BOUNDARY LAYER ON A FLAT PLATE WITH SUCTION

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INTRODUCTION

IN this paper we discuss the problem of evaluating the thickness of boundary layer and skin friction on a thin plate with suction, placed parallel to an otherwise uniform flow. A number of workers have studied this problem. For example, Blasius' classical work discusses the boundary layer on a plate at some distance from the leading edge, while Schlichting has given the solution of the problem taking into account uniform suction in which the velocity distribution is independent of the distance from the leading edge. Recently Carrier and Lin[1] have obtained the solution in the neighbourhood of the leading edge, while Torda[2] has deduced the necessary equations giving the distribution of suction at the plate which will ensure constant thickness of boundary layer. He has represented this suction velocity distribution in a graphical form for an aerofoil for which experimental data are available. We have obtained the stream function by the method of successive approximations assuming a linear law of suction which holds strictly in the neighbourhood of the leading edge. In passing, we may mention that in the absence of suction the present solution reduces to a form similar to that given by Carrier and Lin. We have obtained the thickness of the boundary layer and the skin friction explicitly in terms of suction velocity and distance from the leading edge without putting any restriction on the distribution of suction speed on the plate. To ensure symmetry we have assumed the suction to take place with the same speed at the corresponding points on the two sides of the plate.

It is evident that our aim is in a sense different from that of Torda. While Torda has attempted to specify the distribution of suction speed over the plate to ensure a constant thickness of boundary layer, we have attempted to evaluate the boundary layer thickness in terms of a specified distribution of suction speed. The present point of view may be found more useful in practice besides being more general than that of Torda.

The explicit expressions for velocity boundary layer thickness $\delta(x)$ and the skin friction $\tau_0(x)$ for the downstream flow as obtained in this paper are:

$$\delta(x) = \frac{\sqrt{x}}{a} + \frac{v_0(x)}{2}\left(\frac{\sqrt{x}}{a}\right)^2 + \left\{\frac{1}{48} + \frac{v_0^2(x)}{3}\right\}\left(\frac{\sqrt{x}}{a}\right)^3 + \frac{v_0^3(x)}{4}\left(\frac{\sqrt{x}}{a}\right)^4 + \ldots,$$

and

$$\tau_0(x) = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0} = \mu\left(\frac{a}{\sqrt{x}} - v_0'(x)\right),$$

where

$$a = 0\cdot 332.$$

1. DESCRIPTION OF THE FLOW NEAR THE LEADING EDGE

The equations governing an incompressible, viscous, fluid flow in two-dimensional cartesian co-ordinates x_1, y_1 are:

Equation of continuity:

$$\frac{\partial u_1}{\partial x_1} + \frac{\partial v_1}{\partial y_1} = 0, \tag{1.1}$$

Equations of momentum:

$$u_1\frac{\partial u_1}{\partial x_1} + v_1\frac{\partial u_1}{\partial y_1} + \frac{1}{\rho}\frac{\partial p_1}{\partial x_1} = \nu\nabla^2 u_1, \tag{1.2}$$

$$u_1\frac{\partial v_1}{\partial x_1} + v_1\frac{\partial v_1}{\partial y_1} + \frac{1}{\rho}\frac{\partial p_1}{\partial y_1} = \nu\nabla^2 v_1. \tag{1.3}$$

The introduction of the following dimensionless variables:

$$p = \frac{p_1}{\frac{1}{2}\rho u_0^2}, \quad x = \frac{x_1 u_0}{\nu}, \quad y = \frac{y_1 u_0}{\nu}, \quad u = \frac{u_1}{u_0} = \frac{\partial\psi}{\partial y},$$

$$v = \frac{v_1}{v_0} = -\frac{\partial\psi}{\partial x}, \tag{1.4}$$

where u_0 is the free stream velocity, reduces the above equations to:

$$\nabla^4\psi = \frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}(\nabla^2\psi) - \frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}(\nabla^2\psi), \tag{1.5}$$

where

$$\nabla^2\psi \equiv \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2}. \tag{1.6}$$

