ELASTIC AND PHOTOELASTIC PROPERTIES OF SOME OPTICAL GLASSES

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1. INTRODUCTION

IT is well known that isotropic substances like glasses possess only two elastic constants C_{11} and C_{12} which are related to the Young's modulus, the modulus of rigidity and the Poisson's ratio by the following relations:

$$Y = \frac{C_{11} - C_{12}}{C_{11} + C_{12}} (C_{11} + 2C_{12}),\ n = \frac{C_{11} - C_{12}}{2},\ \sigma = \frac{C_{12}}{C_{11} + C_{12}}$$

Again these substances have only two Pockels' elasto-optic constants p_{11} and p_{12} which are related to the corresponding piezo-optic constants q_{11} and q_{12} by the relations,

$$p_{11} = C_{11} q_{11} + 2C_{12} q_{12}$$

and

$$p_{12} = C_{11} q_{12} + C_{12} (q_{11} + q_{12})$$

On historical grounds and for conformity with the numerical observations of Neumann (1841), it is usual to use, not the parameters p_{11} and p_{12} but the quantities p and q, where $p = \frac{1}{2} \mu p_{12}$ and $q = \frac{1}{2} \mu p_{11}$ μ being the refractive index.

While Filon and Jessop (1924), Adams and Williamson (1920) and others determined the values of $(p - q)$ for a number of glasses, Bergmann and Fues (1936) measured p/q for a set of five entirely different glasses. The only known measurements of the absolute values of p and q are those of Pockels (1902) who determined them for seven optical glasses.

We have in this Laboratory a series of 18 optical glasses, which are of special interest as some of their physical properties have been studied. The object of the present investigation is therefore to determine the elastic and photoelastic constants of these glasses.

2. Principle of the Method

To determine the absolute values of the strain-optical constants, a measure of the absolute change in the refractive index for light polarised parallel and perpendicular to the direction of pressure is needed. The method adopted by Pockels (1902) is to determine the absolute path retardation by an interferometer method using two equal plane parallel prisms of the glass in the two beams of the interferometer, of which one is under stress. As two equal and plane parallel prisms for each variety of glass were not available, the following method has been used by the author. The value of $(p - q)$ has been determined by stressing the glass and measuring the difference between the two refractive indices for light vibrating parallel and perpendicular to the direction of pressure. With light polarised at 45° to the direction of pressure, the light emerging from the glass cube under stress will be elliptically polarised which can be analysed with the aid of a Babinet's compensator. Then it can be shown that x the measured fringe shift for a stress P is given by the relation

$$(p - q) = \frac{(C_{11} - C_{12})}{\mu^2 Pd} \cdot \frac{\lambda}{b} \cdot x \qquad (1)$$

where d is the length of path of light within the glass, b the distance between two successive fringes in the Babinet's compensator for wave-length λ, and μ the refractive index of the glass in the undeformed state. Thus by measuring the fringe shift for a known stress P, the value of $(p - q)$ can be determined provided $(C_{11} - C_{12})$ is known. The principle of the measurement of C_{11} and C_{12} for glasses is described below.

If a piezoquartz is cemented to one side of a glass cube and suitably excited by means of an R.F. oscillator such that the glass cube is thrown into strong resonant vibrations, then stationary waves are set up in the cube. Even though the glass cube is excited along one direction only, owing to the contraction of the cross section of the cube, a large number of waves are created which travel in various directions. If now a point source is used to form an image on a screen (or a photographic plate) in the usual way through the vibrating cube, a diffraction pattern called the Schæfer-Bergmann Pattern (1935) is obtained. This consists of a large number of sharp points lying on two concentric circles (Fig. 1). Each spot is due to a sound wave which travels in the plane containing the incident and the diffracted beams. On the other hand, if a slit is used as the source (Hiedemann, 1935), then only those sound waves having their wave-fronts parallel to the slit are effective in producing the diffraction pattern and the pattern obtained is as shown in Fig. 2. (In both cases the central spot or zero order is cut off by means of suitable obstacles, immediately in front of the photographic plate, to avoid fogging of the plate.)

