SIZE DEPENDENT RESONANCES IN THE CLASSICAL ELECTROMAGNETIC SCATTERING

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Abstract

Some size dependent resonances in the extinction efficiency by small metallic particles have been considered on the basis of the exact calculations on Mie Theory of Scattering. The results may be helpful in explaining some structures in the observed interstellar extinction curve and certain unidentified interstellar diffuse absorption bands.

1. Introduction

The size dependent resonance effect in the Rayleigh scattering region has been seldom considered in the literature. An exception is the elegant work of Savost'yanova (1930, 1939 *a*, *b*) who studied the scattering by colloidal systems Na–NaCl and Cu–NaCl. In the case of particles of Cu in NaCl, he obtained precise agreement between theory and experiment. For yellow salts, however, he discovered two maxima, one at ≈ 4620 Å and another at ≈ 7000 Å. The source of the second maximum remained unexplained at that time. But it can be easily understood from the Mie Theory results.

The size dependent resonance, based on certain approximations to the Mie coefficients a_1, a_2 and b_1, has been considered by van de Hulst (1964). His results for the index of refraction $m = 0 - i\sqrt{2}$ show an unusual resonance such that the extinction efficiency is positive definite of value $4 \cdot 25$ for $x = 0$; here $x = 2\pi a/\lambda$, a being the radius of the scatterer and λ the wavelength of the incident electromagnetic radiation. Besides, the extinction efficiency decreases monotonically with increasing x in the range $0 < x \lesssim 1$.

In what follows some new results on the size dependent resonances, emerging from the exact calculations based on Mie Theory [Mie (1908); van de Hulst (1957)] of scattering by smooth spheres, have been considered. The choice of size to wavelength parameter, x, is arbitrary. The complex index of refraction, denoted by $m = m' - im''$ is such that $m'' \gg m'$ and $m' \ll 1$; it is mainly representative of metallic particles.

2. MIE THEORY RESULTS ON SIZE DEPENDENT RESONANCES

In Fig. 1, the curve *a* is for the index of refraction $m = 0{\cdot}0 - i\sqrt{2}$, a hypothetical case corresponding to the singularity in the Mie coefficient a_1. The extinction efficiency, $Q_{ext.}$, has a fairly flat profile with a slow decrease for increasing x in the range $0{\cdot}1 \lesssim x \lesssim 0{\cdot}75$. This portion of the curve agrees with van de Hulst's results (1964). However, for $x > 0{\cdot}75$, Mie Theory predicts strong resonances with the first maximum in $Q_{ext.}$ at $x \approx 0{\cdot}98$. Furthermore, in the region $0 < x \lesssim 0{\cdot}1$, it has been found from a detailed calculation that $Q_{ext.}$ decreases monotonically with x and

