INVESTIGATION OF SECONDARY FLOW ON AN INFINITE YAWED CYLINDER WITH 15 PER CENT. THICK JOUKOWSKI PROFILE AS SECTION

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ABSTRACT

This paper contains a theoretical investigation of the secondary flow in the laminar incompressible boundary layer on an infinite yawed cylinder with chordwise section as Joukowski profile of 15 per cent. thickness at zero incidence and with homogeneous suction, the suction mass flow coefficient being equal to 0.2085. The secondary flow profiles are obtained at different points of the wing section and for various angles of sweepback. It is found that in favourable pressure gradients and at pressure minimum, the secondary flow profiles have negative values. In regions of adverse pressure gradients after the pressure minimum the secondary flow changes sign from negative to positive values and have points of inflexion. The change of sign starts from the surface and extends to the edge of the boundary layer downstream. At some points in adverse pressure gradients the secondary flow profiles have double points of inflexion and values of both signs simultaneously. It is also found that an adverse pressure gradient produces more powerful secondary flow than a favourable pressure gradient of the same strength.

INTRODUCTION

On an infinite yawed cylinder the boundary layer flow is three dimensional. In the presence of a pressure gradient the chordwise potential velocity varies, whereas the spanwise potential velocity is constant. Therefore, the path of the streamline becomes curved. This path becomes a straight line in the case of a flat plate. In order to maintain this curved potential streamline, there exists a pressure gradient normal to it, which is balanced by the centrifugal force acting on the fluid particle. Due to the boundary layer approximation,
the pressure field of the potential flow is transmitted unchanged through the boundary layer. Under this curved potential streamline as the boundary is approached, velocities of the fluid particles in planes parallel to the span continuously decrease to become zero on the boundary. Consequently the streamlines increase their curvature in planes parallel to the span in order to restore the centrifugal force to its required value. In this process, fluid flows towards the concave side of the potential streamline. At any point in the boundary layer, the velocity component in plane parallel to the span is composed of two components, one parallel to the potential streamline direction, and the other normal to it. The velocity component parallel to the potential streamline direction is called the 'Primary flow', and the velocity component normal to it is called the 'Secondary flow'.

Sowerby\(^2\) has shown analytically that in a three-dimensional boundary layer there must be secondary flow unless the streamlines are straight. He and Loos\(^3\) obtained the solution for the secondary flow in the boundary layer on a plate, where the potential streamlines are parabolic.

The secondary flow on a sweptback wing is of considerable importance, because it has been associated with the inflexional type of instability causing early transition near the leading edge.

In the present paper the approximate numerical solutions of the boundary layer are used to investigate the secondary flow on an infinite yawed symmetrical Joukowski profile of 15 per cent. thickness, at zero incidence and with homogeneous suction, the suction mass flow coefficient being equal to 0.2085.

**Symbols**

\(x, y, z\) = Co-ordinates measured in the tangential direction, spanwise direction and normal to the surface respectively (Fig. 1).

\(u, v, w\) = Velocity components in the boundary layer in \(x, y\) and \(z\) directions respectively.

\(U\) = Chordwise velocity at the edge of the boundary layer.

\(U_0\) = Chordwise component in the free stream.

\(V_0\) = Spanwise component in the free stream.

\(U_1\) = \((U^2 + V_0^2)^{1/2}\) = Resultant velocity parallel to the boundary at the edge of the boundary layer.

\(\bar{U}\) = \(U/U_0\) = Non-dimensional chordwise velocity distribution in the potential flow.
\[ \nu = \text{Kinematic viscosity.} \]

\[ \Lambda = \text{Angle of sweepback.} \]

\[ \theta_e = \tan^{-1}(V_0/U) = \text{Inclination of the edge streamline to the chordwise direction.} \]

\[ v_1 = \text{Velocity component of the secondary flow in the boundary layer.} \]

\[ v_1/U_1 = \text{Non-dimensional velocity of the secondary flow.} \]

\[ k_x = \text{Form parameter for the chordwise profile.} \]

\[ k_y = \text{Form parameter for the spanwise profile.} \]

\[ \theta_x = \text{Momentum thickness of the chordwise boundary layer.} \]

\[ \theta_y = \text{Momentum thickness of the spanwise boundary layer.} \]

