THERMAL DIFFUSIVITY OF A ROOF SLAB FROM
IN SITU TEMPERATURE MEASUREMENTS

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INTRODUCTION

The flow of heat through a roof slab is periodic, on account of the solar insolation, which imposes a diurnal variation of temperature on its exposed surface. The exact periodicity can be vitiated by the small difference that may occur between the initial and final temperatures for the 24-hour interval; but this difference is usually negligible, especially when the day forms part of a stable synoptic regime. The significant thermal property of the roof material is not so much its thermal conductivity (\(K\)), as its thermal diffusivity, \(\alpha\) given by the relation

\[
\alpha = \frac{K}{\rho C_p}
\]

where \(\rho\) and \(C_p\) are its density and specific heat respectively.

In common practice, \(\alpha\) is calculated from the experimental values of \(K\), \(\rho\), and \(C_p\) measured under steady state conditions for oven-dry specimens. The temperature dependence of these constants may be a second order effect only; but it is definitely known that they vary with the moisture content of the material. Since the air in the pores of a moist specimen is replaced by the more conducting water both in its liquid and vapour phases, not only the effective \(\rho\), and \(C_p\) but \(K\) also alters. A vapour pressure gradient is set up due to temperature differences; and the consequent vapour movement inside gives rise to an additional heat transport, both latent and sensible, in parallel to the conduction flow. This reduces the thermal resistance or increases the apparent conductivity of the material.

Considerable errors are therefore likely to be introduced in the analysis of heat transfer through building sections if \(\alpha\) is calculated from the conventionally determined values of \(K\), \(\rho\) and \(C_p\). The correct approach would be to evolve a method of determining \(\alpha\) of the material directly under the existing conditions of moisture.

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An attempt is made in this paper to obtain a value for $\alpha$ for a roof slab from \textit{in situ} temperature measurements of its outside and inside surfaces and of the ambient air inside the room below.

**EXPERIMENTAL**

Temperature data were available already, for the living rooms of about 40 dwelling houses put up for the Low Cost Housing Exhibition at Delhi (Raychaudhuri, 1957). Thermocouples were set up at a number of points in the experimental rooms and hourly temperatures were recorded continuously for a number of days, to an accuracy of $\pm 0.1^\circ \text{F}$.

The data used in this paper were that obtained for one of the rooms, kept completely closed throughout a clear day (22nd April 1955) in the middle of a stable stretch of weather conditions. The roof of the room was a $4\frac{1}{4}$ R.C.C. slab. The room was 10' 6" x 12' x 9' 9" in dimensions and built with 9" brick walls in cement mortar, plastered on both sides.

Curves for the observed surface temperatures of the roof (outside) the ceiling (inside) and the ambient air temperature at the centre of the room, on the chosen day are given in Fig. 1.

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**Fig. 1.** Time-temperature curves for roof, ceiling and indoor ambient air, (a) As observed, (b) As synthesised from the Fourier components.
Houghten (1932) and his co-workers have given a solution for the ceiling temperature, $\theta_c$ consequent on a unidirectional transverse flow of heat from the roof surface where the temperature varies sinusodially, with an amplitude $\theta_1$.

The solution is given below in a slightly altered form

$$\theta_c = R_1 e^{-\beta_1 l} \sin (\omega t - \beta_1 l + \gamma_1)$$

where $l$ is the thickness of the roof slab;

$$\beta_1 = \sqrt{\frac{\omega}{2a}}$$

$$R_1 = \frac{2\sqrt{2}}{\sqrt{2 + 2f_1 + f_1^2}};$$

$$\gamma_1 = \tan^{-1} \frac{f_1}{2 + f_1};$$

$$f_1 = \frac{h}{K \beta_1},$$

and $h$ is the appropriate film transfer coefficient, equal to the ratio between the rate of heat transfer per unit area of the ceiling and the ceiling-to-air difference in temperature.

$h$, $K$ and the ambient air temperature for the room below are assumed constant throughout the period while arriving at the above solution.

This solution can be extended to the case where the roof temperature has any periodic variation other than simple sinusoidal. $\theta_c$ will now be given by the expression

$$\theta_c = \bar{\theta}_c + \sum_{n=1}^{\infty} R_n e^{-\beta_n l} \theta_n \cos (n\omega t - \sigma_n - \beta_n l + \gamma_n)$$

when the roof temperature is given by the Fourier expansion

$$\theta_r = \bar{\theta}_r + \sum_{n=1}^{\infty} \theta_n \cos (n\omega t - \sigma_n)$$

where

$$\beta_n = \sqrt{n} \beta_1;$$

$$R_n = \frac{2\sqrt{2}}{\sqrt{2 + 2f_n + f_n^2}};$$
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\[ \gamma_n = \tan^{-1} \frac{f_n}{2 + f_n} \]

and

\[ f_n = \frac{f_1}{\sqrt{n}}. \]

\( \bar{\theta}_r \) and \( \bar{\theta}_c \) are related to \( \bar{\theta}_a \), the assumed constant air temperature by the steady state equation

\[ \frac{K}{l} (\bar{\theta}_r - \bar{\theta}_c) = h (\bar{\theta}_c - \bar{\theta}_a) \]

\[ \frac{h}{K} = \frac{(\bar{\theta}_r - \bar{\theta}_c)}{(\bar{\theta}_c - \bar{\theta}_a)} \times \frac{1}{l}. \] (3)

