

A REMARK ON $g(n)$.

BY S. CHOWLA.

(Government College, Lahore.)

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PILLAI* proved that, for $n > n_0$,

$$(1) \quad g(n) = 2^n + \left[\left(\frac{3}{2} \right)^n \right] - 2$$

provided

$$3^n = l \cdot 2^n + r \quad (0 < r < 2^n) \text{ where } r \geq 2^n - (l + 3).$$

By combining this with a recent theorem of Mahler (*Acta arithmetica*, Band 3, p. 93, Theorem 4) we obtain the result

THEOREM : *Let A be an arbitrarily large number. Then there exist infinitely many x such that (1) is true for $x \leq n \leq Ax$.*

Another form is the following :

THEOREM : *Let ϵ be an arbitrarily small positive number, then there exist infinitely many x such that the number of values of n for which $2 \leq n \leq x$ and in addition (1) is true, is less than ϵx .*

(All letters, except ϵ , denote positive integers.)

In fact if we denote by $K(x)$, the number of solutions of

$$g(n) = 2^n + \left[\left(\frac{3}{2} \right)^n \right] - 2 \quad \left. \begin{array}{l} 1 \leq n \leq x \\ \end{array} \right\}$$

the above results can be put elegantly in the following form :

THEOREM :

$$\lim_{x \rightarrow \infty} \sup \frac{K(x)}{x} = 1$$

Pillai proved that $\frac{K(x)}{x} > \frac{1}{4} \quad (x > x_0)$.

* *Annamalai University Journal*, March 1936, 5, No. 2. The result mentioned was also proved by Dickson.