ANGULAR DISTRIBUTION OF JETS PRODUCED
BY HEAVY PRIMARY COSMIC-RAY NUCLEI

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ABSTRACT

A statistical method is developed for analyzing the angular distribution of meson jets produced by heavy primary cosmic-ray nuclei, enabling detection and elimination of the proton jet due to evaporation of the residual incident nucleus. The procedure is applied to 50 high-energy collisions of heavy primary nuclei. It is shown that a significant fraction of the heavy primaries produce bi-modal meson jets consistent with the two-centre model.

I. INTRODUCTION

Recent observations by the Polish-Zech group\(^1\) have brought evidence for double-maximum angular distributions of secondaries from meson jets. Thus, for the first time, experimental proof was brought forward for the consistency of theoretical models describing the process of meson production at the collision of two very high-energy nucleons by the formation of two "hot centers" emitting independently mesons, in relative motion with respect to the center of mass system.\(^2\)-\(^4\) Although the statistics gathered up to date as to nucleon-nucleon interactions is relatively poor, the bimodality of such jets has been recently proved beyond any statistical doubt.\(^5\)-\(^8\) These features of high energy jets offer prospects for a direct use of the results obtained by means of the two center-model in the study of nucleon structure. Whereas the first attempts in this direction treated the meson field of the nucleon as some sort of a continuous medium,\(^7\)-\(^8\) recent results at lower energies\(^9\)-\(^14\) hinted at the existence of structural elements, \textit{viz.}, appearance at the moment of the collision of quasi-free pions in the meson cloud surrounding the colliding nucleons.

Similar results have been obtained in cloud-chamber experiments for proton and pion induced jets, at energies around 100 Bev.\(^15\)
As far back as 1957 one of the authors\textsuperscript{16} suggested that high energy heavy nuclei of the primary cosmic radiation have to be preferred as compared to individual nucleons as jet-producing particles in such a study for the following reasons:

(a) A heavy primary can be considered (at energies sufficiently high to neglect the binding energies in complex nuclei) as a monokinetic beam of nucleons. If such a "beam" interacts with a complex nucleus, the occurrence of quite different types of collisions (e.g., central or peripheric) may show up in some way or other in the angular distribution.

(b) Due to its characteristic appearance, a heavy nucleus can be easily identified as a primary particle, in distinction to \( Z = 1 \) particles, whose identification is usually ambiguous.

(c) Detection of heavy primary induced jets can be made easily in an unbiased way by following along the track.

These advantages of the heavy primary induced jets are partly counter-balanced by a spurious effect, viz., partial or total evaporation of that part of the incident nucleus, which has not been directly concerned with meson production. In those cases in which this evaporation leads to a relatively large number of protons which cannot be readily distinguished from the mesons of the jet, the protons can easily distort the apparent angular distribution of the meson jets. Thus, e.g., the superposition of the narrow proton beam, due to the evaporation of the residual heavy primary nucleus, over a meson jet distributed isotropically in its own center of mass system, may give the whole jet the appearance of a double maximum angular distribution, characteristic for the two center model.

The present investigation is concerned with a study of the angular distributions characteristic for jets induced by heavy primary nuclei; after eliminating from such events the influence of the evaporation jets, a picture of the interaction is obtained, which contains a greater wealth of information than is the case of jets induced by individual nucleons or pions.

Anticipating our final conclusions we wish to state here that we succeeded in developing a new method for the detection of the evaporation jet; in this way we were able to show that in some cases the angular distributions of secondaries from interactions of high energy heavy primary nuclei, with the nuclei of photographic emulsion, have a polimodal shape, consistent with that observed in interactions produced by particles of unit charge.

Some preliminary results of this investigation have been published previously.\textsuperscript{17–19}
II. ANGULAR DISTRIBUTION OF THE EVAPORATION JET

Let $\gamma_0$ and $Z$ be the energy per nucleon and the charge of a primary nucleus measured in the laboratory system (L system, see Appendix I), interacting with an emulsion nucleus. Part of the incident nucleus interacts with the target nucleus, while the rest of the primary nucleus is evaporated isotropically in the frame of reference in which the incident nucleus is at rest (the anti-laboratory system denoted hereafter by $\overline{L}$, see Appendix I). The energy distribution of the protons evaporated in this system is well known from measurements on stars observed in photographic emulsion (the mirror picture of the evaporation of the incident nucleus) (for numerical values see e.g., 20).

