

CLASSIFICATION OF EXCITED STATES OF EVEN-EVEN NUCLEI

BY S. M. BRAHMAVAR AND M. K. RAMASWAMY, F.A.Sc.

(*Department of Physics, Karnatak University, Dharwar-3*)

Received October 2, 1963

ABSTRACT

In this paper we have attempted to see how far a classification of excited states of even-even nuclei can be made based on the quantity AE , A being the mass number and E , the energy of excitation. The results show that this may be a useful criterion in the understanding of the excitation mechanism.

EXCITED states of even-even nuclei are interesting from the theoretical point of view. Various models have been put forward to give a description of the excited states. The strong coupling collective model is successfully applied to the heavy nuclei and nuclei in the rare-earth region. In the case of medium-weight nuclei configuration mixing in the shell-model, the weak coupling and intermediate coupling models (collective) as also the asymmetric rotor model of Davydov and Filippov have been proposed.

All these models predict a first excited state which is 2^+ . For the second excited state the weak coupling model predicts a triplet of 0^+ , 2^+ and 4^+ states, whereas the asymmetric rotor model with beta vibration included predicts similar states, as also the intermediate coupling model. 0^+ excited states can be either shell model states or result from the beta vibration band. 3^- levels have been interpreted as the octupole vibrations of the even-even core. On the basis of the shell model, the configurations which give a 3^- state also yield a 5^- state. However, the 5^- states can be interpreted¹ as belonging to the $\lambda = 5$ mode of collective vibration.

Classification of these excited states on the basis of the excitation energy or energy ratios is usually ambiguous. We have attempted in this paper to see how far the product AE , A being the mass number and E , the excitation energy, can be used as a possible criterion for classifying the excited states. Table I shows this attempt. In column 1 are included those even-even nuclei for which 0^+ , 2^+ , 0^+ , 2^+ , 3^- and 5^- levels exist. Columns 2, 4, 6, 8 and 10

give the excitation energies of these states. The 0^{+} second excited state is listed just below the first excited 0^{+} state; columns 3, 5, 7, 9 and 11 indicate the product AE for the respective states. The last column gives the reference.

The following conclusions are drawn from Table I.

A. 0^{+} First Excited State

The product AE is sensitive to the shell structure and magic numbers. Whenever we are in the vicinity of a closed shell the product rises to a high value. The value AE lies between 80 and 200 for light and medium-weight nuclei whereas it assumes a value between 180 and 360 for deformed and heavy nuclei.

B. 0^{+} Second Excited State

The product AE is insensitive to the shell closure. With the exception of Hf-178 the product is almost a constant and lies between 140 and 160. The accumulation of more extensive data may however alter this conclusion. The value is closer to AE (2^{+}) than AE (2^{+}) in the same nuclei.

C. 2^{+} First Excited and 2^{+} Second Excited States

The values of AE are quite sensitive to shell closure and magic numbers. AE has a high value whenever N or Z equals a magic number.

(1) AE (2^{+}) lies between 30 and 140 for light and medium-weight nuclei. AE (2^{+}) ranges from 100 to 180. Further, the value AE (2^{+}) is about twice that of AE (2^{+}).

(2) AE (2^{+}) lies in a small range of 10 to 40 for deformed and heavy nuclei. But AE (2^{+}) does not show any such range. Another point that may be noted in this case is that for heavy nuclei the value of AE (2^{+}) remains almost constant (10) whereas the value of AE (2^{+}) rises rather high.

D. 3^{-} and 5^{-} levels

Ae (3^{-}) and AE (5^{-}) do not show any sensitiveness to shell closure. Both assume values between 160 and 320 for medium weight nuclei. For heavy nuclei they increase by almost a factor of two.

TABLE I

1	2	3	4	5	6	7	8	9	10	11	12
Nucleus	E(0 ⁺) (Mev)	AE(0 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(3 ⁻) ^a (Mev)	AE(3 ⁻)	E(5 ⁻) ^b (Mev)	AE(5 ⁻)	Refer- ence
${}^4\text{Be}_4$	7.55	60.39	2.9	23.2	1
${}^6\text{C}_6$	7.66 10.1	91.91 121.1	4.43	53.16	9.62	115.4	2
${}^8\text{O}_8$	6.06	96.96	6.91	110.6	6.13	98.09	1
${}^8\text{O}_{10}$	3.627	65.28	1.98	35.65	3.915	70.47	3
${}^{10}\text{Ne}_{10}$	6.74 7.22	134.8 144.4	1.63	32.6	7.45	149.0	1
${}^{12}\text{Mg}_{12}$	6.44	154.5	1.367	32.78	4.23	101.5	4
${}^{14}\text{Si}_{14}$	4.98	137.8	1.78	49.83	6.88	152.7	5
${}^{14}\text{Si}_{16}$	(3.765)	112.9	2.23	66.89	5
${}^{16}\text{S}_{16}$	3.78	121.0	2.24	71.66	4.2	137.3	5
${}^{16}\text{S}_{18}$	2.127	72.31	3.22	109.5	1
${}^{18}\text{Ar}_{20}$	2.15	81.70	1
${}^{20}\text{Ca}_{20}$	3.35	134.0	3.37	149.2	1

TABLE I (Contd.)

