ON THE SCATTERING OF SCALAR MESONS

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IN a recent paper (Harish-Chandra, 1944, referred to as A in this paper) the equations of motion of a point particle interacting with a scalar meson field have been derived. The object of this note is to use these equations to calculate the scattering of scalar mesons by neutrons (or protons) on the classical theory, taking into account the radiation damping. This calculation is entirely similar to the corresponding one, done by Bhabha (1939, 1941) for the case of the vector-mesons. On account of the neglect of the quantum effects and the charge of the meson these calculations are subject to the same limitations as those of Bhabha. Since it is as yet not at all certain whether the actual meson has a spin of 1 or 0 unit, the scattering formulæ obtained here are to be looked upon as possible alternatives to those given by Bhabha.

§1. We shall keep to the notation of the previous papers (Bhabha and Harish-Chandra, 1944, Harish-Chandra, 1944). τ is the proper time at any point on the world line of the neutron measured from some fixed point on it. $z_\mu(\tau)$ are the co-ordinates of this point. x_μ denote the co-ordinates of any field-point. The fundamental metric tensor is taken to be

$$g_{\mu\nu} = 0 \quad \mu \neq \nu, \quad g_{00} = -g_{11} = -g_{22} = -g_{33} = 1$$

A dot denotes differentiation with respect to $\tau \cdot v_\mu = \dot{z}_\mu(\tau)$ and $u_\mu = x_\mu - z_\mu(\tau)$.

We shall assume that the neutron has a 'charge' and a 'dipole'. Following Bhabha we consider the scattering due to each of these separately. First we calculate the scattering due to the charge alone.

In the notation of A the equation of motion of the neutron is

$$m v_\mu - g_1 \frac{d}{d\tau}(\mathrm{U}'^{\text{mean}} v_\mu) = -g_1 \mathrm{U}_\mu'^{\text{mean}} \tag{1}$$

where m is the mass and g_1 the 'charge' of the neutron. $\mathrm{U}'^{\text{mean}}$ and $\mathrm{U}'^{\text{mean}}_\mu$ are the modified mean-potential and field respectively. It has been shown in A [Eqs. (3·17) and (3·18)] that

$$\mathrm{U}'^{\text{mean}} = \mathrm{U}^{\text{in}} + \widetilde{\mathrm{U}},$$

$$\mathrm{U}_\mu'^{\text{mean}} = \mathrm{U}_\mu^{\text{in}} - g_1\left(\tfrac{1}{3}\ddot{v}_\mu + \tfrac{1}{3} v_\mu (\dot{v})^2 + \tfrac{1}{2}\chi^2 v_\mu\right) + \widetilde{\mathrm{U}}_\mu \tag{2a}$$

where

$$\widetilde{U} = -g_1 \chi \int_{-\infty}^{\tau} \frac{J_1(\chi u)}{u}\, d\tau', \ \widetilde{U}_\mu = g_1 \chi^2 \int_{-\infty}^{\tau} u_\mu \frac{J_2(\chi u)}{u^2}\, d\tau' \qquad (2b)$$

U^{in} is the ingoing potential and $U_\mu^{in} = \partial_\mu U^{in}$ is the ingoing field. We assume the ingoing field to be a plane wave travelling in the x_1-direction. Assuming that the amplitude of oscillation of the neutron is small compared to the incident wave length we can write

$$U_1^{in} = \gamma \sin \omega_0 t, \ U_2^{in} = 0, \ U_3^{in} = 0 \qquad (3)$$

where $t = z_0(\tau)$. We assume that the velocity of the neutron is small so that

$$v_k \approx \frac{dz_k}{dt}$$

for $k = 1, 2, 3$. In conformity with (3) we put (*cf.* Bhabha, 1939)

$$\left.\begin{aligned} z_1 &= \frac{\beta}{\omega_0} \sin(\omega_0 t + \delta), \ z_2 = 0, \ z_3 = 0 \\ v_1 &= \beta \cos(\omega_0 t + \delta), \ \dot{v}_1 = -\beta\omega_0 \sin(\omega_0 t + \delta) \\ v_0 &\approx 1, \ v_2 = v_3 = 0 \end{aligned}\right\} \qquad (4)$$