As is well known the Lorentz transform of angles from $\overline{L}$ to the L system is:

$$\gamma_0 \sin \psi_L = \frac{\beta_0}{\beta_L + \cos \psi_L}$$

(1)

where $\psi_L$ is the emission angle of the evaporated particles with respect to the direction of motion of the incident nucleus in the L system,

$\psi_L$ is the corresponding angle in $\overline{L}$,

$\gamma_0 \equiv (1 - \beta_0^2)^{-\frac{1}{2}}$ and

$\beta_L$ is the velocity of motion of the evaporated particle in the $\overline{L}$ system.*

By means of the above transform, an estimate can be obtained for $\gamma_0$ in the case of pure evaporation (without meson production) using the approximate formula of Peters21:

$$\gamma_0 \approx \frac{\text{const.}}{\sqrt{\psi_L^2}}$$

(2)

where $\cos \psi_L$ is neglected as compared to the ratio $\beta_0/\beta_L$, and only an average value for $\beta_L$ is used.

The use of this formula is nevertheless limited by the existence of a relatively long high energy tail in the energy spectrum of the evaporation protons,20 which may lead to relatively large transverse momenta and hence destroy the clear delimitation of the evaporation and the meson jets.

* Hereafter $c = 1$. 
In order to obtain a more complete picture of the evaporation jet, it appears useful to characterize its angular distribution in the L system by the variance:

\[ S_{ev} = \left[ \frac{1}{n_{ev}} - 1 \right] \sum (\log \cot \psi_L - \overline{\log \cot \psi_L})^2 \]  

(3)

where \( n_{ev} \) is the number of protons of the evaporation jet. It is well known that in the case of the meson jets the angles \( \theta_L \) of the mesons with respect to the primary direction have such a distribution that the quantity \( \log \cot \theta_L \) is distributed approximately normally; in the case of isotropy in the meson center of mass system (C system, see Appendix I) the corresponding variance \( s_{mes} \) has an expectation value \( \sigma_{mes} \approx 0,39. \uparrow \)

The normal distribution of \( \log \cot \theta_L \) in the L system is a direct consequence of the approximation \( \beta_{\pi'}/\beta_c \approx 1 \) in the expression analogous to equation (1) written for mesons of velocity \( \beta_{\pi'} \) in the C system. This equality is violated in a flagrant manner by the proton and \( \alpha \)-particle velocities in the \( \bar{L} \) system. This led us to the idea that may be the expectation value \( \sigma_{ev} \) of \( S_{ev} \) is also sensibly different from \( \sigma_{mes} \); if so, this fact should in principle allow a clear detection of the evaporation jet.

In order to get an answer to the statistical problems raised by the existence of the evaporation jet, we used a Monte-Carlo procedure, whose block diagram is shown in Fig. 1 (see Appendix II, for a detailed description). The energy spectrum of the protons has been taken from reference (30). The value of \( \gamma_0 \) is irrelevant for the estimation of \( S_{ev} \) as long as \( \gamma_0 \) is large enough (\( \beta_0 \approx 1 \)).

In view of the large frequency of incident carbon nuclei, we simulated evaporation jets consisting of 6 protons; in order to get an idea about the influence of fluctuations in very small samples, we simulated also evaporation jets consisting of 3 protons each. The results of the estimation of \( S_{ev} \) for these two classes of artificial evaporation jets are given in Fig. 2;

\[ \text{In decimal logarithms.} \]
Jets Produced by Heavy Primary Cosmic-Ray Nuclei

The distribution of the estimate $s_{ev}$ for artificial jets consisting of 6 and 3 evaporation protons, respectively.

The corresponding average values are:

$$s_{ev} = 0.18 \pm 0.03 \quad \text{for } Z = 6.$$  

$$s_{ev} = 0.17 \pm 0.03 \quad \text{for } Z \approx 3. \quad (4)$$

In the above computation we used the same approximation as in Peters' formula, viz., $\cos \theta_L \ll \beta_0 / \beta_L$. We also applied the correct transformation formula to a restricted sample of events and obtained only negligible differences as compared to the approximate calculation.

Thus our results show that the evaporation jet has, in the L system, an angular distribution of much narrower shape than the angular distribution of a meson jet with the same Lorentz factor; on a Duller-Walker diagram this is equivalent to a straight line much stronger inclined, with respect to the abscissa axis (slope $\approx 4$). This feature of the evaporation jet has been used in the following for separating it from the meson jet.

