ON THE RADIATION FROM THE BOUNDARY OF DIFFRACTING APERTURES AND OBSTACLES

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1. INTRODUCTION

IT is a well-known fact that the boundary of a diffracting aperture or obstacle appears luminous when it is observed from within the shadow cast by it. This naturally suggests that the problem of diffraction can be reduced to the consideration of the radiation from the boundary. The idea is, however, not at all new ; it was used by Young to explain the diffraction fringes obtained with a straight edge. But, it was not realized until the recent work of Sir C. V. Raman (unpublished) that the same method can be extended with equal success to an aperture or obstacle of arbitrary shape. A detailed description of Raman's ideas will be found in a paper by Y. V. Kathavate, who has applied them to a large number of cases in order to explain the diffraction patterns obtained. In the present paper, a mathematical discussion of the problem is given, which leads to some new results.

On the theoretical side, Rubinowicz (1917, 1923) succeeded in converting the Kirchhoff surface integrals occurring in the theory of diffraction into a line integral along the boundary. Laue (1936) has obtained such a transformation for the case of Fraunhofer diffraction. However, these line integrals cannot be evaluated except for very simple cases. In fact, for boundaries of arbitrary shape, they are probably as complicated as the surface integrals. Sir C. V. Raman (*loc. cit.*) showed that if one neglects the variation of amplitude of the secondary wavelets with direction, which is justified if one restricts oneself to small angle diffraction, then the conversion of the surface integral into a line integral can be done in a very simple manner. This idea of neglecting the obliquity factor was employed by the author (1944) to obtain such a transformation in the case of Fraunhofer diffraction. In this paper, it is extended to the case of Fresnel diffraction by both apertures and obstacles. It is shown that in either case the illumination in the region of shadow can be completely represented as the effect of radiations arising from the boundary. On the other hand, in the region of light, the disturbance at any point is the sum of the effects due to the direct light and the boundary. As suggested by Y. V. Kathavate (1945), the boundary radiation can again be effectively replaced by the radiation arising from a finite number of point-sources situated on the boundary, which may be called 'poles'. The diffraction pattern in the field of observation can then be considered as arising from the mutual interference of the radiations from these poles, and with the direct light if that is also present.

2. The Transformation of the Surface Integral into a Line Integral

Suppose that spherical waves are diverging from a point-source O, and that at a distance D from it they meet a diffracting screen consisting of an aperture or an obstacle of arbitrary form. We shall first consider the case of the aperture.

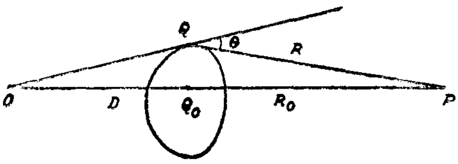


Fig. 1

(*a*) *Aperture.*—If the distance D is large enough, it can be supposed that the boundary of the aperture lies on a spherical wave-front originating from O. In Fig. 1, let Q be any point inside the aperture ; and let the distance QP to the point of observation be denoted by R. Q_0 is the point of intersection of the line joining P and O with the plane of the aperture, and Q_0P is denoted by R_0.

We may represent the disturbance at the observation point P as a summation of the effects of spherical waves having their origins distributed continuously over the area of the aperture. Hence, if the disturbance in the plane of the aperture is A cos $2\pi\nu t$, the disturbance at P is

$$X_P = \frac{A}{\lambda} \iint \frac{1}{R} \sin 2\pi\,(\nu t - R/\lambda)\, dS, \tag{1}$$

according to Kirchhoff's formulation of Huyghens' principle, and neglecting the variation of the amplitude of the secondary wavelets with the direction of propagation, as already stated. Now,

$$dS = R\,dR\, d\epsilon \cdot D/(D + R_0), \tag{2}$$

where ϵ is the angle between the plane OPQ and some fixed reference plane through OP, so that

$$X_P = \frac{AD}{\lambda\,(D + R_0)} \iint \sin 2\pi\,(\nu t - R/\lambda)\, dR\, d\epsilon. \tag{3}$$

