ON THE SPECTRAL STUDY OF SUNSPOT NUMBER AND SOLAR RADIO FLUX BETWEEN 600 AND 9400 MHz

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ABSTRACT

The relationship between the solar activity, as represented by sunspot number and the slowly varying component of solar radio emission at frequencies from 600 to 9400 MHz, has been studied for high solar activity (1957-58), declining phase of moderate solar activity (1961-62) and ascending phase of moderate solar activity (1965-66). Regression and auto-spectrum analyses show that the association of the slowly varying radio emissions with sunspot activity is best around 3750 MHz for high solar activity period, around 2800 MHz for moderate activity during the declining phase and in between 2800 and 3750 MHz for ascending phase of activity. The coherence between radio flux at frequencies from 600 MHz to 9400 MHz was high and free from large fluctuations up to wavelengths as short as 7 days in the year 1961-62. At 9400 MHz, however, it is high and steady for wavelengths in excess of about 15 days. At solar synodic rotation period the phase difference between the sunspot number and the slowly varying component shows little change with the wavelength of the radio emission.

1. INTRODUCTION

The solar regions of origin of the slowly varying component of radiation in the centimetre and decimetre wavelengths appear in the vicinity of facular plages and sunspots. They persist over several solar rotations and the radiation which is of thermal origin shows close association with sunspots and plages. Smerd (1969) showed that the correlation coefficient between the daily flux in the centimetre and decimetre wavelengths and daily relative sunspot number was high for sunspot number greater than 50. From auto and cross-correlations of flux and sunspot number, Smerd (1964) inferred that at short decimetre wavelengths the flux is an excellent index of long-lived slowly varying solar activity and the longevity...
decreases with wavelength in the 30 to 3 cm region. In the region of
10 cm the longevity is same as that of the sunspots. From correlation
analysis for a period of high solar activity, Das Gupta and Basu (1963)
observed that the rate of increase in the flux with increase in sunspot num-
ber was largest for radiation in the region of about 4000 MHz.

In the present investigation the relationship between sunspot number
and slowly varying component of the solar radiation at the frequencies
600 to 9400 MHz has been examined by cross-spectral method for high
and moderate solar activity periods. This technique makes it possible to
ascertain the relationship between periodic components of the two time
series. The method computes the coherence, which is the degree of associa-
tion between the time series, as a function of frequency. It also computes
the relative displacement of the series in terms of phase difference also
as a function of frequency.

2. DATA AND ANALYSES

The period 1957-58, for which the average relative sunspot number
(Rz) was 187, has been considered as the high solar activity period.
1961-62 (average Rz = 45) and 1965-66 (average Rz = 30) are considered
as declining and ascending phases of moderate solar activity periods
respectively. For the homogeneity of the data, the daily flux values in
units of $10^{-22}$ W m$^{-2}$ (C/S)$^{-1}$ at each of the frequencies 600, 1000, 2000,
2800, 3750 and 9400 MHz have been taken from the same station for the
above three periods. The flux data used for the frequency 600 MHz is
from that of Ucc Observatory (Observatoire Royal de Belgique, Uccle,
Belgium) and for 2800 MHz is from Ott station (National Research Council,
Ottawa, Canada). The flux data for the frequencies 1000, 2000, 3750 and
9400 MHz has been obtained from NAG (Research Institute of Atmos-
pheric, Nagoya University, Toyokawa, Japan). Both daily flux data and
the daily mean Rz have been taken from the Quarterly Bulletin of Solar
Activity. The analysis is confined to the period 1957 July to 1958 December
only for the flux values at the frequencies 600, 1000 and 2000 MHz during
high solar activity period due to the non-availability of the data prior to
July 1957.

Linear correlation coefficients between the daily flux values and the
daily mean Rz are determined. Auto and cross-spectra are computed
between flux and Rz for each of the above frequencies, using the standard
procedure as outlined by Panofsky and Brier (1958). The maximum lag
used was 140 for data covering two years (730 data points). It was 100
when the data were for one and half years period (549 data points). This is the limit for reasonable stability of the estimates. The resolution with this lag is also sufficient to resolve the solar synodic line and its harmonics. The phase and coherence as a function of frequency are also computed. This analysis is repeated for the three periods mentioned earlier.