It is found convenient to work through polar co-ordinates defined by

$$x = r\cos\theta, \quad y = r\sin\theta.$$

With these, (1.5) and (1.6) reduce to:

$$\frac{\partial^4\psi}{\partial r^4} + \frac{2}{r}\frac{\partial^3\psi}{\partial r^3} - \frac{1}{r^2}\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r^3}\frac{\partial\psi}{\partial r} + \frac{2}{r^2}\left(\frac{\partial^4\psi}{\partial r^2\partial\theta^2} - \frac{1}{r}\frac{\partial^3\psi}{\partial r\partial\theta^2} + \frac{2}{r^2}\frac{\partial^2\psi}{\partial\theta^2}\right) + \frac{1}{r^4}\frac{\partial^4\psi}{\partial\theta^4}$$
$$= \frac{1}{r}\left\{\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}(\nabla^2\psi) - \frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}(\nabla^2\psi)\right\}, \tag{1.7}$$

where

$$\nabla^2\psi = \frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial\theta} + \frac{1}{r^2}\frac{\partial^2\psi}{\partial\theta^2}. \tag{1.8}$$

Following Carrier and Lin we write this equation as

$$\mathrm{L}(\psi) = \mathrm{L}^*(\psi), \tag{1.9}$$

where L denotes the biharmonic operator and L* denotes the non-linear operator on the right. For a linear law of suction,

$$v(x, 0) = -\mathrm{V}x, \tag{1.10}$$

the boundary conditions are

$$\left.\begin{aligned} \left(\frac{\partial\psi}{\partial r}\right)_{\theta=0} &= \mathrm{V}r, & \left(\frac{\partial\psi}{\partial r}\right)_{\theta=2\pi} &= -\mathrm{V}r, \\ \left(\frac{\partial\psi}{r\partial\theta}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi}{r\partial\theta}\right)_{\theta=2\pi} &= 0 \end{aligned}\right\} \tag{1.11}$$

We assume a solution of (1.9) in the form:

$$\psi = \psi_0 + \psi_1 + \psi_2 + \ldots, \tag{1.12}$$

where

$$\psi_0, \psi_1, \psi_2, \ldots,$$

are defined by

$$L(\psi_0) = 0, \tag{1.13 a}$$

$$L(\psi_1) = L^*(\psi_0), \tag{1.13 b}$$

$$L(\psi_2) = L^*(\psi_0 + \psi_1) - L^*(\psi_0), \tag{1.13 c}$$

. . . .

$$L(\psi_n) = L^*(\psi_0 + \psi_1 + \ldots + \psi_{n-1}) - L^*(\psi_0 + \psi_1 + \ldots + \psi_{n-2}), \tag{1.13 n}$$

We shall choose ψ_0 such that it satisfies the boundary conditions (1.11) completely, *i.e.*,

$$\left.\begin{aligned} \left(\frac{\partial\psi_0}{\partial r}\right)_{\theta=0} &= Vr, & \left(\frac{\partial\psi_0}{\partial r}\right)_{\theta=2\pi} &= -Vr, \\ \left(\frac{\partial\psi_0}{\partial \theta}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi_0}{\partial \theta}\right)_{\theta=2\pi} &= 0 \end{aligned}\right\} \tag{1.14}$$

Besides the velocity distribution (as determined by ψ_0) is to be symmetric about the plate. The boundary conditions to be satisfied by ψ_i for $i > 0$, are

$$\frac{\partial\psi_i}{\partial r} = \frac{\partial\psi_i}{\partial\theta} = 0, \quad \text{at} \quad \theta = 0, \ 2\pi. \tag{1.15}$$

Also the velocity distribution (as determined from each one of the ψ_i's) is to be symmetrical about the plate.

