THE EFFECT OF IONISATION LOSS ON THE ENERGY SPECTRA OF COSMIC RAY NUCLEI UNDERGOING FERMI ACCELERATION

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ABSTRACT

The effect of ionisation loss, on the energy spectra of cosmic ray nuclei undergoing Fermi acceleration, has been studied using an analytic expression derived for the energy spectra of the nuclei undergoing acceleration. The spectra thus obtained have negative slopes which become steeper with increasing charge of the nucleus. In cases where the rate of acceleration is slow, solar modulation effects have been superimposed on the above spectra and it is found that the resulting spectral shapes can be fitted with recent satellite measurements for the energy spectra of various groups of cosmic ray nuclei at low energies.

INTRODUCTION

While studying the effects of ionisation losses suffered by nuclei undergoing acceleration, we found that in a situation where the rate of acceleration is slow, as in the case of Fermi acceleration in interstellar space, ionisation losses become very important and result in an energy spectrum which is steeper for heavy nuclei compared to lighter ones for energies < 500 MeV/n. This observation has prompted us to examine its consequences in more detail in view of some very interesting results obtained by Comstock et al.\textsuperscript{1} from recent satellite experiments. The measurements of Comstock et al., on the energy spectra and relative abundances of various groups\textsuperscript{*} of heavy nuclei of the galactic cosmic radiation, indicate that the M-nuclei have a remarkably flat differential energy spectrum in the interval of 50–200 MeV/n, in contrast to the positive slope of the spectrum for He-nuclei in the same energy range. Since solar modulation affects all nuclei with the same A/Z value in a similar

\begin{itemize}
  \item * Cosmic ray nuclei of \( Z > 3 \) are classified as follows: L-nuclei: \( Z = 3 - 5 \); M-nuclei: \( Z = 6 - 9 \); H-nuclei: \( Z > 10 \); H\textsubscript{p}-nuclei: \( Z = 10 - 15 \); H\textsubscript{q}-nuclei: \( Z = 16 - 19 \) and H\textsubscript{t} nuclei: \( Z > 20 \).
\end{itemize}
manner, the difference between the observed spectra of the helium and M-nuclei implies that their spectra must also be different before entering the solar system. In addition, the same authors find that the relative abundance \( \text{H}_3/\text{M} \) of nuclei with \( Z \geq 20 \) to that of \( Z = 6 - 9 \), also seems to increase with decreasing energy.

These experimental observations are in serious contradiction with those anticipated on the conventional models involving injection of cosmic ray nuclei from the source after acceleration (meaning thereby ionisation losses during acceleration are negligible) and their subsequent motion in interstellar space followed by solar modulation. The consequences of such models on the low energy end of the cosmic ray nuclei have been studied in detail during recent times. All these models predict energy spectra with positive slopes which are larger for heavy nuclei than that for He-nuclei at these very low energies and to our knowledge no satisfactory explanation has been offered yet for the new observations of a contrary nature.

The object of this paper is to calculate the effect of ionisation losses suffered by nuclei of different elements while undergoing Fermi type of acceleration at various rates and see whether in some way it would help us in understanding these new observations.

**ACCELERATION MECHANISM**

In the Fermi type of acceleration, charged particles, on the average gain energy in collisions with randomly moving magnetic irregularities in space. The rate of gain of energy in this process is given as:

\[
\frac{d\epsilon}{dt} = \tilde{a} \epsilon
\]  

(1)

where \( \epsilon \) is the total energy of the particle in rest mass units and \( \tilde{a} \) is the acceleration parameter. If one assumes a constant rate of injection of cosmic ray particles with time \( t \), then the age distribution of cosmic rays will be:

\[
N(t) \, dt = \frac{N_0}{T} \exp\left(-\frac{t}{T}\right) \, dt
\]  

(2)

where \( T \) is the mean lifetime of cosmic rays determined by the escape of particles from the volume under consideration. From (1) and (2) Fermi derived the energy spectrum to be of the form

\[
N(\epsilon) \, d\epsilon = \frac{F}{\tilde{a}T} \, \epsilon^{-(\frac{1}{2}+1/\tilde{a}T)} \, d\epsilon
\]  

(3)
where $F$ is a constant.

Expression (3) will have to be modified if ionisation loss is effective during the time of acceleration. The loss due to ionisation can be represented by the expression

$$\frac{d\varepsilon}{dt} = -\frac{KZ^2}{A\beta}$$

where $Z$ and $A$ are the charge and mass number of the particle whose velocity is $\beta C$ and $K$ is a constant depending on the characteristics of the medium through which the cosmic ray particles travel. From (1) and (4) the net rate of gain of energy is

$$\frac{d\varepsilon}{dt} = \ddot{\varepsilon} - \frac{KZ^2}{A\beta}.$$  

From expression (5), it is clear that acceleration can take place only if

$$\varepsilon > \frac{KZ^2}{A \beta}.$$  

The value of $\varepsilon$ where a positive acceleration is just possible is called the critical energy $\varepsilon_c$. Now expression (5) can be written as

$$dt = \frac{(\varepsilon^2 - 1)^{\frac{3}{2}}}{\varepsilon \left[ \frac{\ddot{\varepsilon} (\varepsilon^2 - 1)^{\frac{3}{2}} - KZ^2}{A} \right]} \frac{d\varepsilon}{\varepsilon}.$$  

