TEMPERATURE FIELDS IN A HOLLOW CYLINDER IN PRESENCE OF HEAT SOURCE UNDER THE BOUNDARY CONDITIONS OF THE SECOND KIND

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ABSTRACT

General expressions have been derived for unsteady temperature distribution in a finite hollow cylinder under the influence of a time dependent volume heat source and prescribed heat fluxes at the boundaries. By introducing certain artificial additional heat source functions, corresponding Pseudo-steady solutions are defined and by means of which the temperature fields are expressed in the form of uniformly convergent series solutions. By the application of integral transform techniques the expressions for temperature distribution are obtained in various forms which may be applicable to various cases of technological importance.

Nomenclature

\( a \) = thermal diffusivity;
\( K \) = thermal conductivity;
\( 2l \) = length of the cylinder;
\( R_i \) = radius of the inner cylinder;
\( R_o \) = radius of the outer cylinder;
\( r, \phi, Z \) = cylindrical coordinates;
\( t \) = time in seconds;
\( T (r, \phi, Z, t) \) = unsteady temperature distribution defined in equations (1), (2) and (3);
\( T_0 \) = characteristic temperature
\( T_{0j} (r, \phi, Z, t), j = 0, 1, 2, 3, 4 \); Pseudo-steady temperature distribution;
\( Q (r, \phi, Z, t) \) = volume heat source function;
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\( f_j(r, \phi, t), \quad j = 1, 2; \) heat fluxes on the surfaces \( Z = -l, Z = l \) respectively;

\( f_i(\phi, Z, t), \quad j = 3, 4; \) heat fluxes on the surfaces \( r = R_1, r = R_2 \) respectively;

\( N'(r, \phi, Z) = \) initial temperature distribution in cylinder;

\( x = \frac{r}{R_1}, \phi, z = \frac{Z}{R_1}; \) non-dimensional cylindrical coordinates;

\( 2b = \frac{2l}{R_1}; \) non-dimensional length of the cylinder;

\( F_0 = \frac{\alpha t}{R_1^2}; \) Fourier number;

\( \theta(x, \phi, z, F_0) = \frac{T(r, \phi, Z, t)}{T_0}; \) non-dimensional temperature distribution defined by equations (4), (5) and (6);

\( \theta_{0j}(x, \phi, z, F_0) = \frac{T_{0j}(r, \phi, Z, t)}{T_0}, j = 0, 1, 2, 3, 4; \) non-dimensional Pseudo-steady temperature distribution defined by equations (18) and (19);

\( P_0(x, \phi, z, F_0) = \frac{R_1^2}{K T_0} Q(r, \phi, Z, t); \) Pomerantsev criterion;

\( K_{ij}(x, \phi, F_0) = \frac{R_1}{K T_0} f_j(r, \phi, t), j = 1, 2; \) Kirpichev criteria for the surfaces \( z = -b, z = b \) respectively;

\( K_{4j}(\phi, z, F_0) = \frac{R_1}{K T_0} f_j(\phi, Z, t), j = 3, 4; \) Kirpichev criteria for the surfaces \( x = 1, x = R \) respectively;

\( I_k(x) = \) modified Bessel function of the first kind of order \( k \) and argument \( x \);

\( K_k(x) = \) modified Bessel function of the second kind of order \( k \) and argument \( x \);

\( \delta_{ij} = \) Kronecker delta;

\( I_k'(\lambda_n R) = \left[ \frac{d}{d\lambda} I_k(\lambda_n x) \right]_{\lambda = \lambda_n}. \)
INTRODUCTION

In the year 1964, Olcer\textsuperscript{1} presented a paper on the theory of conductive heat transfer in finite regions. One year later he\textsuperscript{2} gave another paper on the theory of conductive heat transfer in finite regions with the boundary conditions of second kind.

In this paper we have obtained general expressions for unsteady temperature distribution in a finite hollow cylinder in presence of heat source whose entire surfaces are subjected to boundary conditions of second kind. We have followed here the same method as given by Olcer\textsuperscript{2}.

Statement of the problem.—Consider a three-dimensional problem of unsteady temperature distribution in a hollow cylinder of finite length $2l$. The radius of the inner cylinder is $R_1$ and that of outer is $R_2$ ($R_2/R_1 = R > 1$). Using cylindrical polar coordinates $r$, $\phi$, $Z$ and choosing the $Z$ coordinate along the geometrical axis of the cylinder, the flow of heat by conduction in presence of heat source can be written as:

$$
\left( \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial Z^2} \right) T(r, \phi, Z, t) + \frac{1}{K} Q(r, \phi, Z, t) = \frac{1}{\alpha} \frac{\partial T}{\partial t} (r, \phi, Z, t); (R_1 < r < R_2, 0 \leq \phi \leq 2\pi, |Z| < l, t > 0).
$$

(1)

Associated with (1), the boundary conditions are:

$$
-K \frac{\partial T}{\partial Z} = f_1(r, \phi, t); (R_1 < r < R_2, 0 \leq \phi \leq 2\pi, Z = -l, t > 0);
$$

$$
K \frac{\partial T}{\partial Z} = f_2(r, \phi, t); (R_1 < r < R_2, 0 \leq \phi \leq 2\pi, Z = l, t > 0);
$$

$$
-K \frac{\partial T}{\partial r} = f_3(\phi, Z, t); (r = R_1, 0 \leq \phi \leq 2\pi, |Z| < l, t > 0);
$$

$$
K \frac{\partial T}{\partial r} = f_4(\phi, Z, t); (r = R_2, 0 \leq \phi \leq 2\pi, |Z| < l, t > 0).
$$

(2)

The statement of the problem is completed by specifying the initial condition:

$$
T(r, \phi, Z, 0) = N'(r, \phi, Z); (R_1 \leq r \leq R_2, 0 \leq \phi \leq 2\pi, |Z| < l).
$$