Performing the integration from Q_0 to the boundary,

$$X_P = \frac{AD}{2\pi (D + R_0)} \int [\cos 2\pi (\nu t - R/\lambda)]_R^{R_0} \, d\epsilon, \tag{4}$$

where $R_0 = Q_0P$, and R is the distance from P to the point on the boundary corresponding to each value of ϵ. Since $\cos 2\pi (\nu t - R_0/\lambda)$ is a constant, this may be written in the form

$$X_P = \frac{AD}{2\pi (D + R_0)} \left\{ \cos 2\pi (\nu t - R_0/\lambda) \int d\epsilon - \int \cos 2\pi (\nu t - R/\lambda) \, d\epsilon \right\} \tag{5}$$

Now, $\int d\epsilon = 2\pi$ if the point P lies within the region of light of the aperture, and is zero if it lies outside in the region of shadow. Hence, in the region of light,

$$X_P = \frac{AD}{(D + R_0)} \cos 2\pi (\nu t - R_0/\lambda) - \frac{AD}{2\pi (D + R_0)} \int_0^{2\pi} \cos 2\pi (\nu t - R/\lambda) \, d\epsilon. \tag{6}$$

Now, $AD/(D + R_0)$ is the amplitude of the wave reaching P directly from the source (being inversely proportional to the distance from the source), call it $(X_P)_0$. In the second integral, the integration with respect to ϵ from 0 to 2π may be written as an integration over a complete circuit round the boundary of the aperture. If one denotes by ds an element of arc of the boundary, and by ϕ the angle which it makes with the plane through OP and the element, then,

$$d\epsilon = \frac{(D + R_0)}{D} \cdot \frac{\sin \phi}{R \sin \theta} \, ds, \tag{7}$$

where θ is the angle between the incident ray reaching the element ds, and the ray diffracted from it to the point of observation. Making this substitution, one obtains

$$X_P = (X_P)_0 - \frac{A}{2\pi} \oint \frac{\sin \phi}{R \sin \theta} \cos 2\pi (\nu t - R/\lambda) \, ds. \tag{8}$$

Hence, in the region of light, the total disturbance at P = the disturbance due to the direct wave + a disturbance due to a linear distribution of light sources along the boundary of strength $A \sin \phi / 2\pi \sin \theta$. The latter has a phase *opposite* to that of the direct light at the boundary.

In the region of shadow, $\int d\epsilon = 0$, so that

$$X_P = -\frac{AD}{2\pi (R_0 + D)} \int \cos 2\pi (\nu t - R/\lambda) \, d\epsilon,$$

Here, although ϵ does not have values from 0 to 2π, the corresponding point on the boundary moves completely round it, and the relation between ds and $d\epsilon$ is the same as (7). The sign of $\sin\theta$ however depends on the position of the point on the boundary ; this will be discussed in the next section. Hence,

$$X_P = -\frac{A}{2\pi} \oint \frac{\sin\phi}{R\sin\theta} \cos 2\pi\,(\nu t - R/\lambda)\, ds, \tag{9}$$

so that the disturbance at any point in the region of shadow arises solely from the boundary radiation.

(*b*) *Obstacle.*—An obstacle can be considered as a diffraction screen complementary to an aperture. The wave-front in this case is the complete one from which a finite area bounded by a closed curve of arbitrary shape is cut off. Using the same notation as with the aperture, and considering the case when the point of observation P is within the projection of the boundary on the observation plane, the disturbance at P is

$$X_P = \frac{AD}{\lambda\,(D + R_0)} \int_0^{2\pi}\!\!\int_R^{\infty} \sin 2\pi\,(\nu t - R/\lambda)\, dR\, d\epsilon \tag{10}$$

Here, the upper limit ∞ is used to denote the limit up to which the wave-front is effective at P. For a spherical wave-front, this will correspond to the value of R for which the line from P to the variable point Q is tangential to the wave-front. The above integral can be split up into two as

$$X_P = \frac{AD}{\lambda\,(R_0 + D)} \int_0^{2\pi}\!\!\int_{R_0}^{\infty} \sin 2\pi\,(\nu t - R/\lambda)\, dR\, d\epsilon$$

$$+ \frac{AD}{\lambda\,(R_0 + D)} \int_0^{2\pi}\!\!\int_R^{R_0} \sin 2\pi\,(\nu t - R/\lambda)\, dR\, d\epsilon. \tag{11}$$

