ON THE PLANE COUETTE FLOW OF A VISCOUS
COMPRESSIBLE FLUID WITH TRANSPIRATION COOLING

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ABSTRACT

The exact solution for the plane couette flow of a viscous compressible, heat conducting, perfect gas with the same gas injection at the stationary plate and its corresponding removal at the moving plate has been studied. It is found that the gas injection is very helpful in reducing the temperature recovery factor. Effects of injection on the shearing stress at the lower plate, longitudinal velocity profiles and the enthalpy are shown graphically.

1. NOMENCLATURE

E  Eckert number, \( \frac{(\gamma - 1) M_a^2 T_1}{T_1 - T_0} \);

\( b \)  distance between the plates;

\( h^* \) defined by equation (30);

\( H \)  enthalpy, \( = c_p T \);

\( H^* = \frac{H}{H_1} \);

\( H_{a}^* \) dimensionless adiabatic wall enthalpy;

\( M_a \)  Mach number;

\( P \)  pressure;

\( p^* \)  dimensionless pressure, \( = \frac{p}{\rho_1 U^2} \);

\( Pr \)  Prandtl number;

\( r_f \)  recovery factor, \( = \frac{H_{a}^* - 1}{\frac{1}{2} (\gamma - 1) M_a^2} \);
Greek Symbols:

\( \gamma \)  isentropic exponent, = \( c_p/c_v \);
\( \eta \)  dimensionless transverse distance, = \( y/b \);
\( \lambda \)  injection parameter, = \( \mu_0 v_0 b/\mu_1 \);
\( \mu \)  viscosity;
\( \mu^* \)  dimensionless viscosity, = \( \mu/\mu_1 \);
\( \rho \)  density;
\( \rho^* \)  dimensionless density, = \( \rho/\rho_1 \);
\( \tau_{0*} \)  dimensionless shearing stress defined by equation (16);

Subscripts:

0, 1 values of the variables at the stationary and moving plate respectively.

2. Introduction

It is well known that an exact solution of the Navier-Stokes equations exists for a viscous incompressible fluid in the case of plane Couette flow. This solution has been generalized by Nahme, Hausenblas and Groff when the viscosity of the fluid depends on temperature. The further extension to a compressible fluid was given by Illingworth and Morgan. An
important problem in the development of missiles, satellites and spaceships is the excessive heating of the skin of these vehicles by friction in the high velocity air stream. A method which appears very effective is called transpiration cooling. Although the nature of the problem is purely of boundary layer, effort has also been made by Eckert\textsuperscript{7} to describe the reduction of heat transfer by Couette flow in the case of an incompressible fluid by injecting the fluid into the flow field from the stationary plate and correspondingly removal from the moving plate. Moreover, the Couette flow model proved to be very helpful, as has been reported by Rannie\textsuperscript{8} and Mickley,\textsuperscript{9} to describe the conditions in the laminar sublayer of a turbulent boundary layer. Since at high speeds the compressibility of the gas should also be taken into account and the viscosity of the gas depends on temperature the corresponding Couette flow needs an investigation.

In the present paper an exact solution for the Couette flow of a viscous compressible fluid with the same fluid injection at the stationary plate and its removal at the moving plate has been studied. The assumption made in the present study are (i) the specific heat of the gas is constant (ii) for solid walls the Prandtl number $Pr$ is constant but arbitrary (iii) for porous walls the value of $Pr$ has been taken as $3/4$, which is an important special case since it is very near to the value of air and (iv) the viscosity of the gas is directly proportional to the temperature.