(3)
On introducing the non-dimensional variables defined in the nom
clature list we can write:

\[
\left( \frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} + \frac{1}{x^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} \right) \theta (x, \phi, z, F_0) + P_0 (x, \phi, z, F_0) = \frac{\partial \theta}{\partial F_0} (x, \phi, z, F_0); \quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ |z| < b, \ F_0 > 0);
\]

\[\tag{4}\]

\[- \frac{\partial \theta}{\partial z} = K_t (x, \phi, F_0); \quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ z = -b, \ F_0 > 0);\]

\[- \frac{\partial \theta}{\partial z} = K_t (x, \phi, F_0); \quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ z = b, \ F_0 > 0);\]

\[- \frac{\partial \theta}{\partial x} = K_t (\phi, z, F_0); \quad (x = 1, \ 0 \leq \phi \leq 2\pi, \ |z| < b, \ F_0 > 0);\]

\[- \frac{\partial \theta}{\partial x} = K_t (\phi, z, F_0); \quad (x = R, \ 0 \leq \phi \leq 2\pi, \ |z| < b, \ F_0 > 0)\]

and

\[\theta (x, \phi, z, 0) = N (x, \phi, z); \quad (1 \leq x \leq R, \ 0 \leq \phi \leq 2\pi, \ |z| < b). \tag{6}\]

\textit{Solution of the problem:} To solve the above problem, we shall investigate
the solution of the eigenvalue problem

\[
\left[ \frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} + \frac{1}{x^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2} + \lambda^2 \kappa mn \right] \psi (x, \phi, Z) = 0;
\]

\[\quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ |z| < b); \tag{7}\]

subject to the boundary conditions:

\[- \frac{\partial \psi}{\partial z} = 0; \quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ z = -b);\]

\[- \frac{\partial \psi}{\partial z} = 0; \quad (1 < x < R, \ 0 \leq \phi \leq 2\pi, \ z = b);\]

\[- \frac{\partial \psi}{\partial x} = 0; \quad (x = 1, \ 0 \leq \phi \leq 2\pi, \ |z| < b);\]

\[- \frac{\partial \psi}{\partial x} = 0; \quad (x = R, \ 0 \leq \phi \leq 2\pi, \ |z| < b)\]
The eigenfunction is defined by
\[ \hat{\psi}(x, \phi, z) = \xi_k(\mu_{km} x) \left\{ \cos \frac{k\phi}{2} \right\} \cos \frac{n\pi}{2} \left(1 + \frac{z}{b}\right); \quad (k, m, n = 0, 1, 2, \ldots); \] (9)

where
\[ \xi_k(\mu_{km} x) = -Y_k(\mu_{km}) J_k(\mu_{km} x) + J'_k(\mu_{km}) Y_k(\mu_{km} x). \] (10)

The eigenvalues \( \lambda_{kmn} \) are given by:
\[ \lambda_{kmn}^2 = \mu^2_{km} + \left(\frac{n\pi}{2b}\right)^2; \] (11)

where \( \mu_{km} \geq 0 \) is the \( m \)-th root of
\[ (\mu_{km} R) \xi'_k(\mu_{km} R) = 0 \] (12)
and the prime in (12) denotes differentiation with respect to the argument.

A three-dimensional finite integral transform of \( \theta(x, \phi, x, F_0) \) is defined as
\[ \hat{\theta} = \int_0^{2\pi} \int_{-b}^b \int_{-1}^1 \phi(\lambda_{kmn} x) \theta(x, \phi, z, F_0) x d\phi dz dx; \] (13)

with inversion
\[ \theta(x, \phi, z, F_0) = \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} C_{kmn} \phi(\lambda_{kmn} x) \cdot \hat{\theta}; \] (14)

where \( \phi(\lambda_{kmn} x) \) is taken as the kernel of the transform and \( C_{kmn} \) is defined as
\[ \frac{1}{C_{kmn}} = \int_0^{2\pi} \int_{-b}^b \int_{-1}^1 \xi_k^2(\mu_{km} x) \left\{ \cos^2 \frac{k\phi}{2} \right\} \cos^2 \frac{n\pi}{2} \left(1 + \frac{z}{b}\right) x d\phi dz dx \]
\[ = \frac{\pi b}{2} \left(1 + \delta_{k0}\right) \left(1 + \delta_{n0}\right) \left[ \left(1 + \frac{k^2}{\mu^2_{km}}\right) \xi_k^2(\mu_{km} R) \right. \right.
\[ \left. \left. - \left(1 - \frac{k^2}{\mu^2_{km} m}\right) \xi_k^2(\mu_{km}) \right]. \] (15)

The solution to the system of equations (4), (5) and (6) can now be written down directly from the general expression \([2; (22 \cdot a)]\) The result is
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\[
\theta(x, \phi, z, F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_1^R \int_0^{2\pi} \int_{-b}^b N(x, \phi, z) \, x \, dx \, d\phi \, dz + \sum_{j=0}^4 \{N_j (F_0) + \theta_{0j} (x, \phi, z, F_0)\}
\]

\[
+ \frac{2}{\pi b} \sum_{k=0}^\infty \sum_{m=0}^\infty \sum_{n=0}^\infty \frac{\xi_k (\mu_{km}x) \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right)}{(1 + \delta_{k0})(1 + \delta_{n0})} \int_0^R \int_{-b}^b \int_1^\infty \int_0^\infty \frac{\exp \left( -\lambda^2_{kmn}F_0 \right)}{\left( R^2 - \frac{k^2}{\mu^2_{km}} \right) \xi_k^2 (\mu_{km}x)} - \left( 1 - \frac{k^2}{\mu^2_{km}} \right) \xi_k^2 (\mu_{km}) \right] \int_1^R \int_{-b}^b \xi_k (\mu_{km}x) \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right) \times \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right) \left\{ N(x, \phi', z) - \sum_{j=0}^4 \theta_{0j} (x, \phi', z, 0) \right\} \, x \, dx \, dz
\]

An alternate expression to the above in which the Kirpichev criteria \(K_{ij} (j_i = 1, 2, 3, 4)\) and Pomerantsev criteria \(P_0 (x, \phi, z, F_0)\) appear more explicitly is given by \([2; (22.b)]\).

