CHARGE DISTRIBUTION OF THE PROTON

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ABSTRACT

An expression for the charge density of proton is presented and the form factor calculated on the basis of this is compared with the experimental results. The results are also compared with those of the exponential model and the cut-off model of Hofstadter (1959).

INTRODUCTION

The experiments on the scattering of electrons by protons have yielded valuable information on the structure of the proton.\(^1\) The experimental results are interpreted in terms of the Rosenbluth expression\(^2\) for the scattering cross-section, modified by the introduction of the charge and the magnetic moment form factors \(F_1\) and \(F_2\).

\[
\frac{d\sigma}{d\Omega} = \left( \frac{e^2}{2E_0} \right)^2 \cos^2 \frac{\theta}{2} \frac{\sin^2 \frac{\theta}{2}}{1 + \frac{2E_0}{Mc^2} \sin^2 \frac{\theta}{2}} \\
\times \left\{ F_1^2 + \frac{\hbar^2 q^2}{4Mc^2} \left[ 2 (F_1 + KF_2)^2 \tan^2 \frac{\theta}{2} + K^2 F_2^2 \right] \right\} \tag{1}
\]

where \(E_0\) is the incident electron energy, \(M\) is the mass of the proton, \(q = (2/\hbar) \sin \theta/2\) is the 4-momentum transfer and \(K\) is the anomalous magnetic moment of the proton. This formula is expected to be valid in the range of the electron energies and momentum transfers investigated at present. The phenomenological analysis of the data available is consistent with the simple assumption \(F_1 = F_2 = F\), and that \(F\) may be interpreted in terms of a static charge distribution \(\rho(r)\) as

\[
F = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin qr \cdot rdr. \tag{2}
\]
Several different charge distribution models may be written down, all of which give equally satisfactory fit to the experimental data. One of the best known is the exponential model which gives for the root-mean-square radius of the proton $0.80 \times 10^{-13} \text{cm}$. All other satisfactory models also give similar values. Hofstadter\(^3\) has pointed out that although the simplest meson-theoretic model

$$p(r) = \frac{\rho_0 e^{-\alpha r}}{r^2}$$  \hspace{1cm} (3)$$

is inadequate to fit the experimental data, this distribution modified for small values of $r$ by the introduction of an additional cut-off parameter can satisfactorily fit the data. Hofstadter's model is given by

$$\rho(r) = \begin{cases} \rho_0 h & 0 \leq r \leq d \\ \frac{\rho_0 e^{-\beta r}}{r^2} & d \leq r \leq \infty \end{cases}$$  \hspace{1cm} (4)$$

and contains an unpleasant discontinuity at $r = d$. In view of the theoretical interest of such models we have explored further in an attempt to write down a satisfactory well-behaved charge distribution function. Such a function is described in §2, and the results are discussed in §3.

2. Results

The general aim is to change the expression (3) to

$$p(r) = \frac{\rho_0 e^{-\alpha r}}{r^2} \cdot f(r)$$  \hspace{1cm} (5)$$

where $f(r)$ has suitable analytic properties. We require $f(r) \to 0$ for $r \to 0$, and $f(r) \to 1$ as $r \to \infty$, so that the divergence of $\rho(r)$ at $r = 0$ is removed, and piling up of charge at the centre of the proton is avoided. Our first attempt was to choose

$$f(r) = (1 - e^{-\beta r}) \cdot (1 - e^{-\alpha r}) \cdot \frac{e^{-\alpha r}}{r^2}$$  \hspace{1cm} (6)$$

It was however found that no choice of parameters $\alpha$ and $\beta$ could produce a satisfactory fit to the experimental data.

Next, we have tried a modifying function of the type

$$f(r) = (1 - e^{-\beta r})^2$$  \hspace{1cm} (7)$$

as we wish to keep the charge-density positive at all values of $r$. This combined with (5) and (2) gives
\[ F(q) = \left[ \frac{a}{2q} (1 + n)(2 + n) \right] \left\{ \tan^{-1} \left( \frac{n}{n + 1} \right) \frac{a}{q} \right. \\
+ \left. \tan^{-1} \left( \frac{n}{n + 2} \right) \frac{q}{a} \right\} \]  

(8)

where \( n = \frac{a}{\beta}. \)

Now \( F^2 \) versus \( q^2 \) is plotted for different values of \( a \) and \( \beta \) varied systematically. It is found that a satisfactory fit to the experimental data is obtained for

\[ a = \beta = 2.557 \times 10^{18} \text{ cm}^{-1} \]
\[ a^{-1} = 0.39 \times 10^{-13} \text{ cm}. \]

With these values of the parameters, one easily finds for the root-mean-square radius

\[ a = 0.85 \times 10^{-13} \text{ cm}. \]

Figure 1 shows the comparison of the experimental values of \( F^2 \) and the values calculated on the above model. The errors on the experimental values are \( \approx 10\% \). It can be seen that the agreement is quite good up to the highest momentum transfers yet observed, and a breakdown of the static
model is not yet in sight. The shape of the charge distribution $\rho(r)$ multiplied by $r^2$ is plotted against $r$ in Fig. 2. Also shown $r$ in this figure is the exponential charge distribution.

![Figure 2](image)

**Fig. 2.** Plot of $4\pi r^2 \rho(r)$ vs. $r$. The abscissa is in the units of $10^{-18}$ cm. The solid line represents the charge distribution model given by equations (5) and (7), and the dotted line gives the exponential model.

**DISCUSSION**

We shall now make a few remarks on the results described above. The values of $a$ and $a$ confirm the general trend of the results of Hofstadter. The root-mean-square radius is quite close to that obtained by other models, *viz.*, $0.80 \times 10^{-13}$ cm. for the exponential model, and $0.82 \times 10^{-13}$ cm. for the models given by equation (4). The value of the range parameter $a^{-1}$ is larger than that obtained in the exponential model, but is comparable to the value $b^{-1} = 0.4 \times 10^{-13}$ cm. obtained by Hofstadter for the models of equation (4). It is also clear that whatever the type of the modification introduced in the simple meson-theoretic charge distribution (3) for small
it has to be operative for $r \leq 0.4 \times 10^{-13}$ cm., as indicated by the value $d \approx 0.4 \times 10^{-13}$ cm. found by Hofstadter and $a = \beta$ obtained by us. At such distances, one would expect the simple $\pi$-meson cloud effects to be modified by the effects of the $K$-mesons ($\hbar/\sqrt{M_\pi} \approx 0.4 \times 10^{-13}$ cm.) and the nucleon core ($\hbar/\sqrt{M_N} = 0.21 \times 10^{-13}$ cm.) structure. However, in view of the fact that the simple meson-theoretic model (3) would only be valid for $r \leq 1 \times 10^{-13}$ cm. and the purely phenomenological approach of our model, it would be unwise to stress the discrepancy between the range-parameters obtained here ($0.4 \times 10^{-13}$ cm.) and required by the meson theory ($0.7 \times 10^{-13}$ cm.).

In view of the result $a = \beta$, the expressions obtained from (6) and (7) would appear to be power series in $\exp(-ar)$. However, in view of our observation that (6) is quite unsatisfactory, one is tempted to interpret (7) as indicating a modification of the simple meson-theoretic potential $\psi \sim \exp(-ar)/r$ to $\psi \sim \exp(-ar/2)[1 - \exp(-(ar))]r$ resulting from the effects of the proton structure.

Finally, we note that the charge distribution model obtained from equations (5) and (7) [but not that obtained from equations (5) and (6)] reduces to the exponential form $\exp(-ar)$ for small values of $r$.

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**REFERENCES**