3. Statement of the Problem

Let us consider the two-dimensional laminar flow of a viscous compressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform injection of the same fluid at the stationary plate and a corresponding uniform suction at the moving plate. The axis of $x$ is taken along the stationary plate and $y$ is measured at right angle to it. In such a case all variables of the gas motion can be taken as functions of $y$ only. The equations of continuity, momentum and energy therefore are:

$$\frac{d}{dy} (\rho v) = 0, \quad (1a)$$

or

$$\rho v = \text{constant}, \quad (1b)$$

$$\rho v \frac{du}{dy} = \frac{d}{dy} \left( \mu \frac{du}{dy} \right), \quad (2)$$
\[ \rho v \frac{dv}{dy} = -\frac{dp}{dy} + \frac{d}{dy} \left( \frac{4}{3} \mu \frac{dv}{dy} \right), \]  
(3)

\[ \frac{1}{Pr} \frac{d}{dy} \left( \mu \frac{dH}{dy} \right) + \frac{4}{3} \mu \left( \frac{dv}{dy} \right)^2 + \mu \left( \frac{du}{dy} \right)^2 = \rho v \frac{dH}{dy} - v \frac{dp}{dy}, \]  
(4)

and the equation of state is

\[ p = \rho \frac{\gamma - 1}{\gamma} H, \]  
(5)

where \( u \) and \( v \) are the velocity components along \( x, y \) directions respectively, \( H = c_p T \) and the other symbols have their usual meaning.

The boundary conditions are

\[ y = 0: u = 0, \ v = v_0, \ H = H_0, \ \rho = \rho_0, \]  
(6)

\[ y = b: u = U, \ v = \frac{v_0 \rho_0}{\rho_1}, \ H = H_1, \ \rho = \rho_1. \]  
(6)

The normal velocities \( v_0 \) and \( \frac{v_0 \rho_0}{\rho_1} \) are prescribed in such a manner so that the equation of continuity (1) is satisfied, i.e.

\[ \rho v = \text{constant} = \rho_0 v_0. \]  
(7)

Let us introduce the following non-dimensional quantities:

\[ u^* = \frac{u}{U}, \ v^* = \frac{v}{U}, \ \eta = \frac{y}{b}, \ H^* = \frac{H}{H_1}, \ \rho^* = \frac{\rho}{\rho_1}, \]  
\[ \mu^* = \frac{\mu}{\mu_1}, \ p^* = \frac{p}{\rho_1 U^2}, \ \lambda = \frac{\rho_0 v_0 b}{\mu_1} \]  
(injection parameter),

\[ Re = \frac{\rho_1 U b}{\mu_1} \]  
(Reynolds number),

where \( b \) is the distance between the plates, \( U \) the velocity of the moving plate and the subscripts 1 and 2 denote the values of the variables at the stationary plate and at the moving plate respectively.

Equations (1) to (5) in non-dimensional form are:

\[ \rho^* v^* = \frac{\lambda}{Re}, \]  
(8)

\[ \lambda \frac{d u^*}{d \eta} = \frac{d}{d \eta} \left( \mu^* \frac{d u^*}{d \eta} \right), \]  
(9)
Plane Couette Flow of a Compressible Fluid

\[ \lambda \frac{dv^*}{d\eta} = - Re \frac{dp^*}{d\eta} + \frac{d}{d\eta} \left( 4 \frac{\mu^*}{3} \frac{dv^*}{d\eta} \right), \quad (10) \]

\[ \frac{d}{d\eta} \left[ \frac{1}{Pr (\gamma - 1)} M_\infty^2 \mu^* \frac{dH^*}{d\eta} \right] + \frac{4}{3} \mu^* \left( \frac{dv^*}{d\eta} \right)^2 + \mu^* \left( \frac{du^*}{d\eta} \right)^2 = \frac{\lambda}{(\gamma - 1) M_\infty^2 \frac{dH^*}{d\eta} - Re v^* \frac{dp^*}{d\eta}}, \quad (11) \]

and

\[ p^* = \frac{\rho^* H^*}{\gamma M_\infty^2}, \quad (12) \]

where

\[ \frac{U^2}{H_1} = (\gamma - 1) M_\infty^2. \quad (13) \]

The boundary conditions (6) reduce to

\[ \begin{align*}
 \eta = 0: & \quad u^* = 0, \quad v^* = v_0^*, \quad H^* = H_0^*, \\
 \eta = 1: & \quad u^* = 1, \quad v^* = \frac{\lambda}{Re}, \quad H^* = 1.
\end{align*} \quad (14) \]

4. Analysis

Equation (9), on integration, with proper boundary conditions yields

\[ \mu^* \frac{du^*}{d\eta} = \lambda u^* + \tau_{0^*}, \quad (15) \]

where

\[ \tau_{0^*} = \left( \mu^* \frac{du^*}{d\eta} \right)_{\eta=0}, \quad (16) \]

is the dimensionless shearing stress on the stationary plate, to be determined.

