ON A RELATION BETWEEN TWO CONJECTURES OF THE THEORY OF NUMBERS.

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§1. Let \( r_{k,s}(n) \) denote the number of representations of \( n \) as a sum of \( s \) \( k \)th powers of integers \( \geq 0 \). Since \( r_{k,2}(n) = O\left(n^{\epsilon}\right) \) for every positive \( \epsilon \), we have

\[
r_{k,k}(n) = O\left(n^{1 - \frac{2}{k} + \epsilon}\right)
\]

and no better result is known. Denote by \( (A_s) \) the unproved result that

\[
(A_s) \quad r_{k,k}(n) = O\left(\frac{n^s}{n^k}\right)
\]

where \( k \geq 5 \) and \( s \) is a positive integer \( \leq k - 3 \). Denote by \( (B_s) \) the conjecture that

\[
(B_s) \quad \sum_{m=1}^{k-s} x_m^k = \sum_{m=1}^{k-s} y_m^k \text{ has a non-trivial solution in positive integers } x_1, x_2, \text{ etc., } y_1, y_2, \text{ etc.}
\]

Then we have the

Theorem.* If \( (A_s) \) is false then \( (B_s) \) is true, and conversely.

In other words at least one of (two desirable results) \( (A_s) \) and \( (B_s) \) is true.

Proof. If \( (B_s) \) is false, then

\[
r_{k,k-s}(n) = O\left(1\right)
\]

whence

\[
r_{k,k-s+1}(n) = O\left(\frac{1}{n^k}\right)
\]

\[
r_{k,k-s+2}(n) = O\left(\frac{2}{n^k}\right)
\]

etc.

\[
r_{k,k}(n) = O\left(\frac{s}{n^k}\right)
\]

* The proof shows that the result is trivial; my object is to draw attention to the conjectures themselves.
i.e., if (B,) is false then (A,) is true. Conversely, if (A,) is false, it follows that (B,) must be true.

§2. Note the special case when $s = 3$ ($k \geq 6$), at least one of the following conjectures is true:

(A) \[ a_1^k + a_2^k + a_3^k = b_1^k + b_2^k + b_3^k \]

has a non-trivial solution in positive integers, $a_1, a_2, a_3, b_1, b_2, b_3$.

(B) \[ r_{k,k} (n) = O \left( \frac{k-3}{n^k} \right) \]

It seems to the writer that (A) is not likely to be proved in the near future. On the other hand, it is possible that (B), which asserts so much less than "Hypothesis K" of Hardy and Littlewood, is not a difficult result. We observe also that (A) may be false, for solutions of (A) are not known even for $k = 5$. 