\[
\theta(x, \phi, z, F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_1^R \int_0^{2\pi} \int_{-b}^b N(x, \phi, z) \, x \, dx \, d\phi \, dz + \sum_{j=0}^4 \{N_j (F_0) + \theta_{0j} (x, \phi, z, F_0)\}
\]

\[
+ \frac{2}{\pi b} \sum_{k=0}^\infty \sum_{m=0}^\infty \sum_{n=0}^\infty \frac{\xi_k (\mu_{km}x) \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right) \exp \left( -\lambda^2_{kmn}F_0 \right)}{(1 + \delta_{k0})(1 + \delta_{n0})} \int_0^R \int_{-b}^b \int_1^\infty \int_0^\infty \frac{\exp \left( -\lambda^2_{kmn}F_0 \right)}{\left( R^2 - \frac{k^2}{\mu^2_{km}} \right) \xi_k^2 (\mu_{km}x)} - \left( 1 - \frac{k^2}{\mu^2_{km}} \right) \xi_k^2 (\mu_{km}) \right] \int_1^R \int_{-b}^b \xi_k (\mu_{km}x) \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right) \times \cos \frac{\pi n}{2} \left( 1 + \frac{z}{b} \right) \left\{ N(x, \phi', z) - \frac{P_0 (x, \phi', z, 0)}{\lambda^2_{kmn}} \right\} \, x \, dx \, dz
\]
\[
- \frac{1}{\lambda^2_{kmn}} \int_1^r \xi_k (\mu_{kmn}) \{K_i, (x, \phi', 0) + (-1)^n K_i, (x, \phi', 0)\} \, dx \\
- \frac{1}{\lambda^2_{kmn}} \{R \xi_k (\mu_{kmn}) - \xi_k (\mu_{kmn})\} \int_{-b}^b \cos \frac{n\pi}{2} \left(1 + \frac{z}{b}\right) \, dx \\
\times \{K_i, (\phi', z, 0) + (-1)^n K_i, (\phi', z, 0)\} \, dz - \frac{1}{\lambda^2_{kmn}} \int_0^{r^*} \exp \left(\lambda^2_{kmn} F_0'\right) \\
\times \left\{ \int_1^r \int_{-b}^b \xi_k (\mu_{kmn}) \cos \frac{1}{2} n\pi \left(1 + \frac{z}{b}\right) \hat{P}_0 (x, \phi', z, F_0') \, dx \, dz \\
+ \int_1^r \xi_k (\mu_{kmn}) \left(\hat{K}_{i z} (x, \phi', F_0') + (-1)^n \hat{K}_{i z} (x, \phi', F_0')\right) \, dx \\
+ (R \xi_k (\mu_{kmn}) - \xi_k (\mu_{kmn})) \int_{-b}^b \cos \frac{n\pi}{2} \left(1 + \frac{z}{b}\right) \left(\hat{K}_{i z} (z, \phi', F_0') \right) \\
+ (-1)^n K_i, (z, \phi', F_0') \right) \, dz \right\} \, dF'_0 \cos k (\phi - \phi') \, d\phi'; \quad (17)
\]

where the pseudo-steady temperatures \(\theta_{ij} (x, \phi, z, F_0)\) function are given by
\[
\left(\frac{\partial^2}{\partial x^2} + \frac{1}{x} \frac{\partial}{\partial x} + \frac{1}{x^2} \frac{\partial^2}{\partial \phi^2} + \frac{\partial^2}{\partial z^2}\right) \theta_{ij} (x, \phi, z, F_0) + \delta_{ij} P_0 (x, \phi, z, F_0) \\
= \frac{dN_{ij} (F_0)}{dF_0}; \quad (1 < x < R; \ 0 \leq \phi \leq 2\pi, \ |z| < b, \ j = 0, 1, 2, 3, 4); \quad (18)
\]

subject to the boundary conditions
\[
- \frac{\partial \theta_{ij}}{\partial z} = \delta_{ij} K_{i z} (x, \phi, F_0); \quad (1 < x < R, 0 \leq \phi \leq 2\pi, z = -b, F_0 > 0); \\
\frac{\partial \theta_{ij}}{\partial z} = \delta_{ij} K_{i z} (x, \phi, F_0); \quad (1 < x < R, 0 \leq \phi \leq 2\pi, z = b, F_0 > 0); \\
- \frac{\partial \theta_{ij}}{\partial x} = \delta_{ij} K_{i z} (x, \phi, F_0); \quad (x = 1, 0 \leq \phi \leq 2\pi, \ |z| < b, F_0 > 0); \\
\frac{\partial \theta_{ij}}{\partial x} = \delta_{ij} K_{i z} (x, \phi, F_0); \quad (x = R, 0 \leq \phi \leq 2\pi, \ |z| < b, F_0 > 0); \quad (19)
\]
\[
\int_1^r \int_{-b}^b \theta_{ij} (x, \phi, z, F_0) \, x dx d\phi dz = 0
\]
and

\[
N_0 (F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_{-b}^{b} \int_{0}^{2\pi} \int_{0}^{R} P_0 (x, \phi, z, F'_{0}) x dx d\phi dz dF'_0;
\]

\[
N_1 (F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_{0}^{2\pi} \int_{0}^{R} \int_{-b}^{b} K_{1i} (x, \phi, F'_0) x dx d\phi dF'_0;
\]

\[
N_2 (F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_{0}^{2\pi} \int_{0}^{R} \int_{-b}^{b} K_{i1} (x, \phi, F'_0) x dx d\phi dF'_0;
\]

\[
N_3 (F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_{0}^{2\pi} \int_{0}^{R} \int_{-b}^{b} K_{i1} (\phi, z, F'_0) d\phi dz dF'_0;
\]

\[
N_4 (F_0) = \frac{1}{2\pi b (R^2 - 1)} \int_{0}^{2\pi} \int_{0}^{R} \int_{-b}^{b} K_{i1} (\phi, z, F'_0) d\phi dz dF'_0.
\]

Thus, once the pseudo-steady temperatures \(\theta_{0j} (x, \phi, z, F'_0)\) have been determined, the expression (16) and (17) gives the unsteady temperature field in the cylinder.