\[ \delta_x = \text{A measure of the chordwise boundary layer thickness.} \]

\[ \delta_y = \text{A measure of the spanwise boundary layer thickness.} \]

\[ g_x(k_x) = \frac{\theta_x}{\delta_x} = \text{Ratio of the chordwise momentum thickness to the measure of the chordwise boundary layer thickness.} \]

\[ g_y(k_y) = \frac{\theta_y}{\delta_y} = \text{Ratio of the spanwise momentum thickness to the measure of the spanwise boundary layer thickness.} \]

\[ H_x = \frac{\delta_x}{\theta_x} = \text{Shape factor for the chordwise profile in the boundary layer.} \]

\[ H_y = \frac{\delta_y}{\theta_y} = \text{Shape factor for the spanwise profile in the boundary layer.} \]

\[ H_t = \text{A variable to denote } H_x \text{ or } H_y, \text{ equation (1.6).} \]

\[ K_t = \text{A variable to denote } K_x \text{ or } K_y, \text{ equation (1.6).} \]

\[ g_t = \text{A variable to denote } g_x \text{ or } g_y, \text{ equation (1.7).} \]

\[ c = \text{Length of wing chord.} \]

\[ \bar{\theta}_x = \frac{\theta_x (U_0/cv)^4}{\text{Non-dimensional chordwise momentum thickness.}} \]

\[ \zeta = \frac{z (U_0/cv)^4}{\text{A non-dimensional variable in the case of an airfoil.}} \]

\[ q = \frac{\theta_y}{\theta_x} = \text{Ratio of the momentum thickness of the boundary layer in the spanwise and chordwise directions.} \]

\[ \eta_x = \frac{z}{\delta_x} = \text{A non-dimensional variable.} \]

\[ \eta_y = \frac{z}{\delta_y} = \text{A non-dimensional variable,} \]
Method for the Calculation of the Secondary Flow

A method for the calculation of the secondary flow has been developed by Sinha. He has shown that the secondary flow profiles are given by

\[ \frac{v}{\tilde{U}_0} = \frac{1}{2} \sin 2\theta_e \left( \frac{v}{V_0} - \frac{u}{\tilde{U}} \right) \]  

(1.1)

where,

\[ \frac{u}{\tilde{U}} = \text{Chordwise profile} \]

and

\[ \frac{v}{\tilde{V}_0} = \text{Spanwise profile}. \]

Using Schlichting's approximate profiles, the chordwise profile is given by

\[ \frac{u}{\tilde{U}} = (1 + K_x) (1 - e^{-\eta_x}) - K_x \sin (\pi/6 \eta_x), \quad 0 \leq \eta_x \leq 3, \]

\[ = 1 - (1 + K_x) e^{-\eta_x}, \quad \eta_x \geq 3 \]  

(1.2)

where,

\[ \eta_x = \left\{ \frac{g_x(K_x)}{\tilde{\theta}_x} \right\} \cdot \zeta. \]  

(1.3)

Similarly, the spanwise profile is given by

\[ \frac{v}{\tilde{V}_0} = (1 + K_y) (1 - e^{-\eta_y}) - K_y \sin [(\pi/6) \eta_y], \quad 0 \leq \eta_y \leq 3, \]

\[ = 1 - (1 + K_y) e^{-\eta_y}, \quad \eta_y \geq 3 \]  

(1.4)
where,
\[ \eta_y = \left\{ \frac{g_y(K_y)}{q\theta x} \right\} \cdot \zeta. \] (1.5)

The calculation, then, proceeds as follows:

The chordwise layer being given, \( \eta_x \) and \( H_x \) are known. Hence \( K_x \) is determined from the equation:
\[ 0.02358 H_t K_t^2 - (0.09014 + 0.06656 H_t) K_t + 1 - \frac{1}{2} H_t = 0. \] (1.6)

Consequently \( g_x \) becomes known due to the relation
\[ g_t = \frac{1}{2} + 0.06656 K_t - 0.02358 K_t^2. \] (1.7)

Thus \( \eta_x \) is calculated for given values of \( \zeta \) from the relation (1.3) and \( u/U \) is obtained for those values of \( \zeta \). Similarly the profile \( v/V_0 \) is calculated, because the spanwise layer parameters \( H_y \) and \( q \) are also known, using the same relations (1.6), (1.7) and (1.5).