The first integral is obviously the effect of the total wave-front at P, and from the general theory of diffraction, its value is

$$\frac{AD}{(R_0 + D)} \cos 2\pi\,(\nu t - R/\lambda) = (X_P)_0, \tag{12}$$

The second integral is the same as in the case of the aperture with the limits reversed, and is therefore equal to

$$\frac{AD}{2\pi\,(D + R_0)} \int_0^{2\pi} \cos 2\pi\,(\nu t - R/\lambda)\, d\epsilon - (X_P), \tag{13}$$

Hence, when P lies in the region of shadow,

$$X_P = \frac{AD}{2\pi(D+R_0)} \int_0^{2\pi} \cos 2\pi (\nu t - R/\lambda)\, d\epsilon$$

$$= \frac{A}{2\pi} \oint \frac{\sin\phi}{R\sin\theta} \cos 2\pi(\nu t - R/\lambda)\, ds. \tag{14}$$

Even when the point of observation lies in the region of light, X_P can be put in the form (10): but here R is different from R_0 only for those values of ϵ for which the plane through OP making this angle with the reference plane cuts the boundary of the obstacle. Similarly, it can also be brought into the form (11), with the first term equal to (12). The second term, however, is equal to (9) with the sign changed, so that

$$X_P = (X_P)_0 + \frac{A}{2\pi} \oint \frac{\sin\phi}{R\sin\theta} \cos 2\pi(\nu t - R/\lambda)\, ds. \tag{15}$$

It may be noted that, wherever the point of observation P may be, the sum of the amplitudes due to an aperture and a complementary obstacle at it is always equal to the amplitude $(X)_0$ due to the direct unobstructed wave. This is the extension of Babinet's principle to Fresnel patterns. It could in fact have been used to obtain the expressions (14) and (15).

3. The Phase of the Boundary Radiation

In the previous section, we have seen how it is possible to represent the disturbance at any field-point as the summation of the effects of the direct light and of the boundary radiation. We shall now consider the relation between the phase of the radiation emitted by an element of the boundary and that of the incident wave at that point.

It is evident from Equation (8) that, in the region of light of an aperture, the phase of the boundary radiation is opposite to that of the incident light. In the region of shadow, however, it is not so obvious: but it can be determined by the following method. The integral over ϵ in this case does not cover a cycle of values, *viz.*, from 0 to 2π, but only a limited range of values, say from ϵ_1 to ϵ_2 and back again from ϵ_2 to ϵ_1. This will be clear from Fig. 2, where P is the projection of the point of observation on the plane of the aperture with respect to the point source, and PA and PB are the tangents to the curved boundary from P. As the representative point on the boundary makes a circuit BDAC, ϵ increases from the value ϵ_1 (say) at B to ϵ_2 (say) at A, and then diminishes to ϵ_1 in the path ACB. It is obvious that the above statement implies the usual definitions of the signs of $d\epsilon$ and ds, *viz.*, that the former is positive if it is anti-clockwise when viewed from above, and the latter is positive if it is described in an anti-clockwise sense for an observer inside the curve looking down from above the plane of the paper. Hence, ds and $d\epsilon$ have the same sign along BDA, and are of opposite signs along ACB, and

$$-\frac{\mathrm{AD}}{2\pi\,(\mathrm{R}_0+\mathrm{D})}\int \cos 2\pi\,(\nu t-\mathrm{R}/\lambda)\, d\epsilon$$

$$=-\frac{\mathrm{A}}{2\pi}\oint\frac{\sin\phi}{\mathrm{R}\sin\theta}\cos 2\pi\,(\nu t-\mathrm{R}/\lambda)\, ds$$

$$=\frac{\mathrm{A}}{2\pi}\left[\int_{\mathrm{ACB}}\frac{\sin\phi}{\mathrm{R}\,|\sin\theta|}\cos 2\pi\,(\nu t-\mathrm{R}/\lambda)\, ds - \int_{\mathrm{BDA}}\frac{\sin\phi}{\mathrm{R}\,|\sin\theta|}\cos 2\pi\,(\nu t-\mathrm{R}/\lambda)\, ds\right] \qquad (16)$$

This is equivalent to defining the sign of $\sin\theta$ to be positive or negative according as ds and $d\epsilon$ have the same sign or opposite signs. An examination of Fig. 2 shows that these regions are respectively those for which a ray of light reaching a point of observation in the region of shadow has to pass through the region of light or has not to. For the former, the phase of the boundary radiation is opposite and for the latter the same as that of the incident wave.