III. EXPERIMENTAL

The jets studied in the present work have been collected in three stacks of nuclear emulsions, exposed in the 1953 Sardinian expedition, and in the 1957 and 1958 Po-Valley expeditions respectively.

The $\alpha$-particle induced jets have been obtained by area scanning, while those induced by primaries of $Z \geq 3$ (H P jets) by following along the track heavy nuclei detected by area scanning in the marginal plates.

The charge distribution of the primaries is shown in Fig. 3. All charges have been estimated from $\delta$-ray densities in the course of another investigation at present under way in our laboratory. The jets accepted for measurement had to show a visible collimation and had a median track length per plate of $\sim 3000 \mu$. 
The spatial angles \( \phi_L \) of the secondaries of unit charge with respect to the primary direction have been obtained by co-ordinate measurements on Koristka MS-2 and Leitz-Ortholux microscopes.

The following analysis concerns 50 interactions (17-\( \alpha \) and 33 H P jets). The total number of incident nucleons was 546 and the total number of relativistic secondaries 1158. For each jet we computed the Lorentz factor \( \gamma_c \) of the center of mass system (assuming that all particles have been produced from a single center of mass) by means of the Castagnoli formula \(^{23}\); this formula was corrected for the non-monoenergetic spectrum of mesons in the C system according to references \(^{24-25}\).

The degree of anisotropy of each jet has been characterised as usual by the parameter:

\[
S = \left[ \frac{1}{n_s - 1} \sum (\log \ctg \phi_L - \bar{\log \ctg \phi_L})^2 \right]^{\frac{1}{2}}
\]  

(5)

where \( n_s \) is the number of secondary particles of unit charge.

The values of \( \gamma_c \) spanned the range 2–15, corresponding to energies between 10–500 Bev per incident nucleon. The median energy was approximately 35 Bev per nucleon.

IV. RESULTS

For each jet we constructed the integral Duller-Walker diagram \(^{22}\) and the corresponding differential distribution of the quantity \( \log \ctg \phi_L \). According to the shape of these diagrams the jets have been divided into three classes, viz.,

(A) Unimodal jets.

(B) Bimodal ones.

(C) Jets with a complex structure showing up several maxima in the differential distribution.

\( \dagger \) No matter whether they were mesons or evaporation protons.
Typical examples of such jets, all produced by carbon nuclei, are shown in Figs. 4a, b, c which illustrate the great diversity of types realised by one and the same primary nucleus.

Fig. 4. Typical jets with (a) one center \((7 + 9\ \text{C})\); (b) two centers \((3 + 15\ \text{C})\); (c) three or more centers \((13 + 181\ \text{C})\).
The detailed angular distributions of all our jets are given in Figs. 5a, b and c.

**Class A**

\[ n_{4/2} > 5 \]

<table>
<thead>
<tr>
<th>Type</th>
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<tbody>
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<td>3 + 12</td>
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</tr>
<tr>
<td>9 + 11</td>
<td>2</td>
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<td>3 + 11</td>
<td>2</td>
</tr>
<tr>
<td>6 + 12</td>
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</tr>
<tr>
<td>17 + 36</td>
<td>6</td>
</tr>
<tr>
<td>10 + 12</td>
<td>2</td>
</tr>
<tr>
<td>14 + 4</td>
<td>2</td>
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</tbody>
</table>

\[ n_{3/2} < 5 \]

<table>
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</tr>
<tr>
<td>2 + 9</td>
<td>4</td>
</tr>
<tr>
<td>7 + 11</td>
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<td>4 + 12</td>
<td>5</td>
</tr>
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<td>4 + 6</td>
<td>3</td>
</tr>
<tr>
<td>7 + 10</td>
<td>6</td>
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<td>9 + 11</td>
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<td>4 + 7</td>
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<tr>
<td>5 + 7</td>
<td>2</td>
</tr>
<tr>
<td>3 + 15</td>
<td>10</td>
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</tbody>
</table>

**Fig. 5a**
Class B
\[ \frac{n_s}{z} > 5 \]

\[ \frac{n_s}{z} < 5 \]

FIG. 5b
Class C
$n_f/Z > 5$

$n_f/Z < 5$

**Fig. 5c**

**Fig. 5.** Detailed angular distributions in log $\text{ctg} \phi$ co-ordinates of all the 50 jets analysed in the present work.
(1) First we analysed the jets with a double maximum shape. As already stated we were here faced with the problem of the superposition of the evaporation jet on a single-maximum meson jet, as an explanation for the observed bimodal shape. We chose as a criterium for distinguishing between the two possible types of interactions (with, and without evaporation jet) the ratio $n_s/Z$. The distribution of the observed values of this ratio is shown in Fig. 6, in which there appear two distinct groups of jets characterized by either very large or very low values of the ratio $n_s/Z$.