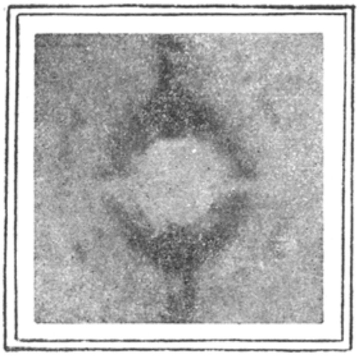


FIG. 1

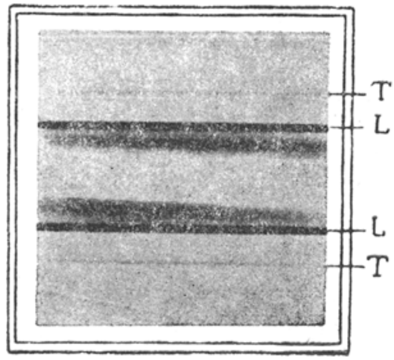


FIG. 2

FIG. 1. Schæfer — Bergmann Pattern in glass
FIG. 2. Hiedemann Pattern in glass
L = Longitudinal *T* + Transverse

When a point source is used, according to the theory of Fues and Ludloff (1935) the radius of the inner circle produced by the longitudinal waves is given by

$$r_l = \frac{D\lambda}{2\pi} \sqrt{\frac{\rho\omega^2}{C_{11}}} \tag{2}$$

where D is the distance between the glass cube and the photographic plate, ρ the density, λ the wave-length of light, $\omega = 2\pi f$, f being the frequency; and the radius of the outer circle produced by transverse waves is given by

$$r_t = \frac{D\lambda}{2\pi} \sqrt{\frac{2\rho\omega^2}{C_{11} - C_{12}}} \tag{3}$$

If instead of a point a slit is used as the source, r_l represents the distance between the zero order and the inner line due to longitudinal waves and r_t that between the zero order and the outer weaker line due to transverse waves. Thus by measuring the values of r_l and r_t, the values of C_{11} and C_{12} can be calculated. Mueller (1938) has shown that the same method with slight modifications can be used to determine the value of p/q of glasses and cubic crystals. Recently, this method has been successfully adopted for cubic crystals by Burstein, Smith and Henvis (1948). Using a horizontal slit and incident light polarised at 45° to it, the direction of polarisation of the first order longitudinal pattern will be oriented at an angle 'θ' given by the relation,

$$tan\ (\theta + 45) = \frac{J_1 (R\nu)}{J_1 (\nu)} \tag{4}$$

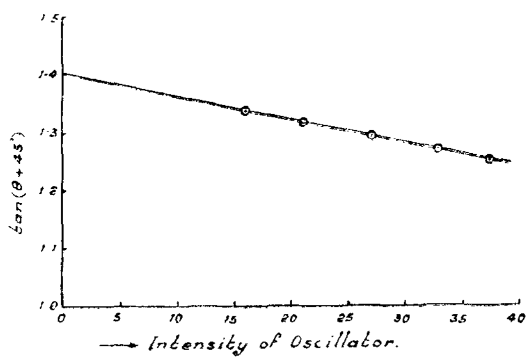
where J_1 is the first order Bessel function, $R = p/q$, $\nu = \frac{2\pi a L}{\lambda}$. '$a$' is a function of the sound amplitude, L the width of the supersonic field, and λ the wave-length of light. If now, the variation of θ with sound amplitude is plotted, θ approaches a limiting value θ_1 given by the relation

$$tan\ (\theta_1 + 45) = R = p/q \tag{5}$$

A slight modification of the above method of calculating R, which gives more accurate results, is given below. Expanding equation (4) and neglecting higher terms, we get

$$tan\ (\theta + 45) = \frac{J_1 (R\nu)}{J_1 (\nu)} = R\left[1 - \frac{\nu^2}{8}(R^2 - 1) + \frac{\nu^4}{192}(R^4 - 3R^2 - 1) + \ldots\right]$$

As the value of ν is less than $\frac{1}{2}$ when only the first order longitudinal pattern is obtained, it is seen that the term involving ν^4 and higher terms are negligible for values of R up to 2·5. Hence if tan $(\theta + 45)$ is plotted against the square of the sound amplitude or the intensity of the oscillator, then we should get a straight line from which the value of R for $\nu = 0$, can be extrapolated. A typical graph obtained from experimental data on glass no. 14 is given in Fig. 3.



