MODULATION OF GALACTIC COSMIC RADIATION IN INTERPLANETARY SPACE BY THE SOLAR WIND

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ABSTRACT

The effect of solar wind on the relative abundances of galactic cosmic rays is assessed using the theories proposed by Parker and Dorman. The modulation factors for the ratios of electrons, deuterons, tritons and helium nuclei to protons, helium-3 to helium-4 and light (Z = 3-5) to medium nuclei (Z = 6-9) are presented.

The effect of the modulation on the differential energy spectra is studied. The form of the spectra, the position of the maximum in the spectra and the relative reduction over a solar cycle place a restriction on the parameters in the solar wind theory. Starting with plausible galactic spectra and by varying the parameters it is possible to fit the form of the solar minimum spectrum but it does not seem possible to produce the relative reduction between minimum and maximum throughout the energy range, with in the framework of the present solar wind theory.

1. INTRODUCTION

The nature of the cosmic radiation is modified by several modulation processes since it starts from the source region. The radiation observed at the earth should bear the effects of the modulation processes both in interstellar and interplanetary space. These effects have to be unscrambled before one can ascertain the nature of the radiation at the source region which in turn will allow one to understand its origin.

The possible modulation processes in interstellar space have been investigated by many authors; however, only the effects of ionization and spallation losses have so far been calculated quantitatively. A large number of investigations on the modulation processes in interplanetary space have also been made. While many of these have tried to explain in a qualitative
manner the 11-year variation of cosmic-ray intensity, some have tried similarly to explain the form of the helium spectrum. It is possible, however, that almost all the modulation processes suggested contribute to the "total modulation" of the galactic radiation. It is necessary then to understand the detailed implications of each of these theories in order to assess their contribution to the "total modulation".

In the present paper we investigate the effects of modulation by solar wind. In Section 2 is presented a brief summary of the theory and formulae for modulation. In Section 3, the effect of modulation on the spectrum of helium nuclei has been determined. As will be shown, the variation of the helium spectrum over a solar cycle places a restriction on the variation of the parameters. In Section 4 is presented the effect of modulation on the relative abundances of galactic cosmic rays and its variation with energy. Finally, in Section 5, the results are discussed.

2. THE SOLAR WIND THEORY

The solar wind theory has been proposed by Parker. The hydrodynamic expansion of the solar corona has been shown to yield a steady outflow of matter which is termed the "solar wind". This solar wind stretches the lines of force of the solar magnetic field in a radial and regular manner up to the region where turbulent magnetic inhomogeneities are set up due to various causes discussed by Parker. This turbulent region extends until the galactic magnetic field becomes dominant. Thus a picture of this model is that there exists a cavity where the magnetic field is more or less regular and this cavity is surrounded by a region of turbulence. The relative dimensions of these two regions have been deduced from the solar flare of February 23, 1956; it is found that the radius of inner cavity is \( \sim 1.4 \) A.U. and the thickness of the turbulent region is \( \sim 5 \) A.U. It is of course true that the actual interplanetary conditions are far from the simple model outlined, but the gross features have been shown to be correct from several solar flare studies. It is also expected that the dimensions of the cavity and of the turbulent region vary with the solar cycle.

The effect of the above model on primary cosmic rays streaming towards the earth from interstellar space has been studied by Parker. The cosmic radiation has to diffuse through the turbulent region; in addition, one also has to take into account convection effects due to a drift of the magnetic inhomogeneities in the turbulence due to the outward flow of the solar wind. The equation for the diffusion and convection is then
where $N(\epsilon)$ is the differential energy spectrum, $\epsilon$ is the total energy of the particles in terms of their rest mass $m_0c^2$, $u$ is the velocity of the solar wind directed radially from the sun and $D$ the diffusion coefficient. The stationary solution, under the assumption that $D$ is dependent only on $\epsilon$ and not on spatial co-ordinates and that the effect of the wind is in a spherical layer of inner and outer radii $r_1$ and $r_2$, is

$$N^z(\epsilon) = N^f(\epsilon) \exp \left\{ - \frac{u}{D} (r_2 - r_1) \right\}$$

