ON THE THERMAL BENDING AND TWISTING OF THIN SKEWED PLATES

By V. PUTTARAJU

(*Department of Mathematics, Bangalore University, Bangalore, India*)

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ABSTRACT

A solution for pure bending and twisting of thin skewed plates of uniform thickness under a constant temperature moment is obtained by using the basic equations for thermal bending. The corresponding expressions for classical case are obtained as particular cases.

1. INTRODUCTION

In a recent paper Reissner (1) has obtained a solution for the problem of pure bending and twisting of thin skewed plates of uniform thickness. Assuming the results for rectangular plates of uniform thickness, he has extended the problem to skewed plates. In this paper we extend his results to thermal bending and twisting under a constant temperature moment. We obtain the influence of the angle skew on the torque-twist relation and the pure thermal bending associated with twisting deformation. Our problem reduces to that of Reissner if the temperature moment is neglected.

2. BASIC EQUATIONS AND BOUNDARY CONDITIONS

Let xy plane coincide with the undeflected middle surface of the plate. The equation for deflection is given by

$$\nabla^4 w + (1 + \nu)\, \alpha\, \nabla^2 m_\theta = 0 \tag{1}$$

where w is the deflection, α is the deflection angle and m_θ is the temperature moment.

If the temperature moment be constant, say M, then (1) takes the form

$$\nabla^4 w = 0. \tag{2}$$

The expressions for stress couples M_x, M_y and M_{xy} are

$$M_x = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} + (1+\nu)\,\alpha M\right]$$

$$M_y = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} + (1+\nu)\,\alpha M\right]$$

$$M_{xy} = -D\left[(1-\nu)\frac{\delta^2 w}{\partial x \partial y} + (1+\nu)\,\alpha M\right] \qquad (3)$$

where D is the flexural rigidity of the plate.

We consider the plate bounded by the straight lines $x = \pm l$ and $y = \pm \frac{1}{2} c - x \tan \phi$, where ϕ is the angle of skew, $2l$ is the span and c is the chord of the plate.

The bending and twisting moments M_n and M_{nt} acting along the edges $y = \pm \frac{1}{2} c - x \tan \phi$ follow by the transformations (1)

$$\left.\begin{aligned} M_n &= M_y \cos^2\phi + M_x \sin^2\phi + 2\,M_{xy}\cos\phi\sin\phi \\ M_{nt} &= (M_x - M_y)\cos\phi\sin\phi + M_{xy}(\cos^2\phi - \sin^2\phi) \end{aligned}\right\}. \qquad (4)$$

We also have Kelvin's and Tait's result (1) that there occur concentrated forces P at the corners of the plate given by

$$P = (M_{xy} + M_{nt})_{\text{corner}}. \qquad (5)$$

The boundary conditions for pure bending are

$$\left.\begin{aligned} c\,M_x &= m \quad \text{at} \quad x = \pm l \\ M_n &= 0 \quad \text{at} \quad y = \pm \tfrac{1}{2} c - x \tan\phi \end{aligned}\right\} \qquad (6)$$

and

$$M_{xy} + M_{nt} = 0 \qquad (7)$$

bwheroe m is the applied bending mment. [The equation (7) has be notained by equating the corner forces P to zero].

The boundary conditions for pure twisting are

$$\left.\begin{aligned} M_x &= 0 \quad \text{at} \quad x = \pm l \\ M_n &= 0 \quad \text{at} \quad y = \pm \tfrac{1}{2} c - x \tan\phi \end{aligned}\right\} \qquad (8)$$

and

$$T = cP \tag{9}$$

where T is the applied torque.