Now eliminating \( p^* \) between equations (10) and (11), we get

\[ \frac{d}{d\eta} \left[ \frac{1}{Pr (\gamma - 1)} M_\infty^2 \mu^* \frac{dH^*}{d\eta} \right] + \frac{4}{3} \mu^* \left( \frac{dv^*}{d\eta} \right)^2 + \mu^* \left( \frac{du^*}{d\eta} \right)^2 - \lambda v^* \left( \frac{dv^*}{d\eta} \right) \]

\[ = \frac{\lambda}{(\gamma - 1) M_\infty^2 \frac{dH^*}{d\eta} - v^* \frac{d}{d\eta} \left( \frac{4}{3} \mu^* \frac{dv^*}{d\eta} \right)}. \quad (17) \]
Taking the help of (15), the equation (17) can be written as

\[
\frac{d}{d\eta} \left[ \frac{1}{\Pr (\gamma - 1) M_{\infty}^2} \mu^* \frac{dH^*}{d\eta} + \frac{4}{3} \mu^* \nu^* \frac{d\nu^*}{d\eta} \right] - \lambda \frac{d}{d\eta} \left[ \frac{H^*}{(\gamma - 1) M_{\infty}^2} + \frac{v^*}{2} \right] = - (\lambda u^* + \tau_0^*) \frac{du^*}{d\eta}.
\]

Integrating equation (18), we get

\[
\frac{4}{3} \mu^* \frac{d}{d\eta} \left[ \frac{3}{4\Pr (\gamma - 1) M_{\infty}^2} + \frac{v^*}{2} \right] - \lambda \left[ \frac{H^*}{(\gamma - 1) M_{\infty}^2} + \frac{v^*}{2} \right] = A - \frac{\lambda u^*}{2} - \tau_0^* u^*.
\]

where \( A \) is a constant of integration.

**Case I.**—For \( \lambda = 0 \) (i.e., in the absence of porosity) and for arbitrary values of the Prandtl number \( \Pr \) the equations (15) and (19) reduce to

\[
\mu^* \frac{du^*}{d\eta} = \tau_0^*.
\]  

and

\[
\mu^* \frac{dH^*}{d\eta} = \Pr (\gamma - 1) M_{\infty}^2 (A' - \tau_0^* u^*).
\]

Equation (21) with the help of (20), becomes

\[
\frac{dH^*}{du^*} = \Pr (\gamma - 1) M_{\infty}^2 (A' - u^*),
\]

where \( A' \) is now the unknown constant.

Integrating the equation (22), and applying the boundary conditions on \( u^* \) and \( H^* \) from (14), we get

\[
H^* - H_0^* = (1 - H_0^*) u^* + \frac{\Pr}{2} (\gamma - 1) M_{\infty}^2 u^* (1 - u^*),
\]
or in terms of temperature

\[
\frac{T - T_0}{T_1 - T_0} = u^* + \frac{E}{2} u^* (1 - u^*),
\]

(24)

where

\[
E = \frac{(\gamma - 1) M_\infty^2 T_1}{T_1 - T_0} \quad \text{(Eckert number).}
\]

(25)

One can easily see that for an incompressible fluid \(u^* = 1\) and therefore from (20), with proper boundary conditions, we get

\[
u^* = \eta \quad \text{and} \quad \tau_0^* = 1.
\]

(26)

In this case the expression in (24) reduces to the well known result for the temperature distribution, Schlichting,\(^1\) when the frictional heat is taken into account.

