HYPERSONIC FLOW PAST A SPHERE

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ABSTRACT

The effects of dissociation or ionization of air on the analytical solution for hypersonic flow past a sphere are considered here, under certain assumptions. It has been assumed that the shock wave is in the shape of a sphere, that the density ratio across the shock is constant, that the flow behind the shock is at constant density and that dissociation or ionization only occurs behind the shock wave. Thus the effects of the compressibility of the air, variation of density ratio along the shock, and the departure of the shock shape from being circular are not taken into account. Here the velocity, pressure, temperature, pressure coefficient and vorticity, etc., at any point between the shock and the surface of the sphere in the presence of dissociation or ionization are obtained. In addition, shock detachment distance, drag coefficient, stagnation point velocity gradient and sonic points on the shock and the surface have also been obtained. The results have been compared with the corresponding results obtained in the case when dissociation or ionization does not occur behind the shock.

SYMBOLS

\[ R_B \] = radius of the sphere.
\[ R_S \] = radius of the shock sphere.
\[ p \] = pressure.
\[ \rho \] = density.
\[ T \] = temperature.
\[ R, \theta \] = polar co-ordinates.
\[ \Delta = R_S - R_B \]
\[ u, v \] = velocity components.
\[ \nu_{\infty} \] = ratio of specific heats (Here it is taken as 1.4).

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\( \nu \) = effective ratio of specific heats just behind the shock in the presence of dissociation or ionization.

\( m_a \) = molecular weight of air (28.97).

\( m \) = effective molecular weight of air just behind the shock in the presence of dissociation or ionization.

\( M_\infty \) = free stream Mach number.

\( a \) = velocity of sound.

\( K \) = density ratio \( \frac{\rho_\infty}{\rho_s} \) (across the shock).

\( W_\infty \) = free stream velocity.

\( \omega \) = Vorticity.

\( \psi \) = stream function.

\( A, B \) = constant of integration.

\( \alpha \) = deflection of the stream at any point \((R_s, \theta)\) on the shock.

\( h \) = specific enthalpy.

\( \beta \) = angle between the sonic line leaving the body and the direction of the body surface downstream of the sonic point.

\( \phi \) = angle between the stream line direction and the direction of the sonic line leaving the shock.

\( \delta \) = deflection of the stream at any point \((R, \theta)\) between the shock and the body.

\( \xi, \eta \) = intrinsic co-ordinates.

\( \psi_\theta, \psi_{\theta\theta}, \text{etc.} \) = \( \frac{\partial \psi}{\partial \theta}, \frac{\partial^2 \psi}{\partial \theta^2} \).

Subscripts

\( \infty \) = condition in free stream.

\( S_\theta \) = condition just behind the shock at the point \((R_s, \theta)\).

\( S_t \) = condition just behind the shock at the point \((R_s, 0)\).

\( O \) = stagnation point condition.

\( B_\theta \) = condition on the surface of the sphere at the point \((R_B, \theta)\).

\( \theta \) = condition at any point \((R, \theta)\).

\( \epsilon \) = condition at any point \((R, O)\) along the axis \((\theta = 0)\).
1. INTRODUCTION

The problem of obtaining aerodynamic information at hypersonic flight speeds has currently received considerable interest in connection with the development of intercontinental ballistic missiles, satellites, and space vehicles. For flight Mach numbers greater than about 10, air can no longer be considered as being a simple mixture of diatomic oxygen and nitrogen. At Mach number 20, the air temperature behind a normal shock wave for a blunt body can be as high as 6500° K. during re-entry. At these hypersonic speeds air is greatly decelerated and undergoes considerable change in composition. Dissociation of oxygen and nitrogen occurs and possibly also thermal ionization. The convective heat transfer will, as a result, be altered from what it was for the "perfect gas". The decelerated gas becomes capable of radiating energy and the radiative heat transfer must generally be considered for hypersonic vehicles, particularly, for long range ballistic rockets. It is not only the aerodynamic heating problems that are affected, but at very high temperature the gas (or air) becomes electrically conductive, and introduces new problems in radio wave transmission and reception. In addition, a conducting gas flow can be influenced by a magnetic field. The magnetic field can alter skin friction, heat transfer, and total drag, etc.

