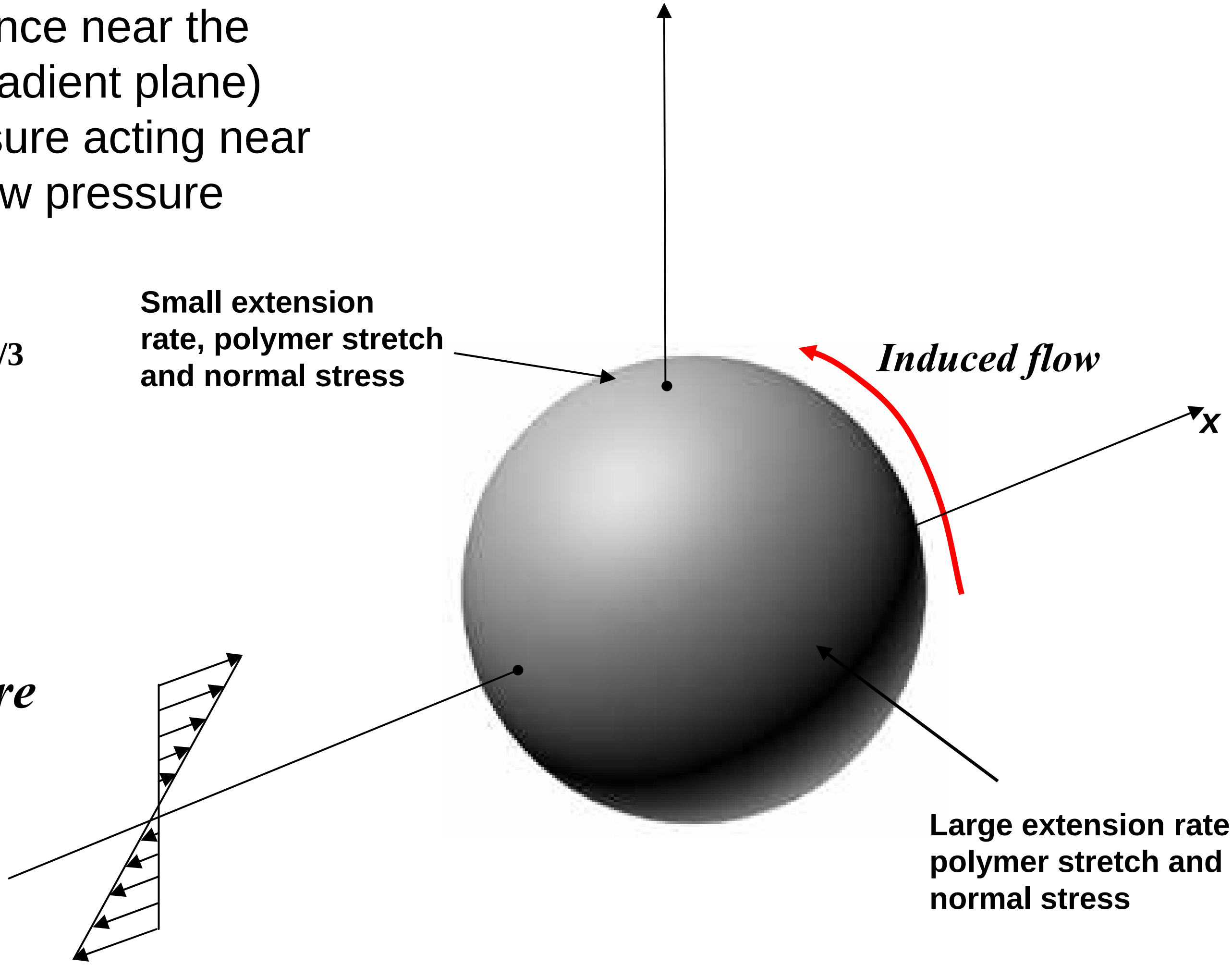
$|E_\omega|$ **: axial extension**

Heat transfer from particle in a viscoelastic fluid

The sphere reduces the extension rate and normal stress difference near the sphere’s equator (flow-gradient plane) leading to a higher pressure acting near the sphere’s poles and low pressure near the equator

$$Nu = 6.01\,(Pe\,De(1+\varepsilon))^{1/3}$$

De : dimensionless measure of a relaxation time



Small extension rate, polymer stretch and normal stress

Induced flow

Large extension rate, polymer stretch and normal stress

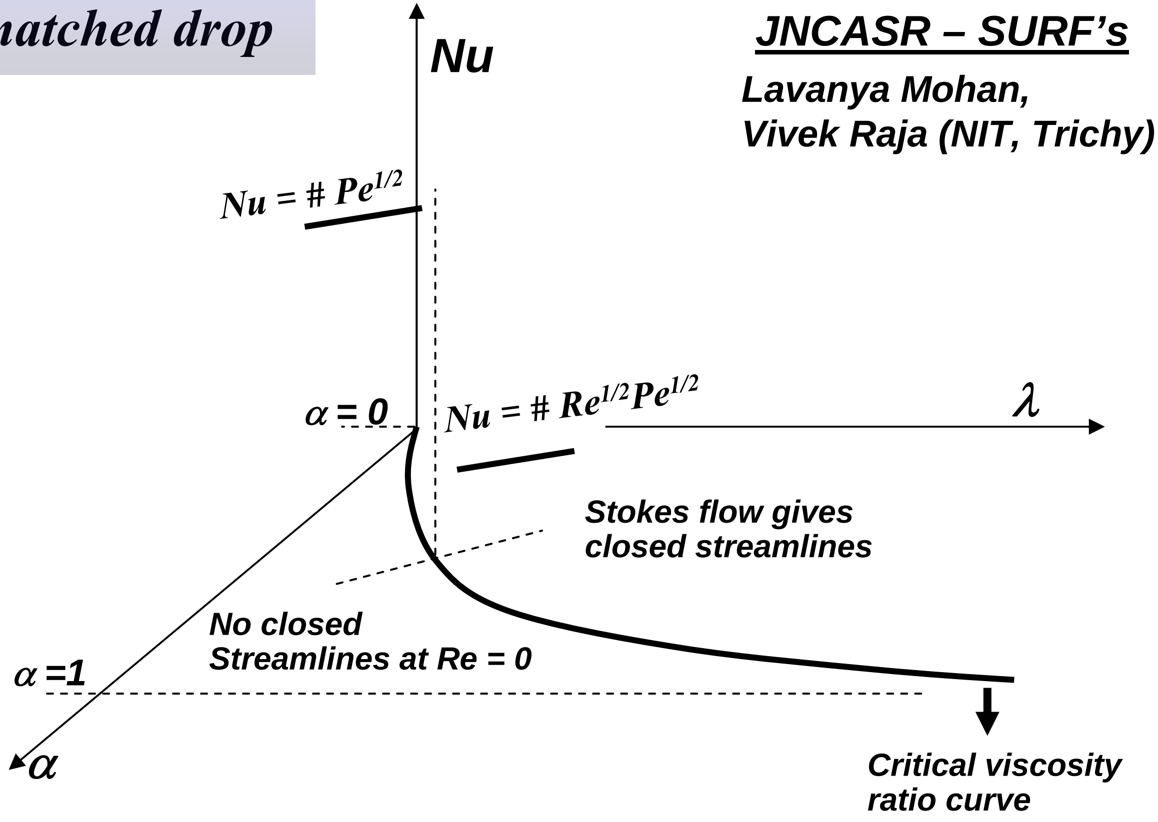
(Subramanian & Koch, JNNFM, 2007)