**Determination of \(\theta_{0j} (x, \phi, z, F'_0)\).**—The \(\theta_{0j} (x, \phi, z, F'_0)\) functions are determined by making use of finite integral transform techniques. We define finite integral transforms as follows:

(i) Finite cosine transform with respect to \(\phi\).

\[
\tilde{\theta}_{0j} (x, k, \phi', z, F'_0) = \int_{0}^{2\pi} \theta_{0j} (x, \phi, z, F'_0) \cos k (\phi - \phi') d\phi;
\]

with inversion

\[
\theta_{0j} (x, \phi, z, F'_0) = \frac{1}{\pi} \sum_{k=0}^{\infty} \tilde{\theta}_{0j} (x, k, \phi', z, F'_0) \frac{1}{(1 + \delta_{k0})}.
\]

(ii) Finite trigonometrical transform with respect to \(z\)

\[
\tilde{\theta}_{0j} (x, \phi, n, F'_0) = \int_{-b}^{b} \theta_{0j} (x, \phi, z, F'_0) \cos \frac{1}{2} n\pi \left(1 + \frac{z}{b}\right) dz;
\]
with inversion
\[
\theta_{0j}(x, \phi, z, F_0) \quad = \quad \frac{1}{b} \sum_{n=0}^{\infty} \frac{\tilde{\theta}_{0j}(x, \phi, n, F_0)}{(1 + \delta_{n0})} \cos \frac{1}{2} n \pi \left( 1 + \frac{z}{b} \right).
\] (24)

_Determination of \( \theta_{00}(x, \phi, z, F_0) \)._—From (18) and (19) with \( j = 0 \), we find the differential equations and boundary conditions defining the Pseudo-steady function \( \theta_{00} \). The corresponding solution is obtained by the simultaneous application of transform (21) and (23) as:

\[
\tilde{\theta}_{00}(x, n, k, \phi', F_0)
\quad = \quad \{E_1(1) - E_1(x)\} \log x + E_2(x) + \frac{1}{(R^2 - 1)} \left\{ R^2 \left( \frac{1}{2} - \log R \right) \right. 
\quad \times \left( E_1(1) - E_1(R) \right) - R^2 E_2(R) + E_2(1) + \frac{1}{2} \int_{r} \tilde{P}_0(x) x^2 \, dx \} 
\quad + \frac{R^2 E_2(R) (x^k + x^{-k}) - (E_3(1) + E_4(1)) (x^k + x^{-k} R^2 k)}{2 k (R^{2k} - 1)} 
\quad + \frac{1}{2k} \left( x^{-k} E_4(x) - x^k E_3(x) \right) + \lambda_n \left[ K_0(\lambda_n x) E_5(x) - I_0(\lambda_n x) E_6(x) \right]
\quad + \frac{\lambda_n}{K_1(\lambda_n R) I_1(\lambda_n R) - I_1(\lambda_n) K_1(\lambda_n R)} \left\{ [I_1(\lambda_n R) E_6(R) + K_1(\lambda_n R) E_6(R)] \right. 
\quad \times [K_1(\lambda_n) I_0(\lambda_n x) + I_1(\lambda_n) K_0(\lambda_n x)] - [I_1(\lambda_n) E_6(1) + K_1(\lambda_n) E_6(1)] 
\quad \times \left. [K_1(\lambda_n R) I_0(\lambda_n x) + I_1(\lambda_n R) K_0(\lambda_n x)] \right\} + \lambda_n \left( E_7(x) K_k(\lambda_n x) \right.
\quad - E_3(x) I_k(\lambda_n x) \right] + \frac{\lambda_n}{[I'_k(\lambda_n R) K'_k(\lambda_n R) - I'_k(\lambda_n) K'_k(\lambda_n)]} \left\{ [E_3(R) I'_k(\lambda_n R) \right.
\quad - E_7(R) K'_k(\lambda_n R)] \{ I'_k(\lambda_n) K_k(\lambda_n x) - K'_k(\lambda_n) I_k(\lambda_n x) \} + [E_3(1) I'_k(\lambda_n) \right.
\quad - E_7(1) K'_k(\lambda_n)] \{ K'_k(\lambda_n R) I_k(\lambda_n x) - I'_k(\lambda_n R) K_k(\lambda_n x) \} \}\].

The inversion follows by using the formula (22) and (24). Thus

\[
\theta_{00}(x, \phi, z, F_0) \quad = \quad \frac{1}{4 \pi b} \left\{ \{E_1(1) - E_1(x)\} \log x + E_2(x) + \frac{1}{(R^2 - 1)} \left\{ R^2 \left( \frac{1}{2} - \log R \right) \right. 
\quad \times \left( E_1(1) - E_1(R) \right) - R^2 E_2(R) + E_2(1) + \frac{1}{2} \int_{r} \tilde{P}_0(x) x^2 \, dx \} \right. 
\quad \left. \times \left( E_1(1) - E_1(R) \right) - R^2 E_2(R) + E_2(1) + \frac{1}{2} \int_{r} \tilde{P}_0(x) x^2 \, dx \right\} \\]
\[
\times \frac{1}{2\pi b} \sum_{k=1}^{\infty} \left[ \left\{ E_4(R) + R^{2k} E_5(R) \right\} (x^k + x^{-k}) - \left( E_3(1) + E_4(1) \right) \frac{x^k + x^{-k} R^{2k}}{2k (R^{2k} - 1)} \right]
\]
\[
+ \frac{1}{2k} \{x^{-k} E_4(x) - x^k E_3(x)\} + \frac{1}{2\pi b} \sum_{n=1}^{\infty} \left[ \lambda_n \left\{ K_0 (\lambda_n x) E_5(x) \right\} \right] \cos \frac{1}{2} \pi (1 + \frac{z}{b})
\]
\[
- I_0 (\lambda_n x) E_6(x) \right) + \frac{\lambda_n}{(K_1 (\lambda_n) I_1 (\lambda_n R) - I_1 (\lambda_n) K_1 (\lambda_n R))} \left\{ (I_1 (\lambda_n R) E_6(R)) \right\} \cos \frac{1}{2} \pi (1 + \frac{z}{b})
\]
\[
+ \frac{1}{nb} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \left[ \lambda_n \left( E_7(x) K_k (\lambda_n x) - E_8(x) I_k (\lambda_n x) \right) \right]
\]
\[
+ \frac{\lambda_n}{(I_k (\lambda_n) K_k (\lambda_n R) - I_k (\lambda_n R) K_k (\lambda_n))} \left\{ (E_8(R) I_k (\lambda_n R) - E_7(R) K_k (\lambda_n R)) \right\} \cos \frac{1}{2} \pi (1 + \frac{z}{b})
\];
\]
\[ E_\phi (x) = \int \tilde{P}_0 (x', n, 0, \phi', F_0) x' K_0 (\lambda_n x') \, dx'; \]
\[ E_\gamma (x) = \int \tilde{P}_0 (x', n, k, \phi', F_0) x' I_k (\lambda_n x') \, dx'; \]
\[ E_\beta (x) = \int \tilde{P}_0 (x', n, k, \phi', F_0) x' K_k (\lambda_n x') \, dx'. \]