For a compressible fluid let us consider a linear relationship between \(\mu\) and \(T\), \(i.e.\)

\[
\mu^* = H^*.
\]

(27)

Therefore from (20), using (23), on integration, we find

\[
\eta \tau_0^* = H_0^* u^* + (1 - H_0^*) \frac{u^*_2}{2} + \frac{Pr}{2} (\gamma - 1) M_\infty^2 \left(\frac{u^*_2}{2} - \frac{u^*_3}{3}\right),
\]

(28)

since at

\[
\eta = 0, \quad u^* = 0.
\]

Now, in order to find out \(\tau_0^*\) we apply the condition \(\eta = 1, u^* = 1\) in (28) and find

\[
\tau_0^* = \frac{1}{2} \left[1 + H_0^* + \frac{Pr}{6} (\gamma - 1) M_\infty^2\right].
\]

(29)

Further details may be found in Liepmann and Roshko.\(^10\)
Case II.—$\lambda \neq 0$. In order to get a compact solution of the equation (19) we make an assumption that $Pr = 3/4$ which is an important special case, since it is very near to the value for air.

Let

$$\frac{H^*}{(\gamma - 1) M^*^2} + \frac{v^{*2}}{\zeta} = h^*.$$  \hspace{1cm} (30)

Hence equation (19), with the help of (15), becomes

$$\frac{dh^*}{du^*} - \frac{3}{4} \lambda^* u^* + \frac{\lambda^*}{4 (\lambda^* u^* + 1)} = \frac{3}{4} \left( \frac{C - \frac{\lambda^* u^{*2}}{2} - u^*}{(\lambda^* u^* + 1)} \right),$$  \hspace{1cm} (31)

where

$$\lambda^* = \frac{\lambda}{\tau^*},$$  \hspace{1cm} (32)

and $C$ is now an unknown constant.

Equation (31) is a linear differential equation and its solution with the boundary conditions (14) is

$$h^* = a_3 (\lambda^* u^* + 1)^{3/4} - a_2 (\lambda^* u^* + 1) - a_1, \ldots$$  \hspace{1cm} (33)

where

$$a_1 = \frac{h_1^* - h_0^* (\lambda^* + 1)^{3/4} - a_2 ((\lambda^* + 1)^{3/4} - (\lambda^* + 1)^2)}{(\lambda^* + 1)^{3/4} - 1},$$

$$a_2 = \frac{3}{10 \lambda^*},$$

$$a_3 = h_0^* + a_1 + a_2,$$

$$h_0^* = \frac{H_0^*}{(\gamma - 1) M^*^2} + \frac{v_0^{*2}}{2},$$

and

$$h_1^* = \frac{1}{(\gamma - 1) M^*^2} + \frac{1}{2} \left( \frac{\lambda}{Re} \right)^2.$$
Now in order to determine the distribution of transverse velocity, we integrate the equation (10),

$$\frac{4}{3} \mu^* \frac{dv^*}{d\eta} = \lambda v^* + \text{Re}p^* + D,$$

(35)

where D is a constant of integration.

Equation (35), with the help of the equations (12), (8), (30) and (33) may be written as

$$\frac{4}{3} (\lambda^* u^* + 1) \frac{dv^*}{du^*}$$

$$= \lambda^* v^* + \frac{\lambda^* \gamma - 1}{\gamma} \left[ a_3 (\lambda^* u^* + 1)^{3/4} - a_2 (\lambda^* u^* + 1)^2 - a_1 \right] + \frac{D}{\tau_{0^*}},$$

(36a)

or

$$\frac{dv^{*2}}{du^*} = \frac{3}{2} \left[ \left( 1 - \frac{\gamma - 1}{2\gamma} \right) v^{*2} + \lambda^* \frac{\gamma - 1}{\gamma} \left\{ a_3 (\lambda^* u^* + 1)^{3/4} - a_2 (\lambda^* u^* + 1)^2 - a_1 \right\} + \frac{Dv^*}{\tau_{0^*}} \right]$$

(36b)