It is well known that the smallest value of \( H_y \) is 2.0 for the asymptotic suction case. This value is also taken as the lower limit for \( H_x \), although Thwaites has obtained some values of \( H_x \) less than 2. If \( H_t \) equals the minimum value 2.0, the product of the two roots of \( K_t \) becomes zero. One root \( K_t = 0 \) gives the asymptotic suction profile and the other root is positive which is excluded by Schlichting. For all \( H_t \geq 2 \) the product of the two roots of \( K_t \) is always negative, and only the negative roots are permissible. Thus one value of \( H_t \) yields only one relevant value of \( K_t \).

Thus the chordwise profile \( u/U \) and the spanwise profile \( v/V_0 \) both being known, the secondary flow profile can be calculated at any point of the yawed cylinder for given \( \Lambda \) and \( \bar{U} \) using the formula (1.1).

**RESULTS OF CALCULATIONS**

The secondary flow is calculated at points \( x/c = 0.0148, 0.1445 \) and 0.734 for the sweep angles 15°, 30°, 45°, 60° and 75° as shown in Figs. 2, 3 and 4. These secondary flow profiles have also been calculated for sweep angles 15°, 45° and 75° at various points of the wing section as shown in Figs. 5, 6 and 7. Figs. 2, 3 and 4 show the variation of the secondary flow at a particular point for various sweep angles whereas Figs. 5, 6 and 7 show the variation of the secondary flow for a given sweep angle at different points of the wing section. The points on the wing section chosen for the cal-
calculation have been taken to lie in regions of different types of pressure gradients.

Fig. 2. Nature of the secondary flow profiles at $x/c = 0.0148$.

Fig. 3. Nature of the secondary flow profiles at $x/c = 0.1445$. 
DISCUSSION OF THE RESULTS

From Figs. 2 and 3, it is clear that in regions of favourable pressure gradient and pressure minimum, the secondary flow is negative. From Fig. 4, it is seen that in adverse pressure gradients, the secondary flow changes direction and becomes positive. Moreover, it is also seen that
Secondary Flow on an Infinite Yawed Cylinder

Fig. 6. Nature of the secondary flow profiles at $A = 45^\circ$.

Fig. 7. Nature of the secondary flow profiles at $A = 75^\circ$. 
the change in the direction starts after the pressure minimum and from the surface. It is also seen from Fig. 4 that at some points after the pressure minimum, the secondary flow profiles are both positive and negative and have points of inflexion for different sweep angles. From Figs. 5, 6 and 7, it is seen that the magnitude of the secondary flow increases at all points of the wing section with the increase in the sweep angles. These figures also show that at some points of the wing section in regions of adverse pressure gradients, the secondary flow profiles have double points of inflexion.

**CONCLUSIONS**

The following is a brief statement of the conclusions to which the present paper leads:—

(1) The secondary flow is zero when the angle of sweepback is zero [Equation (1.1)].

(2) The magnitude of the secondary flow increases with increasing sweep angle at any point of the wing section.

(3) For zero pressure gradient, there is no secondary flow in the boundary layer on an infinite yawed cylinder.

A pressure gradient, whether favourable or adverse, produces a secondary flow which increases with the magnitude of the pressure gradient.

(4) The secondary flow velocities are all negative in the pressure fall domain.

(5) At pressure minimum also the secondary flow profiles have negative values and the values are not the least here.

(6) The secondary flow profiles begin to change their signs (from negative to positive) a little distance downstream of the pressure minimum. The change begins from the surface and the region of change increases to the edge of the boundary layer.

(7) In regions of adverse pressure gradients, some secondary flow profiles are both positive and negative, have points of inflexion and are unstable.

(8) At some points of the wing section in regions of adverse pressure gradients, the secondary flow profiles have velocities of both signs and double points of inflexion. These are likely to be more unstable.

(9) Ultimately in large adverse pressure gradients, the secondary flow profiles become completely positive and become free from any point of inflexion.
(10) Adverse pressure gradients produce more powerful secondary flows than favourable pressure gradients of the same numerical strength.

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REFERENCES