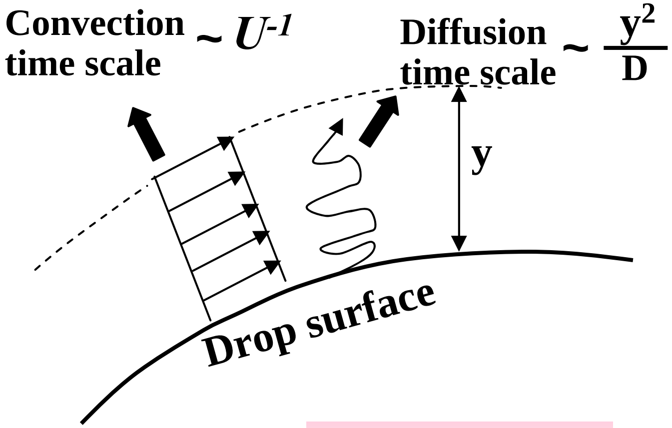
Heat transfer from a density matched drop

JNCASR – SURF's

Lavanya Mohan,
Vivek Raja (NIT, Trichy)

- *Flow inside drop likely to be chaotic*
- *Enhanced drop mixing*
- *Controlling resistance lies outside the drop*





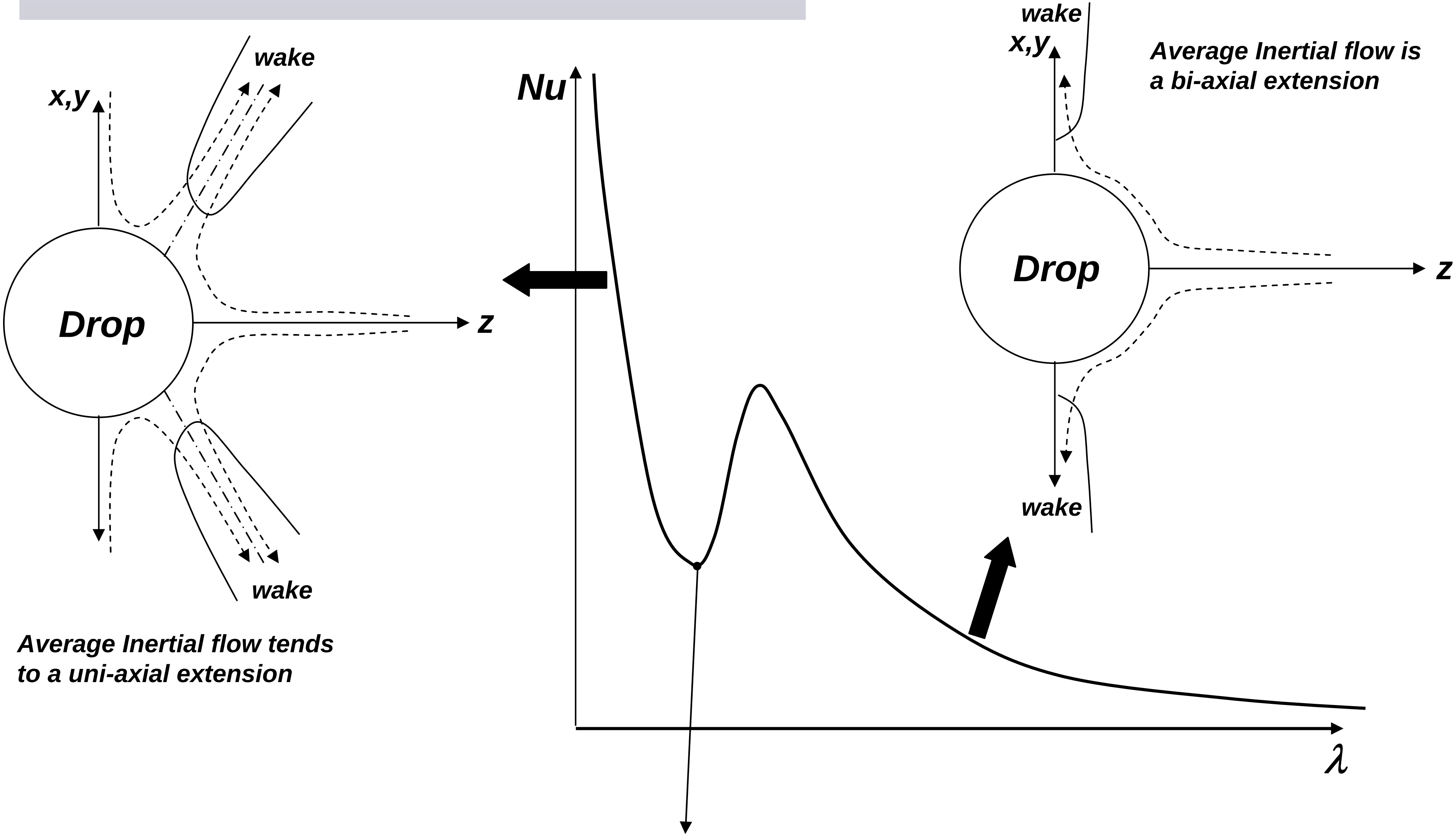
$$Nu = \# Pe^{1/2} \text{ for } Pe >> 1 \quad \textit{Open streamlines}$$

