EVAPORATION FROM WATER-DROPS AND WET SPHERICAL SURFACES*

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1. Introduction

The problem of evaporation of water from cloud droplets, rain-drops, hailstones and snowflakes is one of great meteorological interest. The main factors which determine the evaporation are the size and shape of the condensed particles, their speed of movement relative to the surrounding atmosphere and the difference between the vapour densities at the surface of the particles and in the surrounding atmosphere. In the present paper, we shall only consider the problem of evaporation from spherical bodies.

2. The Size and Speed of Particles which occur in the Atmosphere

Water-drops occur in the atmosphere in sizes varying from an aggregation of a few molecules as in large ions ($r = 10^{-6}$ to $10^{-5}$ cm.) to droplets of cloud or fog (2 to $5 \times 10^{-4}$ cm.), drizzle droplets ($r = 1$ to $5 \times 10^{-4}$ cm. and rain-drops $5 \times 10^{-2}$ cm. to 0.23 cm.). The biggest rain-drops are about 0.45 cm. in diameter. Hailstones vary in size from that of rain-droplets to diameters of 10 cm. or more. The smallest rain-drops are spherical in shape; when the diameter increases to more than 0.1 cm. the drop becomes slightly flattened perpendicular to the direction of movement and the flattening increases with further increase of size, and ultimately when the diameter exceeds 0.45 cm. the drop breaks up into smaller droplets. Hailstones have varying shapes but well-formed ones tend to take up a conical form with a rounded base like a sector of a sphere.

The velocities with which the particles move in the atmosphere also differ considerably, the constant terminal velocity of a fog droplet being of the order of 1 cm. per second, while that of a hailstone may exceed the speed of an express train. The terminal speed of the biggest possible rain-drop is about 8 metres per second. Table I gives the radius $r$, the constant terminal velocity $V$ and the Reynolds numbers $R = \frac{Vd}{\nu}$ of spherical or nearly spherical particles of water and ice. $\nu$ stands for the kinematic viscosity of air.

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Terminal velocities and corresponding Reynolds numbers of water-drops and spheres of specific gravities 1 and 0.915 falling freely in air at sea-level pressure. In the case of water-drops, the shape appreciably departs from the spherical as the size increases.

<table>
<thead>
<tr>
<th>Nature of falling body</th>
<th>Radius r in cm.</th>
<th>Velocity V in cm/sec.</th>
<th>R</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water-drops</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.005</td>
<td>26</td>
<td>2</td>
<td>Lenard and Schmidt, quoted in Hann, Lehrbuch der Meteorologie (1926)</td>
</tr>
<tr>
<td>Do.</td>
<td>0.01</td>
<td>78</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.02</td>
<td>180</td>
<td>48</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.05</td>
<td>430</td>
<td>290</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.10</td>
<td>590</td>
<td>770</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.15</td>
<td>690</td>
<td>1,380</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.225</td>
<td>800</td>
<td>2,400</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
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<th>Radius r in cm.</th>
<th>Velocity V in cm/sec.</th>
<th>R</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid sphere of sp. gr. 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.05</td>
<td>400</td>
<td>270</td>
<td>N. Frössling</td>
</tr>
<tr>
<td>Do.</td>
<td>0.075</td>
<td>550</td>
<td>550</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.10</td>
<td>680</td>
<td>905</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.20</td>
<td>960</td>
<td>2,560</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>0.30</td>
<td>1,190</td>
<td>4,760</td>
<td></td>
</tr>
<tr>
<td>Solid sphere of sp. gr. 0.915</td>
<td></td>
<td></td>
<td></td>
<td>Adapted from Bilham and Reif1</td>
</tr>
<tr>
<td>Do.</td>
<td>0.03</td>
<td>1,130</td>
<td>4,820</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>1.27</td>
<td>2,230</td>
<td>37,800</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>3.81</td>
<td>3,960</td>
<td>200,000</td>
<td></td>
</tr>
<tr>
<td>Do.</td>
<td>4.44</td>
<td>4,360</td>
<td>238,000</td>
<td></td>
</tr>
</tbody>
</table>

ν is assumed to be 0·150 cm.²/sec.

3. Review of Previous Measurements of Evaporation from Water-drops or Wet Porous Spheres

There have been a number of previous measurements of evaporation from water-drops, but they have generally been on small drops. A few measurements of evaporation from moistened porous spheres of large size are also available.

H. Von dem Borne1 investigated the evaporation of water from a wet porcelain ball of diameter 5.9 cm. at different wind speeds and for varying
values of relative humidity of the surrounding atmosphere. He found that the rate of evaporation could be expressed by the following formula:

$$\frac{E'}{\Delta \rho} = C f(V),$$  \hspace{1cm} (1)

where $E'$ is the rate of evaporation from the sphere, $\Delta \rho$ is the difference between the vapour density at the surface of the moist sphere and in the surrounding atmosphere, $C$ is a constant and $f(V)$ is a function of velocity.