(1)

where $N^z(\epsilon)$ is the spectrum inside the cavity (the earth is inside the cavity, therefore $N^z$ is the spectrum at the earth) and $N^f(\epsilon)$ is the spectrum outside. The diffusion coefficient $D$ is given by $D = \frac{1}{2}v \lambda$, where $v$ is the velocity of the particle and $\lambda$ the mean free path for scattering the particle out of the beam. $\lambda$ is derived to be

$$\lambda = \frac{\pi^2 \rho^2}{4l}$$

where

$$\rho = \frac{m_0c^2 (\epsilon^2 - 1)^{\frac{1}{2}}}{ZeH}$$

$l$ is the mean dimension of the field inhomogeneities, $H$ the magnetic field intensity and $Ze$ is the charge of the particle. Substituting the value for $D$ in Equation (1) the modulation factor $M(\epsilon)$ can be written as

$$M(\epsilon) = \frac{N^z(\epsilon)}{N^f(\epsilon)} = \exp \left\{ \frac{\alpha \epsilon}{(\epsilon^2 - 1)^{3/2}} \right\}$$

where

$$\alpha = \frac{12u (r_2 - r_1) lZe^2H^2}{\pi^2 m_0^2 c^6}$$

(2)

with the above expression for the modulation the gross variation of the intensity of cosmic radiation over a solar cycle has been obtained by Parker using $u = 10^8$ cm./sec. $(r_2 - r_1) = 6 \times 10^{11}$ cm., $H = 2 \times 10^{-6}$ gauss and $l = 2 \times 10^{11}$ cm. (We shall call $\alpha_0$ the value of $\alpha$ corresponding to these values.)

Dorman² has studied the implications of the above model on the variations of neutron and $\mu$-meson components at sea-level at various latitudes
and concluded that a single value of $a$ at all energies is not tenable. He has also argued that instead of magnetic inhomogeneities of a single scale length $2 \times 10^{11}$ cm., there might exist a whole hierarchy of inhomogeneities with perhaps the above value of $1$ as the minimum and a maximum scale length $L$ of about $10^{18}$ cm. If so the mean free path for scattering is given by

$$\lambda(\epsilon) = \begin{cases} 
1 & \text{for } \rho \leq 1 \\
\rho & \text{for } 1 \leq \rho \leq L \\
\frac{\rho^2}{L} & \text{for } \rho \geq L.
\end{cases}$$

(3)

Then the modulation factor is given by

$$M(\epsilon) = e^{-b}$$

where

$$b = a \left(\frac{\epsilon}{(\epsilon^2 - 1)}\right)^3$$

(4a)

for

$$\epsilon \leq \sqrt{1 + \frac{Z^2 e^2 \hbar^2}{m_0^2 c^4}}$$

and

$$a = 3u (r_2 - r_1) ;$$

$$b = a \left(\frac{\epsilon}{(\epsilon^2 - 1)}\right)$$

(4b)

for

$$\sqrt{1 + \frac{Z^2 e^2 \hbar^2}{m_0^2 c^4}} \leq \epsilon \leq \sqrt{1 + \frac{Z^2 e^2 \hbar^2 L^2}{m_0^2 c^4}}$$

and

$$a = 3u (r_2 - r_1) \frac{Z e \hbar}{m_0 c^2} ;$$

$$b = a \left(\frac{\epsilon}{(\epsilon^2 - 1)^{3/2}}\right)$$

(4c)

for

$$\epsilon \geq \sqrt{1 + \frac{Z^2 e^2 \hbar^2 L^2}{m_0^2 c^4}}$$
and

$$a = \frac{3u(r_2 - r_1)Z^2e^2H^2L}{m_0^2c^6}.$$  

Dorman finds that with the solar wind velocity $u \approx 3 \times 10^7$ cm./sec., the above formulae yield variations in neutron and $\mu$-meson intensities at various latitudes which are in agreement with experiment.

Parker has modified his original formulae by considering thin scattering centres ($1 < \rho$) and thick scattering centres ($1 > \rho$) and has given formulae similar to the ones according to Dorman. Parker, however, does not consider the possibility of a distribution of the scale lengths of the magnetic clouds. In our consideration we shall use the formulae given by Dorman which are more general than Parker's. The values used for the parameters in calculating $a = a_0$ are $U = 10^8$ cm./sec., $(r_2 - r_1) = 6 \times 10^{18}$ cm., $1 = 2 \times 10^{11}$, $H = 2 \times 10^{-5}$ gauss and $L = 10^{18}$ cm. [The value of $a_0$ is different for (4 a), (4 b), (4 c) and the corresponding scale length of the magnetic clouds should be used]. All values given above are only estimates; however, in case of 1 evi-