As no exact solution of the equation (36b) exists it can be integrated only numerically. Applying Picard's method of successive integration, we find

$$v^{*2} = v_{0^*}^{*2} + \frac{3}{2} \int v^* \left[ \frac{\lambda^* \frac{\gamma + 1}{2\gamma} v^{*2} + \lambda^* \frac{\gamma - 1}{\gamma} \{ a_3 (\lambda^* u^* + 1)^{3/4} - a_2 (\lambda^* u^* + 1)^2 - a_1 \}}{(\lambda^* u^* + 1)} \right]$$

$$+ \frac{Dv^*}{\tau_{0^*}} \right] du^*$$

(37)
whose first approximation is

\[ v^{*2} = v_0^{*2} + \frac{3}{2} \left( \frac{\gamma + 1}{2\gamma} v_0^{*2} \ln \left( \lambda^* u^* + 1 \right) + \frac{\gamma - 1}{\gamma} \left\{ \frac{4}{3} a_2 (\lambda^* u^* + 1)^{3/4} - \frac{1}{2} a_3 (\lambda^* u^* + 1)^2 \right\} \right. \\
\left. - a_1 \ln (\lambda^* u^* + 1) \right\} + \frac{Dv_0^*}{\tau_0^* u^*} \ln (\lambda^* u^* + 1) - \frac{\gamma - 1}{\gamma} \left\{ \frac{4}{3} a_3 - \frac{a_2}{2} \right\} \right] \]  

(38)

Now applying the boundary conditions (14), we finally have

\[ v^{*2} = a_7 \ln (\lambda^* u^* + 1) + a_6 - a_5 (\lambda^* u^* + 1)^2 + a_4 (\lambda^* u^* + 1)^{3/4}, \]

(39)

where

\[ a_4 = 2a_8 \frac{\gamma}{\gamma} - 1, \]
\[ a_5 = \frac{3}{4} a_2 \frac{\gamma}{\gamma} - 1, \]
\[ a_6 = v_0^{*2} - a_4 + a_5, \]
\[ a_7 = \left( \frac{\lambda}{\text{Re}} \right)^2 - a_6 + a_5 (\lambda^* + 1)^2 - a_4 (\lambda^* + 1)^{3/4} \]
\[ \ln (\lambda^* + 1). \]  

(40)

Although second and higher order approximations can be obtained in a similar manner we shall confine ourselves only to the first as the analysis becomes increasingly complicated.

We have yet to determine the longitudinal velocity distribution and for this we again consider the assumption (27). Equation (15) in view of (27) and (30), may be written as

\[ (\gamma - 1) M_\infty^2 \left[ h^* - \frac{v^{*2}}{2} \right] \frac{d u^*}{d \eta} = (\lambda^* u^* + 1) \tau_0^*. \]  

(41)

Substituting the values of \( h^* \) and \( v^{*2} \), from equations (33) and (39) respectively, in equation (41) and on integration with the boundary conditions (14), we find

\[ \frac{\eta \tau_0^*}{(\gamma - 1) M_\infty^2} = \frac{a_{11}}{\lambda^*} ((\lambda^* u^* + 1)^{3/4} - 1) - \frac{a_{10}}{\lambda^*} ((\lambda^* u^* + 1)^2 - 1) \]
\[ - \frac{a_2}{\lambda^*} (\ln (\lambda^* u^* + 1))^2 - \frac{a_9}{\lambda^*} \ln (\lambda^* u^* + 1), \]

(42)
where

\[
a_9 = a_1 + \frac{a_5}{2}, \quad a_8 = \frac{a_7}{4},
\]

\[
a_{10} = \frac{1}{2} \left( a_6 - \frac{a_5}{2} \right), \quad a_{11} = \frac{4}{3} \left( 4a_3 - \frac{a_4}{2} \right).
\]

(43)