It is well known that a blunt body in supersonic or hypersonic flight is accompanied by a detached curved shock with a resultant mixed flow field between the body and the shock. The practical importance of blunt-nosed bodies has led to many recent investigations of hypersonic flow with detached curved shock, and these are both of inverse and direct type. The inverse problem in the absence of dissociation or ionization is considered by Zlotnick and Newman, Garabedian and Lieberstein, and van Dyke and others. The direct problem in the absence of dissociation or ionization is considered by Maslen and Moeckel, Li and Geiger, and Le's and Kubota and others.

The constant density solution to the hypersonic inviscid flow past a sphere (at zero angle of incidence) in the absence of dissociation or ionization has been considered by Lighthill, Li and Geiger, Koon-Sang wan and Linnel.

The present author has considered the above problem in the presence of dissociation or ionization under the following assumptions: (a) The shock wave is in the shape of a sphere of radius $R_s$. (b) The density ratio $K$ along the shock is constant. (c) The flow behind the shock is of constant density. (d) The dissociation or ionization of air occurs behind the shock. It is further
assumed that the constant density assumption is not invalidated by dissociation or ionization and all changes of state take place at thermodynamic equilibrium, i.e., the relaxation times are assumed to be zero.

Equations are obtained giving velocity, pressure, temperature, velocity of air, shock detachment distance, pressure and drag coefficient, vorticity, stagnation point velocity gradient, sonic points, etc., and comparison has been made with the corresponding results when dissociation or ionization does not occur. Further, graphs have been plotted showing pressure, drag, vorticity and stagnation point velocity gradient distributions.

2. BASIC EQUATIONS

Consider a hypersonic flow field over a sphere at zero angle of incidence. The density \( \rho \) of the flow in the shock layer is assumed to be constant, and is equal \( \rho_{\infty}/\rho = K \), where the value of \( K \) depends on the free stream Mach number, \( m_{\infty} \), and the ratio of the specific heats, \( \nu_{\infty} \).

Making use of spherical polar co-ordinates, the radial (normal) component of the velocity is given by

\[
\rho v = \frac{\psi_\theta}{R^2 \sin \theta}.
\]

The tangential component of the velocity is expressed as

\[
\rho u = \frac{\psi_R}{R \sin \theta}.
\]

The stream function immediately behind the shock equals that immediately in front of it.

Hence,

\[
\psi_{\infty} = \frac{1}{2} \rho_{\infty} W_{\infty} R S^2 \sin^2 \theta.
\]

The vorticity is given by

\[
\omega = \frac{W_{\infty}}{KR S^2} R \sin \theta.
\]

The stream function \( \psi \) satisfies the equation

\[
\psi_{RR} + \frac{\sin \theta}{R^2} \left( \frac{1}{\sin \theta} \cdot \psi_{\theta} \right) = \frac{(1 - K)^2 \rho_{\infty} W_{\infty}}{KR^2 S^2} R^2 \sin^2 \theta.
\]

The boundary conditions are: \( u = v = 0 \), at the stagnation point, \( v = 0 \) on the surface of the sphere; \( u = W_{\infty} \sin \theta, v = -K W_{\infty} \cos \theta \) at the shock wave.
3. Conditions Just Behind the Shock Wave

The thermodynamic state of a gas behind a shock wave is calculated from Rankine-Hugoniot equations of the conservation of mass, energy and momentum across a shock wave. For weak shocks which do not cause any dissociation of the gas, the state behind the shock can be calculated easily. But in the case of strong shock which occurs in the case of hypersonic vehicles, the gas behind the shock dissociates or ionizes and the calculation of the state behind the shock becomes complicated. But the equations can be solved by means of successive iterations.

Here, the equilibrium conditions behind the shock in the presence of dissociation or ionization are determined by means of correlation equations developed by Waiter and Choudhury. The equations can be expressed as:

\[ \frac{P_{s0}}{P_\infty} = 1.27 M_\infty^2 \cos^2 \theta \]  
(6)

\[ \frac{\frac{v_\infty a_{s0}^2}{\nu}}{a_{\infty}^2} = \frac{m_\infty}{m} \frac{T_{s0}}{T_\infty} = 0.127 M_\infty^2 \cos^2 \theta \]  
(7)

\[ \frac{h_{s0}}{h_\infty} = 1 + 0.2 M_\infty^2 \cos^2 \theta \]  
(8)

where \( v_\infty = 1.4 \) and \( K = 0.1 \), for all \( M_\infty \) in hypersonic flow when dissociation or ionization occurs. The values of \( \nu \) and \( m \) can be determined from tables of thermodynamic properties of air. It is also assumed that in the flow behind the shock, the conditions between the shock and the surface approximate to the conditions just downstream of the shock. Hence \( m \) and \( \nu \) may be considered to be constant between the shock and surface. In the absence of dissociation or ionization \( \nu = v_\infty = 1.4 \), \( m = m_\infty = 28.97 \). When \( m_\infty = 20 \), \( \nu = 1.17 \).