The first two conditions in (1.14) suggest a solution for (1.13 *a*) in the form

$$\psi_0 = r^2 f(\theta). \tag{1.16}$$

Using this form we find that $f(\theta)$ has to satisfy

$$f^{\text{IV}} + 4f'' = 0, \tag{1.17}$$

with the conditions

$$\left.\begin{aligned} f(0) &= \frac{V}{2}, & f(2\pi) &= -\frac{V}{2}, \\ f'(0) &= 0, & f'(2\pi) &= 0. \end{aligned}\right\} \tag{1.18}$$

With these we easily see that the solution of (1.13 *a*) of the assumed type is

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta) - 2Cr^2 \sin^2\theta, \tag{1.19}$$

where the constant C is yet undetermined.

The symmetry of velocity distribution demands

$$\left.\begin{aligned} \left(\frac{\partial\psi_0}{r\partial\theta}\right)_{\substack{r=\rho\\ \theta=\alpha}} &= \left(\frac{\partial\psi_0}{r\partial\theta}\right)_{\substack{r=\rho\\ \theta=2\pi-\alpha}}, \\ \left(\frac{\partial\psi_0}{\partial r}\right)_{\substack{r=\rho\\ \theta=\alpha}} &= -\left(\frac{\partial\psi_0}{\partial r}\right)_{\substack{r=\rho\\ \theta=2\pi-\alpha}}, \end{aligned}\right\} \tag{1.20}$$

for all ρ and α.

These conditions yield

$$C = 0. \tag{1.21}$$

Thus

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta). \tag{1.22}$$

If we put V = 0 in (1.22), we get,

$$\psi_0 \equiv 0. \tag{1.23}$$

Hence we find that the above stream function is unable to account for the flow without suction.

The boundary conditions for the flow without suction are

$$\left.\begin{aligned} \left(\frac{\partial\psi}{\partial r}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi}{\partial r}\right)_{\theta=2\pi} &= 0, \\ \left(\frac{\partial\psi}{\partial\theta}\right)_{\theta=0} &= 0, & \left(\frac{\partial\psi}{\partial\theta}\right)_{\theta=2\pi} &= 0. \end{aligned}\right\} \tag{1.24}$$

Besides the velocity distribution is to be symmetric about the plate. It is found by trial that the least value of n for which a function of the form $r^n f(\theta)$ satisfies these conditions is 3/2 and the corresponding function $f(\theta)$ is given by

$$f(\theta) = \cos\frac{\theta}{2} - \cos\frac{3\theta}{2}. \tag{1.25}$$

Thus to get a complete expression for ψ_0 which will also account for the flow in the absence of suction without affecting the boundary conditions we must add to (1.22).

$$2Ar^{3/2}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right), \tag{1.26}$$

where A is yet an undetermined constant which is independent of suction.

Hence

$$\psi_0 = \frac{Vr^2}{2} + \frac{Vr^2}{4\pi}(\sin 2\theta - 2\theta) + 2Ar^{3/2}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right). \tag{1.27}$$

The presence of the constant A allows us a choice to fix up the flow without suction.

With this value of ψ_0, (1.13 *b*) becomes

$$\begin{aligned} L(\psi_1) = {} & \frac{2A^2}{r}(2\sin\theta - 3\sin 2\theta) \\ & + \frac{AV}{\pi r^{1/2}}\left(2\pi\sin\frac{\theta}{2} + 7\cos\frac{\theta}{2} - 2\theta\sin\frac{\theta}{2} - 6\cos\frac{3\theta}{2}\right. \\ & \qquad \left. - \cos\frac{5\theta}{2}\right) \\ & + \frac{2V^2}{\pi}\left(1 - \frac{\theta}{\pi} + \frac{1}{2\pi}\sin 2\theta\right) \end{aligned} \tag{1.28}$$