TEXT-FIG. 3

3. SPECIMENS STUDIED

The specimens studied were in the form of rectangular blocks (30 × 30 × 20 mm.) having all but two sides polished. The thickness of each glass did not deviate by more than ·003 cm. in 2·000 cm. These were the same specimens which were used not only for the investigations on the scattering of light by Krishnan (1936), but also for studies on Raman effect by Norris (1941), and on Faraday effect and dispersion by Ramaseshan (1946). Table I gives the list of these glasses, their approximate composition, and density as furnished by the manufacturers. The refractive indices for λ 5893 Å of these specimen as measured by Ramaseshan with the help of a Pulfrich Refractometer, are also given in Table I. All the specimens were found

TABLE I

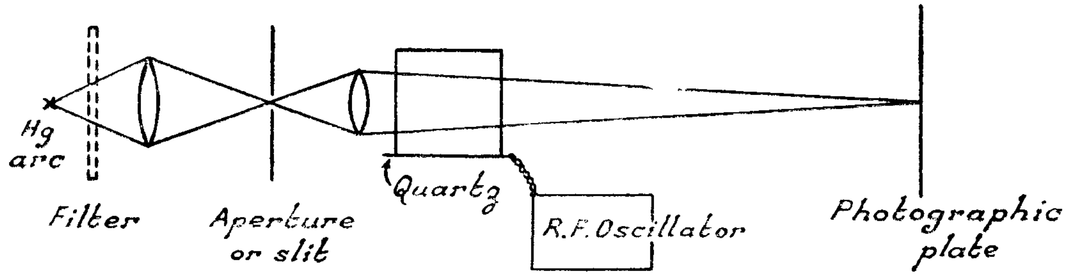
List of Glasses Studied

Sl. No.	Melt. No.	Chemical Composition		Density	μ_{5893}
		More than 10%	Less than 10%		
1	25188	SiO_2, B_2O_3, Al_2O_3, K_2O	Na_2O, F	2·3	1·46693
2	18415	SiO_2, B_2O_3, K_2O	Al_2O_3	2·3	1·49340
3	16776	SiO_2, K_2O	Al_2O_3, Na_2O, CaO	2·4	1·50228
4	23975	SiO_2, B_2O_3, ZnO	Al_2O_3, Na_2O	2·5	1·50933
5	24906	SiO_2, B_2O_3	Na_2O, K_2O, BaO	2·5	1·51714
6	22601	SiO_2, B_2O_3, Sb_2O_3	Al_2O_3, Na_2O, K_2O	2·7	1·52998
7	23125	SiO_2, Na_2O, PbO	ZnO	2·7	1·52686
8	22638	SiO_2, PbO	Na_2O, K_2O	2·9	1·54495
9	24464	SiO_2, ZnO, BaO	B_2O_3, Na_2O, K_2O, PbO	3·1	1·57000
10	19510	SiO_2, PbO	Na_2O, K_2O	3·2	1·56991
11	23355	SiO_2, B_2O_3, BaO	Al_2O_3, Na_2O	3·3	1·58950
12	20672	SiO_2, PbO	B_2O_3, Na_2O, K_2O	3·4	1·59813
13	22986	SiO_2, BaO, PbO	Na_2O, K_2O, ZnO	3·5	1·60620
14	23441	SiO_2, PbO	Na_2O, K_2O,	3·9	1·64712
15	23497	SiO_2, BaO, PbO	Na_2O, K_2O, ZnO	3·9	1·64917
16	23850	SiO_2, PbO	Na_2O, K_2O	4·4	1·7130
17	23590	SiO_2, PbO	Na_2O, K_2O	5·1	1·7850
18	S1075	SiO_2, PbO	B_3O_3, K_2O, Sb_2O_3	6·7	1·8900

to be completely free from natural birefringence when tested with the aid of a Babinet compensator.

4. EXPERIMENTAL DETAILS

The schematic diagram of the experimental arrangement for the measurement of the elastic constants is given in Fig. 4. The glass block was mounted on a quartz plate 4 cm. × 4 cm. × ·08 cm. with a thin layer of transformer oil in between, and the quartz plate was excited at its 5th harmonic by a push-pull T 55 oscillator. Using λ 4358 Å radiations of Hg arc, the Hiedemann pattern was photographed and measured with the aid of a Hilger cross slide micrometer capable of reading up to 1/1000 mm. In the case of glass No. 16 as the transverse pattern was too weak, and as the



TEXT-FIG. 4

frequency of the oscillator was not sufficiently constant throughout the exposure (the variation of 1/200 Mc. in the frequency of the oscillator, is enough to make the transverse pattern disappear), a Fuess micrometer eyepiece reading up to 1/200 mm. was used to measure the values of r_l and r_t directly. The frequency was determined with the help of a G.R. type 724 A precision wave-meter. In the case of glasses numbered 17 and 18, the transverse pattern could not be observed even with the maximum power of the oscillator, as the intensity of the transverse pattern depends on $(p\text{-}q)$ which is very nearly equal to zero. Experiments to measure C_{11} and C_{12} for these glasses by the static method are in progress.