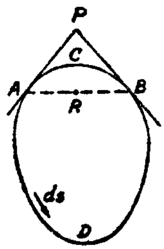


FIG. 2

For simplicity, we have treated above the case of a boundary for which only two tangents PA and PB can be drawn. It can easily be verified that the main results are true even for a boundary to which any number of tangents can be drawn from a point outside. In such a case, the phase of the boundary wave is the same as or opposed to that of the incident wave according as it immediately proceeds into the region of shadow or the region of light. So also, even if the point of observation is inside the region of light, if the boundary radiation reaching it has first to pass through the region of shadow, then the phase is the same as that of the incident wave. In exactly the same way, it can be verified that, for an obstacle also, the phase of the boundary radiation is the same as that of the incident if it proceeds into the region of shadow, and *vice versa.*

In general therefore, we have the theorem:

"*If a ray reaching the point of observation from an element of the boundary proceeds first into the region of light, then its phase is opposed to that of the incident wave at that element of the boundary, and if it passes first into the region of shadow, then the phase is the same.*"

4. Continuity of the Disturbance across the Projection of the Boundary

It will be noticed that the expressions derived in Section 2 for the disturbance at a point of observation are quite different in form according as the point lies in the region of light or of shadow, one of the expressions containing $(X_P)_0$, the illumination due to the direct wave at P, while the other does not. This is so both for an aperture and an obstacle. Besides, the substitution (7) is invalid when $\theta = 0$, *i.e.*, when P lies on the projection of the boundary, so that one has to work with the integral

$$\frac{AD}{2\pi(D+R_0)}\int [\cos 2\pi\,(\nu t - R/\lambda)]\,d\epsilon. \tag{A}$$

Even here, the range of integration of ϵ is not clear. Consequently, it is proposed to show in this section that the disturbance is continuous as one crosses the projection of the boundary in the field of observation. This will be done by showing that the limiting value of the amplitude at a point on the boundary is the same, irrespective of whether the point is brought to the boundary from within or without.

In Fig. 2, C is the projection of the observation point (on the boundary between the region of light and of shadow), and P and R are the projections of two points on either side close to C lying respectively outside and inside the boundary. We shall designate by $(A)_+$ and $(A)_-$ the limits to which the value of the integral (A) tends as R and P respectively are brought into coincidence with C.

Limit $(A)_+$.—Draw a line ARB through R parallel to the tangent at C to cut the boundary at A and B. Choosing RB to correspond to $\epsilon = 0$, the above integral can be split into

$$\int_0^{\pi} [\;]\,d\epsilon + \int_{\pi}^{2\pi} [\;]\,d\epsilon,$$

where the former corresponds to ACB and the latter to BDA. In the limit when R is very near C, the value of [ ] is very nearly a constant, and the first integral becomes equal to $\pi \cos 2\pi (\nu t - R/\lambda)$, where R is the distance from C to the point of observation. The other integral can be written as such, it being understood that the origin is now at C, and the reference line for zero ϵ is the tangent at C.

Hence,

$$(A)_+ = \frac{(X_C)}{2} + \frac{AD}{2\pi (D + R_0)} \int_{\pi}^{2\pi} \cos 2\pi (\nu t - R/\lambda)\, d\epsilon. \tag{17}$$

Limit $(A)_-$.—Let P be a point close to C outside the boundary, and let PA and PB be tangents to the curve. With the same definitions as before, the integral in (A) can be written as

$$\int_{\substack{\pi+\eta_1 \\ \mathrm{ADB}}}^{2\pi-\eta_2} [\ ]\, d\epsilon - \int_{\substack{\pi+\eta_1 \\ \mathrm{ACB}}}^{2\pi-\eta_2} [\ ]\, d\epsilon,$$

where η_1 and η_2 are small angles. Just as in the previous case, the value of [ ] is a constant in the limit when P is very near C. Also, η_1 and η_2 tend to zero in the same limit. Hence,

$$(A)_- = \frac{AD}{2\pi (D + R_0)} \int_{\pi}^{2\pi} \cos 2\pi (\nu t - R/\lambda)\, d\epsilon - \frac{(X_C)_0}{2}. \tag{18}$$