It appears that a solution of (1.28) is of the form

$$\psi_1 = r^3 f_1(\theta) + r^{7/2} f_2(\theta) + r^4 f_3(\theta); \tag{1.29}$$

but such a solution is found not to satisfy boundary conditions (1.15) and the symmetry conditions unless $V = 0$. Thus we cannot proceed with the above form of solution for ψ_1 if we are interested in a solution for the problem with suction. It is found that a function of the form

$$\begin{aligned} \psi_1 = {} & A^2 r^3 \{f_1(\theta) + \phi_1(\theta)\log r\} \\ & + \frac{AV}{\pi} r^{7/2} \{f_2(\theta) + \phi_2(\theta)\log r\} \end{aligned}$$

$$+\frac{2V^2}{\pi} r^4 \{f_3(\theta) + \phi_3(\theta) \log r\}, \tag{1.30}$$

satisfies (1.28) and the conditions (1.15) along with the symmetry of the velocity distribution. Using this we find that f_1, f_2, f_3, ϕ_1, ϕ_2, ϕ_3 have to satisfy

$$\phi_1^{IV} + 10\phi_1'' + 9\phi_1 = 0, \tag{1.31}$$

$$f_1^{IV} + 10f_1'' + 9f_1 = -24\phi_1 - 8\phi_1'' + 4\sin\theta - 6\sin 2\theta, \tag{1.32}$$

$$\phi_2^{IV} + \frac{29}{2}\phi_2'' + \frac{441}{16}\phi_2 = 0, \tag{1.33}$$

$$f_2^{IV} + \frac{29}{2} f_2'' + \frac{441}{16} f_2 = -\frac{105}{2}\phi_2 - 10\phi_2'' + 2\pi \sin\frac{\theta}{2}$$
$$+ 7\cos\frac{\theta}{2} - 2\theta \sin\frac{\theta}{2} - 6\cos\frac{3\theta}{2} - \cos\frac{5\theta}{2}, \tag{1.34}$$

$$\phi_3^{IV} + 20\phi_3'' + 64\phi_3 = 0, \tag{1.35}$$

$$f_3^{IV} + 20f_3'' + 64f_3 = -96\phi_3 - 12\phi_3'' + 1 - \frac{\theta}{\pi}$$
$$+ \frac{1}{2\pi}\sin 2\theta, \tag{1.36}$$

with the conditions

$$\left.\begin{aligned} &f_i(0) = f_i(2\pi) = f_i'(0) = f_i'(2\pi) = 0,\\ &\phi_i(0) = \phi_i(2\pi) = \phi_i'(0) = \phi_i'(2\pi) = 0,\\ &\qquad\qquad\qquad i = 1,\ 2,\ 3. \end{aligned}\right\} \tag{1.37}$$

These equations admit of direct solution and we get

$$\psi_1 = Ar^3\left[\frac{\pi-\theta}{8}(\cos 3\theta - \cos\theta) + \frac{2}{5}(\sin 2\theta - 2\sin\theta)\right.$$
$$\left. + \frac{1}{8}(3\sin\theta - \sin 3\theta)\log \kappa r)\right]$$
$$+ \frac{AV}{\pi} r^{7/2}\left[\frac{1}{144}\left(49\cos\frac{\theta}{2} + 6\cos\frac{5\theta}{2} - 55\cos\frac{7\theta}{2}\right)\right.$$
$$+ \frac{\pi-\theta}{240}\left(20\sin\frac{\theta}{2} + 89\sin\frac{3\theta}{2} - 41\sin\frac{7\theta}{2}\right)$$
$$\left. + \frac{41}{240}\left(\cos\frac{3\theta}{2} - \cos\frac{7\theta}{2}\right)\log \lambda r\right]$$

$$+ \frac{V^2}{96\pi^2} r^4 [(\pi - \theta)(3 - 8\cos 2\theta + 5\cos 4\theta)$$

$$+ 5(2\sin 2\theta - \sin 4\theta)\log \mu r], \tag{1.38}$$

where κ, λ, μ are constants yet undetermined.

It is difficult to find the domain of convergence of the series

$$\psi_0 + \psi_1 + \psi_2 + \dots$$

But the formation of the terms show that there exists a circle of convergence for the series.

