A REMARK ON g (n).

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§ 1. In my previous note[1] with this title I pointed out that

THEOREM

$$\limsup_{x \to \infty} \frac{K(x)}{x} = 1$$

where K (x) *is the number of solutions of*

$$g(n) = 2^n + \left[\left(\frac{3}{2}\right)^n\right] - 2.$$

(Here $[x]$ denotes the greatest integer $\leqslant x$, and $g(n)$ is the least s such that every integer > 0 is a sum of s nth powers of integers $\geqslant 0$) is an easy consequence of known results. The present note contains full details of the proof.

§ 2. Mahler (*Acta Arithmetica,* Band 3, p. 93, theorem 4) proved the

THEOREM. *Suppose that* $\theta \neq 0$ *is an algebraic number and that* u *and* v *are integers with* $u > v > 1$, $v \nmid u$, *that* ϵ *is a positive constant, and that* $n = n_1, n_2, n_3, \cdots$ *is an infinite increasing sequence of positive integers for which*

$$\theta\left(\frac{u}{v}\right)^n - \left[\theta\left(\frac{u}{v}\right)^n\right] \leqslant u^{-\epsilon n}.$$

Then

$$\limsup_{t \to \infty} \frac{n_{t+1}}{n_t} = \infty.$$

In this theorem take

$$u = 3, v = 2, \ \theta = -1, \ \epsilon = \frac{1}{100}.$$

We shall write

$$3^n = l.\, 2^n + r \ (0 < r < 2^n).$$

[1] *Proc. Ind. Acad. Sci.*, Oct. 1938, vol. 8, p. 237.

In that note the word " true " (in the second of the three theorems stated) is to be replaced by " false ".

Further we use $f(x)$ to denote the fractional part of x, and note that if w (> 0) is not an integer, then

$$[-w] = -\{[w] + 1\}.$$

From Mahler's theorem it now follows that, given any $A > 0$, however large, we can find infinitely many integers x such that

$$(1) \qquad 1 - f\left(\frac{3^n}{2^n}\right) \geqslant \frac{1}{2^{\frac{n}{100}}}$$

is true for $x < n < Ax$.

Now (1) means that

$$(2) \qquad f\left(\frac{3^n}{2^n}\right) \leqslant 1 - \frac{1}{2^{\frac{n}{100}}}$$

i.e.,

$$(3) \qquad r \leqslant 2^n - 2^{\frac{99n}{100}}$$

Now it is easily verified that for $n > n_0$

$$(4) \qquad l + 3 < 2^{\frac{99n}{100}}$$

Hence if (3) is true then, for $n > n_0$,

$$(5) \qquad r < 2^n - (l + 3).$$

Thus we conclude that given any $A > 0$, we can find infinitely many x such that (5) is true for $x < n < Ax$.

But Pillai has shown that for $n > n_0$,

$$(6) \qquad g(n) = 2^n + \left[\left(\frac{3}{2}\right)^n\right] - 2$$

if (5) is true. Hence, given any $A > 0$, there exi t infinitely many integers x such that (6) is true for $x < n < Ax$. Thus the theorem stated in § 1 is proved.