We assume the amplitudes β and γ to be small so that quantities quadratic in them may be neglected. Putting $x_\mu = z_\mu(\tau)$ we get for (2)

$$u_1 = z_1(\tau) - z_1(\tau') = \frac{\beta}{\omega_0}[\sin(\omega_0 t + \delta) - \sin(\omega_0 t' + \delta)],$$

$$u_2 = u_3 = 0 \qquad (5a)$$

$$u \approx t - t' = u_0 \qquad (5b)$$

From (2*b*), (4) and (5) we get

$$\widetilde{U} = -g_1 \chi \int_{-\infty}^{\tau} \frac{J_1(\chi u)}{u}\, d\tau'$$

$$= -g_1 \chi \int_{0}^{\infty} \frac{J_1(\chi u)}{u}\, du = -g_1 \chi \qquad (6)$$

$$\widetilde{U}_0 = g_1 \chi^2 \int_{-\infty}^{\tau} u_0 \frac{J_2(\chi u)}{u^2}\, d\tau'$$

$$= g_1 \chi^2 \int^{\infty} \frac{J_2(\chi u)}{u}\, du = \tfrac{1}{2} g_1 \chi^2 \qquad (7)$$

$$\widetilde{U}_1 = g_1 \chi^2 \int_{-\infty}^{\tau} u_1 \frac{J_2(\chi u)}{u^2} d\tau'$$

$$= g_1 \chi^2 \int_0^{\infty} \frac{J_2(\chi u)}{u} u_1 \, du$$

$$= \frac{\beta g_1 \chi^3}{\omega_0} \int_0^{\infty} \frac{J_2(s)}{s} [\sin \alpha - \sin (\alpha - \nu s)] \, ds \tag{8}$$

where $s = \chi u$, $\nu = \frac{\omega_0}{\chi}$ and $\alpha = \omega_0 t + \delta$. Using the well-known result that

$$\int_0^{\infty} ds \frac{J_2(s)}{s^2} e^{-i\nu s} = \begin{cases} \frac{1}{3}[(1-\nu^2)^{3/2} + i\nu^3 - \frac{3}{2} i\nu] & 0 < \nu < 1 \\ \frac{1}{3}[-i(\nu^2-1)^{3/2} + i\nu^3 - \frac{3}{2} i\nu] & \nu > 1 \end{cases} \tag{9}$$

we get

$$\widetilde{U}_1 = -\beta g_1 \omega_0^2 (P \cos \alpha + Q \sin \alpha) \tag{10a}$$

where

$$P = \begin{cases} \frac{1}{3} - \frac{1}{2}\nu^2 & 0 < \nu < 1 \\ -\frac{1}{3}\frac{(\nu^2-1)^{3/2}}{\nu^3} + \frac{1}{3} - \frac{1}{2\nu^2} & \nu > 1 \end{cases}$$

$$Q = \begin{cases} \frac{1}{3}\frac{(1-\nu^2)^{3/2}}{\nu^3} - \frac{1}{3\nu^3} & 0 < \nu < 1 \\ -\frac{1}{3\nu^3} & \nu > 1 \end{cases} \tag{10b}$$

Substituting the value of the various quantities in (1) and putting $m + g_1^2\chi = M$ we get up to terms of the first order in β and γ

$$-\beta\omega_0 M \sin \alpha = -g_1 \gamma \cos(\alpha - \delta) + \beta g_1^2 \omega_0^2 [P \cos \alpha + Q \sin \alpha] - \tfrac{1}{3} g_1^2 \beta \omega_0^2 \cos \alpha + \tfrac{1}{2}\chi^2 g_1^2 \beta \cos \alpha \tag{11}$$

Equating coefficients of $\sin \alpha$ and $\cos \alpha$ on both sides of (11) we get

$$\beta = \frac{\gamma}{g_1 \omega_0 \left\{ P'^2 \omega_0^2 + \left[Q\omega_0 + \frac{M}{g_1^2} \right]^2 \right\}^{\frac{1}{2}}} \tag{12a}$$

$$\cos \delta = \frac{\omega_0 P'}{\left[P'^2 \omega_0^2 + \left(Q\omega_0 + \frac{M}{g_1^2} \right)^2 \right]^{\frac{1}{2}}} \tag{12b}$$

where the positive value of the square root is to be taken. $P' = P - \frac{1}{3} + \frac{1}{2}\nu^2$

so that

$$P' = \begin{cases} 0 & \nu < 1 \\ -\frac{1}{3}\frac{(\nu^2-1)^{3/2}}{\nu^3} & \nu > 1 \end{cases} \tag{12c}$$

It is clear from (12) that the quantity $M + g_1^2 Q\omega_0$ behaves as the effective mass. For very slow oscillations $\omega_0 \ll \chi$ $(\nu \ll 1)$ $Q \approx -\frac{1}{2\nu}$

$$M + g_1^2 Q \omega_0 = M - \tfrac{1}{2} g_1^2 \chi = m + \tfrac{1}{2} g_1^2 \chi.$$

Therefore in this case the field contributes a *positive mass* $\frac{1}{2} g_1^2\chi$. On the other hand in the vector meson case the field adds a *negative mass* $-\frac{1}{2}g_1^2 \chi$ for slow oscillations (*cf.* Bhabha, 1939).