(3.1) Deflection of the Stream as it comes out of the Shock Wave

The deflection at the point \((R_s, 0)\) on the shock is zero. The deflection of the stream at the point \((R_s, \theta)\) in the presence of dissociation or ionization is given by

\[ \tan \alpha = \frac{9 \cot \theta}{10 + \cot^2 \theta}. \]  
(9)

Hence the deflection depends upon \( \theta \), for same \( M_\infty \) and it increases due to dissociation or ionization.
4. Solution for Flow between the Shock and the Sphere

The solution of (5) at any point \((R, \theta)\) can be expressed as,

\[
\psi = \varphi_0 \frac{W_\infty R_s^2 \sin^2 \theta}{30 K^2} \left[ 3 (1 - K)^2 \left( \frac{R}{R_s} \right)^4 + A \left( \frac{R}{R_s} \right)^2 + B \left( \frac{R}{R_s} \right) \right]
\]

(10)

where

\[
A = -5 (1 - 4 K), B = 2 (1 - K) (1 - 6 K), K = 0.1
\]

in the presence of dissociation or ionization.

Hence,

\[
u = \frac{W_\infty \sin \theta}{15 K} \left[ 3 (1 - K)^2 \left( \frac{R}{R_s} \right)^2 + A + B \left( \frac{R}{R_s} \right)^3 \right]
\]

(11)

and

\[
v = -\frac{W_\infty \cos \theta}{15 K} \left[ 3 (1 - K)^2 \left( \frac{R}{R_s} \right)^2 + A + B \left( \frac{R}{R_s} \right)^3 \right].
\]

(12)

From (11) and (12),

\[
\frac{u^2}{(\lambda_1 W_\infty)^2} + \frac{v^2}{(\lambda_2 W_\infty)^2} = 1
\]

(13)

which is an ellipse.

Where

\[
\lambda_1 = \frac{12 (1 - K)^2 \left( \frac{R}{R_s} \right)^2 + 2 A - B \left( \frac{R}{R_s} \right)^3}{30 K}
\]

(11)

and

\[
\lambda_2 = \frac{3 (1 - K)^2 \left( \frac{R}{R_s} \right)^3 + A + B \left( \frac{R}{R_s} \right)^3}{15 K}.
\]

(12)

(4.1) Shock Detachment Distance

The shock detachment distance in the presence of dissociation or ionization is obtained from (10), by putting \(\psi = 0\) at the surface of the sphere. The approximate solution is given by

\[
\frac{\Delta}{R_B} = \frac{(K - 2 K^2) - 2.8 K^{3/2}(1 - K)^{1/2}}{(1 - 8 K + 8 K^2)} = 0.0714
\]

(13)

as \(K = 0.1\). Hence it is independent of \(M_\infty\) and is displaced towards the surface of the sphere due to dissociation or ionization.
(4.2) Velocity, Pressure, Temperature, Velocity of Sound and Mach Number, etc.

The velocity, pressure, temperature, velocity of sound and Mach number, etc., in the presence of dissociation or ionization of air behind the shock are expressed as:

\[ q_{e}^{2} = \frac{W_{e}}{W_{\infty}} \left[ \lambda_{1}^{2} \sin^{2}\theta + \lambda_{2}^{2} \cos^{2}\theta \right] \]  
\[ q_{B\theta}^{2} = W_{\infty}^{2} \sin^{2}\theta \times 0.277729 \]  
\[ q_{S\theta}^{2} = W_{\infty}^{2} \left[ \sin^{2}\theta + 0.01 \cos^{2}\theta \right] \]

\[ \frac{P_{\theta}}{P_{\infty}} = [1.34 + 37.8 \sin^{2}\theta \cdot \lambda_{3} - 7 (\lambda_{1}^{2} \sin^{2}\theta + \lambda_{2}^{2} \cos^{2}\theta)] M_{\infty} \]  

where,

\[ \lambda_{3} = \left\{ 2.43 \left( \frac{R}{R_{S}} \right)^{4} - 3 \left( \frac{R}{R_{S}} \right)^{2} + 0.72 \left( \frac{R_{S}}{R} \right) \right\} \]

\[ \frac{P_{B\theta}}{P_{\infty}} = [1.34 - 7 \sin^{2}\theta \times 0.277729] M_{\infty} \]