The experimental arrangement for the determination of p/q is the same as before, except that the photographic plate is replaced by an analyser, and the incident light (λ 5893 Å) polarised at 45° to the vertical. A suitable diaphragm was used to cut off the zero order so as to reduce the background illumination. Using the crossed position of the analyser in the beginning, the analyser has to be rotated through an angle θ, to get the extinction of the first order longitudinal pattern. θ is taken as positive for a clockwise rotation for an observer looking towards the source. For each glass at least four different values of θ were taken, each value of θ itself being a mean of ten readings.

The value of $(p-q)$ was determined by the usual classical method of Pockels (1902) using the lever arrangement. Normally stresses less than 0·75 kg. wt/mm.^{2} were used. To uniformise the pressure 3 mm. thick rubber washers were used on both sides of the glass block. The measurements of double-refraction generated by pressure for λ 5893 Å were made using a Fuess Babinet compensator at nine different points of the glass block. From the average of these readings and from the observation that 21·80 divisions of the Babinet compensator correspond to a fringe shift for 5893 Å, the value of $(p\text{-}q)$ has been calculated using the formula (1).

5. Results

The values of C_{11}, C_{12} and other elastic constants Y, n and σ as calculated from C_{11} and C_{12} are given in Table II. C_{11} and C_{12} are obtained from the measured values of r_l and r_t using the equations (2) and (3). The values of the elastic constants are accurate to within 1%. Table II contains also the values of p/q which are obtained by plotting ($\tan\theta + 45$) against the intensity of the oscillator, for various values of the plate voltage. Table II contains the values of $(p-q)$ as well, which are obtained as explained in the last section. The values of Neumann's strain-optical constants, and also the Pockels' elasto-optic and piezo-optic constants for these glasses, as calculated from the measured values of (p/q) and $(p-q)$ are also given in Table II. Measurements on $(p-q)$ for glasses numbered 8 and 14 could not be made as they were found to have internal cracks. The values of the photo-elastic constants are accurate only to within 5%.

Table II

	$C_{11} \times 10^{-11}$	$C_{12} \times 10^{-11}$	$Y \times 10^{-11}$	$n \times 10^{-11}$	σ	$(p-q)$	$\frac{p}{q}$	p	q	p_{11}	p_{12}	$q_{11} \times 10^{13}$	$q_{12} \times 10^{13}$
1	5·531	1·775	4·669	1·878	·243	·0762	1·75	·178	·102	·139	·242	·444	3·21
2	6·684	1·608	6·061	2·538	·194	·0918	2·37	·159	·067	·090	·212	·120	2·54
3	7·904	2·180	6·961	2·862	·214	·0774	1·79	·176	·099	·131	·234	·430	2·23
4	8·078	2·244	7·102	2·917	·217	·1049	2·30	·186	·081	·107	·246	·0013	2·38
5	8·734	2·026	7·971	3·354	·188	·0909	2·06	·177	·086	·114	·233	·322	2·11
6	6·485	1·936	5·595	2·275	·230	·0875	1·71	·212	·124	·162	·276	·626	3·14
7	7·238	2·031	6·348	2·604	·219	·0771	1·77	·177	·100	·131	·232	·464	2·40
8	6·648	1·726	5·937	2·461	·206	..	1·46	..	..	..	..	..	..
9	9·085	2·978	7·614	3·053	·247	·0855	1·84	·188	·102	·130	·239	·161	1·95
10	6·891	1·942	6·207	2·474	·220	·0698	1·70	·169	·099	·127	·216	·474	2·23
11	10·078	3·494	8·279	3·292	·257	·0633	1·61	·167	·104	·131	·211	·273	1·48
12	6·694	1·885	5·865	2·404	·220	·0642	1·50	·193	·129	·161	·242	·940	2·61
13	7·680	2·542	6·416	2·569	·249	·0630	1·51	·188	·125	·155	·234	·609	2·14
14	6·647	1·969	5·747	2·339	·228	..	1·41	..	..	..	..	..	..
15	7·867	2·689	6·497	2·589	·255	·0503	1·39	·179	·129	·156	·217	·702	1·88
16	6·637	2·044	5·674	2·296	·235	·0308	1·18	·202	·171	·200	·236	1·581	2·36
17	..	..	..	..	..	..	1·05	..	..	..	..	..	..
18	..	..	..	..	..	..	0·995	..	..	..	..	..	..