He found that in different ranges of wind velocity $\frac{E'}{\Delta \rho}$ had the following values. Here $E'$ is expressed in gm./min., $\Delta \rho$ in gm./metre$^3$ and $V$ in metre/sec.

(a) $V < 2$ metres/sec.

$$\frac{E'}{\Delta \rho} = 4.3 \times 10^{-3} (1 + 1.95 V)$$  \hspace{1cm} (2)

(b) $V > 3$ metres/sec.

$$\frac{E'}{\Delta \rho} = 4.3 \times 10^{-3} (1 + 2.75 \sqrt{V}).$$  \hspace{1cm} (3)

If, instead of $\Delta \rho$, the difference of vapour pressure $\Delta e$ is substituted (in mm. of mercury), (3) becomes

$$\frac{E'}{\Delta e} = 4.55 \times 10^{-3} (1 - a t) \frac{B}{750} (1 + 2.75 \sqrt{V}),$$  \hspace{1cm} (4)

where $B$ is the barometric pressure in millimetres of mercury and $a$ is the coefficient of expansion of air at constant pressure.

K. Büttner$^2$ made measurements on a moist porous sphere of diameter 5 cm. and found that the rate of evaporation from this sphere in a wind stream could be expressed in the form

$$\frac{E}{\Delta e} = 12.2 \times 10^{-3} \times \sqrt{V} \times \frac{B}{750} \text{ gm./cm.}^2 \text{ min. (mm. Hg)}$$

Combining this result with theoretical considerations based on the analogy of evaporation from a wet surface to transfer of heat from a hot body, Büttner generalised the formula for different diameters of spheres and different wind velocities and expressed it in the form

$$\frac{E}{\Delta e} = \frac{0.152}{d} \times \left( \sqrt{V} \times \frac{B}{750} \right)^{0.52} \text{ gm./cm.}^2 \text{ min. (mm. Hg)},$$  \hspace{1cm} (5)

A.3a
where $E$ is the rate of evaporation in grammes per square centimetre per minute,

$d$ is the diameter of the sphere in centimetres,

$V$ is the speed of the air in metres/sec.,

$B$ is the barometric pressure and $B_0$ is the standard barometric pressure.

Since the Reynolds number is $Vd/\nu$, it follows that $\frac{E}{\Delta e}$ is approximately proportional to $R_i$.

H. G. Houghton\textsuperscript{5} made measurements on drops ranging in diameter from 0.025 to 2.6 mm. at several temperatures and relative humidities and found that the theoretical relation deducible from the general diffusion equation, viz.,

$$\frac{dm}{dt} = 2\pi Dd \Delta \rho$$

represents the results well.

In equation (6), $\frac{dm}{dt}$ is the rate of change of mass of a drop in gm./sec., $D$ is the coefficient of diffusion of water vapour in air and $d$ is the diameter of the drop.

Similar work on the evaporation from very small drops has been carried out by Takahasi and N. Frössling. Takahasi\textsuperscript{4} worked on drops 0.2 to 1 mm. in radius with wind velocities ranging from 1.0 to 6.5 m./sec. He found the following relation for the rate of decrease of the radius of the drop with time.

$$\frac{dr^2}{dt} = -0.45 \times 10^{-6} \Delta e (1 + 0.173 V) \text{cm.}^2/\text{sec.},$$

where $r$ is expressed in centimetres, $V$ in metres per second and $\Delta e$ in millimetres of mercury. Takahasi's method of determining the rate of evaporation was by measuring its diameter at successive times from its photographs.

Frössling\textsuperscript{5} followed the same method as Takahasi and worked mainly in the range of diameters 0.1 to 0.9 mm. with wind velocities varying from 0.2 to 7 metres per second in a wind tunnel. He suspended the drops by a glass-fibre or a thermocouple. In addition to water-drops, he also studied drops of nitrobenzene and aniline. From his measurements, he verified the following approximate formula which he derived theoretically for Reynolds numbers below the critical range

$$\frac{dm}{dt} = 2\pi Dd (1 + k\sqrt{R}) \Delta \rho,$$
where \( k \) is a constant characteristic of the evaporating substance (0.229 for water in air at 20°C).

4. Description of Present Experiments

The experiments so far described were made either on small drops of water or on fairly large moist spheres. The measurements of Houghton, Takahasi and Frössling fall in the region of Reynolds numbers 0 to 600, while those of Borne and Büttner fall in the region 4,000 to 25,000. As the investigation of the region lying between Reynolds numbers 600 to 4,000 was considered to be of interest from theoretical and practical points of view, the following work was planned to measure the rates of evaporation from water-drops in the region of Reynolds numbers 200 to 10,000.