2. DETERMINATION OF THE CONSTANT A

In the case of no suction, *i.e.*, $V = 0$, the stream function is given by

$$\bar{\psi} = 2Ar^{3/2}\left(\cos\frac{\theta}{2} - \cos\frac{3\theta}{2}\right)$$

$$+ A^2 r^3 \left[\frac{\pi - \theta}{8}(\cos 3\theta - \cos\theta) + \frac{2}{5}(\sin 2\theta - 2\sin\theta)\right.$$

$$\left. + \frac{1}{8}(3\sin\theta - \sin 3\theta)\log \kappa r\right] \tag{2.1}$$

So for small values of θ the velocity u is given by

$$u \simeq 4Ar^{1/2}\theta + \frac{8}{3}Ar^{1/2}\theta^3 + A^2 r^2 \left[-\pi\theta + \frac{3}{10}(1 + 5\log r)\theta^2\right.$$

$$\left. + \frac{2}{3}\pi\theta^3 - \left(\frac{14}{15} + \frac{1}{2}\log r\right)\theta^4\right] + \dots, \tag{2.2}$$

The velocity profile obtained by Blasius for flow downstream is given by

$$u = a\eta - \frac{a^2\eta^4}{2.4\,!} + \dots, \tag{2.3}$$

where

$$\eta = \frac{y}{x^{1/2}} \quad \text{and} \quad a = 0{\cdot}332.$$

For small values of r and θ,

$$u \simeq ar^{1/2}\theta - \frac{a^2 r^2 \theta^4}{48} + \dots, \tag{2.4}$$

For sufficiently small values of r and θ the leading term of the expansion describes the flow. Hence comparing the coefficients of $r^{1/2}\theta$ in the above expansions (2.2) and (2.4) we get

$$4\mathrm{A} = \alpha = 0{\cdot}332.$$

Therefore

$$\mathrm{A} = 0{\cdot}083. \tag{2.5}$$

Now that A is found we can calculate the vorticity ζ and skin friction τ_0.

We get

$$\begin{aligned}\zeta = &-\frac{2\mathrm{V}}{\pi}(\pi-\theta) - \frac{4\mathrm{A}}{r^{1/2}}\cos\frac{\theta}{2}\\ &+ \mathrm{A}^2 r\left\{(\pi-\theta)\cos\theta + \frac{22}{5}\sin\theta + 2\sin 2\theta\right.\\ &\qquad\left. + 3(\sin\theta)\log\kappa r\right\}\\ &-\frac{\mathrm{AV}}{\pi}r^{3/2}\left\{\left(\pi-\theta\right)\left(\sin\frac{\theta}{2} + \frac{89}{24}\sin\frac{3\theta}{2}\right) + 4\cos\frac{\theta}{2}\right.\\ &\qquad\left. + \frac{1}{12}\cos\frac{3\theta}{2} + \frac{1}{4}\cos\frac{5\theta}{2} + \frac{41}{24}\left(\cos\frac{3\theta}{2}\right)\log\lambda r\right\}\\ &-\frac{\mathrm{V}^2 r^2}{2\pi^2}\left\{(\pi-\theta)(1-2\cos 2\theta) + \sin 2\theta\right.\\ &\qquad\left. + \frac{5}{2}(\sin 2\theta)\log\mu r\right\}.\end{aligned} \tag{2.6}$$

The skin friction in the case of continuously varying suction is given by,

$$\tau_0 = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}. \tag{2.7}$$

Hence we get

$$\tau_0 = \mu\left[\frac{\alpha}{x^{1/2}} - \frac{\alpha^2\pi x}{16} + \frac{\alpha\mathrm{V}}{12\pi}x^{3/2}\left(13 + \frac{41}{8}\log\lambda x\right) - \frac{1}{2}\frac{\mathrm{V}^2 x^2}{\pi}\right]. \tag{2.8}$$

3. Description of the Flow Downstream

In the downstream region the Navier Stokes equations in the dimensionless form are represented approximately by the boundary layer equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2}, \tag{3.1}$$

along with the equation of continuity (1.1). The boundary conditions to be satisfied are

$$\left.\begin{aligned} &u = 0, \quad v = -v_0(x), \quad \text{at} \quad y = 0, \\ \text{and} \quad & \\ &u \to 1, \qquad\qquad\qquad \text{as} \quad y \to \infty, \end{aligned}\right\} \tag{3.2}$$

where $v_0(x)$ is the suction speed.