**PROCEDURE**

The observed experimental time temperature curves of the roof \( (\theta_r) \), the ceiling \( (\theta_c) \) and the indoor ambient air \( (\theta_a) \) are first analysed into their Fourier components as follows:

\[ \theta_r = 86.07 + 23.02 \cos (\omega t - 214.9') + 6.85 \cos (2\omega t - 27.59') + 1.01 \cos (3\omega t - 115.50') + 0.50 \cos (4\omega t - 130.22') + \ldots \ldots \] (4)

\[ \theta_c = 84.33 + 14.95 \cos (\omega t - 242.35') + 3.66 \cos (2\omega t - 70.30') + 0.32 \cos (3\omega t - 163.50') + 0.25 \cos (4\omega t - 139.31') + \ldots \ldots \] (5)

\[ \theta_a = 81.45 + 6.50 \cos (\omega t - 259.54') + 1.76 \cos (2\omega t - 84.48') + 0.71 \cos (3\omega t - 56.44') + 0.34 \cos (4\omega t - 117.17') + \ldots \ldots \] (6)
The curves, summing up the first five terms of the Fourier expansions for $\theta_r$, $\theta_c$ and $\theta_a$, are also given in Fig. 1 along with the observed curves. The correspondence between the two justifies limiting the calculations to the fundamental and the first three harmonics only and neglecting as insignificant the succeeding terms of the Fourier series.

It is to be noticed that the ambient air-temperature is not constant as assumed in the solution (Equation 2). However, if it is taken as constant at its mean value, $\tilde{\theta}_a$ during the entire period, the solution for the ceiling temperature (Equation 2) can be applied as a first approximation, and more so when $\theta_a = \tilde{\theta}_a = 81.45^\circ$ F., at the instant defined by $\omega t = 175^\circ 16'$ (as obtained from Equation 6).

The next step would be to solve for $\alpha$. But due to the inherent difficulty in employing the solution directly, an indirect method is adopted. The values of $\theta_c$, the ceiling temperatures are computed at the instant corresponding to $\omega t = 175^\circ 16'$, for different values of $\alpha$ from Equation 2, and substituting for $h/K$ the value $1.609$ as calculated from Equation 3. The relationship between $\alpha$ and the computed values of $\theta_c$ at the specified instant ($\omega t = 175^\circ 16'$) is shown graphically in Fig. 2. The observed value of $\theta_c$ at the instant is $90.82^\circ$ F. (as obtained from Equation 5), which when interpolated in the curve of Fig. 2 gives the value for $\alpha$ as $0.0322$ sq.ft. per hour.
The ceiling temperature for the full cycle computed for this value of $\alpha$ can now be expressed by the equation:

$$\theta_c = 84\cdot33 + 14\cdot86 \cos (\omega t - 241^\circ33') + 3\cdot59 \cos (2\omega t - 76^\circ50') + 0\cdot44 \cos (3\omega t - 180^\circ15') + 0\cdot18 \cos (4\omega t - 207^\circ34')$$

(7)

The curve of Equation 7 is given in Fig. 3, along with the experimental curve for the ceiling as defined by Equation 5. The correspondence between the two curves is good and the two coincide as they should when $\theta_a = \bar{\theta}_a$. The discrepancy, where it exists, should be ascribed to the fact that the air temperature was not constant throughout as was assumed. Also the value of $h$ can vary with time, being greater during the later hours of the night and early morning on account of a convective regime set up in the room air, $\theta_c$ being less than $\theta_a$ then (vide Fig. 3, where the room air temperature curve is also given for easy reference). During the rest of the period a stratified regime is set up as $\theta_c > \theta_a$ at that time. The reflections on the ceiling temperatures due to variations in $\theta_a$ and $h$ will be greater during nights but in opposing directions. This will displace the observed curve above the calculated curve at first with a possibility of the two crossing over and the former coming below the latter after some time. During the day and up to about midnight the observed curve will be throughout above the calculated curve, as the effects due to $h$ and $\theta_a$ are additive then: but the shift may now be smaller in magnitude than that during nights as the relative variations in $h$ and $\theta_a$ from their average values are less during this period. These inferences, necessarily qualitative, are borne out by Fig. 3.

To a first degree of approximation at least, the value of $\alpha$ for the roof can be taken as 0·032 sq.ft./hr. based on in situ temperature measurements and the method of calculation adopted. This value of $\alpha$ is the average value of the thermal diffusivity of the roof slab for the complete cycle of 24 hours under existing conditions of moisture.

**Summary and Conclusion**

An attempt is made in this paper to compute the value of $\alpha$ for the roof material from in situ temperature measurements of roof and ceiling surfaces and indoor ambient air below, using the Fourier Components of the periodic variations and the available classical methods of solution for the general differential equation of unidirectional heat flow through the structure. Even
though the boundary conditions assumed in the solution are not strictly identical with the experimental ones the computed thermal diffusivity, under in situ conditions, can be considered accurate enough for practical purposes.

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