Considering the difficulty of working on small drops of water, the work was carried out on moistened porous spheres. After trying various substances such as pith, rubber, sponge, porous tiles, etc., it was finally found that blackboard chalk was a suitable substance both as regards porosity and strength. Therefore spheres of chalk of various diameters were prepared. For each series of measurements, a separate sphere of chalk was used. The arrangement of the apparatus for keeping the sphere moist and measuring the rate of evaporation is described below.

A glass tube ABCD (Fig. 1) bent twice at right angles at B and C was mounted with the limbs AB and CD vertical. The end D of the tube was
drawn out into a thin capillary and the porous sphere of chalk was held in position at D by means of a thin horizontal needle. Enough water was poured into AB to come up to D in the capillary and feed the porous sphere. A scale was mounted just behind and in contact with AB to read the level of water in the tube. As the water from the chalk evaporated, the level of water in AB came down and the rate of evaporation could be determined from the rate of fall of water level in the tube. When working with different wind speeds and different spheres, the same fall of level between two fixed marks on tube (distant 0.5 cm. from each other) was used. The diameters of chalk spheres used were 0.30, 0.50, 0.75, 1.00, 1.50 and 2.00 cm.

Two different methods of varying the air speed were employed. In the first, the wind was produced by an electric fan and it was varied by varying the distance of the fan from the ball. The maximum wind velocity obtained with this arrangement was 3.5 metres per second. Higher wind speeds going up to 9 metres per second were obtained by means of an electric blower. The speeds could be varied by adjusting the resistance in the circuit of the motor and also by inserting sheets of wire-gauze in front of the blower. The wind speeds were measured at the place where the ball was mounted by means of a small hand anemometer made by Fuess or a vane anemometer made by Casella.

An attempt was made to measure the temperature of the evaporating sphere by means of a thin copper constant an thermo-couple, but as the thermo-junction was partly in contact with the wet sphere and partly in contact with air, the results were not considered satisfactory. The temperature was taken to be that of the wet bulb at the particular wind speed as calculated from the Pernter formula. The vapour pressure in the air at a distance from the sphere was calculated from the reading of an Assmamm psychrometer.

5. Results

(1) Variation of \( E \) (rate of evaporation) with difference of vapour pressure between the moist surface and the air at a distance from the sphere, keeping \( V \) and \( d \) constant.—The vapour pressure \( e \) of the air in the room was varied by working on different days and also by adding moisture to the air of the room as required from broad pans of warm water and mixing it up by fans. For each diameter of the sphere, the rate of evaporation was found to be proportional to \( e - e_0 \) or \( \Delta e \) where \( e_0 \) is the vapour pressure at the surface of the sphere, assumed to be given by the saturation vapour pressure at the wet bulb and \( e \) is the vapour pressure in the air of the room.
The results obtained with a sphere of diameter 2·0 cm. and wind speeds 0, 3, 6 and 9 metres per sec. are shown in Fig. 2 and are in agreement with Dalton’s law. Similar results were obtained with other spheres.

![Graph showing rate of evaporation against difference of vapour pressure](image1)

**Fig. 2.** Rate of evaporation against difference of vapour pressure

(2) *Variation of E with V when the diameter is kept constant.*—The results of the measurements on six spheres are plotted as curves in Fig. 3. For each sphere, as the speed of air increases, the rate of evaporation divided

![Graph showing variation of rate of evaporation with wind speed](image2)

**Fig. 3.** Variation of rate of evaporation with wind speed
by the vapour pressure difference increases approximately in the form of a parabola, but the evaporation for zero wind does not come down to zero. By plotting log (E/Δe) against log V, it was seen that E/Δe was very nearly proportional to V^3.

(3) Variation of E with d when V is kept constant.—If the wind were kept constant and expressed as a function of the diameter of the sphere, it was found that E/Δe was approximately proportional to kd^n. n varied from 1.5 for no wind to 1.6 for a wind of speed 9 metres per second (see Fig. 4).

If, following Reynolds, momentum transfer and heat transfer from a solid body of given shape in a stream of fluid are considered similar, the simplest way to express the results for different wind speeds and different diameters of spheres would be through the Reynolds number. In Fig. 5 E/Δe·d is plotted against √R. The results obtained by V. d. Borne, Büttner, Houghton, Frössling and the author have all been plotted in the same figure, and except at very low Reynolds numbers, fall into a single curve. The gradually increasing slope of the curve with increasing values of R shows that a constant value of the exponent will not suit the results completely. At small values of R, the observed rate of evaporation is generally higher than that given by the simple diffusion formula and vary from author to author. In this region, greater reliance should be placed on the values obtained from
the experiments on small spheres, because with large spheres, the auto-
convection due to the difference of temperature between the sphere and
the surrounding air and the small air currents accidentally present even in
a closed room will cause the effective value of $R$ to be finite.