From (25), one-dimensional case of Pseudo-steady temperature distribution can be easily obtained. Thus, if \( P_o (x, \phi, z, F_0) \) is independent of both \( \phi \) and \( z \), (25) simplifies to:

\[ \theta_{00} (x, F_0) \]
\[ = \frac{1}{8} \left\{ 2x^2 - (R^2 + 3) - 4 \log x + \frac{4R^2}{R^2 - 1} \log R \right\} \frac{dN_0}{dF_0} \]
\[ + \log x \int \frac{1}{x'} x' P_o (x', F_0) \, dx' \quad + \int x' \log x' P_o (x', F_0) \, dx' \]
\[ + \frac{1}{(R^2 - 1)} \left[ R^2 (\log R - \frac{1}{2}) \int x P_o (x, F_0) \, dx \right. \]
\[ - R^2 \int x \log x P_o (x, F_0) \, dx + \int x \log x P_o (x, F_0) \, dx \]
\[ + \frac{1}{2} \int x^2 P_o (x, F_0) \, dx \right\}. \quad (26) \]

If, in addition, \( P_o (x, F_0) \) is independent of \( x \), (26) gives

\[ \theta_{00} (F_0) = 0. \quad (27) \]

**Determination of \( \theta_{01} (x, \phi, z, F_0) \).**—From (18) and (19) with \( j = 1 \) we find differential equation and boundary conditions defining Pseudo-steady function \( \theta_{01} (x, \phi, z, F_0) \). The corresponding solution is obtained by the simultaneous application of transforms (21) and (23). Thus we obtain

\[ \tilde{\theta}_{01} (x, n, k, \phi', F_0) \]
\[ = (F_1 (1) - F_1 (x)) \log x + F_2 (x) - \frac{1}{(R^2 - 1)} \left\{ R^2 (\log R - \frac{1}{2}) (F_1 (1) \right\}
\[ - F_1 (R)) + R^2 F_2 (R) - F_2 (1) - \frac{1}{2} \int x^3 \tilde{K}_4 (x) \, dx \} \]
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\[\begin{align*}
&\times \left( F_4(R) + R^{2k} F_3(R) \right) \left( x^k + x^{-k} \right) - \left( F_3(1) + F_4(1) \right) \left( x^k + x^{-k} R^{2k} \right) \\
&+ \frac{x^{-k} F_4(x) - x^k F_3(x)}{2k} + \frac{\lambda_n}{K_1(\lambda_n) I_1(\lambda_n R) - I_1(\lambda_n) K_1(\lambda_n R)} \\
&\times \{(I_1(\lambda_n R) F_6(R) + K_1(\lambda_n R) F_6(R)) \left( K_1(\lambda_n) I_0(\lambda_n x) + I_1(\lambda_n) K_0(\lambda_n x) \right) \\
&- (I_1(\lambda_n) F_6(1) + K_1(\lambda_n) F_6(1)) \left( K_1(\lambda_n R) I_0(\lambda_n x) + I_1(\lambda_n R) K_0(\lambda_n x) \right) \} \\
&+ \lambda_n \left\{ K_0(\lambda_n x) F_5(x) - I_0(\lambda_n x) F_6(x) \right\} + \lambda_n \left\{ F_7(x) K_k(\lambda_n x) \\
&- F_8(x) I_k(\lambda_n x) \right\} + \frac{\lambda_n}{K_k(\lambda_n R) - I_k(\lambda_n R) K_k(\lambda_n)} \\
&\times \{(F_8(R) I_k'(\lambda_n R) - F_7(R) K_k'(\lambda_n R)) \left( I_k'(\lambda_n) K_k(\lambda_n x) \\
&- K_k'(\lambda_n) I_k(\lambda_n x) \right) + (F_8(1) I_k'(\lambda_n) - F_7(1) K_k'(\lambda_n)) \\
&\times (K_k'(\lambda_n R) I_k(\lambda_n x) - I_k'(\lambda_n R) K_k(\lambda_n x)) \}. \\
\end{align*}\]

Using formulae (22) and (24), we have the inverse transform of the above as

\[\begin{align*}
\theta_{01}(x, \phi, z, F_0) &= \frac{1}{4\pi b} \left\{ \left( F_1(1) - F_1(x) \right) \log x + F_2(x) - \frac{1}{(R^2 - 1)} \left\{ R^2 \left( \log R - \frac{1}{2} \right) \right. \right. \\
&\times \left( (F_1(1) - F_1(R)) + R^2 F_2(R) - F_2(1) - \frac{1}{2} \int_1^R K_4(x) x^2 \, dx \right) \right. \\
&\left. \left. + \frac{1}{2\pi b} \sum_{k=1}^{\infty} \left[ \left( F_4(R) + R^{2k} F_3(R) \right) \left( x^k + x^{-k} \right) - \left( F_3(1) + F_4(1) \right) \left( x^k + R^{2k} x^{-k} \right) \right] \right. \\
&\left. \left. + \frac{1}{2k} \left( x^{-k} F_4(x) - x^k F_3(x) \right) \right\} + \frac{1}{2\pi b} \sum_{n=1}^{\infty} \left[ \lambda_n \left( K_0(\lambda_n x) F_5(x) - I_0(\lambda_n x) F_6(x) \right) \right. \\
&\left. \left. + \frac{\lambda_n}{K_1(\lambda_n) I_1(\lambda_n R) - I_1(\lambda_n) K_1(\lambda_n R)} \left\{ (I_1(\lambda_n R) F_6(R) \\
&+ K_1(\lambda_n R) F_5(R)) \left( K_1(\lambda_n) I_0(\lambda_n x) + I_1(\lambda_n) K_0(\lambda_n x) \right) - (I_1(\lambda_n) F_6(1) \\
&+ K_1(\lambda_n) F_5(1)) \right\} \cos \frac{1}{2} \pi \left( 1 + \frac{z}{b} \right) \right] \right\}.
\end{align*}\]
\[
\frac{1}{2b} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \left[ \lambda_n \left( F_7(x) K_k(\lambda_n x) - F_8(x) I_k(\lambda_n x) \right) + \frac{\lambda_n}{(I_k'(\lambda_n) K_k'(\lambda_n) - I_k(\lambda_n) K_k(\lambda_n))} \right] \cos \frac{n\pi}{2} \left( 1 + \frac{z}{b} \right);
\]