$$Nu = \# (RePe)^{1/2} \text{ for } Pe >> 1$$

$$U^{-1} \sim \frac{y^2}{D}$$

$\# \longrightarrow$ *Function of flow type and viscosity ratio*

Heat transfer from a density matched drop



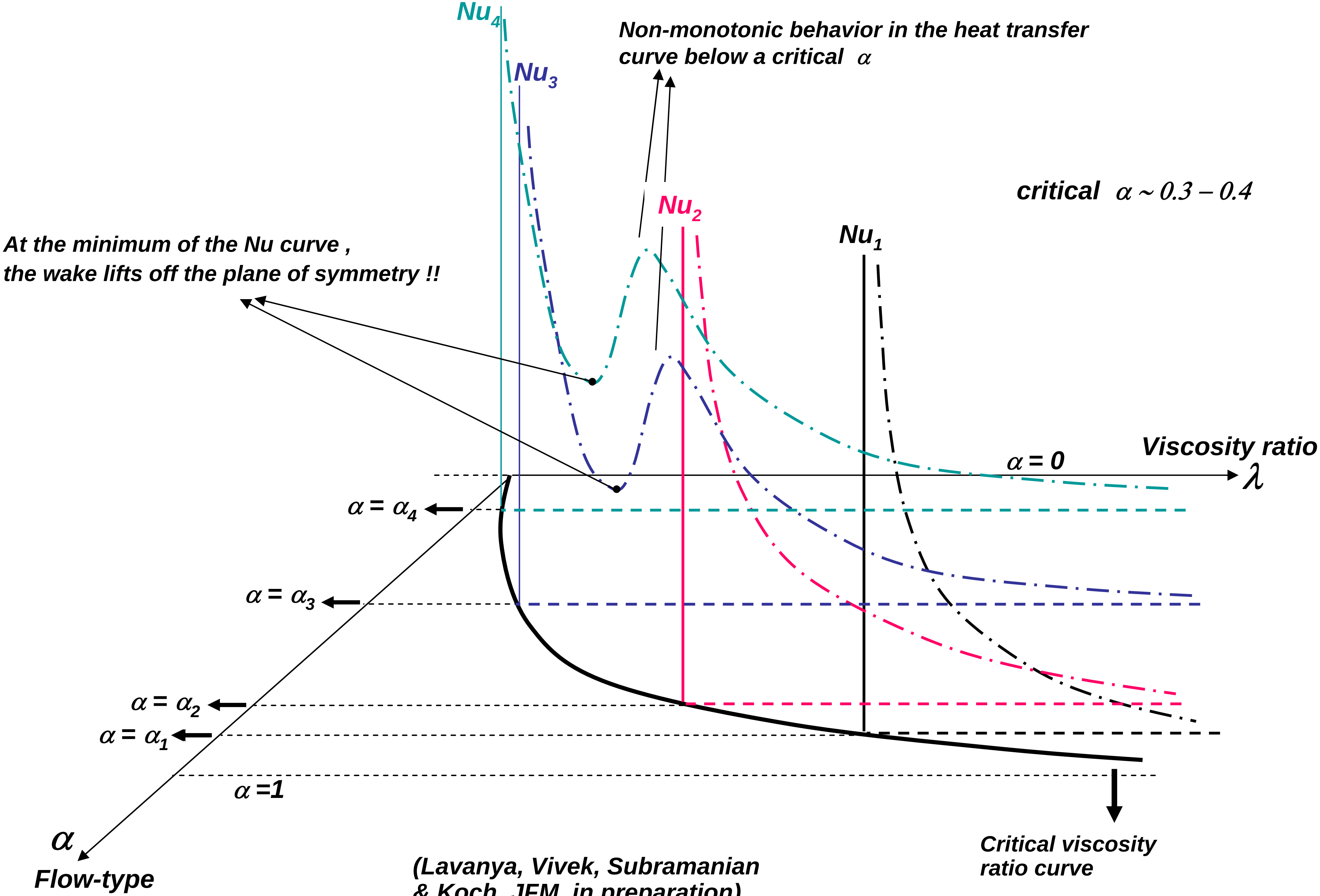
Average Inertial flow is a bi-axial extension

Average Inertial flow tends to a uni-axial extension

At the minimum of the Nu curve ,
the wake lifts off the plane of symmetry !!

Nu **curve for** α **less than critical**

Heat transfer from a density matched drop



Non-monotonic behavior in the heat transfer curve below a critical α

critical $\alpha \sim 0.3 - 0.4$

At the minimum of the Nu curve , the wake lifts off the plane of symmetry !!

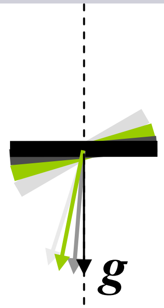
Critical viscosity ratio curve

(Lavanya, Vivek, Subramanian & Koch, JFM, in preparation)

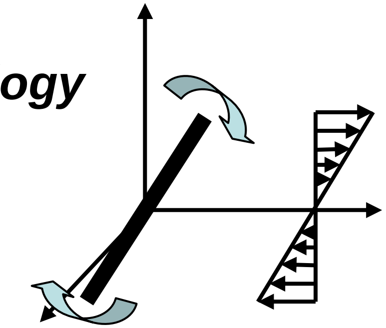
Momentum transfer :

Orientation dynamics of anisotropic particles

Orientation distribution in Sedimentation



Orientation distribution in shearing flows - rheology

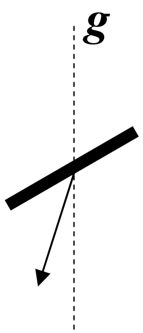


Irreversibility due to

- ***Inertia***
- ***non-Newtonian rheology***
- ***Interactions***
- ***Brownian motion***

Sedimenting particle

Inertial effects in sedimentation



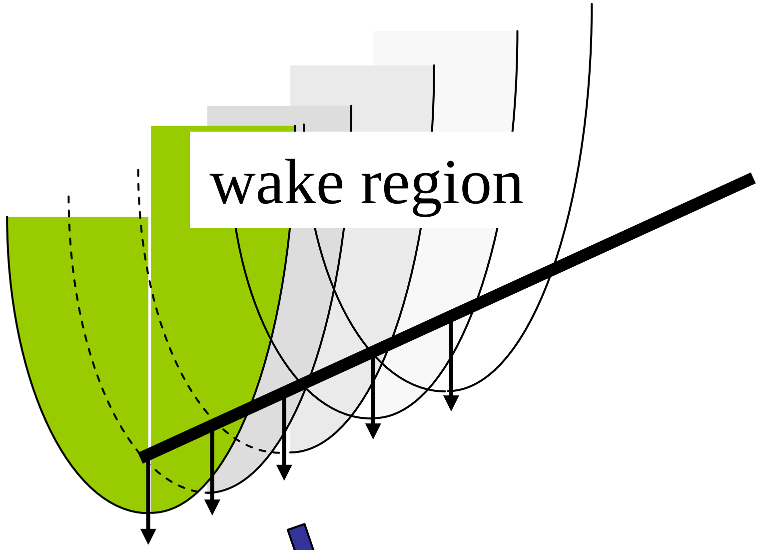
$Re = 0$

Constant orientation (Reversibility)