1	2	3	4	5	6	7	8	9	10	11	12
Nucleus	E(0 ⁺) (Mev)	AE(0 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(3 ⁻) ^a (Mev)	AE(3 ⁻)	E(5 ⁻) ^b (Mev)	AE(5 ⁻)	Reference
$^{76}_{34}\text{Se}$	0.561	42.64	1.22	92.72	1
$^{78}_{34}\text{Se}$	0.615	47.97	1.307	101.7	1
$^{80}_{34}\text{Se}$	0.667	53.35	1.455	116.4	8
$^{80}_{36}\text{Kr}$	0.618	49.44	1.258	100.7	8
$^{82}_{36}\text{Kr}$	0.777	63.71	1.475	121.0	6
$^{84}_{36}\text{Kr}$	0.88	73.92	1.9	159.6	1
$^{86}_{38}\text{Sr}$	1.084	93.24	1.925	165.5	9
$^{88}_{38}\text{Sr}$	1.85	162.9	4.2	369.6	2.76	242.9	1
$^{90}_{40}\text{Zr}$	1.734	156.1	2.18	196.2	2.2	198.0	2.32	208.7	6
$^{92}_{40}\text{Zr}$	2.26	207.9	2.71	249.4	1.81	166.5	10
$^{94}_{42}\text{Mo}$	0.874	82.15	1.57	147.6	6
$^{96}_{42}\text{Mo}$	0.77	73.92	1.61	154.5	11
$^{88}_{44}\text{Ru}$	0.654	64.09	1.39	136.2	6
$^{100}_{44}\text{Ru}$	0.535	53.5	1.358	135.8	1

$^{104}_{46}\text{Pd}_{68}$	1.05	109.1	0.556	57.82	1.34	139.3	1
$^{106}_{46}\text{Pd}_{60}$	1.14	120.9	0.51	54.06	1.13	119.7	2.883	220.8	..	12
$^{108}_{46}\text{Pd}_{62}$	1.05	113.4	0.43	46.44	0.941	101.6	1.987	214.6	..	13
$^{108}_{48}\text{Cd}_{60}$	6.30	2.19	236.4	2.54	274.3
$^{110}_{48}\text{Cd}_{62}$	0.656	72.16	1.474	162.1	2.056	226.2	2.92	321.2
$^{112}_{48}\text{Cd}_{64}$	1.23 1.43	137.7 160.2	0.610	68.31	1.30	145.5	1.96	219.6	..	15
$^{114}_{48}\text{Cd}_{66}$	1.133 1.304	129.1 148.6	0.557	63.51	1.208	137.4	1.946	221.8	..	16
$^{114}_{50}\text{Sn}_{64}$	1.299	148.01	3.5.8	251.1	..	6
$^{116}_{50}\text{Sn}_{66}$	1.72	199.5	1.27	147.2	2.09	242.4	2.24	259.8	..	17
$^{118}_{50}\text{Sn}_{68}$	1.22	144.0	2.23	269.0	2.29	270.2
$^{120}_{50}\text{Sn}_{70}$	1.18	141.6	2.38	285.2	2.29	274.8
$^{122}_{50}\text{Sn}_{72}$	1.14	134.1	2.50	305.4	..	6
$^{124}_{50}\text{Sn}_{74}$	1.13	140.2	2.55	316.1	..	6
$^{122}_{52}\text{Te}_{70}$	0.564	68.82	1.26	153.8	6
$^{124}_{52}\text{Te}_{72}$	0.603	74.76	1.35	167.4	2.253	279.4	..	6
$^{126}_{52}\text{Te}_{74}$	0.66	83.06	1.41	177.6	2.31	191.1	..	6

TABLE I (Contd.)