3. DISCUSSION

3.1. Linear correlations.—The solar radio emission at a frequency can be classified as (1) a basic component of thermal origin, (2) highly variable component of non-thermal origin which is attributed to disturbed emission and (3) slowly varying radiation only at centimetre and lower decimetre wavelengths. The slowly varying component shows a close association with sunspot number and exhibits the 27-day variation. To ascertain the degree of association between the flux at each of the frequencies and the $R_z$, linear correlations are computed and shown in Table I. The correlation coefficients are relatively higher at the frequencies 2800 and 3750 MHz for the period of high solar activity, at 2000, 2800 and 3750 MHz for declining phase of solar activity and at 1000, 2000, 2800 and 3750 MHz for the ascending phase of solar activity. Das Gupta and Basu (1963) observed that the correlation was maximum at the frequency near about 4000 MHz between the daily flux of radio emission and $R_z$ for the period 1957 July to 1958 December. They compiled an index of variability for all the frequencies and showed that both the index of variability and the slope of the regression line was maximum at 4000 MHz. From the power $P$, corresponding to the solar synodic rotation period (about 27 days) obtained in auto spectrum analysis, the amplitude $A$, which is a measure of the average intensity of the flux in 27-day cycle, is calculated by the relationship $A = 2 \sqrt{P}$. The amplitude of flux at each frequency and the slope of the regression line for each of the three periods, plotted against frequency, are shown in Fig. 1. It is noticed that both, the amplitude and slope of the regression line, are maximum at 3750 MHz for the high solar activity period confirming the results of Das Gupta and Basu (1963). For the declining phase of moderate activity both are maximum at the frequency 2800 MHz, but for ascending phase of solar activity the slope is maximum at the frequency 2800 MHz and the amplitude at the frequency 3750 MHz. Thus there is always a relationship between solar radio emission in the 3 to 50 cm wavelength and $R_z$ but the association appears to vary from one phase of the solar cycle to another. Anderson (1965) has also shown that the 10.7 cm flux from the Sun generally accepted
as a good index of the solar extreme ultraviolet radiation (EUV), could not be a precise index of EUV in phases of solar activity other than those of high activity.

**Table 1**

*Correlation coefficients between $Rz$ and the daily flux at different frequencies ($F$) and associated parameters*

<table>
<thead>
<tr>
<th>Frequency MHz</th>
<th>Means with standard errors Flux $10^{-22}$ w m$^{-2}$ (C/S)$^{-1}$</th>
<th>Correlation coefficient</th>
<th>Regression equation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Rz$</td>
<td>$F$</td>
<td></td>
</tr>
<tr>
<td>1957-58</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600*</td>
<td>195±2</td>
<td>58±0·9</td>
<td>-0·04</td>
</tr>
<tr>
<td>1000*</td>
<td>195±2</td>
<td>128±0·6</td>
<td>0·74</td>
</tr>
<tr>
<td>2000*</td>
<td>195±2</td>
<td>178±1·2</td>
<td>0·84</td>
</tr>
<tr>
<td>2800</td>
<td>187±2</td>
<td>231±1·4</td>
<td>0·87</td>
</tr>
<tr>
<td>3750</td>
<td>187±2</td>
<td>227±1·4</td>
<td>0·85</td>
</tr>
<tr>
<td>9400</td>
<td>187±2</td>
<td>334±1·1</td>
<td>0·74</td>
</tr>
<tr>
<td>1961-62</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>46±1</td>
<td>54±0·3</td>
<td>0·61</td>
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<tr>
<td>1000</td>
<td>46±1</td>
<td>50±0·3</td>
<td>0·74</td>
</tr>
<tr>
<td>2000</td>
<td>46±1</td>
<td>71±0·5</td>
<td>0·85</td>
</tr>
<tr>
<td>2800</td>
<td>46±1</td>
<td>97±0·6</td>
<td>0·95</td>
</tr>
<tr>
<td>3750</td>
<td>46±1</td>
<td>107±0·6</td>
<td>0·85</td>
</tr>
<tr>
<td>9400</td>
<td>46±1</td>
<td>255±0·4</td>
<td>0·66</td>
</tr>
<tr>
<td>1965-66</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>31±1</td>
<td>62±0·6</td>
<td>0·74</td>
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<tr>
<td>1000</td>
<td>31±1</td>
<td>48±0·4</td>
<td>0·86</td>
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<td>2000</td>
<td>31±1</td>
<td>64±0·5</td>
<td>0·89</td>
</tr>
<tr>
<td>2800</td>
<td>31±1</td>
<td>89±0·7</td>
<td>0·90</td>
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<tr>
<td>3750</td>
<td>31±1</td>
<td>99±0·6</td>
<td>0·89</td>
</tr>
<tr>
<td>9400</td>
<td>31±1</td>
<td>255±0·4</td>
<td>0·77</td>
</tr>
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</table>

For the Peri 1957 July to 1958.
For moderate solar activity period high correlations are observed in the band of frequencies 1000 to 3750 MHz. This bandwidth becomes narrower as the solar activity increases.