- ***Degenerate scenario***
- ***Mean sedimentation velocity not unique***

Finite Re

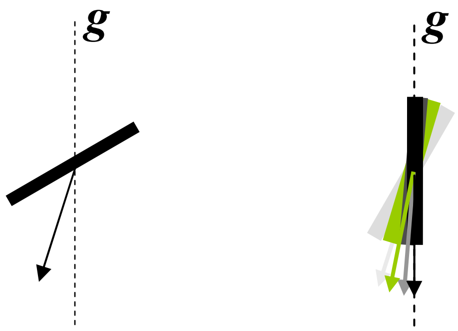
Particle settles broadside-on



Lagging part of the particle is shielded by the wake

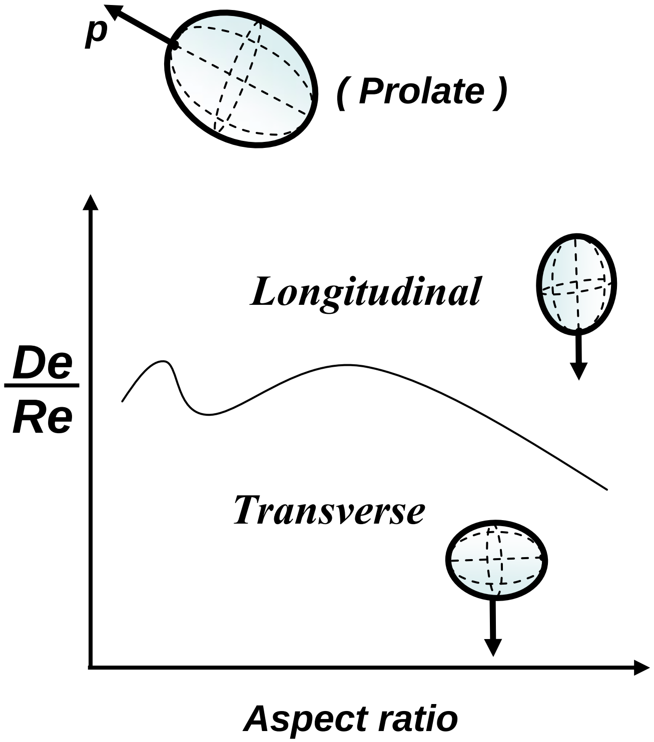
Sedimenting particle

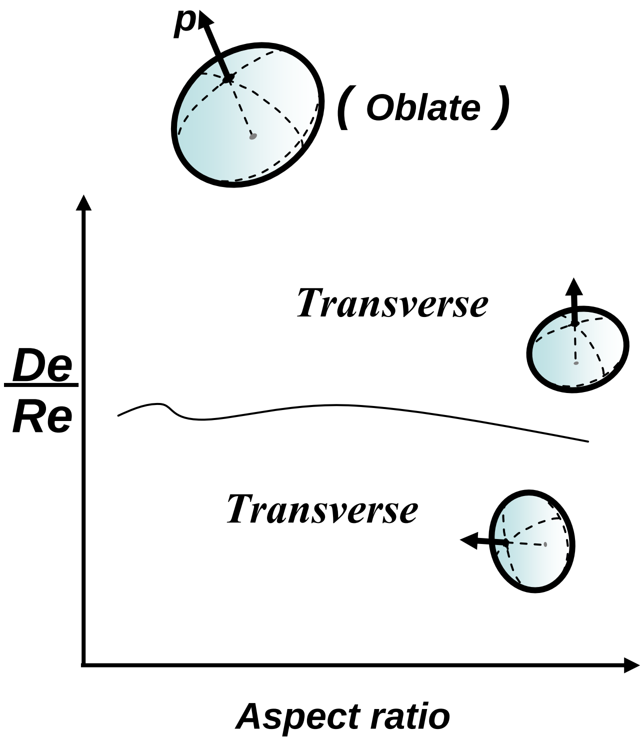
Viscoelastic effects in sedimentation
(Tensioned streamlines) →



De = 0
Constant orientation
(Reversibility)

Finite De
Particle settles
longside-on





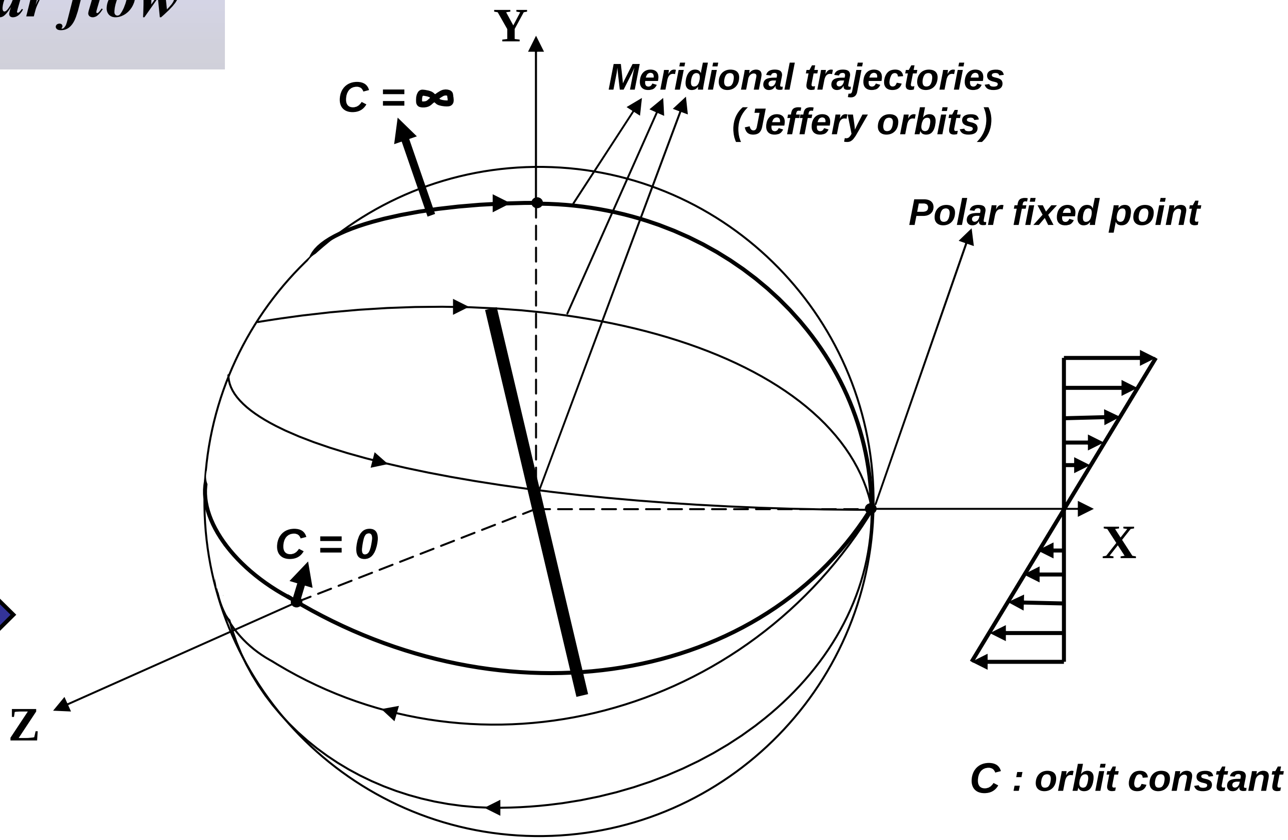
Spheroidal harmonics formalism allows an analytical prediction