1	2	3	4	5	6	7	8	9	10	11	12
Nucleus	E(0 ⁺) (Mev)	AE(0 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(3 ⁻) ^o (Mev)	AE(3 ⁻)	E(5 ⁻) [*] (Mev)	AE(5 ⁻)	Refer- ence
$^{128}_{52}\text{Te}$	0.75	96.01	2.45	313.6..	6
$^{130}_{52}\text{Te}$	0.85	110.5	2.32	301.6	6
$^{126}_{54}\text{Xe}$	0.386	47.53	0.86	108.3	6
$^{128}_{54}\text{Xe}$	0.45	57.59	0.98	125.4	6
$^{130}_{54}\text{Xe}$	0.528	68.63	1.998	259.8	2.34	304.2	6
$^{132}_{54}\text{Xe}$	0.673	88.84	1.26	166.3	6
$^{136}_{54}\text{Xe}$	1.32	179.5	2.6	353.6	6
$^{134}_{56}\text{Ba}$	0.605	81.08	1.168	156.5	6
$^{136}_{56}\text{Ba}$	1.041	141.6	2.98	405.2	1
$^{138}_{56}\text{Ba}$	1.43	197.3	6
$^{140}_{58}\text{Ce}$	1.502	266.3	1.596	223.5	2.349	328.7	2.46	344.3	18
$^{142}_{60}\text{Nd}$	1.57	222.9	2.08	295.4	6
$^{144}_{60}\text{Nd}$	0.69	99.36	1.22	175.7	1.504	216.6	19
$^{146}_{60}\text{Nd}$	0.455	66.43	1.22	178.1	1

${}_{60}\text{Nd}_{88}^{148}$	0.3	44.4	1.01	149.4	6
${}_{60}\text{Nd}_{90}^{150}$	0.131	19.65	0.981	147.2	6
${}_{62}\text{Sm}_{86}^{148}$	0.562	83.18	1.182	175.0	6
${}_{62}\text{Sm}_{88}^{150}$	0.741	111.1	0.334	50.09	0.736	110.4	1.067	159.7	20
${}_{62}\text{Sm}_{90}^{152}$	0.685	104.1	1.22	185.5	1.092	166.0	1.067	159.7	6
${}_{64}\text{Gd}_{88}^{152}$	0.615 (1.047)	93.48 159.2	0.344	52.29	0.929	141.2	6
${}_{64}\text{Gd}_{90}^{154}$	0.123	18.94	0.998	153.7	6
${}_{64}\text{Gd}_{92}^{156}$	0.87	135.7	0.089	13.89	1.14	177.8	21
${}_{66}\text{Dy}_{92}^{158}$	0.0993	15.69	22
${}_{66}\text{Dy}_{94}^{160}$	0.087	13.92	0.964	154.3	6
${}_{66}\text{Dy}_{96}^{162}$	0.081	13.12	6
${}_{68}\text{Er}_{98}^{164}$	0.0728	11.93	1
${}_{68}\text{Er}_{100}^{166}$	1.062	176.3	0.0811	13.46	1.541	255.8	1
${}_{68}\text{Er}_{102}^{168}$	0.080	13.44	1
${}_{69}\text{Tm}_{103}^{173}$	0.538	92.53	6
${}_{70}\text{Yb}_{102}^{173}$	0.079	13.5	6
${}_{72}\text{Hf}_{106}^{176}$	0.089	15.66	1

TABLE I (Contd.)

1	2	3	4	5	6	7	8	9	10	11	12
Nucleus	E(0 ⁺) (Mev)	AE(0 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺)	E(2 ⁺) (Mev)	AE(2 ⁺) ^a	E(3 ⁻) (Mev)	AE(3 ⁻)	E(5 ⁻) ^b (Mev)	AE(5 ⁻)	Refer- ence
${}_{72}\text{Hf}_{106}^{178}$	1.197 (1.44)	213.0 256.9	0.0932	16.56	1.277	226.8	23
${}_{72}\text{Hf}_{108}^{180}$	0.0933	16.8	1
${}_{74}\text{W}_{108}^{182}$	0.1009	18.35	1.222	222.4	1.374	250.1	1.62	294.8	1
${}_{74}\text{W}_{110}^{184}$	0.111	20.42	0.904	166.3	6
${}_{74}\text{W}_{112}^{186}$	0.125	23.25	0.73	135.8	1
${}_{76}\text{Os}_{110}^{186}$	0.137	25.48	0.768	142.9	6
${}_{76}\text{Os}_{112}^{188}$	1.086	204.2	0.155	29.14	0.633	119.2	24
${}_{76}\text{Os}_{114}^{190}$	0.187	35.53	0.558	106.0	6
${}_{76}\text{Os}_{116}^{192}$	0.206	39.56	0.489	93.89	6
${}_{78}\text{Pt}_{114}^{192}$	0.317	60.87	0.613	117.7	6
${}_{78}\text{Pt}_{116}^{194}$	1.265	245.4	0.328	63.63	0.621	120.5	6
${}_{78}\text{Pt}_{118}^{196}$	0.356	69.77	0.689	135.1	6
${}_{80}\text{Hg}_{118}^{198}$	1.082	244.2	0.412	81.58	1.09	215.8	6