To calculate the scattering we have to calculate the retarded field at a very distant point $x_\mu = (X, Y, Z, T)$ lying on the future light cone from the point τ on the world line.

$$U^{ret}_{(x_\mu)} = \frac{g_1}{\kappa(\tau)} - \chi g_1 \int_{-\infty}^{\tau} \frac{J_1(\chi u)}{u} d\tau'$$

$$= g_1 \int_{-\infty}^{\tau} \frac{J_0(\chi u)}{u} \frac{d}{d\tau}\left(\frac{1}{\kappa}\right) d\tau'$$

$$= g_1 \int_{-\infty}^{\tau} J_0(\chi u) \left\{\frac{1}{\kappa^2} - \frac{\kappa'}{\kappa^2}\right\} d\tau' \tag{13}$$

where $\kappa = u^\mu(\tau') v_\mu(\tau')$ and $\kappa' = u^\mu(\tau') \dot{v}_\mu(\tau')$. For a very distant point we can neglect the first term $\frac{1}{\kappa^2}$ and write

$$U^{ret} \approx -g_1 \int_{-\infty}^{\tau} J_0(\chi u) \frac{\kappa'}{\kappa^2} d\tau' = -g_1 \int_0^{\infty} J_0(\chi u) \frac{\kappa'}{\kappa^3} du \tag{14}$$

Writing $R = \sqrt{X^2 + Y^2 + Z^2}$ we get up to terms of the first order in β

$$\left.\begin{aligned} \kappa' &= -X\dot{v}_1 = \beta\omega_0 X \sin(\omega_0 t' + \delta) \\ u^2 &= (T - t')^2 - R^2 + \frac{2X\beta}{\omega_0}\sin(\omega_0 t' + \delta) \\ \kappa &= (T - t') - Xv_1 = (T - t') - \beta X \cos(\omega_0 t + \delta) \end{aligned}\right\} \tag{15}$$

so that in the same approximation

$$\frac{\kappa'}{\kappa^3} = \frac{\beta\omega_0 X \sin(\omega_0 t' + \delta)}{(u^2 + R^2)^{3/2}}$$

$$= \frac{\beta\omega_0 X \sin(\omega_0 T + \delta - \omega_0\sqrt{u^2 + R^2})}{(u^2 + R^2)^{3/2}}$$

$$= \frac{\beta\omega_0 X \sin(a' - \omega_0\sqrt{u^2 + R^2})}{(u^2 + R^2)^{3/2}} \tag{16}$$

if we write a' for $\omega_0 T + \delta$. Thus from (14) and (16)

$$U^{ret} = - g_1 \beta \omega_0 X \int_0^\infty J_0(\chi u) \frac{\sin(a' - \omega_0 \sqrt{u^2 + R^2})}{(u^2 + R^2)^{3/2}} u \, du$$

$$= - \frac{g_1 \beta \omega_0 X}{\chi} \int_0^\infty J_0(s) \frac{\sin(a' - \nu \sqrt{s^2 + r^2})}{(s^2 + r^2)^{3/2}} s \, ds \qquad (17)$$

where $s = \chi u$, $r = R\chi$ and $\nu = \frac{\omega_0}{\chi}$. On evaluating (17) and retaining only the terms of the lowest order in $\frac{1}{R}$ we find

$$U^{ret} = \begin{cases} g_1 \beta \frac{X}{R} \frac{\sqrt{\chi^2 - \omega_0^2}}{R\omega_0} e^{-R\sqrt{\chi^2 - \omega_0^2}} \sin(\omega_0 T + \delta), & \omega_0 < \chi \\ g_1 \beta \frac{X}{R} \frac{\sqrt{\omega_0^2 - \chi^2}}{R\omega_0} \cos(\omega_0 T + \delta - R\sqrt{\omega_0^2 - \chi^2}), & \omega_0 > \chi \end{cases} \qquad (18)$$

For the case $\omega_0 < \chi$ the potential falls off exponentially with distance and obviously there is no scattering. We therefore consider the case $\omega_0 > \chi$. In this case the incident wave is consistently with (3)

$$U^{in} = \frac{-\gamma}{\sqrt{\omega_0^2 - \chi^2}} \sin(\omega_0 t - \sqrt{\omega_0^2 - \chi^2} \, x_1). \qquad (19)$$

