NUCLEAR STRUCTURE EFFECT IN THE INTERNAL CONVERSION OF THE 81 Kev. TRANSITION IN Cs^{133}

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INTRODUCTION

IN an earlier publication (Subba Rao, 1961), experimental details and results of the 358 Kev. gamma $\rightarrow$ K-conversion electron of 81 Kev. transition angular correlation were presented together with a brief account of the 358 Kev. gamma $\rightarrow$ 81 Kev. gamma angular correlation. At that time the particle parameters for internal conversion electrons were known from calculations (Biedenharn and Rose, 1953) based on the assumption of the nucleus to be a point. However, as discussed in the earlier work (Subba Rao, 1961), the spin of the 439 Kev state could be inferred to be 3/2 and the mixing amplitudes of the 358 Kev. gamma-ray $\delta_{I} = -0 \cdot 71$ and 81 Kev. gamma ray $\delta_{II} = -0 \cdot 06$ could also be inferred from those measurements. The gamma-K-conversion electron angular correlation could not be utilised directly to deduce the nuclear structure parameter due to the unavailability at that time of nuclear structure-dependent particle parameters. It is the purpose of this paper to present these nuclear structure-dependent particle parameters calculated by the procedure outlined below and then, utilising these particle parameters, to calculate the nuclear structure effect parameter λ which is the ratio of the matrix element in internal conversion due to the penetration of the electron into the region of the distribution of source current (nucleus) to the gamma-ray matrix element.

METHOD OF CALCULATION OF PARTICLE PARAMETERS INCLUDING NUCLEAR STRUCTURE EFFECTS

Internal conversion process is described by second order perturbation theory together with the description of the electron as a Dirac particle in a Thomas-Fermi-Dirac screened potential. By this method the K-shell internal conversion coefficient for an M 1 transition of energy km_0c^2 is written finally in the form,

$$\beta_1^K = \frac{\pi a k}{6} \sum B_{\chi\chi'} \mid R_\chi(m) \mid^2, \tag{1}$$

where $B_{\chi\chi'}$ are parameters characterising the conversion process with initial state specified by quantum numbers χ' and the final continuum states by χ. For the K-shell ($s_{1/2}$) electron $\chi = -1$ and there are two final continuum states after the M1 conversion process, $s_{1/2}(\chi = -1)$ and $d_{3/2}(\chi = +2)$. For the $s_{1/2} \rightarrow s_{1/2}$ part $B_{-1-1} = 2L(L+1)^2 = 8$ and for the $s_{1/2} \rightarrow d_{3/2}$ part $B_{-12} = 2L^2(L+1) = 4$. Here L is the multipolarity of the transition. The $R_\chi(m)$ are the radial integrals:

$$R_\chi(m) = \int_0^\infty h_l\,(f_\chi g_{\chi'} + g_\chi f_{\chi'})\, r^2 dr \tag{2}$$

where h_1 is the spherical Hankel function of the first kind of order kr and f and g are the radial functions. The K-conversion electron parameter for an M1 transition entering the coefficient of the $P_2(\cos\theta)$ term in the angular correlation function $b_2^{(1)}(m)$ is (Biedenharn and Rose, 1953),

$$b_2^{(1)}(m) = 1 - \frac{2\mid 1 - T_m \mid^2}{2 + \mid T_m \mid^2}, \tag{3}$$

where

$$T_m = \frac{e^{i\delta_2}}{e^{i\delta_{-1}}} \frac{R_2(m)}{R_{-1}(m)}. \tag{4}$$

$R_2(m)$ is the radial integral for $s_{1/2} \rightarrow d_{3/2}$ transition and $R_{-1}(m)$ is that for the $s_{1/2} \rightarrow s_{1/2}$ transition, as defined by (2). These two components are characterised by phase factors $e^{i\delta_2}$ and $e^{i\delta_{-1}}$ respectively. By this procedure, under the point nucleus assumption, conversion electron parameters have already been calculated (Biedenharn and Rose, 1953) and these were used in the analysis of the gamma-K-conversion electron angular correlation results reported earlier (Subba Rao, 1961).