![Fig. 6. The distribution of the ratio $n_s/Z$ for experimental bimodal jets.](image)

The average multiplicity per incident nucleon of the jets with $n_s/Z > 5$ (Table I) is in good agreement with the results obtained for protons of comparable energy. The jets with $n_s/Z < 5$ have a low average multiplicity per incident nucleon, of the same order as that observed in accelerator experiments at energies below 10 Bev, although the average values of the Lorentz factor $\gamma_e$ for the two groups of jets are very near to one another (see Table I).

This contradiction can be explained in the easiest way by the assumption that in the jets with $n_s/Z < 5$ the estimate $\log \cot \phi_L$ for the Lorentz factor, is distorted (enhanced) most probably by the influence of the evaporation jet. In order to check this assumption we studied the angular distribution cumulated from all the corresponding jets in reduced co-ordinates $S$:

$$t = \frac{\log \cot \phi_L - \log \cot \phi_L}{S}$$

separately for jets of each of the two classes (Fig. 7a, b).

For each of these distributions we computed by means of a Monte-Carlo procedure a group of artificial jets under the following assumptions:

(a) the meson jet is unimodal and has an isotropic distribution in its own center of mass system (C system, see Appendix I). In the L system this jet appears as the wide cone with a Lorentz factor $\gamma_-$.

(b) the narrow cone is due to the evaporation of the incident nucleus so that its Lorentz factor $\gamma_+$ is actually a measure of $\gamma_0$ (i.e., of the velocity of the L system). The number of tracks chosen for simulating the evaporation...
<table>
<thead>
<tr>
<th></th>
<th>( n_s/Z &gt; 5 )</th>
<th>( n_s/Z &lt; 5 )</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>( N )</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>( Z = 2 )</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>( Z \geq 3 )</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>( \overline{Z} )</td>
<td>2.6 ± 0.5</td>
<td>2.8 ± 0.4</td>
</tr>
<tr>
<td>( \overline{\rho} )</td>
<td>15.4 ± 3.2</td>
<td>24.5 ± 2.7</td>
</tr>
<tr>
<td>( \overline{N} )</td>
<td>8.4 ± 1.8</td>
<td>6.6 ± 1.6</td>
</tr>
<tr>
<td>( \overline{n_s/Z} )</td>
<td>6.0 ± 0.2</td>
<td>8.6 ± 0.8</td>
</tr>
<tr>
<td>( Z )</td>
<td>18</td>
<td>34</td>
</tr>
<tr>
<td>( Z_{ee} )</td>
<td>2</td>
<td>16</td>
</tr>
<tr>
<td>2 (( Z - Z_{ee} ))</td>
<td>32</td>
<td>36</td>
</tr>
<tr>
<td>( nπ/nucleon )</td>
<td>( nπ/nucleon )</td>
<td>( \bar{\gamma} )</td>
</tr>
<tr>
<td>----------------</td>
<td>----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>( 108 )</td>
<td>( 294 )</td>
<td>( 334 )</td>
</tr>
<tr>
<td>( 3.4 \pm 0.1 )</td>
<td>( 7.2 \pm 0.2 )</td>
<td>( 9.2 \pm 0.5 )</td>
</tr>
<tr>
<td>( 4.0 \pm 0.8 )</td>
<td>( 5.6 \pm 0.8 )</td>
<td>( 5.3 \pm 0.7 )</td>
</tr>
<tr>
<td>( 14.4 \pm 3.7 )</td>
<td>( 25.3 \pm 7.0 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( 2.1 \pm 0.2 )</td>
<td>( 1.6 \pm 0.3 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( 5.9 \pm 0.8 )</td>
<td>( 17.8 \pm 5.5 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( 1.39 \pm 0.08 )</td>
<td>( 2.10 \pm 0.29 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( 0.52 \pm 0.10 )</td>
<td>( 0.49 \pm 0.08 )</td>
<td>( 0.54 \pm 0.05 )</td>
</tr>
<tr>
<td>( 0.15 \pm 0.03 )</td>
<td>( 0.02 \pm 0.07 )</td>
<td>( \ldots )</td>
</tr>
<tr>
<td>( 40 \pm 13 )</td>
<td>( 77 \pm 22 )</td>
<td>( 61 \pm 18 )</td>
</tr>
</tbody>
</table>
The angular distribution (in reduced co-ordinates) of bimodal experimental jets (full line) and artificial jets from a mixed population (dotted line). (a) Jets with $n_s/Z > 5$; (b) jets with $n_s/Z < 5$.