The flow of energy per unit area per unit time in the x_1 direction as calculated from the energy-momentum tensor

$$4\pi T_{\mu\nu} = U_\mu U_\nu - \tfrac{1}{2} g_{\mu\nu} (U_\rho U^\rho - \chi^2 U^2) \qquad (20)$$

is

$$\frac{\overline{U_0 U_1}}{4\pi} = \frac{\gamma^2 \omega_0}{8\pi \sqrt{\omega_0^2 - \chi^2}} \qquad (21)$$

where the bar denotes the average over time. Similarly the flow of energy due to (18) in the direction of the radius R across an element of surface subtending a solid angle $d\Omega$ is

$$\frac{g_1^2 \beta^2}{8\pi} \cos^2\theta \frac{(\omega_0^2 - \chi^2)^{3/2}}{\omega_0} d\Omega \qquad (22)$$

where θ is the angle between the incident and the scattered wave *i.e.*, $X/R = \cos\theta$. The differential-scattering cross-section in the direction θ is therefore

$$g_1^2 \frac{\beta^2}{\gamma^2} \cos^2\theta \frac{(\omega_0^2 - \chi^2)}{\omega_0^2} d\Omega$$

$$= \frac{\cos^2\theta}{\frac{1}{9}(\omega_0^2 - \chi^2) + \frac{\omega_0^4}{(\omega_0^2 - \chi^2)^2}\left[a + \chi - \frac{\chi^3}{3\omega_0^2}\right]^2} \qquad (23)$$

from (12a) if we put $\frac{m}{g_1^2} = a$. Integrating over all directions θ we find that the total cross-section is

$$12\pi \frac{1}{(\omega^2 - \chi^2) + \frac{\omega_0^4}{(\omega_0^2 - \chi^2)^2}\left[3a + 3\chi - \frac{\chi^3}{\omega_0^2}\right]^2} d\Omega \tag{24}$$

For the spin 1 case the scattering of the transverse mesons is given by (Bhabha, 1939)

$$6\pi\left(1 + \frac{1}{2}\frac{\chi^2}{\omega_0^2}\right)\frac{1}{\left(\omega_0^2 - \frac{3\chi^4}{4\omega_0^2} - \frac{1}{4}\frac{\chi^6}{\omega_0^4}\right) + \left(\frac{3}{2}a + \frac{1}{2}\frac{\chi^3}{\omega_0^2}\right)^2}$$

For high energy this is half of (24) independently of the value of g_1. For $\chi = 0$ (24) becomes

$$\frac{12\pi}{9a^2 + \omega_0^2}$$

which is of the same form as the corresponding formula of Dirac (1938), *viz.*, $\frac{24\pi}{9a^2 + 4\omega_0^2}$ for the scattering of light by an electron. As found in other cases the cross-section (24) decreases with increasing ω_0 as $\frac{1}{\omega_0^2}$ for high frequencies.

§2. We shall now treat the scattering by the neutron due to the dipole-moment alone. We denote the dipole-vector by $g_2 S^\alpha$. As already pointed out in A we have to assume

$$S^\alpha v_\alpha = 0 \tag{25}$$

The equation of rotational motion is [Eqn. (5·36b) of A]

$$I\epsilon_{\mu\nu\rho\sigma} v^\rho \dot{S}^\sigma = g_2 [S_\mu \bar{U}_\nu - S_\nu \bar{U}_\mu] \tag{26}$$

where $\bar{U}_\nu = U_\nu'^{\text{mean}} - v_\nu (v^\mu U_\mu'^{\text{mean}})$. $\epsilon_{\mu\nu\rho\sigma}$ is the tensor which is anti-symmetric in each pair of indices and $\epsilon_{0123} = -1$. Following Bhabha we put m equal to infinity to simplify the problem. In this case we find from the translational equation that $\dot{v}_\mu = \ddot{v}_\mu = \ldots. = 0$, so that we can consider the dipole in the rest system. In the usual three-dimensional vector notation (26) can be written as

$$I \dot{S} = g_2 [S \cdot U'^{\text{mean}}] \tag{27}$$

where the components of S and U'^{mean} are S_k and U'^{mean}_k ($k = 1, 2, 3$) respectively. The bracket denotes the vector product.