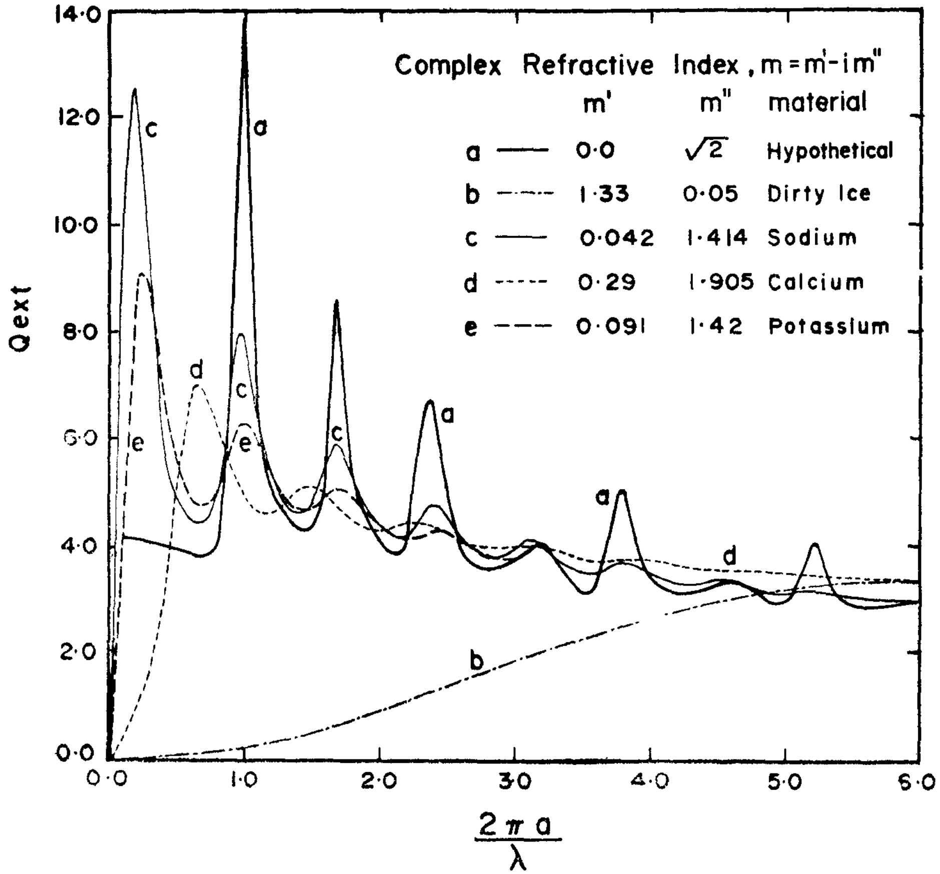

FIG. 1. Theoretical extinction efficiencies ($Q_{ext.}$) for spheres of various materials. Abscissa represents size-to-wavelength parameter, $x = 2\pi a/\lambda$.

tends to zero as x goes to zero. This feature is indeed physically understandable because it reflects the absence of any intervening scattering agency between the source and the receiver.

The curves c, d and e in Fig. 1 are plotted for the particles of sodium, calcium and potassium respectively. The relevant optical constants in the visual wavelengths for vacuum deposited Na and K and evaporated Ca, taken from the measurements by Ives and Briggs (1936, 1937) and Bolle (1955) respectively, are listed in Table I. Some hypothetical cases are given for illustration. Table I also includes the location (x_0) and magnitude of the first maximum in extinction. In Fig. 1, curiously enough, even before the first major resonanee for dieletric material (curve b) has been complete, there appear several resonance peaks for real materials like Na, Ca and K. The peaks recur at nearly regular interval, $\triangle x \approx 0{\cdot}7$, for Na, Ca and K.

TABLE I

Extinction efficiency at the first maximum

No.	Material	Index of refraction		x_0	$Q_{ext.}$ at the 1st Max.
		m' re-part	m'' Im-part		
1	*	0·0	1·414	0·98	12·58
2	Sodium	0·042	1·414	0·17	12·47
3	Calcium	0·29	1·905	0·66	7·01
4	Potassium	0·091	1·42	0·55	9·05
5	*	0·003535	1·41775	0·07	76·80
6	*	0·003536	1·41450	0·05	46·77
7	Dirty Ice	1·33	0·05	6·4	3·31
8	*	0·0	1·3	0·7	7·735
9	*	0·0	1·4	1·0	10·13
10	*	0·0	1·5	0·3	55·85

* = Hypothetical. Case 1 corresponds to singularity at $m^2 = -2$ in the approximation to Mie coefficient, a_1.

The real parts of the complex Mie coefficients a_n and b_n for $n \leqslant 3$ and $x \leqslant 1$ in the case of sodium are given in Table II in order to show the relative contributions to the total extinction. The values smaller than 10^{-8} have been neglected. The scattering efficiency ($Q_{sca.}$) and albedo have also been included in Table II. It seems that for nearly pure imaginary m and $x < 1$, the contribution from the electric dipole mode (coefficient a_1) is predominant upto a certain value of x beyond which the electric quadrupole mode (coefficient a_2) is no longer negligible. In fact, the latter may contribute to the extinction even more than the former.

It may be mentioned that for strictly pure imaginary index of refraction ($m' = 0$) the albedo is unity, independent of x. This is also the case for pure real m ($m'' = 0$) and for $m = \infty$. The maximum value of albedo in Table II for sodium is 0·7681 and occurs almost simultaneously with the minimum in the $Q_{ext.}$ of $\approx 4 \cdot 4$ at $x \approx 0 \cdot 69$. No such correlation is found near the maximum of $Q_{ext.}$ at $x \approx 0 \cdot 17$. This feature does not occur for other indices of refraction studied here.