Fig. 1. Shock detachment distance for a sphere.
The deflection of the stream at any point \((R, \theta)\) between the shock and the surface is given by,

\[
\tan \delta = \frac{\left(\frac{\lambda_2}{\lambda_1} - 1\right) \tan \theta}{1 + \frac{\lambda_2}{\lambda_1} \tan^2 \theta}.
\]

(24)

The velocity, pressure, temperature, Mach number, etc., in the region along the axis including the stagnation in the presence of dissociation or ionization are special cases of the results given above and can be obtained by putting \(\theta = 0\) in equations (14) to (23).

(4.3) Pressure Coefficient

The pressure coefficient at any point \((R_B, \theta)\) on the surface of the sphere in the presence of dissociation or ionization is obtained from (18) and can be expressed as:

\[
C_{p\theta} = \frac{2}{\nu \infty M^2} \left[1 - \sin^2 \theta \times 0.277729\right] M_{\infty}^2.
\]

(25)

The derivative of the pressure coefficient is zero at the stagnation point, i.e.,

\[
\left(\frac{\partial C_{p\theta}}{\partial \theta}\right)_{\theta=0} = 0.
\]

(26)
Similarly, the pressure coefficient at the stagnation point is given by,

$$C_{p0} = \frac{2}{\nu_{\infty}} \frac{1}{M_{\infty}^{-2}} [1 \cdot 34 M_{\infty}^{-2} - 1].$$

The pressure coefficient on the surface of the sphere in the case of dissociation of air is given below and comparison has been made with the results obtained when air is not dissociated or ionized.

In both the cases the pressure coefficient $C_p$ decreases as $\theta$ increases when $M_{\infty}$ remains constant. Further due to dissociation or ionization it increases for same $M_{\infty}$ and $\theta$. Again it increases as $M_{\infty}$ increases, others remaining constant, but it is rather insensitive to the variation of $M_{\infty}$.
(4.4) Vorticity

The vorticity on the body surface can influence both skin-friction and heat transfer and shock radius of curvature is an essential parameter for the calculation of the vorticity. The vorticity in the presence of dissociation or ionization can be obtained from (4) and (13) and can be expressed as:

\[ \omega_\theta = -\frac{W_\infty (7.05637) R \sin \theta}{R_B^2} \]  \hspace{1cm} (28)

\[ \omega_B \theta = -\frac{W_\infty (7.05637) \sin \theta}{R_B} \]  \hspace{1cm} (29)

\[ \omega_S \theta = -\frac{W_\infty (7.5602) \sin \theta}{R_B} \]  \hspace{1cm} (30)

The vorticity is zero along the axis (\( \theta = 0 \)) including the stagnation point and increases as \( \theta \) increases. It is independent of free-stream Mach number. Comparing the present results with the results obtained in the case when no dissociation or ionization is present, it can be easily verified that the vorticity increases due to dissociation or ionization. Moreover to distinguish between these two cases, graphs have been drawn showing the vorticity distribution.
(4.5) Drag Coefficient

The drag coefficient (neglecting the skin friction) in the presence of dissociation or ionization can be obtained from (25) and expressed as:

$$C_D = \frac{4}{\nu_\infty M_\infty^2} \left[ \{1.34 - \frac{7 \times 0.277729}{2}\} M_\infty^2 - 1 \right]. \quad (31)$$

The drag coefficient in the presence and absence of dissociation or ionization is given below:

In both the cases, the drag coefficient very slowly increases as $M_\infty$ increases. For same $M_\infty$, $C_D$ increases due to dissociation or ionization. Graphs have been plotted showing the drag distribution in both the cases.
(4.6) Stagnation Point Velocity Gradient

It is well known that the heat transfer rate depends on the magnitude of the velocity gradient along the body surface. Hence the velocity gradient at the stagnation point is a quantity of considerable importance in the blunt body heating problem and is particularly important for designers of the inter-continental ballistic missiles and other hypersonic vehicles.