where

\[
K_{i_1}(x) = \bar{K}_{i_1}(x, 0, \phi', F_0) - 4\pi b \frac{dN_1}{dF_0};
\]

\[
F_1(x) = \int x' \bar{K}_{i_1}(x') dx';
\]

\[
F_2(x) = \int \bar{K}_{i_1}(x') x' \log x' dx';
\]

\[
F_3(x) = \int \bar{K}_{i_1}(x', k, \phi', F_0) x'^{3-k} dx';
\]

\[
F_4(x) = \int \bar{K}_{i_1}(x', k, \phi', F_0) x'^{3-k} dx';
\]

\[
F_5(x) = \int \bar{K}_{i_1}(x', 0, \phi', F_0) x' I_0(\lambda_n x') dx';
\]

\[
F_6(x) = \int \bar{K}_{i_1}(x', 0, \phi', F_0) x' K_0(\lambda_n x') dx';
\]

\[
F_7(x) = \int \bar{K}_{i_1}(x', k, \phi', F_0) x' I_k(\lambda_n x') dx';
\]

\[
F_8(x) = \int \bar{K}_{i_1}(x', k, \phi', F_0) x' K_k(\lambda_n x') dx'.
\]

If \(K_{i_1}(x, \phi, F_0)\) is independent of both \(x\) and \(\phi\), (28) simplifies to

\[
\theta_{01}(z, F_0) = \frac{1}{4} \frac{(b - z)^2}{b^3} K_{i_1}(F_0).
\]

**Determination of \(\theta_{02}(x, \phi, z, F_0)\).** Differential equation and boundary conditions defining \(\theta_{02}(x, \phi, z, F_0)\) is obtained by substituting \(j = 2\) in (18) and (19). By making use of transform given in (21) and (23), we get
\( \tilde{\theta}_{02} (x, n, k, \phi', F_0) \)

\[
= \{ G_1 (1) - G_1 (x) \} \log x + G_2 (x) + \frac{1}{(R^2 - 1)} \left\{ \left( \frac{1}{2} R^2 - R^2 \log R \right) (G_1 (1) - G_1 (R)) \right. \\
- R^2 G_2 (R) + G_2 (1) + \frac{1}{2} \int_1^x x^2 \tilde{K}_n (x) \, dx \right\} + \frac{1}{2k} \left( x^{-k} G_4 (x) - x^k G_3 (x) \right) \\
+ \frac{G_4 (R) + R^{2k} G_3 (R)}{2k \left( R^{2k} - 1 \right)} \left( x^k + x^{-k} \right) - \left( G_3 (1) + G_4 (1) \right) \left( x^k + R^{2k} x^{-k} \right) \\
+ \frac{\lambda_n}{K_1 (\lambda_n) I_1 (\lambda_n R) - I_1 (\lambda_n) K_1 (\lambda_n R)} \left\{ (I_1 (\lambda_n) R) G_6 (R) + K_1 (\lambda_n R) G_6 (R) \right\} \\
\times \left( K_1 (\lambda_n) I_0 (\lambda_n x) + I_1 (\lambda_n) K_0 (\lambda_n x) \right) - \left( I_1 (\lambda_n) G_6 (1) + K_1 (\lambda_n) G_6 (1) \right) \\
\times \left( K_1 (\lambda_n R) I_0 (\lambda_n x) + I_1 (\lambda_n R) K_0 (\lambda_n x) \right) + \lambda_n \{ K_0 (\lambda_n x) G_6 (x) - I_0 (\lambda_n x) G_6 (x) \} \\
+ \frac{\lambda_n}{I_k' (\lambda_n) K_k (\lambda_n R) - I_k (\lambda_n R) K_k (\lambda_n R)} \left\{ (G_8 (R) I_k' (\lambda_n R) - G_7 (R) K_k' (\lambda_n R)) \right. \\
\times \left( I_k' (\lambda_n) K_k (\lambda_n x) - K_k' (\lambda_n) I_k (\lambda_n x) \right) + \left( G_8 (1) I_k' (\lambda_n) - G_7 (1) K_k' (\lambda_n) \right) \\
\times \left( K_k' (\lambda_n R) I_k (\lambda_n x) - I_k' (\lambda_n R) K_k (\lambda_n x) \right) \left\} + \lambda_n \{ G_7 (x) K_k (\lambda_n x) - G_8 (x) I_k (\lambda_n x) \}. \]

The inversion follows by the use of formulae (22) and (24). Thus,

\( \theta_{02} (x, \phi, z, F_0) \)

\[
= \frac{1}{4 \pi b} \left[ (G_1 (1) - G_1 (x)) \log x + G_2 (x) + \frac{1}{(R^2 - 1)} \left\{ \left( \frac{1}{2} R^2 - R^2 \log R \right) (G_1 (1) \\
- G_1 (R)) \right. \\
- R^2 G_2 (R) + G_2 (1) + \frac{1}{2} \int_1^x x^2 \tilde{K}_n (x) \, dx \right\} \right] \\
+ \frac{1}{2 \pi b} \sum_{k=1}^{\infty} \left[ \left( G_4 (R) + R^{2k} G_3 (R) \right) \left( x^k + x^{-k} \right) - \left( G_3 (1) + G_4 (1) \right) \left( x^k + R^{2k} x^{-k} \right) \right] \\
\times \frac{1}{2k} \left( x^{-k} G_4 (x) - x^k G_3 (x) \right) \right] + \frac{1}{2 \pi b} \sum_{n=1}^{\infty} \left[ \lambda_n \left( K_0 (\lambda_n x) G_6 (x) - I_0 (\lambda_n x) G_6 (x) \right) \right. \\
\times \left( K_1 (\lambda_n) I_1 (\lambda_n R) - I_1 (\lambda_n) K_1 (\lambda_n R) \right) \left\{ (I_1 (\lambda_n) R) G_6 (R) + K_1 (\lambda_n R) G_6 (R) \right\} \]
\[ \times \left( K_1(\lambda_n) I_0(\lambda_n x) + I_1(\lambda_n) K_0(\lambda_n x) - I_1(\lambda_n) G_6(1) + K_1(\lambda_n) G_6(1) \right) \]
\[ \times \left( K_1(\lambda_n R) I_0(\lambda_n x) + I_1(\lambda_n R) K_0(\lambda_n x) \right) \]
\[ \left[ \cos \frac{1}{2} \pi \left( 1 + \frac{z}{b} \right) \right] \]
\[ \frac{1}{\pi b} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \lambda_n \left[ (G_7(x) K_k(\lambda_n x) - G_8(x) I_k(\lambda_n x)) \right] \]
\[ \frac{1}{I_k(\lambda_n) K_k(\lambda_n R) - I_k'(\lambda_n R) K_k'(\lambda_n)} \left[ (G_8(1) I_k(\lambda_n) - G_7(1) K_k(\lambda_n)) \right] \]
\[ \times \left( I_k(\lambda_n) K_k(\lambda_n x) - K_k(\lambda_n) I_k(\lambda_n x) \right) + \left( G_8(1) I_k(\lambda_n) - G_7(1) K_k(\lambda_n) \right) \]
\[ \times \left( K_k(\lambda_n R) I_k(\lambda_n x) - I_k'(\lambda_n R) K_k(\lambda_n x) \right) \]
\[ \cos \frac{1}{2} \pi \left( 1 + \frac{z}{b} \right); \quad (30) \]