Vivekanand Dabade: MS

Particle in a shear flow

- ***Distribution across jeffery orbits is not unique***

$Re = 0$ ➡

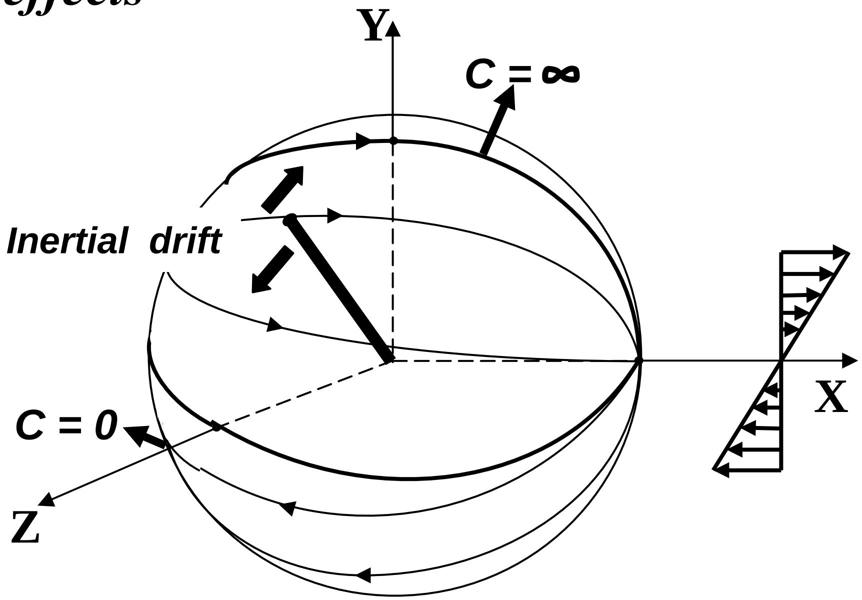


- ***Orientation distribution of the suspended particles is a key determinant of macroscopic properties***
- ***The distribution of orientation of a single particle in simple shear flow under inertialess conditions is indeterminate!***
- ***‘ Microscale inertia ’ eliminates this indeterminacy.***

Particle in a shear flow : Inertial effects

C *: orbit constant*

Finite Re ⇒



- ***Inertial drift leads to a drift across Jeffery orbits***
- ***A determinate orientation distribution***
- ***Direction of drift depends on aspect ratio***

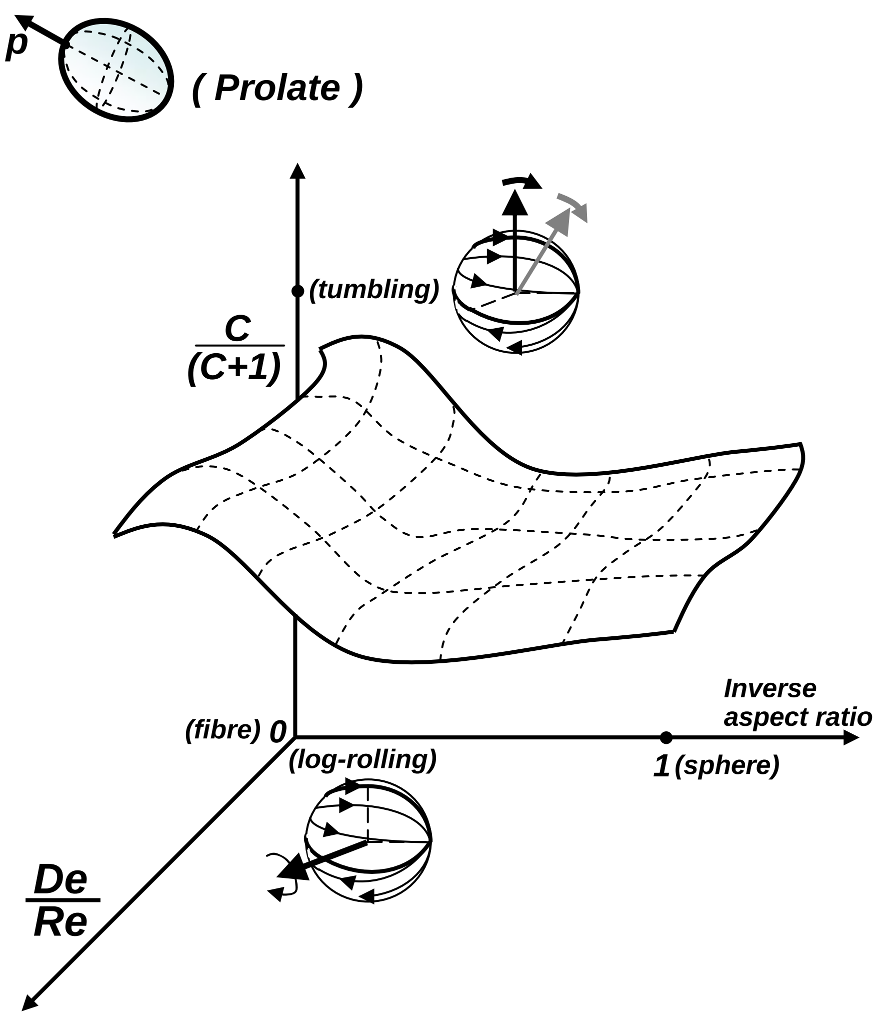
- ***For a fibre, first effects of inertia lead to a drift towards the shearing (XY) plane***
  (C = ∞ : tumbling)
- ***For a nearly spherical particle, drift is towards the vorticity axis***
  (C = 0 : log-rolling)

