ON TWO-BODY CHANNELS IN THE INTERACTIONS OF NEGATIVE PIONS WITH PROTONS IN THE FEW GeV REGION

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Abstract

A compilation on the reactions \(\pi^- p \rightarrow \pi^- p, \pi^- p \rightarrow \rho^0 n\) and \(\pi^- p \rightarrow p^- p\) at nine laboratory momenta between 1.59 and 4.16 GeV/c is presented and certain common features of these channels brought out. Relevant basic parameters for production angular distribution are obtained and their values interpreted. It is found that the diffraction peaks do not shrink as the centre of mass energy increases.

This note presents a compilation on some aspects of certain two-body channels produced in the interactions of negative pions with hydrogen. Exclusively liquid hydrogen bubble chamber data is used in the interest of uniformity; this insures a meaningful comparison of the relevant parameters at different energies since most bubble chamber experiments use more or less similar criteria in the analysis of events. The present study covers essentially the channels involving elastic scattering and the production of rho meson in the few GeV region. We have attempted a methodical organisation of the data from experiments\(^1\)\(^-\)\(^9\) at laboratory momenta 1.59 GeV/c, 2.0 GeV/c, 2.26 GeV/c, 2.7 GeV/c, 2.75 GeV/c, 3.0 GeV/c, 3.3 GeV/c, 4.0 GeV/c and 4.16 GeV/c and investigated their systematics. The present work aims at making a phenomenological analysis of the experimental data in which model dependent specific assumptions are avoided as far as possible.

The cross-sections due to the elastic scattering and the inelastic scattering involving the production of a single pion constitute a significant proportion of the total cross-section in this energy region. A major fraction of the single pion production events consists of quasi two-body final states. The more common of these final states comprise of a pion resonant state along
with a nucleon, a nucleon isobar along with a pion or a nucleon isobar along
with a pion resonant state. The actual reactions we have studied are:

\[(1) \pi^- p \rightarrow \pi^- p; \quad (2) \pi^- p \rightarrow \rho^0 n; \quad (3) \pi^- p \rightarrow \rho^- p.\]

Table I shows the relevant cross-sections from the various experiments. It
is seen that towards the end of the energy region considered the cross-section
in each one of the channels drops rather slowly. With the limited energy
interval that we have at our disposal one is perhaps not justified in asking
whether or not the cross-sections tend towards an asymptotic limit.

**Table I**

*Partial cross-section for two prong production in \(\pi^- p\) interactions*

<table>
<thead>
<tr>
<th>Channel</th>
<th>Cross-section, mb</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\pi^- p \rightarrow \pi^- p)</td>
<td>1.59</td>
</tr>
<tr>
<td>(\pi^- p \rightarrow \rho^0 n)</td>
<td>0.05</td>
</tr>
<tr>
<td>(\pi^- p \rightarrow \rho^- p)</td>
<td>1.0</td>
</tr>
<tr>
<td>(\pi^- p \rightarrow \pi^- n)</td>
<td>0.01</td>
</tr>
</tbody>
</table>

A common feature of all these two-body reactions is their strong peri-
pheral nature manifested particularly in the sharp forward peaking of the
scattering angular distribution. It is instructive to plot the elastic differ-
ential cross-section on a semi-logarithmic scale as a function of \(\cos \theta\) where
\(\theta\) is the scattering angle of the pion in the centre of mass system. The
elastic scattering differential cross-section exhibits a noticeable second maxi-
mum in all the experiments, while some of the experiments also suggest
the presence of another maximum in the backward hemisphere. The posi-
tion of the second maximum is rather poorly defined in most experiments
and sometimes seems to be noticed as a mere shoulder of the fast drop in
the forward peak (as appears to be the case for the data at 4.16 GeV/c)
thus suggesting an apparent variation in the structure of the scattering angular
distribution beyond the first minimum as a function of incident momentum.
The position of secondary maximum in terms of $-t$ (the squared four momentum transferred to the nucleon) however seems to be essentially the same at different energies as seen from Table II and Fig. 1.