where

\[ \hat{K}_{i*}(x) = \bar{K}_{i*}(x, 0, \phi', F_0) - 4\pi b \frac{dN}{dF_0}; \]
\[ G_1(x) = \int x' \hat{K}_{i*}(x') dx'; \]
\[ G_2(x) = \int x' \log x' \hat{K}_{i*}(x') dx'; \]
\[ G_3(x) = \int \bar{K}_{i*}(x', k, \phi', F_0) x'^{3-k} dx'; \]
\[ G_4(x) = \int \bar{K}_{i*}(x', k, \phi', F_0) x'^{3+k} dx'; \]
\[ G_5(x) = (-1)^n \int \bar{K}_{i*}(x', 0, \phi', F_0) x' I_0(\lambda_n x') dx'; \]
\[ G_6(x) = (-1)^n \int \bar{K}_{i*}(x', 0, \phi', F_0) x' K_0(\lambda_n x') dx'; \]
\[ G_7(x) = (-1)^n \int \bar{K}_{i*}(x', k, \phi', F_0) x' I_k(\lambda_n x') dx'; \]
\[ G_8(x) = (-1)^n \int \bar{K}_{i*}(x', k, \phi', F_0) x' K_k(\lambda_n x') dx'. \]

If \( K_{i*}(x, \phi, F_0) \) is independent of both \( x \) and \( \phi \), (30) simplifies to:

\[ \theta_{02}(z, F_0) = \sqrt{\left( \frac{b + z}{2} \right)^2 - \frac{b^2}{3}} \frac{K_{i*}(F_0)}{b}. \quad (31) \]

**Determination of** \( \theta_{03}(x, \phi, z, F_0)\).—Substituting \( j = 3 \) in (18) and (19), we get equations defining \( \theta_{03}(x, \phi, z, F_0) \). Applying transform (21) and then (23) we get after a little simplification:
\[ \tilde{\theta}_{03}(x, n, k, \phi', F_0) \]

\[
= \pi b \frac{dN_3}{dF_0} \left[ x^2 - 2R^2 \log x - \frac{1}{2} (3R^2 + 1) + \frac{2R^4}{R^2 - 1} \log R \right] + \frac{x^k + x^{-k} R^{2k}}{k (R^{2k} - 1)} \]

\[
\times \tilde{K}_i (0, k, \phi', (F_0) + \frac{K_1 (\lambda_n R) I_0 (\lambda_n x) + I_1 (\lambda_n R) K_0 (\lambda_n x)}{\lambda_n \left( - I_1 (\lambda_n) K_1 (\lambda_n R) + K_1 (\lambda_n) I_1 (\lambda_n R) \right)} \tilde{K}_i (n, 0, \phi', F_0) \]

\[
+ \frac{K_k' (\lambda_n R) I_k (\lambda_n x) - I_k' (\lambda_n R) K_k (\lambda_n x)}{I_k' (\lambda_n R) K_k' (\lambda_n) - I_k' (\lambda_n) K_k (\lambda_n R)} \tilde{K}_i (n, k, \phi', F_0); \]

with inversion,

\[ \theta_{03} (x, \phi, z, F_0) \]

\[
= \frac{1}{4} \frac{dN_3}{dF_0} \left[ x^2 - 2R^2 \log x - \frac{1}{2} (3R^2 + 1) + \frac{2R^4}{R^2 - 1} \log R \right] \]

\[
+ \frac{1}{2\pi b} \sum_{k=1}^{\infty} \frac{x^k + x^{-k} R^{2k}}{k (R^{2k} - 1)} \tilde{K}_i (0, k, \phi', F_0) \]

\[
+ \frac{1}{2\pi b} \sum_{n=1}^{\infty} \frac{K_1 (\lambda_n R) I_0 (\lambda_n x) + I_1 (\lambda_n R) K_0 (\lambda_n x)}{K_1 (\lambda_n) I_1 (\lambda_n R) - I_1 (\lambda_n) K_1 (\lambda_n R)} \tilde{K}_i (n, 0, \phi', F_0) \]

\[
\times \cos \frac{1}{2} n\pi \left( 1 + \frac{z}{b} \right) + \frac{1}{b} \sum_{k=1}^{\infty} \sum_{n=1}^{\infty} \frac{K_k' (\lambda_n R) I_k (\lambda_n x) - I_k' (\lambda_n R) K_k (\lambda_n x)}{I_k' (\lambda_n R) K_k' (\lambda_n) - I_k' (\lambda_n) K_k (\lambda_n R)} \]

\[
\times \tilde{K}_i (n, k, \phi', F_0) \cos \frac{1}{2} n\pi \left( 1 + \frac{z}{b} \right). \quad (32) \]

If \( K_i (\phi, z, F_0) \) is independent of both \( \phi \) and \( z \), the problem becomes one-dimensional in the radial direction, and (32) reduces to

\[ \theta_{03} (x, F_0) = \frac{K_{i_3} (F_0)}{4 (R^2 - 1)} \left[ 2x^2 - (3R^2 + 1) - 4R^2 \log x + \frac{4R^4}{R^2 - 1} \log R \right]. \quad (33) \]