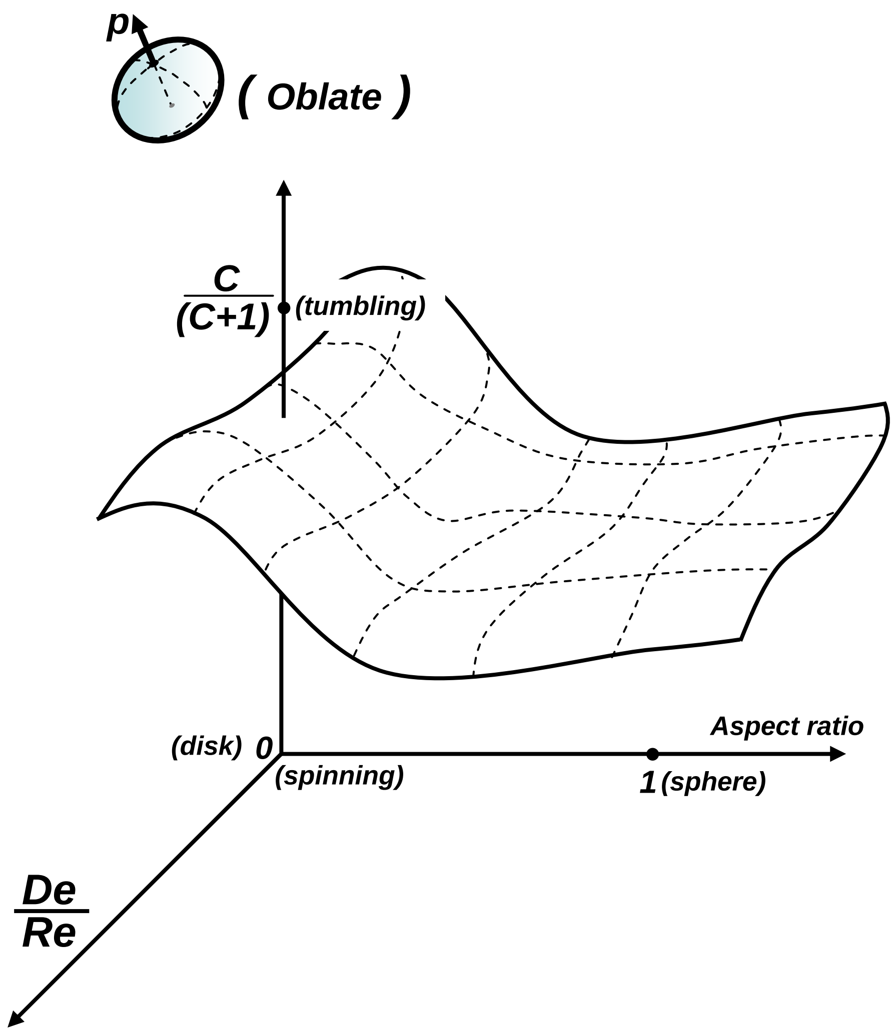
- *Implied bifurcation in orientation dynamics for a critical aspect ratio !*

(Subramanian & Koch, JFM, 2005)
(Subramanian & Koch, JFM , 2006)

Particle in a shear flow : Inertial + Viscoelastic effects

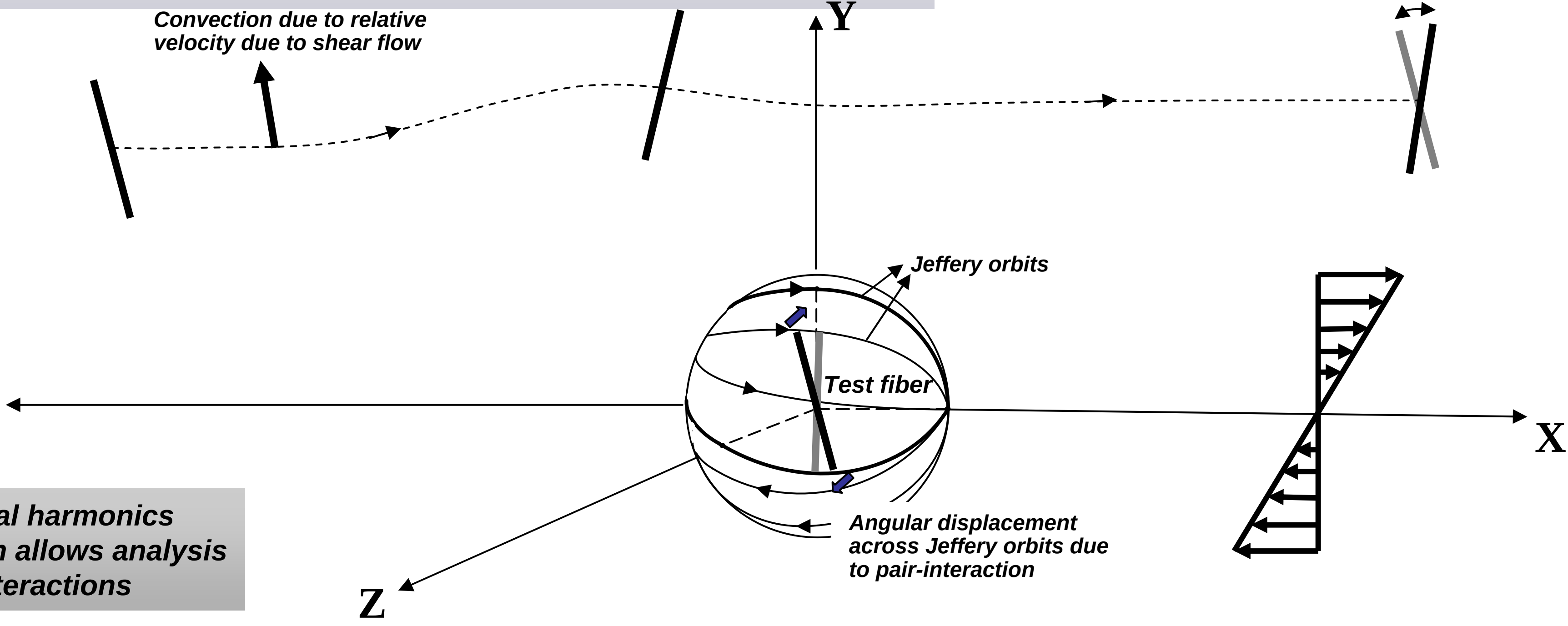
- Competing inertial and viscoelastic effects may lead to intermediate orbits
- Orientation distribution is a function of (De/Re) and the particle aspect ratio





Vivekanand Dabade: MS

Particle interactions in a shear flow



Spheroidal harmonics formalism allows analysis of pair-interactions

- *Orientation de-correlation for very slender particles is a local rotary diffusion process*

Inertialess suspension of orientable particles : *[Position-orientation space]*

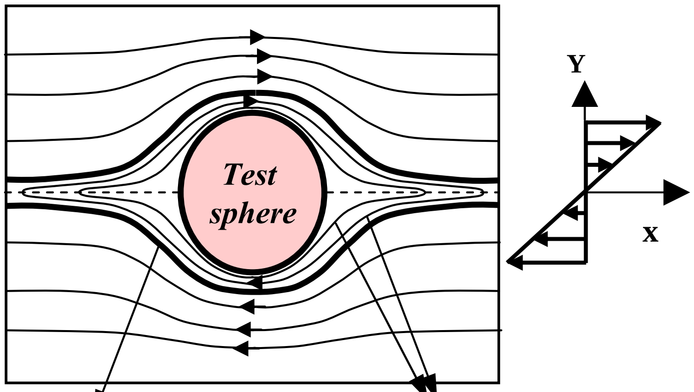
- *Non-local interactions in orientation space for an arbitrary aspect ratio*
- *Complex distributed (long-ranged) pair-interactions lead to orientation de-correlation*
- *Orientation de-correlation leads to a determinate single-particle orientation distribution*

Molecular gas of (elastic) hard-spheres or Granular gases : *[Position-velocity space]*

- *Non-local interactions in velocity space - Boltzmann equation*
- *Impulsive pair-interactions lead to velocity de-correlation*
- *Velocity de-correlation leads to a Maxwellian distribution*