The introduction of the static effects of the finite size of the nucleus on the internal conversion process may be represented by writing:

$$R_\chi' = R_\chi - i\epsilon_\chi. \tag{5}$$

The value of ϵ_χ may be calculated by replacing R_χ with R_χ' in using β_1^k (Sliv) instead of the point-nucleus value. The radial integrals for the point-nucleus approximation are obtained from extrapolations of the computed values (Rose, 1958) as in Fig. 1. The accuracy of the radial integrals obtained in this way were tested by evaluating equation (1) and comparing with the point-nucleus value of the K-conversion coefficient ($k = 0{\cdot}158\, m_0c^2$ and

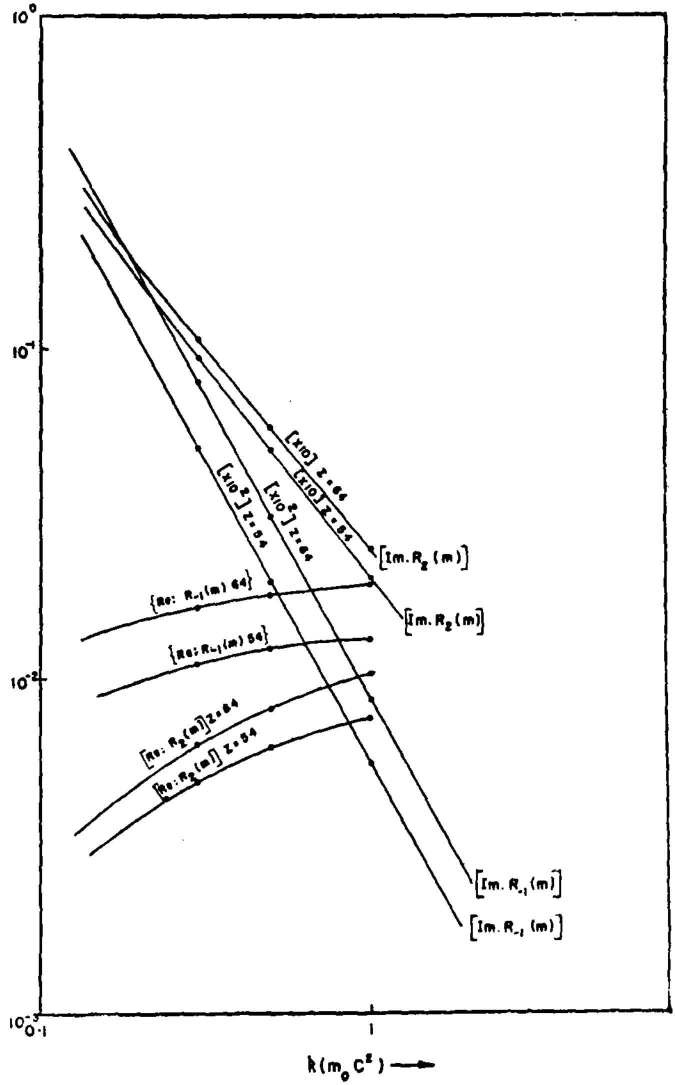

FIG. 1

$Z = 55$) for M1 transition of $1 \cdot 48$. These values of the radial integrals are $R_2(m) = (3 \cdot 3 \times 10^{-3}) + i(2 \cdot 15)$ and $R_{-1}(m) = (9 \cdot 5 \times 10^{-3}) + i(17 \cdot 42)$. With the above-mentioned procedure to take into account the static effects of nuclear structure, $R_{-1}(m) = (9 \cdot 5 \times 10^{-3}) + i(17 \cdot 42 - 0 \cdot 28)$ was deduced so that $\epsilon_{-1} = 0 \cdot 28$ in (5). ϵ_2 is assumed to be zero, since the recognition of the finite nuclear size is known to seriously affect only $s_{1/2}$ electron wave functions and are known to be negligible for the $d_{3/2}$ electron wave functions, because of their lower density inside the nucleus than that of the $s_{1/2}$

electrons. This is further justified because of the smaller contribution of $s_{1/2} \rightarrow d_{3/2}$ component than the $s_{1/2} \rightarrow s_{1/2}$ component of internal conversion as may be seen from an evaluation of the weight factor defined in equation (4).