Using (25) we find from (3·19) of A that

$$U'^{\text{mean}}_\mu = U_\mu^{\text{in}} + g_2\left(\frac{1}{3}\dddot{S}_\mu + \frac{\chi^2}{2}\dot{S}_\mu\right) + \tilde{U}_\mu \tag{28}$$

where

$$\widetilde{U}_\mu = g_2 \chi^2 \int_{-\infty}^{t} S_\mu \frac{J_2(\chi u)}{u^2} dt' \tag{29}$$

Here we have used the fact that $\tau = t$ and $u_1 = u_2 = u_3 = 0$ so that $S^\sigma u_\sigma = 0$. Also

$$u = u_0 = t - t'$$

so that

$$\widetilde{\mathbf{U}} = g_2 \chi^2 \int_0^\infty \mathbf{S}(t-u) \frac{J_2(\chi u)}{u^2}. \tag{30}$$

It is to be noted that the right side of (30) is the same as $\frac{1}{2}\widetilde{\mathbf{G}}$ of Bhabha [1941, Eqn. (68)] if we replace **M** by **S** there. (27) can now be written as

$$\mathrm{I}\,\dot{\mathbf{S}} = g_2 [\mathbf{S}\cdot\mathbf{U}^{\mathrm{in}}] + \tfrac{1}{3} g_2^2 [\mathbf{S}\cdot\dddot{\mathbf{S}}] + \tfrac{1}{2} g_2^2 \chi^2 [\mathbf{S}\cdot\dot{\mathbf{S}}] + g_2 [\mathbf{S}\cdot\widetilde{\mathbf{U}}] \tag{31}$$

This equation is the same as Eqn. (66) of Bhabha (1941) if we replace **H** by $2\,\mathbf{U}^{\mathrm{in}}$, **M** by **S** and I by 2I in the latter and put $K = 0$. Thus corresponding to equations (69) and (70) of Bhabha we put

$$\mathbf{U}^{\mathrm{in}} = \tfrac{1}{2}\mathbf{H}_0 \cos \omega_0 t \equiv \tfrac{1}{2}\mathbf{H} \tag{32}$$

$$\mathbf{S}(t) = \mathbf{S}_0 + \mathbf{S}_1 \sin \omega_0 t + \mathbf{S}_2 \sin(\omega_0 t + \delta) \tag{33}$$

where $\mathbf{S}_0$ is the initial direction of the dipole and $\mathbf{S}_0$, $\mathbf{S}_1$ and $\mathbf{S}_2$ are mutually perpendicular and such that $\mathbf{S}_2$ is along $[\mathbf{S}_0\cdot\mathbf{S}_1]$. We assume $\mathbf{S}_0{}^2 = 1$. Substituting (32), (33) in (30) and (31) we get corresponding to Eqn. (74) of Bhabha

$$\begin{aligned}\omega_0 \mathbf{S}_1 &\left[\alpha \cos \omega_0 t + \frac{|\mathbf{S}_2|}{|\mathbf{S}_1|}\{\xi \sin(\omega_0 t + \delta) - \zeta \cos(\omega_0 t + \delta)\}\right] \\ + \omega_0 \mathbf{S}_2 &\left[\alpha \cos(\omega_0 t + \delta) - \frac{|\mathbf{S}_1|}{|\mathbf{S}_2|}\{\xi \sin \omega_0 t - \zeta \cos \omega_0 t\}\right] \\ &= \frac{3}{2g_2^2}[\mathbf{S}_0\cdot\mathbf{H}_0]\cos\omega_0 t\end{aligned} \tag{34}$$

where $\alpha = \frac{3I}{g_2^2}$ and

$$\xi \equiv \begin{cases} -\dfrac{\chi^3}{\omega^0} + \dfrac{(\chi^2 - \omega_0^2)^{3/2}}{\omega_0} & \omega_0 < \chi \\ -\dfrac{\chi^3}{\omega_0} & \omega_0 > \chi \end{cases} \tag{35}$$

$$\zeta \equiv \begin{cases} 0 & \omega_0 < \chi \\ \dfrac{(\omega_0^2 - \chi^2)^{3/2}}{\omega_0} & \omega_0 > \chi \end{cases} \tag{36}$$

Here the value of α is twice that used by Bhabha [Eqn. (73) l.c.], while ξ and ζ are the same as in his case. Since (34) is identical with Eqn. (74) of Bhabha, all the further results derivable from it in terms of α, ξ, ζ remain the same. Thus we get

$$\tan\delta = -\frac{\xi}{\zeta} \tag{37a}$$

$$\frac{|S_1|}{|S_2|} = \frac{\alpha}{\sqrt{\xi^2+\zeta^2}} \tag{37b}$$

$$|S_1| = \frac{3\alpha\,|H_0|\sin\theta}{2g_2\,\omega_0\,[(\alpha^2-\xi^2-\zeta^2)^2+4\alpha^2\zeta^2]^{\frac{1}{2}}} \tag{37c}$$