![Diagram](image-url)

**Fig. 1.** The modulation factors for protons as a function of total energy (in terms of rest mass), as predicted by solar wind theory according to Parker and Dorman. Curves 1, 2, 3 correspond to different values of the parameter $a$ (see text).
Fig. 2. The modulation factors for He\(^4\) nuclei as a function of total energy (in terms of rest mass), as predicted by solar wind theory according to Parker\(^4\) and Dorman.\(^3\) Curves 1, 2, 3 correspond to different values of the parameter \(\alpha\) (see text).

The modulation factors \(M(\epsilon)\) have been calculated for the various nuclear species in the cosmic radiation for different values of \(\alpha\). Typical curves for protons and alpha particles are shown in Figs. 1 and 2, for \(\alpha = 0.1 \alpha_0, 0.4 \alpha_0\) and \(\alpha_0\). Also shown in the figure is the curve using Parker's formula (2).

3. THE DIFFERENTIAL SPECTRA OF COSMIC RAY NUCLEI

(a) The helium spectrum.—The differential spectrum of the helium nuclei has been measured at various stages of the solar cycle by several workers and is by far the best determined differential spectrum among the various nuclei in the primary cosmic radiation at present. The differential spectrum shows a maximum both at solar minimum (around 1.7 BV rigidity) and solar maximum (around 2.2 BV); the position of the maximum depends on the time of the solar cycle and shifts to higher rigidity at solar maximum.
Thus the shape of the spectra at solar minimum and maximum, the relative decrease in the intensities as a function of energy and the positions of the maxima in the two spectra are the features which any modulation theory has to explain. Some of the above features depend on the galactic spectrum one starts with. In the present consideration, the following three plausible spectra have been tried:

(i) Power spectrum in energy

\[ N(\epsilon) = \frac{K}{\epsilon^{\gamma+1}} ; \quad K \text{ is a constant; } \gamma = 1.5 \]

(ii) Power spectrum in rigidity

\[ N(\rho) = \frac{K}{\rho^{\gamma+1}} ; \quad K \text{ is a constant; } \gamma = 1.5 \]

(iii) The spectrum obtained by Apparao\(^7\) by starting with a power spectrum of the form (i) at the "source" of cosmic rays and by taking into account the ionization and interaction losses during the traversal of cosmic rays in interstellar space. (For further details reference may be made of the paper by Apparao.)

The modulation factors calculated in Section 2 were applied and the resulting spectra are displayed in Figs. 3, 4 and 5. In these figures, the solid points indicate the experimental measurements of various observers\(^8\)\textsuperscript{14} at solar minimum and the open points the measurements at solar maximum. For the curve corresponding to solar minimum the modulated curve which has the maximum around 1.8 BV has been chosen and the normalisation with the experimental value made at a rigidity of 5 BV, the value at this rigidity being known from integral flux measurements. It must be emphasized at this point that this procedure of choosing the solar minimum curve is not entirely arbitrary and that the position of the maximum is an important parameter in fixing the proper curve. In Figs. 3, 4, 5 are also shown the galactic spectra assumed and the modulated curves for different values of \(\alpha\).

From Fig. 3 it is seen that starting with a power spectrum in energy (i), it is possible to fit approximately the experimental data at solar minimum with the calculated curve for \(\alpha = 0.4 a_0\), but it is not possible to fit the solar maximum data with any of the calculated curves. Starting with a power spectrum in momentum (ii), one gets a reasonable fit for the solar minimum data (see Fig. 4) for \(\alpha = 0.5 a_0\), while for the solar maximum data the low rigidity part can be fitted if \(\alpha = 0.9 a_0\). However, the calculated modulation is not sufficient to give a fit with the data at the high rigidity end. In Fig. 5
are shown curves obtained by starting with spectrum (iii) as the galactic spectrum. Here a value of $\alpha = 0.15 \ a_0$ gives a reasonable fit for the solar minimum data. On the other hand, for the solar maximum data a value of $\alpha = 0.5 \ a_0$ will give a fit at the low rigidity part while for the high rigidity end we have the same situation as for (ii). The important point that emerges from these graphs, however, is that variability of $\alpha$ is between $0.1 \ a_0$ and $a_0$, though it is not possible to obtain a complete fit to the data with these values.
Thus in spite of the lack of the knowledge of the individual parameters we have an estimate of the variability of the modulation possible in the solar wind theory.