**Table II**

*Position of the secondary maximum in $\pi^-p$ elastic scattering angular distribution at different momenta*

<table>
<thead>
<tr>
<th>Laboratory momentum (GeV/c)</th>
<th>Position $\cos \theta$</th>
<th>Position $(-t)$ (GeV/c)$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.59</td>
<td>0.05</td>
<td>1.12</td>
</tr>
<tr>
<td>2.0</td>
<td>0.22</td>
<td>1.18</td>
</tr>
<tr>
<td>2.7</td>
<td>0.27</td>
<td>1.72</td>
</tr>
<tr>
<td>3.0</td>
<td>0.47</td>
<td>1.29</td>
</tr>
</tbody>
</table>

**Fig. 1.** Position of the secondary maximum in the elastic scattering angular distribution in terms of $(-t)$ in units of (GeV/c)$^2$ versus the laboratory momentum.

Various aspects of this obvious diffraction-like structure have been studied. The most elementary of these is to fit the forward, $|t| < 0.3$ (GeV/c)$^2$, angular distribution to a purely exponential diffraction pattern of the type:

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_{t=0} e^{-A|t|}.$$

Table III lists the values of the best fit slope $A$ at the various energies for the three different channels. In the light of the experimental errors one
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Table III

Slope A in \((GeV/c)^{-2}\) for \(d\sigma/dt = (d\sigma/dt)_{t=0} e^{-A|t|}\)

<table>
<thead>
<tr>
<th>Lab. momentum GeV/c</th>
<th>1.59</th>
<th>2.0</th>
<th>2.26</th>
<th>2.7</th>
<th>2.75</th>
<th>3.0</th>
<th>3.3</th>
<th>4.0</th>
<th>4.16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channel</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. (\pi^-p \rightarrow \pi^-p)</td>
<td>7.29</td>
<td>7.5</td>
<td>8.7</td>
<td>7.77</td>
<td>..</td>
<td>6.8</td>
<td>..</td>
<td>8.53</td>
<td>7.36</td>
</tr>
<tr>
<td></td>
<td>(\pm 0.25)</td>
<td>(\pm 0.6)</td>
<td>(\pm 0.2)</td>
<td>(\pm 0.05)</td>
<td>..</td>
<td>(\pm 0.7)</td>
<td>..</td>
<td>(\pm 0.49)</td>
<td>(\pm 0.14)</td>
</tr>
<tr>
<td>2. (\pi^-p \rightarrow \rho^0n)</td>
<td>9.1</td>
<td>7.7</td>
<td>..</td>
<td>10.26</td>
<td>9.8</td>
<td>7.4</td>
<td>9.3</td>
<td>8.7</td>
<td>..</td>
</tr>
<tr>
<td>(\rightarrow \pi^-n)</td>
<td>(\pm 0.6)</td>
<td>(\pm 0.9)</td>
<td>..</td>
<td>(\pm 0.06)</td>
<td>(\pm 0.7)</td>
<td>(\pm 0.7)</td>
<td>(\pm 0.9)</td>
<td>(\pm 0.7)</td>
<td>..</td>
</tr>
<tr>
<td>3. (\pi^-p \rightarrow \rho^-\rho)</td>
<td>6.8</td>
<td>5.8</td>
<td>..</td>
<td>9.32</td>
<td>7.6</td>
<td>5.9</td>
<td>9.1</td>
<td>8.6</td>
<td>..</td>
</tr>
<tr>
<td>(\rightarrow \pi^-n^0)</td>
<td>(\pm 9.9)</td>
<td>(\pm 9.8)</td>
<td>..</td>
<td>(\pm 0.08)</td>
<td>(\pm 0.5)</td>
<td>(\pm 0.9)</td>
<td>(\pm 1.2)</td>
<td>(\pm 0.7)</td>
<td>..</td>
</tr>
</tbody>
</table>