θ being the angle between S_0 and H_0. The work done by the external force on the dipole is on the average

$$-\tfrac{1}{2}g_2\,\overline{(\mathbf{H}\cdot\dot{\mathbf{S}})} = \frac{3\,H_0^2\sin^2\theta\,\zeta\,(\alpha^2+\xi^2+\zeta^2)}{8\,(\alpha^2-\xi^2-\zeta^2)^2+4\alpha^2\,\zeta^2}. \tag{38}$$

To obtain the scattering we have to calculate as before the retarded potential U^{ret} at a very distant point X, Y, Z, T lying on the future light cone from the point $z_\mu(\tau) = (0, 0, 0, t)$.

$$U^{ret}_{(x\mu)} = \partial_\alpha\left\{g_2\int_{-\infty}^{\tau} J_0(\chi u)\left(\frac{S^\alpha}{\kappa}\right)d\tau\right\} \tag{39}$$

$$u^2 = (T-t')^2 - R^2 \tag{40a}$$

$$\kappa = T - t' = \sqrt{u^2+R^2} \tag{40b}$$

Retaining only terms of the lowest order in $\frac{1}{R}$ we get

$$U^{ret} = g_2\,\partial_\alpha\int_0^\infty J_0(\chi u)\,\frac{S^\alpha}{(u^2+R^2)}\,du$$

$$= -g_2\,div\left\{\int_0^\infty \frac{\omega_0 J_0(\chi u)}{u^2+R^2}\{S_1\cos(\omega_0 T-\omega_0\sqrt{u^2+R^2}) + S_2\cos(\omega_0 T+\delta-\omega_0\sqrt{u^2+R^2})\}\cdot u\,du\right. \tag{41}$$

where the components of divergence are $\partial_k = \left(\frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z}\right)$. The term inside the divergence can be evaluated and up to the terms of lowest order in $\frac{1}{R}$ it is equal to

$$\frac{e^{-R\sqrt{\chi^2-\omega_0^2}}}{R}\,[S_1\sin\omega_0 T + S_2\sin(\omega_0 T+\delta)] \text{ for } \omega_0 < \chi$$

$$\frac{1}{R}\{S_1\sin(\omega_0 T - R\sqrt{\omega_0^2-\chi^2}) + S_2\sin(\omega_0 T+\delta-R\sqrt{\omega_0^2-\chi^2})\} \text{ for } \omega_0 > \chi$$

so that to the same approximation

$$U^{ret} = \begin{cases} g_2 \frac{\sqrt{\chi^2 - \omega_0^2}}{R} e^{-R\sqrt{\chi^2-\omega_0^2}} \left[\frac{(\mathbf{R}\ \mathbf{S}_1)}{R} \sin \omega_0 T + \frac{(\mathbf{R}\ \mathbf{S}_2)}{R} \sin(\omega_0 T + \delta)\right] & \text{for } \omega_0 < \chi \\ g_2 \frac{\sqrt{\omega_0^2 - \chi^2}}{R} \left[\frac{(\mathbf{R}\ \mathbf{S}_1)}{R} \cos(\omega_0 T - R\sqrt{\omega_0^2 - \chi^2}) + \frac{(\mathbf{R}\ \mathbf{S}_2)}{R} \cos(\omega_0 T + \delta - R\sqrt{\omega_0^2 - \chi^2})\right] & \text{for } \omega_0 > \chi \end{cases} \tag{42}$$

where $\mathbf{R} = (X, Y, Z)$. (42) would agree completely with the expression (84) of Bhabha for U_k^{ret} if we replace the scalar products with $\mathbf{R}$ by vector products. When $\omega_0 < \chi$ there is no radiation. For $\omega_0 > \chi$ the average rate of radiation in the direction $\mathbf{R}$ inside a solid angle $d\Omega$ is from (20)

$$\frac{g_2^2}{8\pi} (\omega_0^2 - \chi^2)^{3/2} \omega_0 \left[\frac{(\mathbf{R}\ \mathbf{S}_1)^2}{R^2} + \frac{(\mathbf{R}\ \mathbf{S}_2)^2}{R^2} + 2\frac{(\mathbf{R}\ \mathbf{S}_1)(\mathbf{R}\ \mathbf{S}_2)}{R^2} \cos\delta\right] d\Omega \tag{43}$$

corresponding to (84) of Bhabha. The total radiation obtained by integrating (43) over all directions is

$$\tfrac{1}{6} g_2^2 (\omega_0^2 - \chi^2)^{3/2} \omega_0 (|\mathbf{S}_1|^2 + |\mathbf{S}_2|^2) \tag{44}$$

which is the same as (38) due to (37) and (36). The energy flow due to the incident wave (32) is

$$\frac{|\mathbf{H}_0|^2}{32\pi} \frac{\omega_0}{\sqrt{\omega_0^2 - \chi^2}} \tag{45}$$