Allen (1957) and Covington and Harvey (1960) have found that the quiet-sun radio flux should have a significant variation with solar activity.
by a factor of about 2 for the minimum to maximum variation in 10 cm 
flux. The intercept in the regression equation, given in Table I, can be 
considered as the quiet-sun component of the flux at that frequency. 
For the flux at 2800 MHz, the ratio of the quiet-sun component for 1957- 
58 to that of 1965-66 is 1.57. This ratio for the flux at the frequency 
9400 MHz is only 1.03. Thus for lower wavelengths the increase in the 
magnitude of variation in the quiet-sun component from minimum to 
maximum is very insignificant.

3.2. Auto Spectrum.—The primary purpose of computing the auto-
spectra for each of the wavelengths is to resolve the 27-day line. The 
spectra are not presented here but some of the salient features are 
pointed out below:

(a) The auto-spectra for the period 1957-58 show a steep fall of 
continuum power between wavelengths of 27-day and 7.7 days. The solar 
synodic rotation line appears as a small increase over the continuum. For 
wavelengths shorter than about 7.7 days, the power levels decrease slowly 
and the random fluctuations dominate this part of the spectrum.

(b) The 27-day line appears in all the spectra but it is most prominent 
in 1961-62, a period of moderate activity during the descending phase of 
solar activity.

(c) During the ascending phase of solar activity, 1965-66, a sharp 
dip in power is noticed in the spectrum of Rz corresponding to a wave-
length of 8 days. This reduction in power also appears in spectra of 
radio flux but is most marked at frequencies 3750 and 2800 MHz. In 
1957-58 also a similar reduction occurs at all frequencies at a wavelength 
of 7 days and it is most prominent at 2000 MHz.

(d) There are large random variations at 600 MHz, except during 
high activity period, 1957-58, in the power as compared to the spectra at 
other wavelengths.

(e) Harmonics of 27-day signal appear in all the spectra.

The amplitudes corresponding to 27-day period are calculated from 
the peak powers in auto-spectra. These amplitudes are given in Table II. 
It is noticed from the table that there is a systematic variation of the 
amplitude with a maximum between 2800 and 3750 MHz in all phases of 
solar activity. These results are in agreement with those of Das Gupta 
and Basu (1963) for high activity period and of Nicolet (1963). Varia-
tions of amplitude similar to high solar activity period are also observed for both declining and ascending phases of solar activity. Nicolet (1963), however, showed that the amplitude of 27-day component at 15 cm was higher than that at 8 cm for low solar activity period (1961–62).

3.3. Coherence and phase.—Coherence is a measure of degree of association between two time series as a function of frequency. It is ana-