One can qualitatively say that the value of A for a particular channel is essentially the same at different laboratory momenta and there is not much difference in the A-values for the three channels thus implying the existence of a more or less definite interaction size. The value of \(A = 7.6 \pm 0.5\) \((GeV/c)^{-2}\) for the elastic scattering channel obtained by the least squares method as the best fit to the data discussed here is in agreement with the value \(7.4 \pm 0.2\) \((GeV/c)^{-2}\) found by a spark chamber experiment\(^{10}\) in the interval 2.3–6.0 GeV/c. We would like to observe however that A has certainly not reached the rather constant value of 9 \((GeV/c)^{-2}\) suggested by high energy experiments.\(^{11}\) Further, the A-value corresponding to channel 2 is consistently higher than that for the other two channels. This might have something to do with the peculiar nature of the exchange trajectories in case of this channel as contrasted with those for the other two channels. The question of exchange trajectories is further discussed in one of the following paragraphs.

One can compute the square of the imaginary part of the forward scattering amplitude from the optical theorem:

\[
\text{Im } f(o) = \frac{p_{eM}}{4\pi} \times \text{(total cross-section)}.
\]

These optical theorem predictions for the elastic scattering channel are compared with the experimental forward scattering cross-section in Table IV and Fig. 2. They indicate that for most of the data considered in this energy region the contribution to the scattering cross-section from the imaginary part dominates over that from the real part of the amplitude. The
experimental value of the forward cross-section is however always appreciably larger than the theoretical one thus being consistent with the presence of some real part. Column 4 of Table IV lists the magnitude of the ratio of the real part of the forward scattering amplitude to the imaginary part of the forward scattering amplitude. These numbers must be interpreted in the light of the errors coming from uncertainties in the total cross-section, limitations in obtaining the experimental forward cross-section by an extrapolation procedure and the fact that the effect of Coulomb scattering has been neglected. Saxer has computed this ratio on the basis of the predictions of dispersion relations. In our energy interval his value ranges from 13 to 21%. A set of counter measurements by Foley et al. yield an effective ratio of about 30% in the momentum range of 8 to 10 GeV/c with a fast energy dependence indicated by the near vanishing of this ratio around 12 GeV/c. Optical model with constant opacity, $a$, gives for the elastic and total cross-section, respectively, the expressions $\pi R^2 (1-a)^2$ and $2\pi R^2 (1-a)$. The ratio (elastic cross-section)/(total cross-section) = $(1-a)/2$ is presented in Table V and Fig. 3. Our compilation is consistent with an essentially constant 50% absorption in the region of investigation. Our value of the ratio is considerably larger than the rather constant value of $0.172 \pm 0.004$ quoted for the energy region 8 to 20 GeV.

![Fig. 2. Forward differential cross-section computed from the optical theorem, and experimental values, as a function of incident momentum.](image)

![Fig. 3. Momentum dependence of the ratio of the elastic to the total cross-section. The dotted line is the upper limit of this ratio in the Van Hove model.](image)
In the model of Van Hove\textsuperscript{15} elastic scattering amplitude asymptotically becomes pure imaginary when a large multiplicity and a weak correlation among secondary particles produced in inelastic collisions is assumed together with a Gaussian form for the inelastic overlap function. Several quantities of interest can then be determined in terms of the exponent of the overlap function and a parameter called $f_0$ (range: 0 to 0.5). The ratio