Substituting the values in the expressions for the disturbance at a point in the field of observation, and denoting the limits to which this tends as the point approaches C from the region of light and the region of shadow as $(X_C)_+$ and $(X_C)_-$ respectively, we get, for an aperture,

$$(X_C)_+ = (X_C)_0 - (A)_+ = \frac{(X_C)_0}{2} - \frac{AD}{2\pi (D + R_0)} \int_{\pi}^{2\pi} \cos 2\pi (\nu t - R/\lambda)\, d\epsilon,$$

$$(X_C)_- = -(A)_- = \frac{(X_C)_0}{2} - \frac{AD}{2\pi (D + R_0)} \int_{\pi}^{2\pi} \cos 2\pi (\nu t - R/\lambda)\, d\epsilon,$$

so that $(X_C)_+ = (X_C)_-$.

In the same way, for a point on the boundary of an obstacle also,

$$(X_C)_+ = (X_C)_0 + (A)_- = (A)_+ = (X_C)_-.$$

It is thus seen that in both cases the two limits are identical, so that the disturbance is continuous across the projection of the boundary.

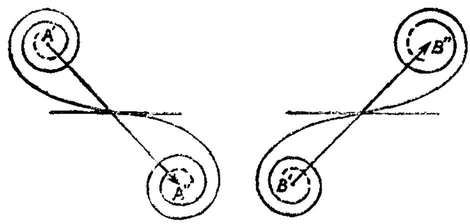
5. Application of the Boundary Radiation Method to Practical Problems in Diffraction

In Section 2, we have seen how it is possible to consider the diffracted radiation as arising from the boundary of an aperture or an obstacle. There it was shown that each line-element of the boundary is a source of radiation, whose strength is inversely proportional to the sine of the angle of diffraction and directly proportional to the sine of its inclination with the plane of diffraction. The phase of the radiation reaching the point of observation, however, varies as the source moves round the boundary. It follows then that the resultant effect would be contributed mainly by those parts of the edge for which R is a maximum or a minimum, and the phase consequently stationary. The radiations from contiguous parts of the boundary in these parts reinforce each other, producing a large amplitude. The phase of the radiations from other portions of the boundary varies rapidly, so that they contribute little to the total disturbance at the field point. It can be shown that for those points on the boundary for which R is a maximum or a minimum, sin ϕ is a maximum numerically. In fact, the projections of these points on the observation plane are the feet of the normals from the observation point P to the projection of the boundary. These points may be called the 'poles' of the point of observation P. The bulk of the disturbance at P is contributed by narrow regions on the boundary on either side of the poles. The radiations from the various poles interfere and produce the diffraction pattern.

We shall now consider whether the phase of the resultant radiation reaching the point of observation from regions of the boundary including and lying on either side of a pole is the same as the phase of the light wave radiated by the pole itself. This can be discussed in somewhat general terms by employing a vector diagram similar to the Cornu spiral. We shall begin the discussion by a consideration of the straight edge, which may, as an extrapolation of the ideas proposed in this paper, be considered as a very large aperture with one edge straight, while the remaining parts of the boundary are at infinity. Consequently, one pole will be on the straight edge, the other one being at infinity and therefore ineffective. Now, the path to the point of observation P *via* a point on the boundary varies with the position of the latter, being a minimum for the pole of the observation point. Consequently, if we sum up the resultant amplitude by means of a vector diagram, we would obtain a curve of the shape shown in Fig. 3 (*a*). It can be shown by a simple calculation that the shape of the curve is identical with that of the Cornu spiral, so that the resultant disturbance is $\overrightarrow{A'A''}$, and the phase of the resultant lags by $\pi/4$ behind that of the radiation coming directly from the pole. Making use of this and of the result that the phase of the boundary radiation is π in advance of the direct light, it is seen that, in the region of light, the resultant pole radiation is $3\pi/4$ in advance of a ray of light reaching it *via* the pole. Consequently, there would be interference bands in this region, the positions of whose maxima and minima are given by

$$2\pi\delta/\lambda = (2n\pi + 3\pi/4) \text{ and } (2n\pi + 7\pi/4)$$

respectively, where $2\pi\delta/\lambda$ is the phase difference between the direct ray to the point of observation and that *via* the edge. ($\delta = s^2(a+b)/2ab$ according to the usual notation). These values are in quantitative agreement with those deduced from the usual theory (see Drude, 1929).