3. Case of Pure Bending

Suppose we take the deflection function in the form (1)

$$w = \tfrac{1}{2} A x^2 + \tfrac{1}{2} B y^2 + C xy \tag{10}$$

where A, B and C are suitable constants to be determined by the boundary conditions. Calculating

$$\frac{\partial^2 w}{\partial x^2}, \frac{\partial^2 w}{\partial y^2} \text{ and } \frac{\partial^2 w}{\partial x \partial y}$$

and substituting in the expressions (3) we get

$$\begin{aligned} M_x &= - D [A + \nu B + (1 + \nu) \alpha M] \\ M_y &= - D [B + \nu A + (1 + \nu) \alpha M] \\ M_{xy} &= - D [(1 - \nu) c + (1 + \nu) \alpha M]. \end{aligned} \tag{11}$$

The bending and twisting moments given by (4) then assume the form

$$\begin{aligned} M_n = &- D [(\nu \cos^2 \phi + \sin^2 \phi) A + (\cos^2 \phi + \nu \sin^2 \phi) B \\ &+ 2 (1 - \nu) \cos \phi \sin \phi C + (1 + \nu) \alpha M (\cos \phi + \sin \phi)^2] \\ M_{nt} = &- D [(1 - \nu) \cos \phi \sin \phi A - (1 - \nu) \cos \phi \sin \phi B \\ &+ (1 - \nu) (\cos^2 \phi - \sin^2 \phi) C + (1 + \nu) \\ &\times \alpha M (\cos^2 \phi - \sin^2 \phi)] \end{aligned} \tag{12}$$

Substituting for M_x, M_n, M_{xy} and M_{nt} from (11) and (12) in (6) and (7) we get

$$- c D [A + \nu B + (1 + \nu) \alpha M] = m \tag{i}$$

$$\begin{aligned} (\nu + \tan^2 \phi) A + (1 + \nu \tan^2 \phi) B + 2 (1 - \nu) \tan \phi C \\ + (1 + \nu) \alpha M (1 + \tan \phi)^2 = 0 \end{aligned} \tag{ii}$$

$$\begin{aligned} (1 - \nu) \tan \phi - (1 - \nu) \tan \phi B + 2 (1 - \nu) C \\ + (1 + \nu) \alpha M = 0 \end{aligned} \tag{iii}$$

solving (i) (ii) and (iii) we get

$$A = \frac{-m - (1-\nu^2)\,\alpha\, McD}{cD\,(1-\nu^2)}$$

$$B = \frac{\nu\, m - (1-\nu^2)\,\alpha\, McD}{cD\,(1-\nu^2)}$$

$$C = \frac{m \tan\phi - (1+\nu)\,\alpha\, McD}{2\, cD\,(1-\nu)}\,.$$

Substituting these values of A, B and C in (10) we get

$$w = -\frac{m}{2\,cD\,(1-\nu^2)}\,[x^2 - \nu\, y^2 - (1+\nu)\tan\phi\; xy]$$

$$-\frac{\alpha M}{2}\left[x^2 - y^2 + \frac{1+\nu}{1-\nu}\, xy\right] \tag{13}$$

If we introduce a new chord wise co-ordinate η counted from the centre line $y = -x\tan\phi$ of the plate by setting

$$y = \eta - x\tan\phi \tag{14}$$

then the equation (13) assumes the form

$$w = -\frac{m}{cD\,(1-\nu^2)}\,[(1+\tan^2\phi)\, x^2 - (1-\nu)\tan\phi\; x\eta - \nu\eta^2)]$$

$$-\frac{\alpha M}{2}\left[\left(1 - \tan^2\phi - \frac{1+\nu}{1-\nu}\tan\phi\right) x^2\right.$$

$$\left. + \left(\frac{1+\nu}{1-\nu} + 2\tan\phi\right) x\eta - \eta^2\right]. \tag{15}$$

We readily verify from (15) that the skewed plate has a smaller bending stiffness than the unskewed plate and that the applied bending moment produces a torsional deformation.

4. Case of Pure Twisting

Substituting for M_x M_y M_{xy} and M_{nt} from (11) and (12) in (8) and (9) we get

$$A + \nu B + (1+\nu)\,\alpha M = 0 \tag{iv}$$

$$(\nu + \tan^2 \phi) A + (1 + \nu \tan^2 \phi) B + 2 (1 - \nu) \tan \phi\, C + (1 + \nu) \alpha M (1 + \tan \phi)^2 = 0 \qquad (v)$$

$$- cD (1 - \nu) \tan \phi\, A + cD (1 - \nu) \tan \phi\, B - 2\, cD (1 - \nu)\, C - 2 (1 + \nu) \alpha M cD = T (1 + \tan^2 \phi) \qquad (vi)$$