The block diagram of the Monte-Carlo calculation is given in Fig. 8, and the procedure is described in Appendix II. The unimodal meson jets were simulated by means of a procedure described in a previous paper. Then we superposed an evaporation jet constructed under the exaggerated assumption that it contains exactly $Z$ protons. Actually both initial assumptions (a and b) are exaggerated in so far as they enhance the influence of the evaporation jet in the angular distribution of the resulting jet.

The angular distribution of these artificial jets is shown by a dotted line in Fig. 7. As can be seen for $n_s/Z < 5$ the experimental and the artificial distribution are in good agreement, while for $n_s/Z > 5$ there appears a strong discrepancy. The consistency of each pair of distributions was checked by means of Pearson tests, the results of which are given in Table II. These results prove that interactions with $n_s/Z < 5$ are mostly
evaporation jets superposed on unimodal meson jets, while events with $n_s/Z > 5$ are mostly true bimodal jets.

It is interesting to note that as ought to be expected, the average charge of the incident nuclei is sensibly larger for the group $n_s/Z < 5$ (Table I).

For each experimental jet with $n_s/Z < 5$ we computed the quantity $s_{ev}$ separately for the narrow cone. As was to be expected the average value of $s$ coincides with that obtained by the Monte-Carlo method for artificial evaporation jets, viz.,

$$s = 0.17 \pm 0.03.$$

Now, the true bimodal jets ($n_s/Z > 5$) have been analysed from the point of view of the two center model. Before processing the experimental data we eliminated those (few) tracks which most probably were evaporation protons. For this purpose we computed for each jet observed in experiment three artificial evaporation jets with the same $\gamma_0$ and $Z$ as the experimental jet. The whole jet was processed as a meson jet only if not one single track of the experimental jet fell inside the angular interval covered by the whole of the artificial evaporation jets. Otherwise all tracks falling inside the doubtful interval were eliminated. It is easily seen that in this way the bias introduced has only a tendency to underestimate the true bimodality.
Table II

Consistency tests: $\chi^2$ and $P_{\chi^2}$ values; $n' = 9$ except for values denoted by an asterisc where $n' = 4$

<table>
<thead>
<tr>
<th>Experimental jets</th>
<th>Artifical jets from mixed population</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test against normality</td>
</tr>
<tr>
<td>( n_s/Z &gt; 5 )</td>
<td>A</td>
</tr>
<tr>
<td></td>
<td>B</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Artificial jets from a truly bimodal population</th>
<th>Test against normality</th>
<th>Mutual consistency with experimental jets of class B (( n_s/Z &gt; 5 ))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4*</td>
<td>6*</td>
</tr>
<tr>
<td></td>
<td>41%</td>
<td>20%</td>
</tr>
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</table>

In a previous paper it has been shown that the interpretation of the two centers as the result of a pair of cuasi-independent peripheric collisions between nucleon cores and pions of the meson cloud of the collision partners, leads to a well-defined relationship between the Lorentz factors $\gamma_+$ and $\gamma_-$ of the two cones; this interpretation allowed also computation of the Lorentz factor $\tilde{\gamma}$ of relative motion of the two centers with respect to the
C system and implicitly the computation of $\sigma$. Table III gives the result of the corresponding computation and shows the values of $\bar{\gamma}$, $\sigma$, $\gamma_+ / \gamma_-$ and $D$ (the bimodality coefficient defined in reference (6)). As can be seen within the experimental errors, good agreement between our results and the theoretical expectation is obtained. It should nevertheless be stressed that just because of the relatively low energy of our primaries the expected effects in the case of each of the estimated parameters are relatively low and hence considerably larger statistics than the ones available here are necessary in order to prove this effect beyond any statistical doubt.