6. Effect of Radiation on Evaporation

In the above discussion, the effect of radiation between the evaporat-
ing sphere and the surrounding objects has not been considered. Owing to
evaporation, the temperature of the sphere will tend to get down to that of
the wet bulb, while owing to exchange of radiation with the surroundings,
it will tend to become equal to that of the surrounding objects. When both
processes are operating, the temperature of the sphere will be slightly greater
than that of the wet bulb and the rate of evaporation increased. It is easy
to calculate the rates of evaporation due to the two processes separately.
The rate of absorption of radiant energy $Q_R$ by unit area of a body is given
by

$$Q_R = k \sigma (T_0^4 - T^4),$$

where $k$ is the absorption coefficient of the surface of a body. $\sigma$ is Stefan's
constant $= 1.37 \times 10^{-12}$ cal./cm.$^{-2}$/sec.$^{-1}$/deg.$^{-4}$, $T$ is the absolute tempera-
ture of the body and $T_0$ the absolute temperature of the surrounding objects.
The rate of evaporation from a sphere of radius $r$ due to the absorption of radiant energy will be given by

$$E_R = \frac{K \sigma (T_0^4 - T^4) \cdot 4\pi r^2}{L}$$

where $L$ is the latent heat of evaporation of water.

For a wet sphere placed in a room, $k$ may be taken as 1 and assuming, for example, $r = 1.0$ cm., $T = 20^\circ$ C. and $T_0 = 28^\circ$ C., $E_R = 1.60 \times 10^{-3}$ gm./min.

In Table II below, are given the values of the rates of evaporation from spheres of different radii corresponding to a wet bulb temperature of $20^\circ$ C. and air temperature of $28^\circ$ C. at three different wind-speeds.

**Table II**

**Comparison of amounts of evaporation due to convection and radiation**

Wet bulb temperature = $20^\circ$ C.

Temperature of surrounding objects and of air = $28^\circ$ C.

Difference between vapour pressure at the surface of the wet bulb and at a distance = $4.1$ mm. of mercury

<table>
<thead>
<tr>
<th>Radius of sphere in cm.</th>
<th>$E_R$ gm./min. (calc.)</th>
<th>$E$ gm./min. (from curve)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>No wind</td>
</tr>
<tr>
<td>1.00</td>
<td>$1.6 \times 10^{-3}$</td>
<td>$4.1 \times 10^{-3}$</td>
</tr>
<tr>
<td>0.50</td>
<td>$4.0 \times 10^{-4}$</td>
<td>$1.43 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.25</td>
<td>$1.0 \times 10^{-4}$</td>
<td>$4.9 \times 10^{-4}$</td>
</tr>
<tr>
<td>0.15</td>
<td>$3.6 \times 10^{-5}$</td>
<td>$2.5 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

It will be seen that the evaporation due to radiation is quite a considerable part of that due to convection, especially when there is no wind or the wind is weak. It was therefore thought desirable to calculate the rates of evaporation due to convection alone after allowing for the effects of radiation. The values of $E_{Total} - E_R$ instead of $E$ (total) are used in Fig. 6 and shows that the general conclusions reached before are not modified in any essential respect. From the graph of log $(E_{Total} - E_R)$ against log $d$, the following slopes of the lines for different wind speeds were obtained.
Evaporation from Water-Drops & Wet Spherical Surfaces

This suggests that the value of the exponent \( n \) in the equation

\[
\frac{E}{\Delta e \cdot d} = cR^n
\]

slightly increases with the Reynolds number and has an average value of 0.55.

My thanks are due to Dr. K. R. Ramanathan under whose direction the investigation was carried out and to the authorities of the Fergusson College in whose laboratories the measurements were made.

7. Summary

After a review of the sizes and velocities of free fall of water-drops and of solid spheres in the atmosphere and their appropriate “Reynolds” numbers, a summary of the previous work on evaporation from wet spherical surfaces is given. The paper then describes experiments made in the laboratory by the author on the evaporation from wet spheres of porous chalk in the range of Reynolds numbers 600 to 10,000. The
results of the present experiments and of the previous workers show that when $0 < R < 25,000$, the rate of evaporation from spheres of different sizes and in different wind speeds can be expressed as a function of $R$. When a suitable correction has been made for the effects of radiation, it is found that $E/(e_s - e)d$ can be expressed in the form $cR^n$ where $n$ increases from $0.48$ to $0.60$ when $R$ increases from about $100$ to $10,000$.

REFERENCES

1. A. v. d. Borne  
2. K. Büttner, Veröff  
3. H. G. Houghton  
4. Takahasi  
5. N. Frössling  
6. E. G. Bilham and E. F. Relf

. Physics, 1933, 4, 419.