The process of aerodynamics heat transfer at hypersonic velocities is complicated by dissociation or ionization of air due to high static temperature encountered by the body when the air is decelerated by shock wave and by viscous forces in the boundary layer or at stagnation point. As dissociation or ionization (and their reverse process recombination) proceed at finite rates, thermochemical equilibrium is not necessarily achieved throughout the flow field, and such rates are therefore a necessary ingredient of the flow process. Moreover, diffusion of atoms or ions, which recombine with a high specific energy release, may appreciably add to the heat transferred by normal molecular conduction. In addition, the high stagnation temperature accompanying flight at extreme hypersonic Mach number renders the air sufficiently ionized behind the bowshock so that it may be regarded as an electrically conducting fluid. Under these circumstances, the presence of a magnetic field will tend to modify both the outer inviscid flow and the boundary layer. Since the magnetic body force always tends to retard the fluid motion, the velocity gradient at the outer edge of the boundary layer will be reduced, thus reducing the heat transfer rate. A complete heat transfer
analysis must account for these effects. In the present analysis only the
effect of dissociation or ionization is considered.

In the present case, the velocity gradient at the stagnation point in the
presence of dissociation or ionization can be determined from (10).

$$\left(\frac{\partial u}{\partial \theta}\right)_{\theta=0} = W_\infty \times 0.527$$

which is independent of Mach number $M_\infty$. Thus the velocity gradient at
the stagnation point is reduced by dissociation or ionization.

(4.7) Sonic Line, Limiting Characteristic and Transonic Region

The difficulty in solving the mathematical problem of determining the
detached shock and the flow behind the shock for a given body lies principally
in the fact that the flow around the body is a non-linear mixed supersonic-
subsonic flow, with unknown curved shock in front of it as a free boundary. However, in the present case it is known. But the location of the “limiting characteristic” which generally intersects the sonic line at one point is unknown. The disturbance in the region between the sonic line and the “limiting
characteristic” would affect the shape of the sonic line and the subsonic flow field upstream. This supersonic region may be referred as the “transonic zone”. Moreover, the presence of dissociation or ionization will affect the sonic line, sonic points, etc.

\[(4.71) \text{Sonic points on the shock and the surface of the sphere and the pressure at the sonic point on the surface of the sphere.}\]

The sonic points on the shock in the presence of dissociation or ionization are determined from (23) and can be expressed as:

\[
\cos^2 \theta_s = \frac{1}{0.99 + \frac{\nu}{\nu_\infty} \times 0.127}.
\]

Similarly, the sonic points on the surface of the sphere is determined from (22) and can be written as:

\[
\sin^2 \theta_B = \frac{\nu}{\nu_\infty} \times 0.134
\]

\[
\frac{0.277729 \left[ 1 + \frac{\nu}{\nu_\infty} \times 0.7 \right]}{0.277729 \left[ 1 + \frac{\nu}{\nu_\infty} \times 0.7 \right]}
\]

\[
\text{Similarly, the critical pressure } p_{B_{\text{sonic}}/p_0} \text{ is determined from (18) and (34). Hence,}
\]

\[
\frac{p_{B_{\text{sonic}}/p_0}}{p_0} = 1.34 - 7 \times 0.277729 \sin^2 \theta_{B_{\text{sonic}}}.
\]

The sonic points on the shock and the surface of the sphere and \(p_{B_{\text{sonic}}/p_0}\) are given below:

In both the cases (with or without dissociation), the sonic points on the shock and the surface and \(p_{B_{\text{sonic}}/p_0}\) are rather insensitive to the variation of \(M_\infty\). The sonic points move towards the axis due to dissociation or ionization. But \(p_{B_{\text{sonic}}/p_0}\) increases due to dissociation or ionization.

\[(4.72) \text{Angle between the sonic line and the surface of the sphere.—The geometrical quantity which characterizes the behaviour of the sonic line near the body is the angle it makes with the body. The main point is to find out whether the angle is acute or obtuse, as this is important in determining whether the body shape has influence on the transonic zone.}\]
In the presence of dissociation or ionization, the expression which determines the angle which sonic line makes with the body is given by:

\[
\tan \beta = \left( \frac{-\frac{\partial q}{\partial \epsilon}}{\frac{\partial q}{\partial \eta}} \right)_{\text{sonic}}
\]  