Integrating and using $\varepsilon = \varepsilon_c$ at $t = 0$ we obtain

$$t = \frac{b^2}{\ddot{a} (\ddot{a}^2 + b^2)} \ln \left( \frac{\ddot{a} u - b}{(\ddot{a} u_c - b)} \right) - \frac{B}{2} \ln \left( \frac{1 + u^2}{1 + u_c^2} \right)$$

$$- C (\tan^{-1} u - \tan^{-1} u_c)$$  

where

$$b = \frac{KZ^2}{A} ; \quad B = \frac{-\ddot{a}}{(\ddot{a}^2 + b^2)}$$

$$C = \frac{-b}{(\ddot{a}^2 + b^2)} ; \quad \mu = (\varepsilon^2 - 1)^{\frac{3}{2}} \text{ and } u_c = (\varepsilon_c^2 - 1)^{\frac{3}{2}}.$$
Substituting in (2) for \( t \) from (7) and \( dt \) from (6) we get

\[
N (\epsilon) \, d\epsilon
= N (t) \, dt = M (\epsilon_c) \frac{u \exp \left( \frac{C}{T} \tan^{-1} u \right) d\epsilon}{T \epsilon \left\{ 1 + \left( \frac{a}{a^2 + b^2} \right) \frac{1}{T} \right\} \left( \frac{\alpha u - b}{\alpha u - b + a^2 + b^2} \right)}
\]

(8)

where \( M (\epsilon_c) \) is a function dependent only on the critical energy and hence is a constant for a particular charge.

RESULTS AND DISCUSSION

Expression (8) gives the energy spectrum of particles immediately after the acceleration in any particular region being considered and has been derived on very general grounds without any reference to the region in which the acceleration is taking place. It can be seen easily that when the ionisation loss is negligible during Fermi acceleration, that is, when \( b \ll \alpha \) the above expression reduces to that derived by Fermi, namely (3).

Acceleration of particles can, in principle, take place in various regions in space, typical examples being supernova envelopes, solar chromosphere and interstellar space. Let us consider the effect of ionisation losses suffered by nuclei while undergoing acceleration in the different accelerating regions on their energy spectra.

(a) Supernova Envelopes and Solar Chromosphere

Probable values\(^6\) of \( \alpha, K, T \) and \( \epsilon_c \) associated with supernova envelopes and solar flare regions are summarised in Table I. From various considerations it is inferred that these values cannot be in serious error.\(^6\) Under these conditions, it can be seen from eq. (8), that ionisation losses can be important only at energies \(< 1 \, \text{MeV} \) and hence they are not considered further here.

(b) Interstellar Space

The acceleration parameter \( \alpha \) appropriate to interstellar space is very poorly known at present. Some indirect estimates are available from the computation of magnetic field strength and matter density in interstellar space. These estimates necessarily suffer from very large uncertainties. Hence we have attempted to estimate the effect of ionisation loss during acceleration by assuming two extreme values for the acceleration parameter \( \alpha \). These extreme values have been selected from those suggested by Brunstein...
Ionisation Loss on Energy Spectra of Cosmic Ray Nuclei

and Cline and Hayakawa et al. and are included in Table I. The constant K appearing in (8) has been calculated by assuming a density of \(10^{-25}\) g. cm.\(^{-3}\) given by Hayakawa et al., for interstellar space. The lifetime \(T\) has been calculated from the consideration that at high energies the spectral shape is \(\sim F e^{-2.6}\) and hence \(1/aT\) should be equal to 1.5. The values of all parameters used in the present computation are summarized in Table I. Corresponding to these values the critical energy can be calculated using the formula

\[
\epsilon_c = \left[ 1 + \left( \frac{K}{\bar{a}} \right) \left( \frac{Z^2}{A} \right) \right]^{1/4}
\]

The critical energy for Fe is given in the last column of Table I.

TABLE I

<table>
<thead>
<tr>
<th>Region</th>
<th>(\bar{a}) Sec.(^{-1})</th>
<th>K Sec.(^{-1})</th>
<th>T Sec.</th>
<th>((\epsilon_c - 1)) for (Z^2/A = 12)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interstellar</td>
<td>2.0 \times 10^{-16}</td>
<td>1.0 \times 10^{-17}</td>
<td>3.33 \times 10^{15}</td>
<td>175 MeV/n.</td>
</tr>
<tr>
<td>Space</td>
<td>1.0 \times 10^{-15}</td>
<td>1.0 \times 10^{-17}</td>
<td>6.66 \times 10^{14}</td>
<td>20 MeV/n.</td>
</tr>
<tr>
<td>Supernova</td>
<td>3.16 \times 10^{-12}</td>
<td>1.0 \times 10^{-15}</td>
<td>2.11 \times 10^{11}</td>
<td>36 KeV/n.</td>
</tr>
<tr>
<td>Solar flare</td>
<td>3.2 \times 10^{-8}</td>
<td>1.0 \times 10^{-7}</td>
<td>\sim 30</td>
<td>37 eV/n.</td>
</tr>
</tbody>
</table>