(*a*) (*b*)

FIG. 3

Next, let us consider a boundary having the shape of a smooth closed curve with two poles. Then, the path will be a minimum for one of them (say A) and a maximum for the other (B). If the boundary is not too small, the vibration curve will consist of two spirals of the form shown in Figs. 3 (*a*) and (*b*), one for each pole. The spiral for the pole A would however not extend to the points A′ and A″; yet, the phase of the resultant will be lagging behind that of the wave direct from the pole by nearly $\pi/4$. For the pole B having maximum path, the phase difference is nearly $+\pi/4$, the resultant leading over the direct wave. In this, we have taken the domain of each pole to extend up to those points on the boundary on either side at which the rate of change of path reaches a maximum numerically (or $d^2\delta/ds^2 = 0$). The vibration curve is thus split up into two parts, one corresponding to the pole A and the other to B.

In the case of a boundary having more than two poles, an extension of the above reasoning shows that for all poles corresponding to a minimum path the phase of the resultant radiation lags by $\pi/4$ behind that of the wave directly from the pole, and for those corresponding to a maximum path, it leads by $\pi/4$.

6. Application to the Diffraction Pattern of a Circular Disc

In this section, we shall consider the application of the above considerations to the calculation of the radii of the circular fringes formed inside the geometric shadow of a circular disc. It is obvious that at the centre, the radiation from all the portions of the boundary are exactly in phase, so that there must be a bright spot there, as is indeed observed. If the point of observation is moved away from the centre, the illumination at it can be supposed to arise from two poles, which (by the definition given above) are situated at the ends of a diameter through the projection P′ of the point of observation on the plane of the circular disc. Now, the pole which is on the same side of the centre as P′ is obviously one of minimum path, so that the other pole corresponds to maximum path. Hence, there is a phase difference of $\pi/2$ between the two radiations at the boundary, the pole on the further side leading by this amount. Now, as P moves away from the centre of the geometric shadow, the phase of the radiation from the farther pole lags behind that from the other one owing to the increase in path. Consequently, the first dark fringe, which corresponds to a phase difference of $-\pi$, must correspond to an extra path of $3\lambda/4$. The successive dark fringes similarly occur at path retardations δ of $(n + 3/4)\,\lambda$, while the bright fringes occur at $\delta = (n + 1/4)\,\lambda$, where n is an integer. From this, the radii of the dark rings can be calculated to be

$$R = (n + 3/4)\,\lambda b/2r, \tag{19}$$

where b is the distance of the screen from the disc of radius r. The coefficient within the brackets in (19) can also be calculated from Lommel's theory. In the following table, they are compared with the values given by the present theory. The agreement is remarkably good.

From present theory	From Lommel's theory	Deviation
0·75	0·766	0·016
1·75	1·754	0·004
2·75	2·754	0·004
3·75	3·753	0·003
4·75	4·753	0·003

My best thanks are due to Prof. Sir C. V. Raman for the suggestion of the problem and for the kind encouragement which he gave me during the investigation.

Summary

Neglecting the obliquity factor, which is justified when one is considering only small angle diffraction, it is shown that the surface integral which

is usually employed for the determination of the disturbance at any point can be easily converted into a line integral along the boundary of the diffracting screen. The formulæ thus obtained show that with either an aperture or an obstacle the illumination in the region of shadow can be completely represented as the effect of radiations arising from the boundary, while in the region of light the disturbance due to the direct light is superposed on this. The phase of the boundary radiation is determined by the region (of light or shadow) to which the ray towards the point of observation proceeds from the boundary, being opposite to that of the incident light in the former case, and being the same in the latter case. It is however shown that this leads to no discontinuity in the illumination as the point of observation passes from the region of light into the region of shadow. The boundary radiation can again be effectively replaced by the radiations arising from a finite number of point-sources situated on the boundary called 'poles', for which the path to the observation point *via* the boundary is a maximum or a minimum. The phase of the resultant disturbance due to regions of the boundary including and lying on either side of a pole is shown to lead over or lag behind that of the wave from the pole by the quantity $\pi/4$, according as the pole is one of maximum or minimum path. Applying these ideas to the diffraction pattern of a circular disc, it is shown that the calculated radii of the rings in the region of shadow agree well with those deduced from Lommel's theory.

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