3. DISCUSSION

The present results can have an important bearing on some astrophysical problems. For instance, they can contribute significantly to the structures in the observed interstellar extinction curve [Johnson and Borgman (1963); Boggess and Borgman (1964); Nandy (1964 *a*, *b*, 1965, 1966); Stecher (1969); Bless and Savage (1970)]. They might also have some relevance in explaining the unidentified interstellar diffuse absorption bands. Some results pertaining to unidentified interstellar diffuse band at 4430 Å have been summarized below.

The case 5 in Table I corresponds to $m^2 + 2 = -(3\gamma/\omega_0)\, i$, where γ = the damping constant and ω_0 = the resonance frequency for the diffuse band near 4430 Å. The value of $\omega_0/\gamma = 300$, suggested by Unsöld (1964), is an average estimate for the centre of the resonance peak. Figure 2 shows a remarkable resonance for this case; the first maximum of $Q_{ext.} \approx 76 \cdot 8$ is at $x \approx 0 \cdot 07$. If one assumes that the first maximum for $m = 0 \cdot 003535 - i\, 1 \cdot 41775$ occurs at $\lambda = 4430$ Å, then one can make rough estimate of the size (a_0) of the absorbing particles which we shall call granules. This turns out to be $a_0 \approx 50$ Å. Similarly for Na, $a_0 \approx 120$ Å. It is likely that materials with similar properties exist in interstellar space. We can roughly estimate the mass requirement for such particles by assuming (1) the model of dirty ice grains with number density and radius equal to 10^{-13} cm^{-3} and

TABLE II

Mie coefficients for sodium spheres, $m = 0{\cdot}042 - 1{\cdot}414\,i$

x	Re a_1	Re b_1	Re a_2	Re b_2	Re a_3	Re b_3	$Q_{ext.}$	$Q_{sca.}$	Albedo
0·01	$1{\cdot}684\times10^{-5}$	0·0	0·0	0·0	0·0	0·0	1·009	$1{\cdot}700\times10^{-5}$	$1{\cdot}683\times10^{-5}$
0·1	$1{\cdot}601\times10^{-2}$	$2{\cdot}620\times10^{-8}$	$3{\cdot}839\times10^{-7}$	0·0	0·0	0·0	9·610	0·1588	$1{\cdot}652\times10^{-2}$
0·2	$8{\cdot}037\times10^{-2}$	$8{\cdot}211\times10^{-7}$	$1{\cdot}287\times10^{-5}$	0·0	0·0	0·0	12·06	1·421	0·1178
0·3	0·1272	$6{\cdot}034\times10^{-6}$	$1{\cdot}064\times10^{-4}$	$1{\cdot}59\ \times10^{-8}$	$1{\cdot}170\times10^{-7}$	0·0	8·496	2·618	0·3082
0·4	0·1627	$2{\cdot}446\times10^{-5}$	$5{\cdot}133\times10^{-4}$	$1{\cdot}163\times10^{-7}$	$9{\cdot}041\times10^{-7}$	0·0	6·135	3·115	0·5078
0·5	0·2060	$7{\cdot}213\times10^{-5}$	$1{\cdot}92\ \times10^{-3}$	$5{\cdot}361\times10^{-7}$	$4{\cdot}493\times10^{-6}$	0·0	5·024	3·299	0·6565
0·6	0·2603	$1{\cdot}767\times10^{-4}$	$6{\cdot}464\times10^{-3}$	$1{\cdot}843\times10^{-6}$	$1{\cdot}699\times10^{-5}$	$1{\cdot}103\times10^{-8}$	4·522	3·360	0·7429
0·7	0·3243	$3{\cdot}889\times10^{-4}$	$2{\cdot}165\times10^{-2}$	$5{\cdot}173\times10^{-6}$	$5{\cdot}359\times10^{-5}$	$4{\cdot}280\times10^{-8}$	4·420	3·393	0·7676
0·8	0·3958	$8{\cdot}046\times10^{-4}$	$7{\cdot}699\times10^{-2}$	$1{\cdot}250\times10^{-5}$	$1{\cdot}491\times10^{-4}$	$1{\cdot}371\times10^{-7}$	4·925	3·556	0·7220
0·9	0·4723	$1{\cdot}598\times10^{-3}$	0·2600	$2{\cdot}699\times10^{-5}$	$3{\cdot}809\times10^{-4}$	$3{\cdot}797\times10^{-7}$	6·728	4·304	0·6398
1·0	0·5513	$3{\cdot}061\times10^{-3}$	0·4584	$5{\cdot}351\times10^{-5}$	$9{\cdot}224\times10^{-4}$	$9{\cdot}353\times10^{-7}$	7·924	5·223	0·6591