(36)

Now

\[
\left( \frac{\partial q_\theta}{\partial \eta} \right)_B = -\omega_{B\theta} - \left( \frac{q_{B\theta}}{R_B} \right).
\]  

(37)

From (4) and (37)

\[
\left( \frac{\partial q_\theta}{\partial \eta} \right)_{\text{sonic}} = \left( \frac{q_{B\theta}}{R_B} \right)_{\text{sonic}} \left[ \frac{(1 - K)^2 W_{\infty} \sin \theta}{K q_{B\theta}} \right]_{\text{sonic}} \left( \frac{R_B}{R_S} \right)^2 - 1 \right]_{\text{sonic}}
\]  

(38)

where,

\[
\frac{(1 - K)^2}{K} = 8.1 \left( \frac{R_B}{R_S} \right)^2 = 0.871048, \left( \frac{W_{\infty} \sin \theta}{q_B} \right)_{\text{sonic}} = \frac{1}{0.527}.
\]

Hence \(\frac{\partial q_\theta}{\partial \eta}\)_{sonic} is positive and \(\frac{\partial q_\theta}{\partial \epsilon}\)_{sonic} is always positive. Thus for a sphere in the presence of dissociation or ionization, the angle \(\beta\) is obtuse in air \((\nu_\infty = 1.4)\) whereas in the two-dimensional case (a cylinder) it is acute. For a sphere in the absence of dissociation or ionization, \(\beta\) is obtuse in air \((\nu_\infty = 1.4)\) if \(M_\infty > 3\).

Similarly, the angle between the sonic line and the free-stream at the shock wave in the presence of dissociation or ionization is given by:

\[
\tan \phi = \frac{K^3}{\tan \theta_{\text{sonic}}} \left[ 3 (\nu_\infty + 1) K^2 + (5 - \nu_\infty) \tan^2 \theta_{\text{sonic}} \right] \left[ 3 (\nu_\infty + 1) K^2 + 2 \tan^2 \theta_{\text{sonic}} \right]^{-1}
\]

(39)
### Table IV

\((v_\infty = 1.4)\)

<table>
<thead>
<tr>
<th>Without dissociation</th>
<th>With dissociation</th>
<th>Percentage difference</th>
<th>Without dissociation</th>
<th>With dissociation</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(\theta)</td>
<td>0°</td>
<td>0°</td>
<td>0°</td>
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<td>15°</td>
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<tr>
<td>(K)</td>
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<td>0.1687</td>
<td>0.1</td>
<td>-40.7</td>
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<tr>
<td>(\frac{q_s}{W_\infty})</td>
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<td>0.1</td>
<td>-40.7</td>
<td>0.3058</td>
<td>0.27625</td>
</tr>
<tr>
<td>(\frac{q_s}{W_\infty})</td>
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<td>0.1</td>
<td>-40.7</td>
<td>0.3058</td>
<td>0.27625</td>
</tr>
<tr>
<td>(\frac{P_s}{P_\infty})</td>
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<td>536</td>
<td>4.3</td>
<td>463.464</td>
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<td>(\frac{P_s}{P_s})</td>
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<tr>
<td>(\frac{P_s}{P_\infty})</td>
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<td>508</td>
<td>8.89</td>
<td>436.1132</td>
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<td>1.7263</td>
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<tr>
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<td>zero</td>
<td>42° 48.3'</td>
<td>54° 32'</td>
</tr>
</tbody>
</table>

A5
where $K = 0 \cdot 1$, $v_\infty = 1 \cdot 4$ and $\theta_{\text{sonic}}$ can be determined from (33). Comparing results with dissociation with results without dissociation, the present investigation shows that for the same $M_\infty$, dissociation or ionization of air reduces the value of $\theta_{\text{sonic}}$.

5. NUMERICAL CALCULATIONS

To illustrate the effects of dissociation or ionization on the inviscid hypersonic flow of air past the sphere, numerical calculations have been made, taking $M_\infty = 20$, $\theta = 0^\circ$ and $15^\circ$ and the results are compared with the corresponding results when dissociation or ionization is absent.