**Table III**

Comparison of the true bimodal jet parameters with the values expected from the pion-exchange two center model (in brackets results of the Monte-Carlo computation)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Experiment</th>
<th>Theoretical expectations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{\gamma}$</td>
<td>$1.39 \pm 0.08$</td>
<td>$1.41$</td>
</tr>
<tr>
<td>$\gamma_+ / \gamma_-$</td>
<td>$5.9 \pm 0.8$</td>
<td>$5.9$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>$0.49 \pm 0.08$</td>
<td>$0.54$</td>
</tr>
<tr>
<td>$D$</td>
<td>$0.15 \pm 0.03$</td>
<td>$0.09$ $^{(0.11 \pm 0.06)}$</td>
</tr>
</tbody>
</table>

As a supplementary check of the reality of the bimodal jets ($n_d/Z > 5$) we computed by means of a Monte-Carlo procedure a sample of artificial bimodal jets of the same multiplicity as the experimental ones. (The block diagram is given in Fig. 9 and the detailed description is again in Appendix II).

![Fig. 9. Block diagram of the Monte-Carlo procedure for simulating true bimodal meson jets.](image-url)
The angular distribution of these artificial jets is compared in Fig. 10 with the experimental data; the mutual consistency of these distributions has been checked by means of a Pearson test, the result of which is also given in Table II.

Table III gives also the value of the bimodality coefficient $D$ of the artificial jets ($n_s/Z > 5$) which is in as good agreement with the theoretical expectations, as were the experimental data.

(2) As already stated in § 1, collisions of heavy primary nuclei with emulsion nuclei offer favourable conditions for the simultaneous observation of peripheral and central collisions in one and the same jet, in view of the larger number of participating nucleons. As was the case in accelerator experiments⁹ one might expect here again an angular distribution structured into several maxima. The extreme maxima are expected to be due to peripheral collisions ($\pi - N$ and $N - \pi$), while $N - N$, $N - 2N$ and $\pi - \pi$ collisions should be located at intermediate positions. This kind of angular distribution (with several maxima) has been actually observed in the jets of class C, characterized by high values of the incident charge as well as by a larger number of evaporation tracks from the target nucleus (see Table I).

Here again one had to solve first the problem of the influence of the evaporation jet; actually the events of this class could also be divided into two subclasses according to the value of the ratio $n_s/Z$. Then the events were processed in an analogous way to the one used for the bimodal jets. The angular distribution in reduced co-ordinates⁵ is shown in Fig. 11 a, b together with the distribution of the corresponding artificial jets. The mutual consistency of these distributions has been checked by Pearson tests, the results of which are given in Table II. These results show that the
class C jets, with \( n_8/Z > 5 \), cannot be interpreted as a superposition of an evaporation jet on a bimodal meson jet as well.

\[ \text{Fig. 11. Angular distribution (in reduced co-ordinates) for experimental and artificial jets of class C (full line experimental jets, dotted line artificial ones). (a) Jets with } n_8/Z > 5; \]
\[ (b) \text{ jets with } n_8/Z < 5. \]

This characteristic feature of the class C jets (the appearance of several maxima in the differential angular distribution) appears especially clearly in a very high multiplicity jet (13 + 181 C, Fig. 12). Similar results have been obtained recently on jets of comparably high multiplicity.\(^{29-30}\)

\[ \text{Fig. 12. Angular distribution (in reduced co-ordinates) of the } 13 + 181 \text{ C jet.} \]
(3) The angular distribution of the apparently unimodal jets (class A) is given in Fig. 13 a, b. Here the Pearson tests (Table II) show that the angular distribution in the C system cannot be distinguished from an iso-

![Diagram](image)

**Fig. 13.** Angular distribution (in reduced co-ordinates) of unimodal experimental jets. (a) Jets with $n_d/Z > 5$; (b) jets with $n_d/Z < 5$.

tropic one (see Fig. 14). Once again dividing the jets according to the criterion $n_d/Z$ it appeared that some jets were built up almost entirely from evaporation protons. The very low multiplicity of these jets and the rela-

![Diagram](image)

**Fig. 14.** Angular distribution (in reduced co-ordinates) of all unimodal jets (dotted line —normal distribution).

tively low value of the Lorentz factor $\gamma_c$ show that most probably the events of class A are a mixture of the following types of events:

(a) Pure evaporation jets.