$0{\cdot}2\,\mu$ respectively and (2) the ratio of the λ 4430 extinction to the total interstellar extinction to be 0·05 [Wampler (1967); Greenberg (1968)];

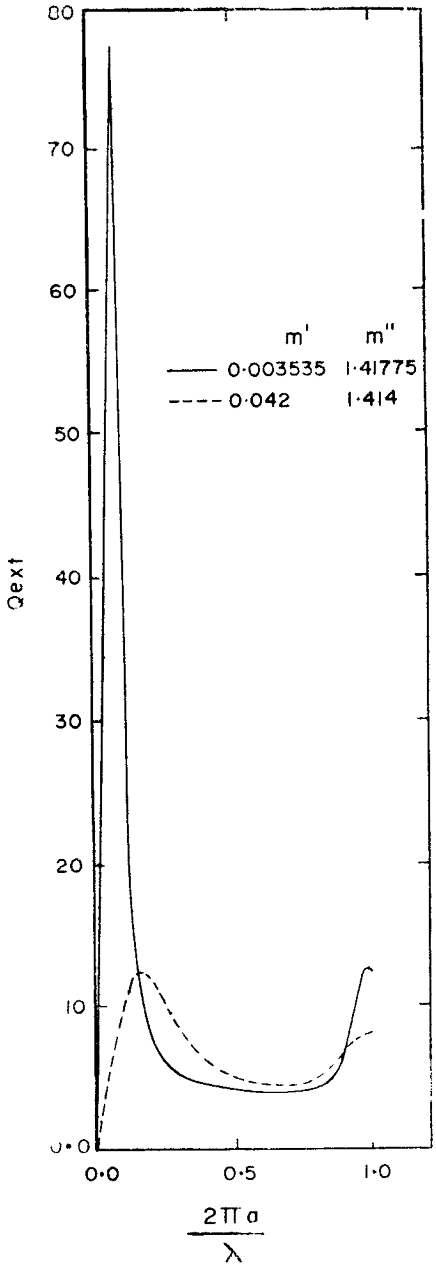

FIG. 2. Comparison of the theoretical extinction curves for sodium and the hypothetical case corresponding to absorption near the centre of the λ 4430 band.

Then with the above sizes of the granules it can be shown that the mass requirements are $\approx 2 \times 10^{-31}$ and $\approx 3 \times 10^{-30}$ gm/cm^{-3} for case 5 and Na respectively. The corresponding ratios of the granule mass to the grain mass are $\approx 3 \times 10^{-5}$ and $\approx 4 \times 10^{-4}$ respectively. In particular, the mass requirement on Na granules can certainly be satisfied because it constitutes only a few per cent of the total mass of sodium thought to exist in interstellar space.

According to Code (1970), the OAO-2 observations of the interstellar dark clouds appear bright shortward of 1600 Å. For example, in the Horsehead nebula, for $\lambda < 1600$ Å, the dark cloud appears brighter than the illuminated part of the nebula. It has been suggested by Code that perhaps the aledo of the scattering particles in this region of the spectrum is very high. In fact, if one considers a fraction of the grains composed of the materials similar to those studied here or a small contamination of such materials on the surface of the grains, then one can account for high scattering because of the resultant predominant imaginary part of the complex index of refraction.

4. CONCLUSION

It would be very interesting to examine the possibility of interstellar grains consisting of materials similar to those studied here. Even a small fraction of such grains, by virtue of their resonance characteristics, may contribute significantly to the structures in the wavelength dependence of the interstellar extinction and polarization. Because of their relatively high absorptivity they can be equally important in the considerations of the infrared emission from the circumstellar envelopes and other celestial sources.

5. ACKNOWLEDGEMENT

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