To solve this equation we assume a stream function

$$\psi = \sum_{n=0}^{\infty} a_n(x) y^n, \tag{3.3}$$

The first two of the boundary conditions (3.2) give

$$a_0'(x) = v_0(x), \quad a_1(x) = 0. \tag{3.4}$$

Substituting ψ from (3.3) in (3.1) we get,

$$a_3(x) = -\tfrac{1}{3} v_0(x) a_2(x), \tag{3.5}$$

$$a_4(x) = \frac{1}{12} v_0^2(x) a_2(x), \tag{3.6}$$

with the general recurrence relation

$$a_{n+3}(x) = -\frac{1}{n+3} v_0(x) a_{n+2}(x) + \frac{1}{(n+1)(n+2)(n+3)}$$

$$\times \sum_{r=2}^{n} (n+3-2r) r a_r(x) a'_{n+2-r}(x), \quad n > 1, \tag{3.7}$$

From the recurrence relation, we get

$$a_5(x) = -\frac{1}{60} v_0^3(x) a_2(x) + \frac{1}{30} a_2'(x) a_2(x), \tag{3.8}$$

$$a_6(x) = \frac{1}{360} v_0^4(x) a_2(x) - \frac{1}{60} v_0(x) a_2(x) a_2'(x)$$

$$- \frac{1}{90} v_0'(x) a_2^2(x), \tag{3.9}$$

$$a_7(x) = \frac{-1}{2520} v_0^5(x)\, a_2(x) + \frac{1}{210} v_0^2(x)\, a_2(x)\, a_2'(x)$$

$$+ \frac{1}{126} v_0(x)\, v_0'(x)\, a_2^2(x), \qquad (3.10)$$

$a_2(x)$ is left undetermined. This is as to be expected since one boundary condition, *viz.*,

$$u \to 1 \quad \text{as} \quad y \to \infty.$$

has not been used so far.

u, as determined from (3.3), is

$$u = 2a_2(x)\, y + 3a_3(x)\, y^2 + \ldots, \qquad (3.11)$$

We determine $a_2(x)$ by comparing the leading terms in the present velocity profile with $v_0(x) = 0$ and the Blasius' solution (2.3).

From (2.3) we get

$$u = \frac{\alpha y}{x^{1/2}} - \frac{\alpha^2}{2.4!} \frac{y^4}{x^2} + \ldots, \qquad (3.12)$$

Equating the coefficients of y from (3.12) and (3.13) we get

$$a_2(x) = \frac{\alpha}{2x^{1/2}}, \quad \text{where } \alpha = 0{\cdot}332. \qquad (3.13)$$

With this value of $a_2(x)$, if we put $v_0(x) = 0$ throughout the series, we get the Blasius solution.

Now that $a_2(x)$ is known we get

$$a_3(x) = -\frac{\alpha}{6x^{1/2}} v_0(x), \qquad (3.14)$$

$$a_4(x) = \frac{\alpha}{24x^{1/2}} v_0^2(x), \qquad (3.15)$$

$$a_5(x) = -\frac{\alpha}{120x^{1/2}} v_0^3(x) - \frac{\alpha^2}{240x^2}, \qquad (3.16)$$

$$a_6(x) = \frac{\alpha}{720x^{1/2}} v_0^4(x) + \frac{\alpha^2}{480x^2} v_0(x) - \frac{\alpha^2}{360x} v_0'(x), \qquad (3.17)$$

$$a_7(x) = -\frac{a}{5040x^{1/2}}v_0^5(x) - \frac{a^2}{1680x^2}v_0^2(x)$$
$$+\frac{a^2}{504x}v_0(x)\,v_0'(x), \tag{3.18}$$