Pair-hydrodynamic interactions of spherical particles in shear flow

- ***Reversibility again leads to closed pair-particle pathlines in the absence of inertia***
- ***Closed pathlines lead to an indeterminacy in the pair-distribution function : g(r)***
- ***Similar to the original heat transfer problem, slow Brownian diffusion across ' path-surfaces ' determines pair-probability across closed pathlines in dilute suspensions***
- ***Brownian diffusion of particles/drops across pathlines also controls the rate of coagulation***

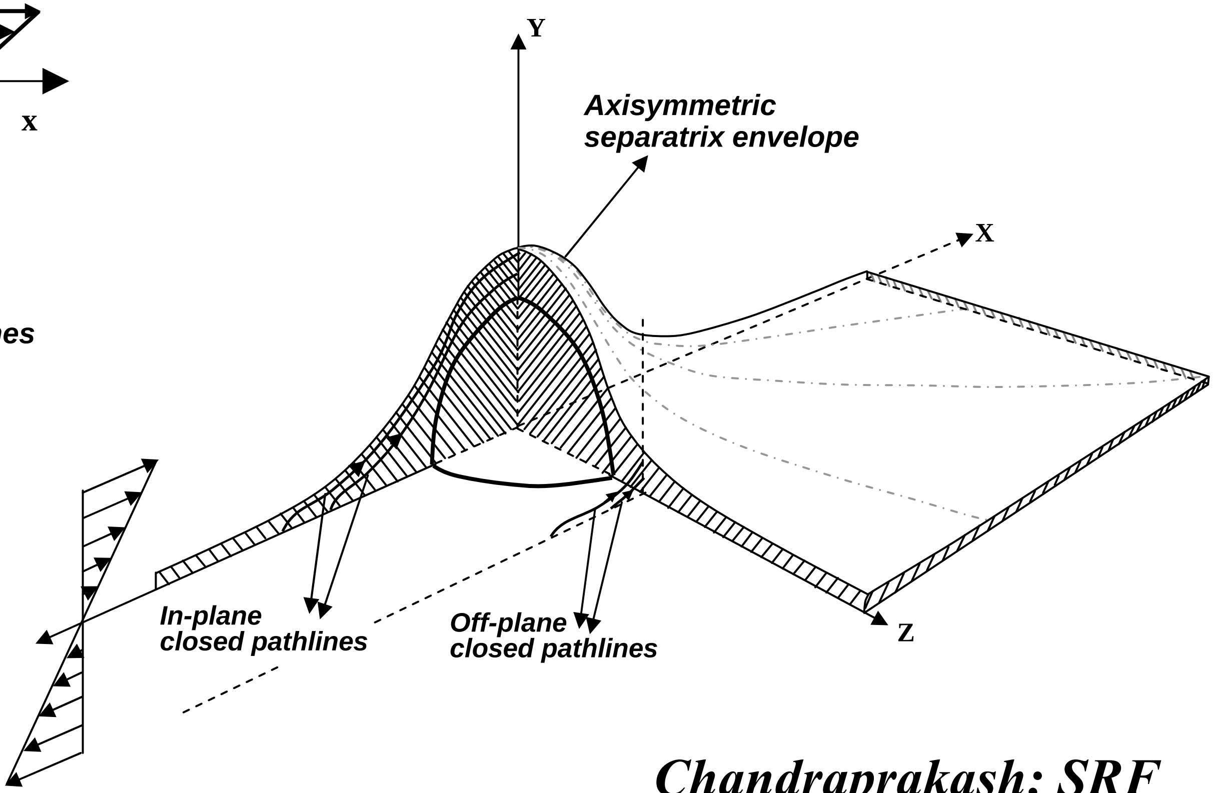


Limiting zero-Stokes Trajectories (separatices)