(b) Unimodal meson jets due to medium energy central collisions.
(c) Bimodal meson jets of such low energy that the two cones cannot be distinguished.

(d) Mixed events.

For the class A events any clear distinction between these types of interactions is practically impossible and hence a more detailed analysis of these jets does not appear to be justified.

V. CONCLUSIONS

Our results show that the main obstacle in the way of any use of the heavy primary induced jets for the study of the elementary process, viz., the presence of the evaporation jet of the incident nucleus, can be eliminated in a significant manner by means of the pair of criteria \( n_s/Z \) and \( s_{ev} \). Thus there appears the possibility of studying with better statistics in an unbiased manner peripheral nucleon-nucleon interactions at high energies by means of heavy primary nuclei. The results obtained in the present work as to the true bimodal jets—which should be considered as preliminary in view of their relatively poor statistics—are in qualitative agreement with the results obtained up to now in nucleon-nucleon collisions.

VI. ACKNOWLEDGMENTS

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VII. REFERENCES


26. Hansen, L. F. and Fretter, W. B.  

27. Friedländer, E. M.  

28. ———  


30. Malhotra, P. K. and Tsuzuki, Y.  
APPENDIX I
REFERENCE FRAMES

A. Before collision

\[ \text{primary nucleus} \quad \xrightarrow{v_0} \quad \text{target nucleus} \]

\[ \text{L system} \quad \xrightarrow{v_0} \quad \text{L system} \]

APP. I, 1

B. After collision

1. Pure evaporation of the two nuclei seen from the L and \( \bar{L} \) systems.

\[ \text{L system} \quad \xrightarrow{v_0} \quad \text{L system} \]

APP. I, 2

2. Unimodal meson jet (c.m.s. isotropical, full line) + evaporation jet (dotted line).

\[ \text{Fire ball} \quad \xrightarrow{v_0} \quad \text{The evaporated part of the primary nucleus} \]

\[ \text{C system} \quad \xrightarrow{v_0} \quad \text{L system} \]

APP. I, 3
3. Pure bimodal meson jet (no accompanying evaporation).

\[ \vec{r}_L \]

\[ \vec{r}_C \]

\[ c \] system

\[ L \] system

APPENDIX II

MONTE-CARLO PROCEDURE

I. Simulation of the evaporation jet

In order to obtain the values of \( s_{ev} \) in artificial jets we proceeded as follows:

The emission angle \( \psi_L \) has been chosen from an isotropical population (uniform in \( \cos \psi_L \)). A second random number gave \( \psi_L \) the integral energy distributions of the protons (computed from the data of reference (20)) the value of \( \beta_L \). These two values were then used in the standard Lorentz transform for angles and the values of \( \psi_L \) obtained in this way were treated exactly as any experimentally obtained ones. (Eq. 3) for the computation of \( s_{ev} \).

II. Simulation of mixed jets (superposition of an evaporation jet on an unimodal meson jet)

For each double maximum experimental jet, an artificial jet of the same multiplicity was computed. The evaporation jet was constructed according to Fig. 8 (see also preceding point). As initial data we took from experiment the values of \( Z \) (the maximum number of the evaporation protons) and the Lorentz factor \( \gamma_\perp \) (of the wide cone) assumed to pertain to the meson jet. \( \gamma_\perp \) was used as well as a measure for \( \gamma_0 \) (Eq. 7). The transform \( \psi_L \rightarrow \psi_L \) was thus performed by means of \( \gamma_0 \); the \( (n_s - Z) \) mesons have been chosen from an isotropic distribution in \( C \). (\( \log \text{ctg} \theta_L \) has been chosen from a normal population of zero mean and variance 0.39) and displaced by \( \gamma_\perp \) for the transform to the \( L \) system.
III. Simulation of pure bimodal meson jets

The procedure has been exactly the same as the one used in reference (27). The experimental values yielded the multiplicity $n_s$ and the Lorentz factor $\gamma_c$ of the center of mass system. We assumed validity of the cloud collision two center model\textsuperscript{28} which leads at the relatively low energies studied here to a Lorentz factor $\gamma \approx 1.4$. By means of this value we computed the separation $2\Delta$ of the two centers in a log $\log\phi_L$ scale, which was used to displace the isotropic forward jet by $\log\gamma_c + \alpha$ and the isotropic backward jet by $\log\gamma_c - \alpha$.\textsuperscript{29}