Solving (iv), (v) and (vi) we get

$$A = \frac{- \nu T \tan \phi - (1 - \nu^2) \alpha McD}{cD (1 - \nu^2)}$$

$$B = \frac{T \tan \phi - (1 - \nu^2) \alpha McD}{cD (1 - \nu^2)}$$

$$C = \frac{- T - 2 (1 + \nu) \alpha McD}{2\, cD (1 - \nu)}.$$

Substituting these values of A, B and C in (10) we get

$$w = - \frac{T}{2\, cD (1 - \nu)} \left[xy - \frac{\tan \phi}{1 + \nu} (y^2 - \nu x^2) \right] + \frac{\alpha M}{2 (1 - \nu)} [2 (1 + \nu) xy - (1 - \nu) (x^2 + y^2)]. \qquad (16)$$

In terms of the chordwise variable η defined by (14), (16) assumes the form

$$w = - \frac{T}{2cD (1 - \nu)} \left[\left(1 + \frac{2 \tan^2 \phi}{1 + \nu}\right) x\eta - \frac{\tan \phi}{1 + \nu} \eta^2 - \frac{(1 + \tan^2 \phi) \tan \phi}{1 + \nu} x^2 \right] + \frac{\alpha M}{2} \left[\left(\frac{1 + \nu}{1 - \nu} + \tan \phi\right) x\eta - \eta^2 - \left\{ \frac{2 (1 + \nu)}{1 - \nu} \tan \phi + (1 + \tan^2 \phi) \right\} x^2 \right]. \qquad (17)$$

Equation (17) indicates that the skewed plate has a smaller torsional rigidity than the unskewed plate and the way in which the torsional rigidity varies with the angle of skew ϕ.

If we define the effective angle of twist θ per unit length by (1)

$$\theta = \frac{(w)_{\eta = c/2} - (w)_{\eta = -c/2}}{cx} \qquad (18)$$

we get for the effective angle of twist the expression

$$\theta = -\frac{T}{2cD(1-\nu)}\left(1+\frac{2\tan^2\phi}{1+\nu}\right)+\frac{\alpha M}{2}\left(\frac{1+\nu}{1-\nu}+\tan\phi\right) \tag{19}$$

where we have used (17) and (18)

If the temperature moment is neglected, the expressions (13), 15), (16), (17) and (19) reduce to those obtained by Reissner for the classical case.

5. A Particular Case

If the angle of skew ϕ tends to zero, the skewed plate reduces to a rectangular plate whose edges are $x = \pm\, l$ and $y = \pm\, \frac{1}{2}\, c$.

Then the expressions for the thermal bending, twisting and angle of twist respectively take the form

$$\left.\begin{aligned} w &= -\frac{m}{2\,cD\,(1-\nu^2)}\,(x^2-\nu\, y^2)-\frac{\alpha\, M}{2} \\ &\quad\times\left(x^2-y^2+\frac{1+\nu}{1-\nu}\,xy\right) \\ w &= -\frac{T}{cD\,(1-\nu)}\,xy+\frac{\alpha\, M}{2\,(1-\nu)} \\ &\quad\times[2\,(1+\nu)\,xy-(1-\nu)\,(x^2+y^2)] \\ \text{and}\quad & \\ \theta &= -\frac{T}{2\,cD\,(1-\nu)}+\frac{\alpha\, M}{2}\frac{1+\nu}{1-\nu} \end{aligned}\right\} . \tag{20}$$

In the absence of the temperature moment, (20) reduce to

$$\left.\begin{aligned} w &= -\frac{m}{2\,cD\,(1-\nu^2)} \\ w &= -\frac{T}{cD\,(1-\nu)} \\ \text{and}\quad & \\ \theta &= -\frac{T}{2\,cD\,(1-\nu)} \end{aligned}\right\} \tag{21}$$

which are valid for the classical case.

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REFERENCES

1. Reissner, E. .. *Quarterly of Applied Mathematics*, 1952, **10**, 395–97.
2. Parkus, H. .. *Thermo-elasticity*, Ginn/Blaisdel, 1968.