The calculations of Sliv not only include the effect of a uniform charge distribution over the finite volume of the nucleus on the electron wave functions, but also include the dynamic nuclear structure effects appearing through nuclear currents in an approximate way. Sliv's assumption of uniform distribution of currents over the nuclear surface corresponds to $\lambda = +1$ in the notation of Church and Weneser (1956) defined through the relation,

$$\beta_1^{\mathrm{K}}(\lambda) = \beta_1^{\mathrm{K}}(\lambda = +1)\,[1 - (\lambda - 1)\,\mathrm{C}(\mathrm{Z}, k)]^2, \tag{6}$$

where $\mathrm{C}(\mathrm{Z}, k)$ are parameters depending on the physical variables Z, and k (Church and Weneser, 1956; Church, Rose and Weneser, 1958). These parameters are of such accuracy as to permit an evaluation of the nuclear structure parameters λ to better than 10%, apart from the experimental errors in $\beta_1^{\mathrm{K}}(\lambda)$. From these definitions and concepts, the dynamic structure dependence is introduced into the conversion electron parameters $b_2^{(1)}(m)$ through the radial integrals R''_{-1} which are related to R'_{-1} of (5) by

$$\mathrm{R}''_{-1}(m) = \mathrm{R}'_{-1}(m) - i(\lambda - 1)\,\mathrm{C}(\mathrm{Z}, k) \times \frac{\beta_1^{\mathrm{K}}(\lambda = +1)\sqrt{\frac{3}{4\pi a k}}}{\sqrt{\beta_1^{\mathrm{K}}(\lambda = +1) - \beta_1^{\mathrm{K}}(x = +2)}}\,. \tag{7}$$

Using these values of radial integrals obtained as a function of λ, the particle parameters were calculated from (3) with $\mathrm{R}''_{-1}(m)$ in place of $\mathrm{R}_{-1}(m)$ in (4).

The static effects of finite nuclear size for conversion electron parameter for E 2 transition, $b_2^{(2)}(e)$, were calculated in a way similar to that outlined above for magnetic conversion but are negligibly small perhaps because of the low Z-value (Z = 55) and low energy of the transition.

For this particular transition, the term in the coefficient of $\mathrm{P}_2(\cos\theta)$ term involving the interference parameter b_2 is very small. As a result the small effects of static and dynamic structure effects do not affect it significantly.

RESULTS

The particle parameters $b_2^{(1)}(m)$ calculated in this way are presented as a function of λ in Table I.

TABLE I

Conversion electron parameter for $Z = 55$, $k = 0\cdot158\ m_0c^2$

λ	$b_2^{(1)}(m)$	λ	$b_2^{(1)}(m)$
0	0·0433	..	..
+ 1	0·0435	− 1	0·0429
+ 5	0·0451	− 5	0·0412
+ 10	0·0469	−10	0·0402
+ 20	0·0514	−20	0·0374
+ 50	0·0713	−50	0·0310
+100	0·2008	−70	0·0279
+150	−0·2403	..	..
+200	−0·0752	..	..
+300	−0·0434	..	..

The angular correlation function for the $3/2+ \xrightarrow{M1+E2} 5/2+ \xrightarrow{M1+E2} 7/2+$ cascade is written (Subba Rao, 1961) as:

$$W(\theta;\ \gamma e_\mu^-) = 1 + A_2(\gamma)\,A_2(e_K^-)\,P_2(\cos\theta) + A_4(\gamma)\,A_4(e_K^-)\,P_4(\cos\theta).$$

$$W(\theta;\ \text{exp.}) = 1 + (0\cdot014 \pm 0\cdot008)\,P_2(\cos\theta) + (0\cdot007 \pm 0\cdot011)\,P_4(\cos\theta)$$

$$A_2(\gamma) = \frac{1}{1+\delta_1^2}\,[\delta_1^2\,F_2(22\ 3/2\ 5/2) + 2\delta_1 F_2(12\ 3/2\ 5/2) + F_2(11\ 3/2\ 5/2)]$$

$$= \frac{1}{1\cdot50}\,[0\cdot50F_2(22\ 3/2\ 5/2) - 1\cdot42F_2(12\ 3/2\ 5/2) + F_2(11\ 3/2\ 5/2)]$$

$$= 1\cdot084.$$

$$A_2^{\text{exp.}}(e_K^-) = 0\cdot013 \pm 0\cdot007$$

and

$$A_2(e_K^-) = \frac{1}{1+p^2}\,[p^2 b_2^{(2)}(e)\,F_2(22\;7/2\;5/2) + 2pb_2F_2(12\;7/2\;5/2) + b_2^{(1)}(m)\,F_2(11\;7/2\;5/2)].$$