C_{11} C_{12} Y and n are given in dynes/cm². For q_{11} and q_{12}, 1 dyne/cm². is taken as unit of stress.

6. Discussion of Results

As all the glasses studied here are of widely varying composition and as the exact percentage compositions of these glasses are also not known, it is not possible to compare the experimental results either amongst themselves or with those obtained by the other authors. However, a few general conclusions can be drawn from these results. Comparing the values of the elastic constants for the different glasses given in Table II, one finds that there is very little correlation between these values and the density or the refractive index. The same generalisation is applicable to quantities p_{11}, p_{12}, q_{11} and q_{12}. But when glasses of similar composition are taken, for example glasses numbered 10, 12, 14, 16 and 17, it is seen that as the lead content increases, the elastic constants decrease, while the photoelastic constants increase. A similar effect was observed by Pockels (1902).

From Table II, it is seen that in general the value of p/q decreases as the refractive index increases, which is very evident when glasses of similar composition are compared. Another interesting fact worthy of mention here is that the value of p/q has become less than unity in the case of glass No. 18, that is, the value of q has become greater than that p, in conformity with Pockel's observations.

TABLE III

Observer's glass No.	Composition		μ	Y $\times 10^{-11}$	n $\times 10^{-11}$	σ	$(p-q)$	p	q	Observer
	More than 10%	Less than 10%								
Barium Flint 13	46 SiO_2, 24 PbO, 15 BaO	8 ZnO, 4 K_2O, 3 Na_2O	1·606	6·466	2·6	·243	·0625	..	..	Adams and Williamson
	SiO_2, PbO, BaO	ZnO, K_2O, Na_2O	1·60620	6·416	2·569	·247	·0630	·188	·125	Author
0500	29·3 SiO_2 67·5 PbO	3 K_2O, 2 As_2O_5	1·7510	5·500	2·220	·239	·017	·202	·185	Pockels
16	SiO_2, PbO	Na_2O, K_2O	1·7130	5·674	2·296	·235	·031	·202	·171	Author

In Table III, the values of the elastic and photoelastic constants for two glasses are given as determined by Adams and Williamson (1920) and Pockels (1902) and also for two glasses No. 13 and 16 as measured by the author. The glass No. 13 has nearly the same composition and refractive index as the "Barium flint" glass used by Adams and Williamson. The same is the case with glass No. 16 and Pockels' glass 0500. It is seen that the results are in good agreement within the limits of experimental error. For the other glasses, the results obtained by the author cannot be compared with the results of Adams and Williamson or of Pockels, because they are of different composition and refractive index.

It is interesting to note that glass No. 4 is in many ways exceptional compared to the other glasses. This glass has the lowest values for the depolarisation factors ρ_u, ρ_v, ρ_h and also for the ratio ρ_v/ρ_h. Moreover the scattering power is maximum for this glass (Krishnan, 1936). Norris (1941) could not get a satisfactory Raman spectrum of this glass. The same specimen has the maximum value for $(p-q)$ and an extremely low value of q_{11} and also that the value of p/q is very high.

In conclusion the author desires to express his grateful thanks to Prof. R. S. Krishnan for suggesting the problem and for valuable help rendered during the course of this investigation. The author's thanks are also due to Dr. G. N. Ramachandran for the useful discussions he had with him.

7. Summary

The elastic and photoelastic constants for a set of 18 optical glasses of widely varying composition have been measured. The elastic constants and the quantities p_{11}, p_{12}, q_{11} and q_{12} have in general no correlation with the density or the refractive index. However, when glasses of similar composition are compared, the photoelastic constants are found to increase, as the lead content in the glass increases; also the value of p/q is found to diminish progressively thus confirming Pockels' observations.

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