\begin{table}[h]
\centering
\caption{Forward cross-section (mb): optical theorem and experimental points.}
\begin{tabular}{cccc}
\hline
Momentum (GeV/c) & Optical & Experimental & $|\text{Re} f(o)/\text{Im} f( )|$ \\
\hline
1.59 & 59.1 ± 0.8 & 60 ± 4 & 0.1 \\
2.0 & 64.3 ± 1.1 & 91 ± 7 & 0.6 \\
2.26 & 62.1 ± 1.0 & 68 ± 3 & 0.3 \\
2.7 & 56.1 ± 0.9 & 73 ± 3 & 0.5 \\
3.0 & 52.0 ± 0.8 & 79 ± 6 & 0.7 \\
4.0 & 48.9 ± 0.7 & 53 ± 5 & 0.3 \\
4.16 & 44.5 ± 1.0 & 47 ± 1 & 0.2 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{Ratio of elastic to total cross-section, viz., $(1 - a)/2$}
\begin{tabular}{ccc}
\hline
Momentum (GeV/c) & $(1-a)/2$ \\
\hline
1.59 & 0.30 ± 0.01 \\
2.0 & 0.22 ± 0.01 \\
2.7 & 0.23 ± 0.01 \\
2.75 & 0.22 ± 0.01 \\
3.0 & 0.20 ± 0.01 \\
4.0 & 0.23 ± 0.01 \\
4.16 & 0.19 ± 0.01 \\
\hline
\end{tabular}
\end{table}
of the elastic cross-section to the total cross-section is predicted to increase with $f_0$ and attains the value 0.185 for the upper limit. Figure 3 shows the experimental values of the ratio along with the upper bound predicted by the model. Though the experimental values have not quite reached the limit, there is not much difference between the two even at a relatively low momentum of about 4 GeV/c.

### TABLE VI

Cross-section at fixed value of $t$ for different channels at the various laboratory momenta

<table>
<thead>
<tr>
<th>Channel</th>
<th>Momentum (GeV/c)</th>
<th>$t$ (Units of $\mu^2$)</th>
<th>$1.59$</th>
<th>$2.0$</th>
<th>$2.7$</th>
<th>$2.75$</th>
<th>$3.0$</th>
<th>$4.0$</th>
<th>$4.16$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-p \rightarrow \pi^-p$</td>
<td>$2$</td>
<td>0.99</td>
<td>1.75</td>
<td>1.10</td>
<td>1.71</td>
<td>1.20</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6$</td>
<td>0.51</td>
<td>0.96</td>
<td>0.61</td>
<td>0.99</td>
<td>0.00</td>
<td>0.36</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$10$</td>
<td>0.26</td>
<td>0.44</td>
<td>0.36</td>
<td>0.58</td>
<td>0.24</td>
<td>0.20</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14$</td>
<td>0.14</td>
<td>0.28</td>
<td>0.20</td>
<td>0.32</td>
<td>0.20</td>
<td>0.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^-p \rightarrow \rho^0\pi^-$</td>
<td>$2$</td>
<td>0.26</td>
<td>0.26</td>
<td>0.19</td>
<td>0.17</td>
<td>0.17</td>
<td>0.21</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6$</td>
<td>0.13</td>
<td>0.13</td>
<td>0.09</td>
<td>0.08</td>
<td>0.12</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$10$</td>
<td>0.06</td>
<td>0.07</td>
<td>0.04</td>
<td>0.04</td>
<td>0.07</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14$</td>
<td>0.03</td>
<td>0.04</td>
<td>0.02</td>
<td>0.03</td>
<td>0.04</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\pi^-p \rightarrow \rho^0\rho^0$</td>
<td>$2$</td>
<td>0.11</td>
<td>0.12</td>
<td>0.12</td>
<td>0.09</td>
<td>0.13</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$6$</td>
<td>0.07</td>
<td>0.06</td>
<td>0.06</td>
<td>0.05</td>
<td>0.08</td>
<td>0.07</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>$10$</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$14$</td>
<td>0.02</td>
<td>0.02</td>
<td>0.02</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The behaviour of the width of the diffraction pattern is now examined in terms of the gross predictions of Regge Pole Theory. Channel 1 admits the exchange of the following trajectories: $P$ [Vacuum trajectory with $\alpha_p (0) \sim 1$], $P'$ (having same quantum numbers as $P$ but with $\alpha_p (0) \sim 0.5$), $\rho$ [with $\alpha_\rho (0) \sim 0.3$] and $\Delta_{3/2; 3/2}^{\ast+}$ (which is required to understand the presence of a backward peak). Channels 2 and 3 can both accommodate a $\pi$ as well as $A_2$. $A_2$ trajectory is not very different from $\rho$ trajectory. At higher
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energies the effect of small $J$ trajectories, like $\pi$, presumably becomes unimportant. Channel 3 can in addition proceed through the exchange of an $\omega$ \([\alpha_\omega(0) \sim 0.2]\) or a $\phi$. The two trajectories are so close that they act as only one pole. The two-body channels 2 and 3 have also been examined from another point of view. The experimental decay angular distributions for $\rho$ have been extensively studied in the light of peripheral model corrected for absorption and suggest that both the reactions are dominated by pion exchange. In case of channel 3, however, the best fit to data requires the inclusion of some contribution from the vector exchange. By analysing $\rho^-$ data from several experiments in the momentum range of 2 to 8 GeV/c, Yen et al. have found that the parameter measuring the relative strength of $\omega$ to $\pi$ exchange has a value of $-(0.31 \pm 0.03)$, if negative or $+(0.39 \pm 0.94)$, if positive. Some predictions can be made about the behaviour of the forward cross-section when a single pole has a dominating role to play. In case of process 1, vacuum trajectory would obviously have such a role. This being an essentially flat trajectory implies that the value of $A$ for this process ought to be the smallest. Of course, we have no reason to believe that only one Regge pole contributes to the forward cross-section for channels 2 and 3. For fixed values of $t$ the cross-section is computed at different laboratory momenta for the three channels. This is summarised in Table VI and shows that for a given $t$ the cross-section stays essentially constant with increasing energy for each one of the three channels. The result is consistent with the non-shrinkage of the diffraction peak. This is in contrast with the observations on two-body channels produced in $K^-P$ interactions (3-5 GeV/c) where the distributions shrink as the centre of mass energy increases. A one pole Regge analysis would suggest that the diffraction pattern must shrink logarithmically with energy. In the high energy limit