The total effective cross-section for the scattering of the scalar meson wave is therefore from (45), (44) and (38) and (36)

$$12\pi \sin^2\theta \frac{(\omega_0^2 - \chi^2)^2}{\omega_0^2} \frac{a^2 + \xi^2 + \zeta^2}{(a^2 - \xi^2 - \zeta^2)^2 + 4a^2\zeta^2}. \tag{46}$$

In terms of a, ξ, ζ (46) is the same as Eqn. (82) of Bhabha except for a factor 2. Substituting the value of ξ and ζ the scattering cross-section becomes

$$12\pi \sin^2\theta (\omega_0^2 - \chi^2)^2 \frac{a^2\omega_0^2 + (\omega_0^2 - \chi^2)^3 + \chi^6}{[a^2\omega_0^2 + (\omega_0^2 - \chi^2)^3 + \chi^6]^2 - 4a^2\omega_0^2\chi^6}. \tag{47}$$

Except for a factor 2, (45) agrees entirely with Eqn. (88) of Bhabha in which K has been put equal to zero. For small g_2 and not very high frequencies we can expand (47) as a series in ascending powers of g_2. The first term is

$$\frac{4\pi}{3} \sin^2\theta \frac{g_2^4}{I^2} \frac{(\omega_0^2 - \chi^2)^2}{\omega_0^2} \tag{48}$$

which is exactly half of the corresponding expression (86) of Bhabha. Thus for large ω_0 the scattering is double (independently of the value g_2) and for small ω_0 half that of transverse vector-mesons. The formulæ (47) and (48) have already been discussed by Bhabha in detail.

§3. The above theory will now be compared with the quantum theory of neutral mesons. For simplicity we put $c = 1$, $\hbar = 1$. The total Lagrangian for the neutron and the meson fields together is taken to be

$$L = \frac{i}{2}(\psi^+ \beta \gamma^\mu \partial_\mu \psi - \partial_\mu \psi^+ \beta \gamma^\mu \psi) - \mu \psi^+ \beta \psi + g_1 \psi^+ \beta \psi U + g_2 \psi^+ \beta \gamma^\mu \psi \partial_\mu U$$
$$+ i \frac{g_2'}{3!} \psi^+ \beta \gamma^\mu \gamma^\nu \gamma^\sigma \psi \epsilon_{\mu\nu\sigma\rho} \partial^\rho U + \frac{g_1'}{4!} \psi^+ \beta \gamma^\mu \gamma^\nu \gamma^\sigma \gamma^\rho \psi \epsilon_{\mu\nu\sigma\rho} \cdot U$$
$$+ \tfrac{1}{2}(\partial_\mu U \cdot \partial^\mu U - \chi^2 U^2) \tag{49}$$

Here γ^μ are the 4-rowed square matrices satisfying

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$$

and $\beta = \gamma^0$. ψ refers to the neutron field and ψ^+ is its hermitian conjugate. γ's are related to the usual Dirac-matrices $\underline{\alpha}$ and β by

$$\underline{\alpha} = (\alpha^1, \alpha^2, \alpha^3) = \beta(\gamma^1, \gamma^2, \gamma^3)$$

We assume α and β to be Hermitian. μ is the mass of the neutron. The terms containing g_1 and g_2 in (49) represent the usual 'scalar' interactions for the charge and the dipole respectively while those containing g_1' and g_2' represent the corresponding 'pseudo-scalar' interactions. Evidently in the classical theory there is no distinction between the scalar and pseudo-scalar interactions since this distinction arises only from the γ-matrices. To simplify (49) we use the usual representation of $\underline{\alpha}$ and β through two sets of mutually independent Pauli matrices.