In the earlier analysis with point-nuclear values, p^2 had been calculated from

$$p^2 = \delta^2\left(\frac{a_2^K}{\beta_1^K}\right)$$

but now the finite nuclear size effects on internal conversion coefficients may be included, so that

$$p = \sqrt{\frac{a_2^K}{\beta_1^K(\lambda = +1)\,[1-(\lambda-1)\,C(Z,K)]^2}}\cdot\delta$$

with $\delta_{II} = -0\cdot065$, $a_2^K = 2\cdot250$ and $\beta_1^K(\lambda = +1) = 1\cdot434$, the values of p were calculated also as a function of the nuclear structure parameter λ. Utilising these nuclear structure-dependent values, $A_2(e_K^-)$ was calculated as a function of λ and the plot is shown in Fig. 2. The correspond-

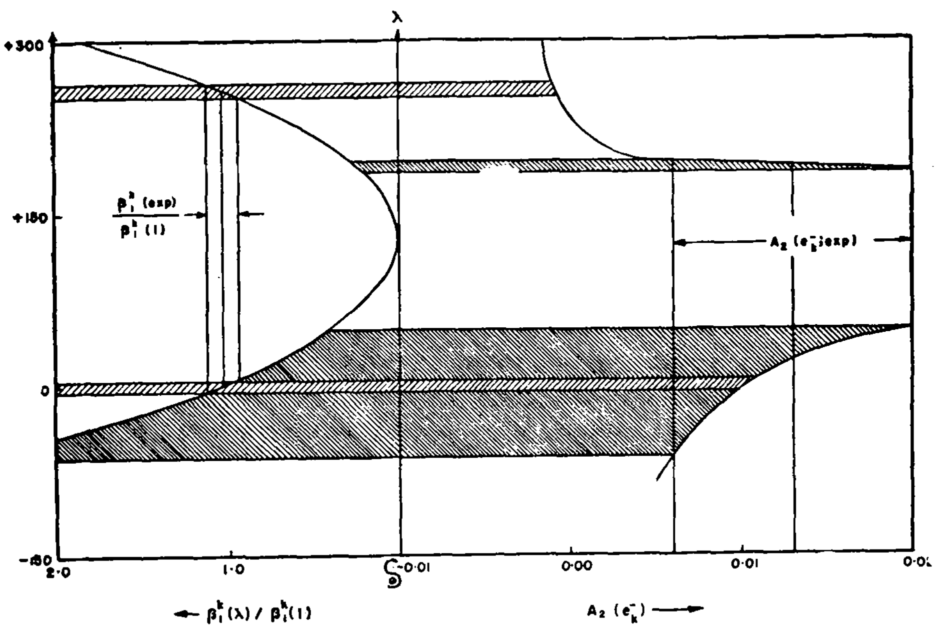

FIG. 2

ing $\beta_1^K(\lambda)/\beta_1^K(1)$, from (6), is also a function of λ and this is also shown in Fig. 2. It is seen from this figure that the experimental value of $\beta_1^K(\lambda)/$

$\beta_1^K(1) = 1\cdot024 \pm 0\cdot096$ leads to $\lambda = -1\cdot28$ or $+256$. The error indicated in the above parameter includes the uncertainty in $\beta_1^K(1)$ of $0\cdot05$ due to the uncertainty in the transition energy of 81 ± 1 Kev. There could be a small systematic uncertainty in the numerical value of C (z, k) which, however, cannot be exactly taken into the error estimate.

The two plots together show (Fig. 2) that the gamma-K-conversion electron angular correlation does not permit the larger value of $\lambda = +256$ suggested by the conversion coefficient, so that it can be concluded that $-5\cdot97 < \lambda < +3\cdot41$. This is to be compared with the value of $|\lambda| = 5$ to 10 deduced by Church and Weneser (1956) for odd $-$ Z nuclei and 1-forbidden transitions. In their calculation they have used the Weiskopf values for the radial integrals. This lowering an be understood as due to the hindrance of the internal conversion process in the nuclear volume. If the error in the transition energy is neglected, the result will be $-4\cdot6 < \lambda < 2\cdot3$.

Summary

Experimental results of angular correlations involving K-conversion electrons in Cs^{133} reported earlier (Subba Rao, 1961) have been interpreted with nuclear structure-dependent conversion electron parameters and conversion electron mixing amplitudes. The calculations and results of these nuclear structure-dependent conversion electron parameters is presented. The interpretation leads to the nuclear structure parameter $-5\cdot97 < \lambda < +3\cdot41$ indicating that the intranuclear part of the internal conversion process is hindered with respect to Weiskopf estimates.

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