(5 1) Discussion of the Results of Tables IV and V

In the above tables, the results have been obtained for both the conditions (i.e., with or without dissociation or ionization of air) for $M_\infty = 20$ on the surface of the sphere and the shock wave, when $\theta = 0^\circ$ and $15^\circ$. In the case of density of air and velocity of sound, there is marked difference between the results of the two cases in both the regions, although the difference between the pressure is less pronounced. Due to dissociation or ionization

<table>
<thead>
<tr>
<th>Without dissociation</th>
<th>With dissociation</th>
<th>Percentage difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_\infty$</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$K$</td>
<td>0.1687</td>
<td>0.1</td>
</tr>
<tr>
<td>$R_0$</td>
<td>0.1185</td>
<td>0.0714</td>
</tr>
<tr>
<td>$\frac{1}{W_\infty} \left( \frac{\delta u}{\delta \theta} \right)_{\theta = 0}$</td>
<td>0.6726</td>
<td>0.527</td>
</tr>
<tr>
<td>$p_{\text{sonic}}$</td>
<td>0.5882</td>
<td>0.6311</td>
</tr>
<tr>
<td>$p_0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{l}$</td>
<td>0.9808</td>
<td>1.04428</td>
</tr>
<tr>
<td>$\theta_{l, \text{sonic}}$</td>
<td>$\pm 32^\circ 4'$</td>
<td>$30^\circ 17^\prime$</td>
</tr>
<tr>
<td>$\theta_{s, \text{sonic}}$</td>
<td>$\pm 22^\circ 24'$</td>
<td>$17^\circ 12^\prime$</td>
</tr>
</tbody>
</table>
of air, while the pressure increases, the density, the temperature and the velocity of sound decrease, $M_\infty$ remaining same. While the velocity $(q_b/W_\infty)_{\theta}$ at the stagnation point in the presence of dissociation or ionization remains the same as before, $(q_s/W_\infty)$ decreases at the shock wave ($\theta = 0$). Similarly, in the region when $\theta = 15^\circ$, due to dissociation or ionization, the velocity both at the surface of the body $(q_b/W_\infty)$ and at the shock $(q_s/W_\infty)$ decreases. Due to dissociation or ionization, vorticity rapidly increases both at the shock and the surface in the region, when $\theta = 15^\circ$. The Mach number at the stagnation point is zero in both the cases. While due to dissociation, the Mach number at the shock in the region, when $\theta = 0^\circ$, decreases, the Mach number both at the shock and the surface of the sphere in the region, when $\theta = 15^\circ$, increases. Similarly, the pressure coefficient increases due to dissociation in both the regions ($\theta = 0^\circ$ and $15^\circ$).

From Table V, it can be seen that due to dissociation or ionization, the shock wave is displaced towards the body, the velocity at the stagnation point decreases, the drag coefficient and $p_{R\text{ sonic}}/p_0$ increase, and the sonic points on the shock and the body are displaced towards the axis.

6. Conclusions

The interest in hypersonic flow around blunt bodies in the presence of dissociation or ionization has been stimulated in the last few years by intercontinental ballistic missile and satellite programmes. High speed Re-Entry Missiles, especially those in the intercontinental class, re-enter the earth's atmosphere at such tremendous speeds that the gas behind the shock may become highly dissociated and ionized. Ionization renders the gas electrically conducting and hence capable of being affected by electromagnetic fields. One of the chief problems associated with the re-entry of an intercontinental ballistic missile is that of convective heating. As the gas near the stagnation region is conducting, the presence of magnetic field influences the skin friction, heat transfer and total drag and the heat transfer rate depends upon the stagnation point velocity gradient.

The present paper considers the effects of dissociation or ionization of air on inviscid hypersonic flow past a sphere, at zero angle of incidence, with a detached curved shock. Expressions have been obtained giving velocity, pressure, temperature, Mach number, shock detachment distance, vorticity pressure and drag coefficient sonic points and stagnation point velocity gradient, etc., in the presence of dissociation or ionization. While pressure coefficient is maximum at the stagnation point, the pressure gradient vanishes.
at that point. While velocity is zero at the stagnation point, the stagnation point velocity gradient does not vanish at it. Due to dissociation or ionization, the pressure, the pressure and drag coefficient, vorticity, etc., increase, but the velocity, the velocity of sound, the density, the shock detachment distance, etc., decrease.

7. REFERENCES