$$\underline{\alpha} = \rho_1(\sigma^1, \sigma^2, \sigma^3), \ \beta = \rho_3$$

With this substitution (49) simplifies to

$$L = \frac{i}{2}(\psi^+ \beta \gamma^\mu \partial_\mu \psi - \partial_\mu \psi^+ \beta \gamma^\mu \psi) - \mu \psi^+ \beta \psi + g_1 \psi^+ \beta \psi \cdot U$$
$$+ g_2 \psi^+ [(\underline{\alpha}\, \mathbf{U}) + U_0] \psi + g_2' \psi^+ [(\underline{\sigma}\, \mathbf{U}) + \rho_1 U_0] \psi - g_1' \psi^+ \rho_2 \psi \cdot U$$
$$+ \tfrac{1}{2}(\partial_\mu U \cdot \partial^\mu U - \chi^2 U^2) \tag{50}$$

where $\mathbf{U} = (\partial_1 U, \partial_2 U, \partial_3 U)$, $U_0 = \partial_0 U$. The Hamiltonian of the system for only one neutron present is found as usual and on ignoring the infinite self-energy of the neutron terms out to be

$$H = (\underline{\alpha}\, p) + \mu\beta - \frac{i}{\sqrt{V}} \Sigma (2k_0)^{-\frac{1}{2}} (g_1 \rho_3 - g_1' \rho_2) \{a_k e^{i(kx)} - a_k^\times e^{-i(kx)}\}$$
$$+ \frac{1}{\sqrt{V}} \Sigma (2k_0)^{-\frac{1}{2}} (g_2 \rho_1 + g_2') (\underline{\sigma}\, \mathbf{k}) - \rho_1 k_0\} \{a_k e^{i(kx)} + a_k^\times e^{-i(kx)}\}$$
$$+ \Sigma (N_k + \tfrac{1}{2}) \tag{51}$$

where $k_0 = +\sqrt{\chi^2 + |\boldsymbol{k}|^2}$, N_k is the number of mesons in the momentum state $\boldsymbol{k}$ and a_k and $a_k^{\times}$ are the corresponding absorption and emission operators respectively.

$$[a_k, a_k^{\times}]_- = 1$$

As usual V is a large volume in which the field quantities are periodic.

Corresponding to the treatment of §2 we put $g_1 = g_1' = 0$ and regard the neutron as an infinitely heavy particle ($\mu \to \infty$). A straightforward calculation shows that the scattering due to a pure 'scalar' dipole-interaction ($g_2' = 0$) is zero in the usual second order (g_2^4) approximation. This result holds even when the calculation is performed taking into account the finite mass of the neutron. Also the terms containing both g_2 and g_2' vanish for $\mu \to \infty$, so that in this limit the contribution to the scattering comes purely from the pseudo-scalar interaction. The differential scattering cross-section is thus found to be for $\mu \to \infty$

$$d\sigma = 4\, d\Omega \left(\frac{g_2'}{\sqrt{4\pi}}\right)^4 \frac{p^4}{E^2} \sin^2 \phi \tag{52}$$

where p is the momentum, E the energy and ϕ the angle of scattering of the meson. The total cross-section is therefore

$$\frac{32\,\pi}{3}\left(\frac{g_2'}{\sqrt{4\pi}}\right)^4 \frac{p^2}{E^2} \tag{53}$$

$\frac{g_2'}{\sqrt{4\pi}}$ is the value of the dipole-moment in the usual units as against its value g_2' in the 'Heaviside-units' which were employed till now. On putting $I = \frac{\hbar}{2}$ ($= \frac{1}{2}$ since $\hbar$ is put equal to 1) and averaging over all θ, (48) becomes identical with (53) except for a factor 3. This discrepancy is due to the well-known (*cf.* Bhabha, 1941, footnote on page 340; also see Bhabha and Madhava Rao, 1941) difference in the classical and the quantum average over the direction of the spin of the neutron. In fact (52) also agrees to the same extent with the corresponding differential cross-section obtained by dividing (43) by (45), expanding in powers of g_2 and retaining only the lowest term.

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Summary

The classical formulæ for the scattering of scalar mesons by a neutron are obtained taking account of the radiation damping. The neutron is assumed to possess a 'charge' and a 'dipole moment'. The scattering due to each of these is treated separately. It is found that the formulæ for

the scattering due to the dipole has exactly the same form as the one obtained by Bhabha for the transverse mesons. Due to numerical factors the scattering for large energies of the incident mesons is double, and for small energies half that of transverse vector-mesons.

The scalar and pseudo-scalar charge and dipole interactions are considered in the quantum theory. The scalar dipole interaction does not give rise to any scattering at all, the whole of the scattering being due to the pseudo-scalar interaction. In this case the quantum-theoretical formulæ agree with the corresponding classical ones if the effect of radiation reaction is neglected in the latter.

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