Another way of looking at the modulation predicted by the solar wind theory is by looking at the ratio of the modulation factors for the solar minimum ($a \approx 0.15 a_0$, the lowest value of $a$ tenable) and the modulation factors
Fig. 5. The modulated differential spectra of helium nuclei for different values of the parameter $a$ (see text), starting with the spectrum given by Apparao as the galactic spectrum. Also shown is the spectrum predicted from Parker's formula.

obtained with values of $a = 0.4 a_0$, $0.6 a_0$, and $a_0$, and compare the results with the ratio of solar minimum intensity of cosmic ray alpha particles to that at solar maximum as a function of rigidity. This has been done in Fig. 6; the experimental values for the ratio of solar minimum intensity to solar maximum intensity are obtained from Webber.³ It is seen from the figure that it is not possible to fit the points throughout the rigidity range by a single value of $a$. The same statement can be made even if it is assumed that there is no modulation at solar minimum. Thus it seems that using the
solar wind theory one cannot produce the necessary reduction of intensity at higher rigidities even by varying the parameters.

![Graph showing the ratio of intensity of helium nuclei at solar minimum to that at solar maximum, compared with values predicted by the solar wind theory for different values of $\alpha$.](image)

**Fig. 6.** Comparison of the experimental values of the ratio of intensity of helium nuclei at solar minimum to that at solar maximum, with the values predicted by the solar wind theory for different values of $\alpha$.

(b) *The proton spectrum.*—The proton spectrum for rigidities $> 3$ BV is mostly obtained by integral flux measurement and is found to be similar in form to that of helium nuclei. Recently, it has been found that the differential spectrum for protons below 2 BV differs from that of the spectrum for helium nuclei. Indeed, the solar wind theory predicts the similarity of proton and helium spectra above 3 BV and marked difference below 2 BV. Details of this aspect is published elsewhere.

(c) *The spectra of heavy nuclei ($Z \geq 3$)*.—The experimental errors on the spectra of heavy nuclei are so large that the spectral shapes are not well

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* It is the usual practice to classify the heavy nuclei as follows: L-nuclei ($Z = 3-5$); M-nuclei ($Z = 6-9$); H-nuclei ($Z > 10$).
defined. However, it is known that they have a maximum between rigidities 1.5 BV and 2.5 BV and that the position of the maximum shifts to higher rigidities with increase in solar activity. Because of the paucity of experimental data we have not presented the curves predicted by solar wind theory together with the experimental data. However, it can be stated that the situation is the same as with the helium spectra.

![Diagram](image)

**Fig. 7.** The ratio modulation factors of α-particles and protons as a function of energy as predicted by solar wind theory for different values of α.

4. **Effect of Solar Wind on Relative Abundances of Primary Cosmic Ray Nuclei**

It is of interest to see how much the modulation in interplanetary space by solar wind can modify the relative abundance of primary cosmic ray nuclei. For this purpose we have calculated the modulation factors for protons $p$, tritons $t$, He$^3$, deuterons $d$ (same as He$^4$ and medium nuclei because of same charge to mass ratio), and light nuclei L. The modulation factors have been calculated for $\alpha = 0.1 \alpha_0$, 0.4 $\alpha_0$ and $\alpha_0$ which is the range of variability. In order to ascertain the effect on relative abundances we have calculated
the ratio of modulation factors $M_i/M_j$ for two species $(i, j)$ as a function of energy. This ratio gives the factor by which the observed relative abundance at the earth has to be divided in order to obtain the relative abundance in interstellar space. The ratios of modulation factors $M_i/M_j$ for $(\alpha, p)$ [which is the same for $(d, P)$ and $(M, P)$ and $(H, P)$], $(t, p)$, $(e, P)$. $(L, M)$ and $(He^8, He^4)$ are presented in Figs. 7 to 11.