Now, in order to determine the dimensionless shearing stress \( \tau^* \) at the stationary plate, we apply the boundary condition \( \eta = 1, u^* = 1 \) in (42) and find

\[
\frac{\tau^*}{(\gamma - 1) M_{\infty}^2} + \frac{a_{11}}{\lambda*} \{ (\lambda* + 1)^{3/4} - 1 \} - \frac{a_{10}}{\lambda*} \{ (\lambda* + 1)^2 - 1 \}
\]

\[
- \frac{a_6}{\lambda*} \{ \ln (\lambda* + 1) \}^2 - \frac{a_5}{\lambda*} \ln (\lambda* + 1).
\]

(44)

Thus for a given values of \( \lambda*, M_{\infty}, \gamma, v_0*, \lambda/Re \) and \( H_0* \) the value of \( \tau^*_0 \) can be calculated from (44) and then (32) will give the corresponding value of the parameter \( \lambda \) from which follows the value of Re. The longitudinal and transverse velocity profiles can be calculated from equations (42) and (39) respectively. After knowing \( u* \) and \( v* \) the enthalpy distribution will be given by (30) and (33).

It can be easily seen that for an incompressible fluid, \( (\rho^* = 1, \mu^* = 1, \)

\( i.e., H^* = 1) \ M_{\infty} \to 0, \) from (30) and (33)

\[
\lim_{M_{\infty} \to 0} a_1 (\gamma - 1) M_{\infty}^2 = -1.
\]

(45)

Therefore, from (43) and (44), we get

\[
\eta \lambda* \tau^*_0 = \ln (\lambda* u^* + 1),
\]

(46)

and

\[
\lambda* \tau^*_0 = \ln (\lambda* + 1).
\]

(47)

Now, in view of equation (32), from equations (46) and (47), we get

\[
u^* = \frac{e^{\gamma} - 1}{e^{\gamma} - 1} \quad \text{and} \quad \tau^*_0 = \frac{\lambda}{e^{\gamma} - 1},
\]

(48)

which are in conformity with the results obtained by Eckert.7
5. RECOVERY FACTOR

The recovery factor \( r_f \) is defined as

\[
rf = \frac{H_a^* - 1}{\frac{1}{2}(\gamma - 1)M_\infty^2},
\]

where \( H_a^* \) being the dimensionless adiabatic wall enthalpy.

**Case I.** \( \lambda = 0 \) and for arbitrary values of the Prandtl number \( Pr \), we find from equation (23) that \( (\partial H^*/\partial \eta)_{\eta=0} = 0 \) if

\[
H_a^* = 1 + \frac{Pr}{2}(\gamma - 1)M_\infty^2,
\]

where \( H_0^* \) has been replaced by \( H_a^* \).

Hence

\[
rf = Pr.
\]

**Case II.** \( \lambda \neq 0 \) and \( Pr = \frac{1}{2} \). From equation (30), with the help of equations (33) and (37), we find that \( (\partial H^*/\partial \eta)_{\eta=0} = 0 \) if

\[
\frac{1}{2}[a_7 - 2a_5 + \frac{3}{4}a_4] = \left[ \frac{3}{4}a_3 - 2a_2 \right].
\]

Substituting the values of \( a's \) from (34) and (43) in equation (52) we will get the value of \( H_a^* \) and hence the recovery factor in this case, is given by

\[
rf \left[ \frac{\gamma - 1}{\gamma} \ln \left( \lambda^* + 1 \right) + \frac{3}{4} \frac{1}{\gamma} \left( \frac{1}{\lambda^* + 1} \right)^{3/4} - 1 \right]
= \frac{3}{10\lambda^{*2}} \left( \frac{3}{4} \frac{\gamma - 1}{\gamma} - 2 \right)
- \frac{1}{2} \left[ \frac{\left( \frac{\lambda}{Re} \right)^2 - v_0^{*2}}{\ln (\lambda^* + 1)} + \frac{3}{4} \frac{3}{10\lambda^{*2}} \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{1}{\lambda^* + 1} \right)^{3/4} - 1 \right]
- \left[ \frac{v_0^{*2}}{2} - \frac{1}{2} \left( \frac{\lambda}{Re} \right)^2 - a_2 \left( \frac{1}{\lambda^* + 1} \right)^2 - 1 \right]
\]

\[
\left[ \frac{\gamma - 1}{\gamma} \ln \left( \lambda^* + 1 \right) + \frac{3}{4} \frac{1}{\gamma} \left( \frac{1}{\lambda^* + 1} \right)^{3/4} - 1 \right].
\]
6. **Numerical Discussion**