$a_8(x)$, $a_9(x)$, etc., can be similarly calculated. With the use of these values we get

$$u = \frac{a}{x^{1/2}}y - \frac{av_0(x)}{2x^{1/2}}y^2 + \frac{av_0^2(x)}{6x^{1/2}}y^3 - \left(\frac{av_0^3(x)}{24x^{1/2}} + \frac{a^2}{48x^2}\right)y^4$$
$$+\left(\frac{av_0^4(x)}{120x^{1/2}} + \frac{a^2v_0(x)}{80x^2} - \frac{a^2v_0'(x)}{60x}\right)y^5$$
$$-\left(\frac{av_0^5(x)}{720x^{1/2}} + \frac{a^2v_0^2(x)}{240x^2} - \frac{a^2v_0(x)\,v_0'(x)}{72x}\right)y^6$$
$$+\ldots\ldots\ldots\ldots\ldots, \tag{3.19}$$

Using Lagrange's reversion formula we get

$$y = \frac{\sqrt{x}}{a}u + \frac{v_0(x)}{2}\left(\frac{\sqrt{x}}{a}\right)^2u^2 + \frac{v_0^2(x)}{3}\left(\frac{\sqrt{x}}{a}\right)^3u^3$$
$$+\left\{\frac{v_0^3(x)}{4}\left(\frac{\sqrt{x}}{a}\right)^4 + \frac{1}{48}\left(\frac{\sqrt{x}}{a}\right)^3\right\}u^4 + \ldots, \tag{3.20}$$

Since at the edge of the boundary layer the velocity is approximately the free stream velocity we have

$$u \simeq 1, \quad \text{at} \quad y = \delta. \tag{3.21}$$

Hence

$$\delta(x) \simeq \frac{\sqrt{x}}{a} + \frac{v_0(x)}{2}\left(\frac{\sqrt{x}}{a}\right)^2 + \frac{v_0^2(x)}{3}\left(\frac{\sqrt{x}}{a}\right)^3$$
$$+\left\{\frac{v_0^3(x)}{4}\left(\frac{\sqrt{x}}{a}\right)^4 + \frac{1}{48}\left(\frac{\sqrt{x}}{a}\right)^3\right\} + \ldots, \tag{3.22}$$

Thus an explicit analytic expression for $\delta(x)$ has been derived valid for any suction velocity distribution. The authors are not aware of any explicit expression having been given earlier by anyone.

The skin friction τ_0 is given by

$$\tau_0(x) = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0} = \mu\left(\frac{a}{\sqrt{x}} - v_0'(x)\right). \tag{3.23}$$

The solution near the leading edge has been given only for a linear law of suction, but from the method it is clear that any other power law can be used. Or in general any suction law can be dealt with by using the power series expansion for the function. The detailed solution for the leading edge with any suction is being worked out.*

SUMMARY

In this paper we have discussed the boundary layer on a plate with suction. The problem is solved near the leading edge as well as far downstream. A linear suction law is assumed near the leading edge for simplicity, whereas no restriction is placed on the suction law in the region downstream. An explict expression for boundary layer thickness in terms of suction speed and distance from leading edge is derived. It is found that the thickness of the boundary layer depends on the derivative of the suction speed. The skin friction also has been evaluated. Though near the leading edge a linear law of suction is assumed, the method used in the paper can be easily generalised for any other power law, for example, we may use a power series expansion for the function defining the suction velocity.

REFERENCES

1. Carrier, G. F. and Lin, C. C. .. *Quart. Appl. Math.*, 1948, **6**, 63.
2. Torda, T. P. .. *Jour. Math. and Physics*, 1952, **31**, 206.

* One of the authors (A. C. J.) has found out the solution at the leading edge with constant suction and his solution will be published in a separate paper along with other results.