**Determination of \( \theta_{04} (x, \phi, z, F_0) \).**—Substituting \( j = 4 \) in (18) and (19), we get equations defining \( \theta_{04} (x, \phi, z, F_0) \). By making use of the transform (21) and (23) we obtain,
If $K_i(\phi, z, F_0)$ is independent of both $\phi$ and $z$, the problem becomes one-dimensional in the radial direction, and (34) reduces to:

$$\theta_{04}(x, F_0) = \left[ 2x^2 - 4 \log x - (3 + R^2) + \frac{4R^2}{R^2 - 1} \log R \right] \frac{R K_i(n, \phi', F_0)}{4(R^2 - 1)} \tag{35}$$
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This concludes the determination of the $\theta_{ij}(x, \phi, z, F_0)$ functions appearing in (16) and (17).

Numerous special cases of heat conduction problem, with boundary conditions of the second kind, follow from the general expression (17). As an example, consider a cylinder insulated at the faces $z = \pm b$, viz., $K_{i+} = K_{i-} = 0$. Further consider that the surfaces $x = 1$, $x = R$ are subjected to heat fluxes proportional to generalised time $F_0$, viz., $K_{i+} (\phi, z, F_0) = K_{i+} F_0$, $K_{i-} (\phi, z, F_0) = K_{i-} F_0$. The initial temperature being zero i.e., $N(x, \phi, z) = 0$ and the internal heat source being proportional to generalised time $F_0$, viz., $P_0(x, \phi, z, F_0) = P_0 F_0$. Substituting these values in the equations (20), (25), (32) and (34), we obtain,

\[
N_0 (F_0) = \frac{1}{2} P_0 F_0^2;
\]

\[
N_8 (F_0) = \frac{1}{R^8 - 1} K_{i+} F_0^2;
\]

\[
N_4 (F_0) = \frac{1}{R^8 - 1} K_{i-} F_0^2;
\]

\[
\theta_{oo} (x, F_0) = 0;
\]

\[
\theta_{os} (x, F_0) = \frac{1}{4} \left[ 2x^2 - 4R^2 \log x - (1 + 3R^2) 
+ \frac{4R^4}{R^8 - 1} \log R \right] \frac{K_{i+} F_0}{R^8 - 1};
\]

\[
\theta_{os} (x, F_0) = \frac{1}{4} \left[ 2x^2 - 4 \log x - (3 + R^2) 
+ \frac{4R^2}{R^8 - 1} \log R \right] \frac{K_{i-} F_0}{R^8 - 1}.
\]

Substituting these values in the general expression (17), we get

\[
\theta (x, F_0) = \left\{ \frac{1}{2} P_0 + \frac{1}{R^8 - 1} (K_{i+} + K_{i-}) \right\} F_0^2 + \left\{ 2x^2 - 4R^2 \log x 
- (1 + 3R^2) + \frac{4R^4}{R^8 - 1} \log R \right\} \frac{K_{i+} F_0}{4(R^8 - 1)} + \left\{ 2x^2 - 4 \log x - (3 + R^2) \right\} \]

\[
+ \frac{4R^2}{R^8 - 1} \log R \]
Fig. 1. Characteristic Roots of Equation $Y_1(2\mu_m)J_1(\mu_m) - J_1(2\mu_m)Y_1(\mu_m) = 0$.

Fig. 2. Variation of $\frac{\theta(X, F_0)}{K_1}$ versus $F_0$ at the inner and the outer surfaces.
\[+ \frac{4R^2}{R^2 - 1} \log R \bigg\} \frac{R K}{4 (R^2 - 1)} - 2 (K_i + K_i) \sum_{m=1}^{\infty} \frac{\xi_0 (\mu_m x) (1 - e^{-\mu_m x})}{\mu_m^4 (R \xi_0 (\mu_m x) + \xi_0 (\mu_m))} \]

\[- 2 P \sum_{m=1}^{\infty} \frac{\xi_0 (\mu_m x)}{\mu_m^4} (1 - e^{-\mu_m x}) \text{;} \quad (36)\]

where, in view of (12), the eigenvalues \(\mu_m\) are determined from the positive roots of:

\[\xi_0' (\mu_m R) = 0. \quad (37)\]

To discuss the above problem, consider \(R = 2\). In this case the characteristic equation becomes

\[\xi_0' (2\mu_m) = 0. \quad (38)\]

Fig. 3. Variation of \(\frac{\theta (X, F_0)}{K_i}\) versus \(F_0\) at the inner and the outer surfaces.
For determination of the roots of the characteristic equation we set
\[ y = Y_1 (2\mu_m) J_1 (\mu_m) \] and
\[ y = J_1 (2\mu_m) Y_1 (\mu_m). \]
\[ y = Y_1 (2\mu_m) J_1 (\mu_m) \] and
\[ y = J_1 (2\mu_m) Y_1 (\mu_m) \] are plotted versus \( \mu_m \) as given in Fig. (1). The abscissa of the points of intersection of the curve \( y = Y_1 (2\mu_m) J_1 (\mu_m) \) with the curve \( y = J_1 (2\mu_m) Y_1 (\mu_m) \) determine the values of the characteristic roots. From Fig. (1), we observe that \( \mu_1 < \mu_2 < \mu_3 < \cdots \). Further, we assume:
\[ P_{0e} = K_{i_1} = K_{i_2} = K_4. \] (39)

The quantity \( \theta (x, F_0)/K_4 \) is plotted versus \( F_0 \) for \( x = 1 \) and \( x = 2 \) (Fig. 2 and Fig. 3). From curves in Fig. (2) we observe that in the interval \( 0 < F_0 < 0.6 \) the temperature of the outer surface decreases and attains a minimum at \( F_0 = 0.6 \) (approximately) and then it continuously goes on increasing. From Fig. (3) we see that near \( F_0 = 1.5 \), the temperature of both surfaces becomes equal; after that the temperature of the outer surface increases.

The boundary conditions (2) prescribed on the curved surfaces of the hollow cylinder reflect in their physical content a large scale of heat exchange phenomenon including the radiation effect also. Such type of problems has special significance in cylindrical cooling fin.

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