The calculations have been carried out for air for which \( \gamma = 1.4 \). The Prandtl number \( Pr \) is already fixed in the analysis and has the value 0.75. Taking \( \lambda/Re = 0.01 \), \( H_0^* = 1.0 \), \( v_0^* = 0.001 \) and \( M_m = 2.0 \) the method adopted is as follows:

(i) We have calculated first the values of \( a', h'_a \) and \( h_0 \) for various values of \( \lambda^* \) and then evaluated \( \tau_0 \) from equation (44). Once for a given value of \( \lambda^*, \tau_0^* \) is known, the corresponding value of \( \lambda \) is obtained from the relation \( \lambda = \lambda^* \tau_0^* \) [cf. equation (32)] and are given in Table I.

<table>
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<th>( \lambda^* )</th>
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<th>5.0</th>
<th>10.0</th>
<th>30.0</th>
<th>80.0</th>
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<td>2.6279</td>
<td>3.7482</td>
<td>4.7025</td>
</tr>
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<td>0.3934</td>
<td>0.2627</td>
<td>0.1249</td>
<td>0.0588</td>
</tr>
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</table>

(ii) The velocity distribution across the channel, i.e., against \( \eta \) is easily determined from equation (42) for a given value of \( \lambda^* \) or \( \lambda \).

(iii) For a given value of \( \lambda^* \) or \( \lambda \) the value of \( h^* \) for different values of \( u^* \) or the corresponding \( \eta \) is calculated from equation (33) between 0 and 1s. The temperature distribution then follows from equation (30).

In Fig. 1 the dimensionless shearing stress is plotted against the injection parameter \( \lambda \) for the values given in Table I. For the sake of comparison a curve from Eckert’s analysis for incompressible flow is also drawn. It is observed that the compressibility of the fluid increases the shearing stress on the stationary plate, which goes on decreasing as the value of injection parameter \( \lambda \) increases and tend to zero asymptotically.

The longitudinal velocity profiles are plotted against \( \eta \) in Fig. 2 for various values of the injection parameter \( \lambda \) and are compared with the profiles of incompressible flow. It is being noted that except \( \lambda = 0 \) the dimensionless velocity \( u^* \) in the compressible flow at any point between the plates is more as compared to the incompressible flow and this difference increases with the increase in \( \lambda \), whereas for \( \lambda = 0 \), it is less near the upper plate.
Fig. 1. The variation of the shearing stress plotted against the injection parameter for various values of the Mach number.

Fig. 2. The velocity distribution plotted against the perpendicular distance for various values of the injection parameter $\lambda$. 
In Fig. 3 the enthalpy is plotted across the section of the channel when the walls are kept at the same temperature \((H_0^* = 1.0)\), for various values of the injection parameter \(\lambda\). It is seen that the enthalpy first increases

Fig. 3. The enthalpy distribution plotted against the perpendicular distance from the stationary plate for various values of the injection parameter \(\lambda\).

Fig. 4. The variation of the temperature recovery factor plotted against the injection parameter \(\lambda\).
attain its maximum and then decreases. Moreover, the maximum value of the enthalpy increases as the injection parameter \( \lambda \) increases and the maxima point shifts towards the upper plate. For \( \lambda = 0 \), the maximum enthalpy occurs in the middle of the channel.

One of the important characteristic parameter of the compressible flow is the temperature recovery factor \( r_f \), which has been plotted against \( \lambda \) in Fig. 4. It may be observed that a considerable reduction in \( r_f \) even for small fluid injection at the stationary plate and its corresponding removal at the moving plate takes place. For \( \lambda = 0 \), \( r_f = Pr \) which is well known.

REFERENCES