\[
\frac{d\sigma}{dt} (s_1) = \left(\frac{s_1}{s_2}\right)^2 (\alpha_i(t) - 1)
\]

where $\alpha_i(t)$ is the leading Regge trajectory contributing to a particular process $i$. Our analysis gives essentially constant $d\sigma/dt$ at different values of energy for a fixed momentum transfer and would thus require an unusually large value $\sim 1$ for $\alpha_i(t)$. For process 1 this is consistent with single Regge pole dominance since vacuum trajectory has a very small slope and as such $\alpha_i(t)$ stays close to 1 for a large range of $t$. For processes 2 and 3 no single
trajectory can yield such a large value for $a_i(t)$. One possible simple explanation is that unlike the $K^+p$ interactions case, the processes here cannot be understood in terms of the exchange of a limited number of important trajectories; further unlike the $K^+p$ case the high energy range of applicability of the Regge pole analysis may not have been reached in $\pi^-p$ scattering at laboratory momenta considered in this work.

We conclude that the three reactions examined are all characterised by strong peripheral production. They fail to exhibit, within statistics, any shrinkage of the diffraction peaks and one is thus not able to describe them qualitatively in terms of a simple Regge pole model.

ACKNOWLEDGEMENT

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REFERENCES

1. Saclay-Orsay-Bari-Bologna Collaboration

2. Tuli, S. K.

3. Reynolds, B. G. \textit{et al.}

4. Miller, D. H., Gutay, L.,
   Johnson, P. B., Loeffler,
   F. J., Mellwain, R. L.,
   Sرافka, R. J. and
   Willmann, R. B.

5. Saclay-Orsay-Bari-Bologna Collaboration

6. Hagopian, V.

7. Guirgossian, Z. G. T.

8. ABBHLM Collaboration

9. Eisner, R. L., Johnson, P. B.,
   Klein, P. R., Peters, R. E.,
   Sahni, R. J., Yen, W. L.,
   and Tautfest, G. W.

10. Coffin, C. T. \textit{et al.}


12. Saxer, H. I.