Inertialess closed pathlines

- ***2D-pathline topology***

- ***3D-pathline topology***



Axisymmetric separatrix envelope

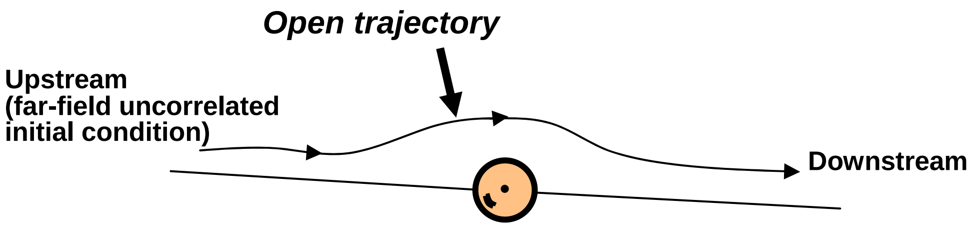
In-plane closed pathlines

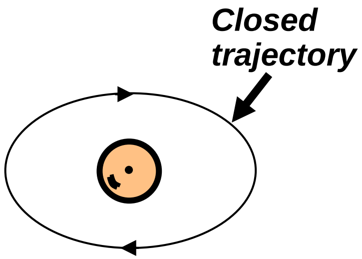
Off-plane closed pathlines

Re = 0

Chandraprakash: SRF

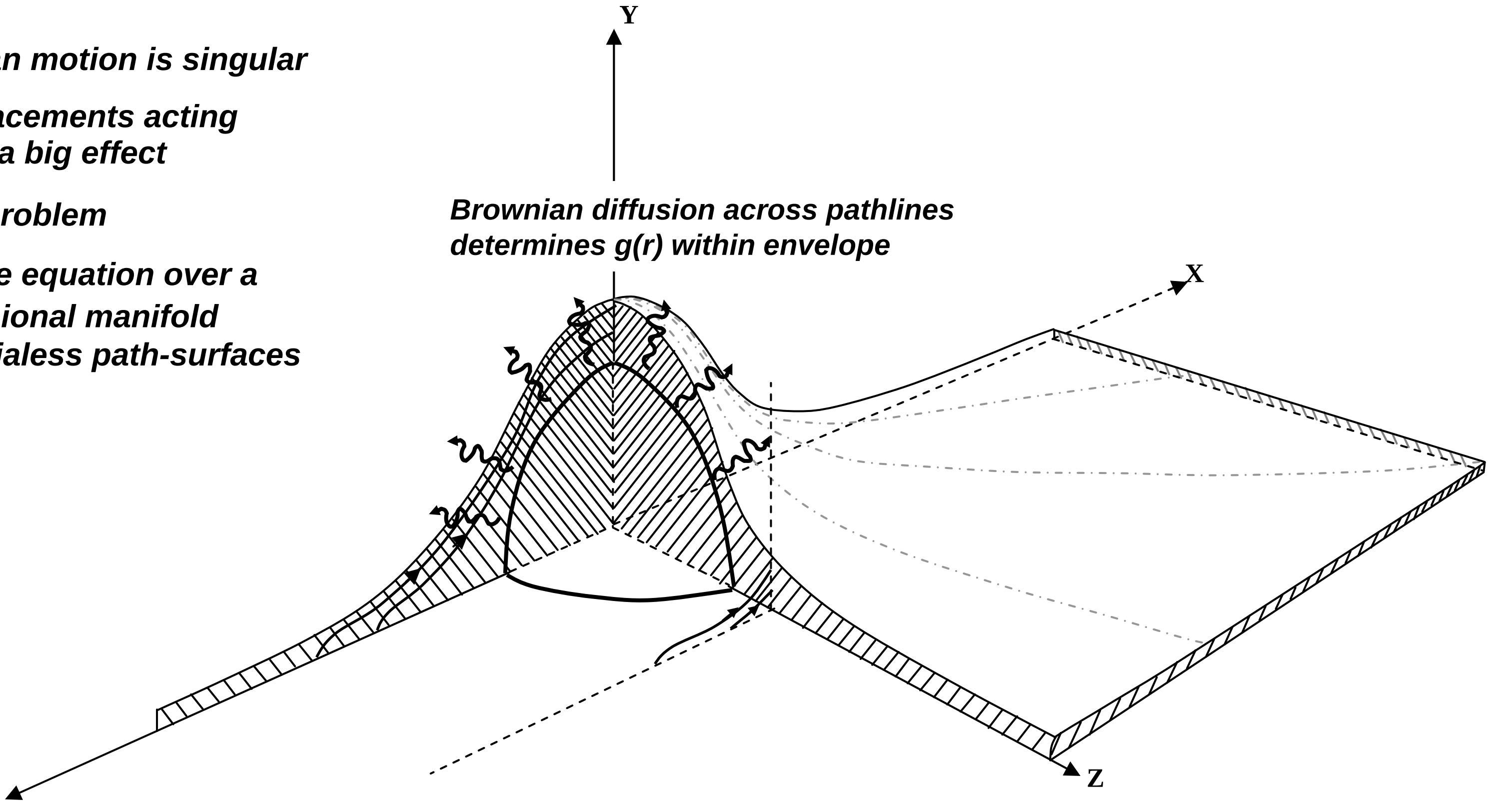
Pair-hydrodynamic interactions of spherical particles in shear flow





Where and what does one
Specify as the initial condition ? Indeterminacy

- *Limit of weak Brownian motion is singular*
- *Small Brownian displacements acting over a long time have a big effect*
- *Interesting diffusion problem*
- *To solve a Laplace- like equation over a curvilinear two-dimensional manifold corresponding to inertialess path-surfaces*



Brownian diffusion across pathlines determines g(r) within envelope

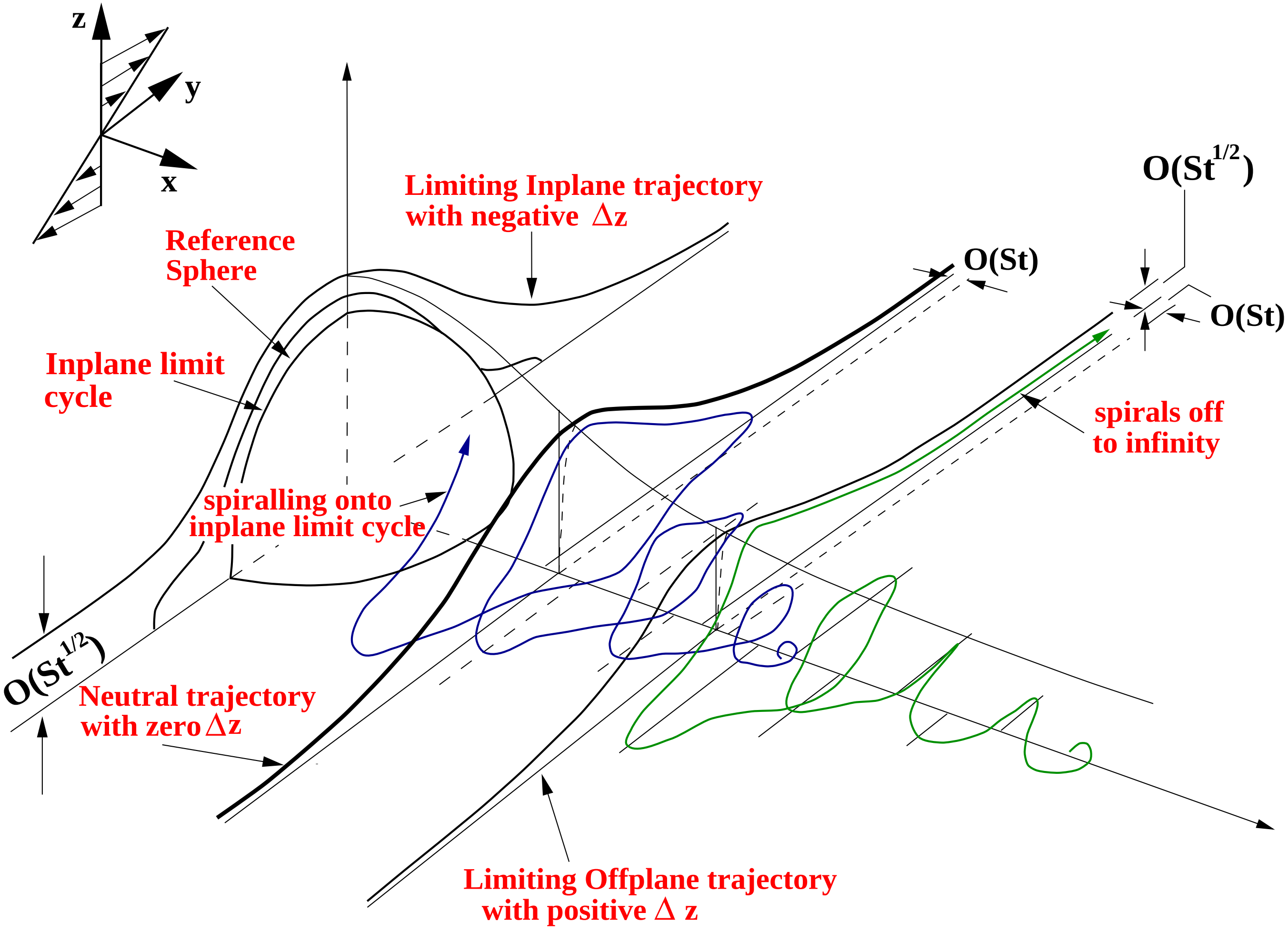
Chandraprakash: SRF

Pair-hydrodynamic interactions of spherical particles in shear flow

Inertial effects: Particle or fluid inertia

Particle → **Finite *St***

fluid inertia → **Finite *Re***



Finite *St* trajectory space

Convection across inertialess Pathlines determines suspension viscosity

(Subramanian & Brady, JFM , 2006)

Conclusions

A SMALL amount of inertia, or in general, a small source of irreversibility at the micro-scale can make a BIG difference

Questions