$^{200}_{80}\text{Hg}_{120}$	0.368	73.57	1.575	315.0	6	
$^{202}_{82}\text{Pb}_{122}$	0.899	181.7	1.374	..	2.04	412.1	6
$^{206}_{82}\text{Pb}_{124}$	0.803	165.4	2.525	520.2	2.8	576.9	6
$^{208}_{82}\text{Pb}_{126}$	2.614	543.8	3.2 3.71	665.6 771.8	6
$^{210}_{84}\text{Po}_{126}$	1.181	248.0	2.91	611.0	6
$^{212}_{84}\text{Po}_{128}$	1.68	356.1	1.27	269.3	1.513	320.7	25
$^{216}_{84}\text{Po}_{130}$	1.416	303.0	0.609	130.3	1.847	395.4	6
$^{210}_{84}\text{Po}_{132}$	0.544	117.0	6
$^{218}_{86}\text{Rn}_{132}$	0.235	70.86	(0.65)	14.14	6
$^{220}_{86}\text{Rn}_{134}$	0.241	53.02	(0.53)	116.6	6
$^{222}_{86}\text{Rn}_{136}$	0.187	41.52	(0.447)	99.24	6
$^{224}_{88}\text{Ra}_{136}$	0.084	18.81	6
$^{226}_{88}\text{Ra}_{138}$	0.068	15.37	6
$^{228}_{90}\text{Th}_{138}$	0.057	12.99	(0.965)	220.0	6
$^{230}_{90}\text{Th}_{140}$	0.053	12.19	(1.06)	243.8	6
$^{232}_{92}\text{U}_{140}$	0.693	160.8	0.0476	11.04	0.735	170.5	26
$^{234}_{92}\text{U}_{142}$	0.811	189.8	1.045	244.5	0.0435	10.18	27

TABLE I—(Contd.)

1	2	3	4p	5	6	7	8	9	10	11	12
Nucleus	E (0 ⁺) (Mev)	AE (0 ⁺)	E (2 ⁺) (Mev)	AE (2 ⁺)	E (2 ⁺) (Mev)	AE (2 ⁺)	E (3 ⁻) ^a (Mev)	AE (3 ⁻)	E (5 ⁻) ^b (Mev)	AE (5 ⁻)	Refer- ence
$^{238}_{92}\text{U}_{144}$	0.045	10.86	6
$^{238}_{94}\text{Pu}_{144}$	0.94	223.8	0.044	10.23	1.03	245.1	6
$^{240}_{94}\text{Pu}_{146}$	(0.94)	225.6	0.0428	10.27	(1.02)	244.8	6
$^{242}_{94}\text{Pu}_{148}$	0.045	10.89	6
$^{246}_{96}\text{Cm}_{150}$	0.043	10.58	6
$^{248}_{96}\text{Cm}_{152}$	0.043	10.67	6
$^{250}_{98}\text{Cf}_{152}$	0.042	10.5	(1.2)	300	6

Note.—Data for E (3⁻) and E (5⁻) are taken from the following references:—

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The above-mentioned systematics may possibly help to classify excited levels by means of the product AE and the range in which this product lies. The above discussion can be summarised as shown in Table II.

TABLE II

Excited state	Range of AE for		Remarks
	Light-weight and Medium-weight nuclei	Deformed and heavy-weight nuclei	
0 ⁺	80-200	180-360	Sensitive to shell-closure and magic numbers.
0 ⁺ '		140-160	(i) Insensitive to shell-closure and magic numbers. (ii) AE (0 ⁺ ') closer to AE (2 ⁺ ') than AE (2 ⁺).
2 ⁺	30-140	10-40	(i) Sensitive to shell-closure and magic numbers. (ii) AE (2 ⁺) remains almost constant (≈ 10) for deformed and heavy-weight nuclei.
2 ⁺ '	100-180	No definite range	(i) Sensitive to shell-closure and magic numbers. (ii) AE (2 ⁺ ') is twice that of AE (2 ⁺) approx. for light and medium-weight nuclei (iii) AE (2 ⁺ ') goes on increasing for deformed and heavy nuclei.
3 ⁻ and 5 ⁻	160-320	500-700	(i) Insensitive to shell closure and magic numbers. (ii) The value of AE for heavy nuclei is nearly twice that for medium-weight nuclei.

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