It is seen from Fig. 7 that the ratio $M_\alpha/M_P$ according to energy per nucleon depends on the solar activity. If one assumes that $\alpha = 0.1\, a_0$ represents the solar minimum and $\alpha = 0.4\, a_0$ the solar maximum, then it is found that around $\epsilon = 3$ the modulation has a pronounced effect at solar maximum while it is not appreciable at solar minimum. On the other hand, the ratio $M_\alpha/M_P$ below $\epsilon = 2$ is appreciably affected by the modulation both at solar

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**Fig. 8.** The ratio of modulation factors of tritons and protons as a function of energy as predicted by solar wind theory for different values of $\alpha$. 
minimum and maximum. The ratio $M_e/M_p$ according to rigidity however is not affected for rigidities $> 3$ BV while below 2 BV there is an appreciable effect.

The ratio $M_e/M_p$ according to energy per nucleon is enhanced by a factor of 2 by solar modulation as seen from Fig. 8. Experimentally this ratio seems to be close to zero for $\epsilon < 1 \cdot 5^{17}$ implying a galactic ratio almost equal to zero.

The ratio of modulation factors $M_e/M_p$ for electrons and protons is not appreciably affected for energies above a BeV. This can be understood because above an energy of a BeV the rigidities are not too different for electrons and protons. This would mean that the electron to proton ratio observed at the earth is almost the galactic ratio.

The ratio of light to medium nuclei for $\epsilon \approx 1 \cdot 5$ is affected slightly by the modulation due to solar wind. From the considerations in Section 3...
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Fig. 10. The ratio of modulation factors of light nuclei to medium nuclei as a function of energy as predicted by solar wind theory for different values of $\alpha$.

about $\alpha$ it seems that the maximum enhancement of the ratio in a solar cycle is about 15 to 20%. This enhancement is not enough to explain the higher value of light nuclei to medium nuclei ratio observed experimentally ($\sim 40\%$) at $\varepsilon \approx 1.5$, as compared to the value observed ($\sim 25\%$) at higher energies ($\varepsilon \sim 4$).

The ratio $\text{He}^9/\text{He}^4$ is considerably modified by the modulation due to solar wind, especially at low energies. This is to be expected due to the different masses of the two isotopes. From Fig. 11, for $\varepsilon \sim 1.5$, where most of the experimental observation exists, the galactic ratio is reduced by about 25% around solar maximum and by about 10% at solar minimum.

5. Discussion

The theory of modulation of the cosmic radiation as given by Parker and Dorman is necessarily approximate due to the simplified structure and extent of the magnetic irregularities assumed. Several of the parameters involved are unknown and can only be reasonably guessed at the present time.
though these will be determined in the near future by satellites. The considerations of the spectra of cosmic ray nuclei, in particular, the differential spectrum of helium nuclei has shown that, even though the parameters in the theory are not known, the variability of their combination ($a$) is limited to a range of a factor of about ten. It should be pointed out that this observation is somewhat dependent on the initial ‘galactic spectra’ assumed. It is true that synthetic ‘galactic spectra’ can be constructed to yield better fits to the experimental data and perhaps increase the variability of $a$ than what is indicated. It seems, however, that within the framework of the present theory the modulation both at high and low rigidities cannot be simultaneously produced to account for observations. This means that solar wind theory by itself cannot account for the modulation in the interplanetary space and hence other mechanisms have to be invoked.
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It is seen from Section 4, that the interplanetary modulation indeed has an effect on the relative abundance of nuclei in primary cosmic radiation, especially at low energies ($\epsilon < 1.5$). In the case of the ratio of helium nuclei to protons, the observed ratio has to be reduced by a factor of 1.5 to 1.8, depending on solar activity, to give the galactic ratio. The same is the case for the ratio of heavy nuclei ($Z > 6$) to protons. This increases the discrepancy between cosmic ray ratio and the cosmic abundance ratio in the case of $a/p$, while it reduces the discrepancy in the case of heavy nuclei.

The ratio $\text{He}^3/\text{He}^4$ is also affected considerably by the modulation; it is reduced by 10 to 25% depending on solar activity. Any attempt at obtaining the amount of interstellar matter from the $\text{He}^3/\text{He}^4$ ratio should bear this in mind. The experimental results on the ratio $\text{He}^3/\text{He}^4$ are not quite in agreement at present though the disparity is narrowing due to better techniques. It will be interesting to see if the experimental determinations of this show a variation of this ratio over a solar cycle.

In conclusion, it seems that the solar wind theory of interplanetary modulation cannot account for all the modulation effects over the 11-year cycle and other modulation effects have to be invoked.

REFERENCES

7. Apparao, M. V. K. (To be published).