**Table II**

*Amplitude, phase and coherence corresponding to solar synodic rotation period*

<table>
<thead>
<tr>
<th>Frequency MHz</th>
<th>Harmonic No.</th>
<th>Period (days)</th>
<th>Peak power (Flux unit)$^2$</th>
<th>Amplitude (Flux unit)</th>
<th>Phase lead of flux over Rz (day-1)</th>
<th>Coherence</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>7</td>
<td>28·6</td>
<td>4·382</td>
<td>4·187</td>
<td>5·119</td>
<td>0·582</td>
</tr>
<tr>
<td>1000</td>
<td>7</td>
<td>28·6</td>
<td>16·226</td>
<td>8·056</td>
<td>0·508</td>
<td>0·822</td>
</tr>
<tr>
<td>2000</td>
<td>7</td>
<td>28·6</td>
<td>57·428</td>
<td>15·169</td>
<td>0·175</td>
<td>0·927</td>
</tr>
<tr>
<td>2800</td>
<td>10</td>
<td>28·0</td>
<td>65·941</td>
<td>16·240</td>
<td>0·295</td>
<td>0·861</td>
</tr>
<tr>
<td>3750</td>
<td>9</td>
<td>31·1</td>
<td>66·723</td>
<td>16·330</td>
<td>0·467</td>
<td>0·916</td>
</tr>
<tr>
<td>9400</td>
<td>9</td>
<td>31·1</td>
<td>36·720</td>
<td>12·120</td>
<td>0·164</td>
<td>0·881</td>
</tr>
<tr>
<td>1957-58</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>10</td>
<td>28·0</td>
<td>8·048</td>
<td>5·674</td>
<td>1·416</td>
<td>0·951</td>
</tr>
<tr>
<td>1000</td>
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<td>28·0</td>
<td>11·731</td>
<td>6·850</td>
<td>0·583</td>
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<tr>
<td>2000</td>
<td>10</td>
<td>28·0</td>
<td>28·068</td>
<td>10·590</td>
<td>0·342</td>
<td>0·937</td>
</tr>
<tr>
<td>2800</td>
<td>10</td>
<td>28·0</td>
<td>38·395</td>
<td>12·390</td>
<td>0·591</td>
<td>0·952</td>
</tr>
<tr>
<td>3750</td>
<td>10</td>
<td>28·0</td>
<td>33·962</td>
<td>11·650</td>
<td>0·288</td>
<td>0·984</td>
</tr>
<tr>
<td>9400</td>
<td>10</td>
<td>28·0</td>
<td>11·371</td>
<td>6·744</td>
<td>0·303</td>
<td>0·965</td>
</tr>
<tr>
<td>1961-62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>600</td>
<td>11</td>
<td>25·5</td>
<td>1·147</td>
<td>2·141</td>
<td>1·551</td>
<td>0·753</td>
</tr>
<tr>
<td>1000</td>
<td>10</td>
<td>28·0</td>
<td>2·565</td>
<td>3·203</td>
<td>0·008</td>
<td>0·880</td>
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<tr>
<td>2000</td>
<td>10</td>
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<td>7·888</td>
<td>5·616</td>
<td>0·047</td>
<td>0·915</td>
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<tr>
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<td>28·0</td>
<td>12·094</td>
<td>6·955</td>
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<td>28·0</td>
<td>13·543</td>
<td>7·368</td>
<td>0·062</td>
<td>0·804</td>
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<tr>
<td>9400</td>
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<td>6·808</td>
<td>5·218</td>
<td>0·715</td>
<td>0·858</td>
</tr>
</tbody>
</table>
logous to the square of the correlation coefficient, and can vary from zero to one. The average coherence between the flux and sunspot number increased from 600 MHz to a maximum between 2000 and 3750 MHz and reduced from then onwards. A smooth parabola can be drawn through these points. The increase in the average coherence was sharp during the moderate activity period (1961–62) from 0·35 at 600 MHz to 0·79 at 2000 MHz. The highest average coherence was observed at 3750 MHz in 1957–58, at 2000 MHz during 1961–62 and at 2800 MHz during 1965–66.

The coherence is high and free from large fluctuations over wide range of frequency in the spectra. The wavelength over which the coherence is high and steady is different for different frequencies (except at 600 MHz when the coherence is marked by a large scatter). During high solar activity, high coherence is observed up to the wavelength 15·6 days for the flux at the frequency 9400 MHz and up to the wavelength 10·4 days for 2800 MHz. The other wavelengths lie within these limits. For moderate solar activity (1961–62), it is maximum up to 14·7 days wavelength for the flux at 9400 MHz and up to the wavelength 7·0 days for 2800 MHz. Similarly it varies between 25·5 and 11·2 days for 9400 MHz and 1000 MHz respectively for 1965–66. Amongst the three periods the coherence is highest during declining phase of moderate solar activity when its value is in between 0·97 and 1·00 for frequencies from 1000 to 9400 MHz. In the individual cross-spectra the highest coherence occurs at a wavelength of 28 days. The coherence decreases sharply at around wavelengths of 56–47 days for emissions at all the frequencies. This is most prominent during 1961–62.

The phase measures the displacement of one time series relative to the other as a function of frequency which can be converted to represent the lead or lag in terms of the data interval. The phase angle is also steady and nearly zero from very long to shorter wavelengths given above for high coherence at various frequencies for different phases of solar activity. The phase angle corresponding to the solar synodic line given in Table II denotes the lead of flux at that frequency over the sunspot number. It can be noticed from the table that the phase variations are very small except in case of flux at 600 MHz. The small phase differences indicate the origin of the radiation in the vicinity of facular plages and sunspots.

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