The energy spectra for the cosmic ray nuclei of various charge groups were obtained using the values given in Table I and expression (8). These are shown in Fig. 1; the curves given in this figure have been calculated for \(6C^{12}\), \(18Al^{27}\) and \(26Fe^{56}\) which have been taken as representative of M, H\(_1\) and H\(_3\) groups of nuclei respectively. Curve F in Fig. 1 is due to Fermi acceleration without taking ionisation losses into consideration.

The calculated post-acceleration energy spectra of different kinds of nuclei given in Fig. 1, illustrate in a striking manner, the progressively increasing negative slope of the spectrum with increasing charge of the nuclei. The foregoing observation is equivalent to saying that for \(Z \geq 2\) the ratio of the flux of any group of nuclei to that of a lighter group will increase with decreasing energy. Since the spectra given in Fig. 1, refer to the situation outside the solar system, let us consider the effect of solar modulation on
these spectra so that a comparison can be made with experimental observations.

![Graph of differential energy spectra of cosmic ray nuclei normalized at high energies. The dash-dot line (---) represents the energy spectrum derived from Fermi's formula without taking into account ionization loss during acceleration. The spectra denoted by \( \text{He}^4, \text{M}, \text{H}_1 \) and \( \text{H}_8 \) represent the energy spectra obtained for the \( \text{He}^4, \text{M}, \text{H}_1 \) and \( \text{H}_8 \) nuclei taking into account ionization loss during acceleration in interstellar space. The solid curves represent the spectra for an acceleration parameter \( a = 2 \times 10^{-16} \text{ sec}^{-1} \) and dashed curves for \( a = 10^{-15} \text{ sec}^{-1} \).](image)

There is as yet no single model explaining satisfactorily all the observed features of solar modulation. However the solar wind modulation\(^9\) in which the modulation depends on the rigidity and velocity of the particles\(^{10-12}\) has been favoured recently. According to this model the spectrum outside the solar system \( N_\infty (E) \, dE \) is related to the one at the earth \( N_e (E) \, dE \) by the relation\(^{10}\)

\[
N_e (E) \, dE = N_\infty (E) \, dE e^{-\eta R_B}
\]  

(10)

where \( \eta \) is a parameter solely dependent on the level of solar activity. This parameter is not known directly, but can be estimated by assuming a power law spectrum in energy at high energies \( (E \gg 1 \text{ BeV/n}) \) and comparing it with the observations on He-nuclei. Using this procedure a value of \( \eta \sim \)
0.23 for 1965 has been obtained; this value has been used here for modifying the spectra of various charge groups according to eqn. (10). The calculations have been made for two assumed values of \( \dot{\alpha} \) and the spectra obtained near the earth are given in Fig. 2. In order to illustrate clearly the discrepancy between the recent experimental results of Comstock et al.\(^1\) and the predictions of the conventional models, we have made use of the spectra obtained by Apparao and Kaplon\(^1\) by assuming a power spectrum in total energy at the source and allowing it to pass through 2 g. cm\(^{-2}\) of neutral hydrogen wherein they suffer ionisation loss.

Fig. 2. The differential energy spectra of \( \text{He}^4 \), \( M \) and \( \text{H}_1 \) nuclei at the earth taking into account solar modulation. Curves (a) are for the acceleration parameter \( \dot{\alpha} = 5 \times 10^{-16} \) sec\(^{-1}\) and curves (b) are for \( \dot{\alpha} = 4 \times 10^{-16} \) sec\(^{-1}\). The dashed curves represent the spectra which will be obtained by the assumption of a single power spectrum in energy at the 'source' for all nuclei and then allowing them to pass through 2 g. cm\(^{-2}\) of neutral hydrogen wherein they suffer ionisation loss.

at the source and allowing it to pass through 2 g. cm\(^{-2}\) of neutral hydrogen without any associated acceleration during propagation; these spectra were then modified using eqn. (10) to take into account solar modulation and are given in Fig. 2. It is evident from the different curves given in Fig. 2 that the general behaviour of the low energy spectra experimentally observed for the different groups of cosmic ray nuclei near the earth, results naturally from a model in which the cosmic ray nuclei are accelerated by Fermi mechanism at such a slow rate that ionisation losses are appreciable during acceleration. While such conditions do exist in interstellar space, it is generally believed, at present, that acceleration by Fermi mechanism in interstellar space cannot be responsible for the bulk of cosmic rays even at low energies. Whether such favourable conditions exist in any other type of source region
is also a matter worth investigating. However, if no better explanation is forthcoming for the experimental observations, it would then become quite necessary to consider the mechanism suggested in the present paper much more seriously.

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