ON FERMI'S THEORY OF β-DECAY.

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THE introduction of the factor

$$\frac{\nu^2 \, \Delta \nu}{\pi^2 \, c^3}, \tag{1}$$

the number of oscillators per unit volume within the frequency interval $\Delta\nu$ for both neutrino and electron waves in Fermi's theory of the β-decay[1], seems to require some closer investigation. As in the treatment of analogous problems in the theory of radiation, the result of the integration, after multiplication by (1), over the whole frequency range of the neutrino waves is determined by a resonance term; the integration over all possible electron frequencies gives rise to a factor of the order of magnitude of the total number of oscillators contained in the frequency interval from 0 to ν_0, the maximum frequency of the electrons emitted,[2]

$$\frac{\nu_0{}^3}{\pi^2 \, c^3} \tag{2}$$

Since an electron is emitted with a definite energy, and since the total transition probability is the average of the transition probabilities for the electrons of different energies, the appearance of a factor of the order of magnitude of the total number of oscillator seems to be somewhat surprising. We should rather expect a much smaller numerical factor to appear in the final result, corresponding to the average number of oscillators actually interacting with the electrons, having different, but definite, energies.

To make this point clear, let us consider Planck's derivation of (1). The energy of an oscillator of characteristic frequency ν_0 in interaction with radiation of the frequency ν is

$$\mathrm{T}(\nu) = \frac{e^2 \, \overline{\mathrm{E}_\nu{}^2}}{8m} \, \frac{1}{(\nu_0 - \nu)^2 + \gamma^2} \tag{3}$$

Remembering that $\gamma = \frac{1}{3} \frac{e^2 \nu^2}{mc^3}$, and further that the density of radiation with frequencies in the interval $\Delta\nu$

$$\rho_\nu = \frac{3 \, \overline{\mathrm{E}_\nu{}^2}}{8 \, \pi} = \mathrm{U}_\nu \, \Delta\nu,$$

we obtain on integration the average oscillator energy

$$\mathrm{T}_\nu = \frac{\pi^2 c^3}{\nu^2} \mathrm{U}_\nu = \rho_\nu \cdot \frac{\pi^2 c^3}{\nu^2 \triangle \nu} \tag{4}$$

Hence follows (1) as the ratio $\frac{\rho_\nu}{\mathrm{T}_\nu}$.

In the case of interaction of an oscillator with monochromatic radiation, having its own characteristic frequency, we obtain in a similar way

$$\mathrm{T}_\nu = \rho_\nu \cdot \frac{c^3}{\nu^2} \frac{\pi}{\gamma}, \tag{5}$$

and therefore

$$\frac{\rho_\nu}{\mathrm{T}_\nu} = \frac{\nu^2 \gamma}{\pi c^3} \tag{6}$$

This expression becomes identical with (1), if

$$\triangle \nu = \pi \gamma \tag{7}$$

We might have expected a result like this, for 2γ is the so-called natural line width. We obtain of course the same transition probability for the interaction with both "white" and monochromatic radiation, but the same is not necessarily the case for multiple processes.

In case of interaction of light quanta and electrons we have for simple processes

$$\gamma \sim \alpha^3 \nu, \tag{8}$$

α being the fine structure constant.[3] Subtituting this expression in (6), we get

$$\frac{\rho_\nu}{\mathrm{T}_\nu} \sim \frac{1}{\pi}\left(\frac{\nu \alpha}{c}\right)^3 \tag{9}$$

In the theory of β-decay the process of emission of every electron independently may have to be treated in a similar way. Let us assume that (8) is applicable to β-radiation also. This conjecture does not seem to be entirely unfounded, since the assumption implied in (8) that the wave-length of the radiation emitted or absorbed is much greater than the dimensions of the system of particles concerned, is also valid for β-radiation, and since, further, (8) does not depend upon the mass of the particles. Fermi's expression for the transition probability has then to be multiplied by a factor of the order of magnitude α^3, and hence a different value for the constant g in Fermi's interaction formula is obtained. As the transition probability is proportional to g^2, we get

$$g^2 \sim \alpha^{-3} g_F^2 \tag{10}$$

$g_F \sim 4 \cdot 10^{-50}$ gr. cm.^{5} sec.$^{-2}$. being Fermi's original constant, and thus

$$g \sim 6 \cdot 10^{-47} \text{ gr. cm.}^5 \text{ sec.}^{-2} \tag{10'}$$

While the order of magnitude of Fermi's g_F, when applied to the problems of shower production and the interaction of heavy particles, appeared to be too small, this is no longer the case with the value (10).

Let us first consider the interaction of heavy particles. In this problem no integrations like those discussed above are involved. Except for the larger value for the constant g, Iwanenko's results[2] remain thus valid. He derived the following expression for the interaction energy

$$|V| = \hbar\, c \cdot \frac{(g/\hbar\, c)^2\, (\hbar/m\, c)^{2(m+n)} \cdot (2\, m + 2\, n + 2)\,!}{\pi^3\, (2\, r)^{2m+2n+5}} \tag{11}$$

m and n being the order of the derivatives of the characteristic functions ot the neutrino and electron waves respectively, introduced in Fermi's interaction formula. This expression can be written in a very simple form. Using (10′) we obtain $g/\hbar c \sim 2 \cdot 10^{-30}$ cm.^{2}, while $\alpha e^2/mc^2 \sim 4,1 \cdot 10^{-30}$ cm.^{2} Thus we see that

$$\frac{g}{\hbar\, c} \sim \tfrac{1}{2} \left(\alpha\, \frac{e^2}{m\, c^2} \right)^2 \tag{12}$$

Introducing further $2r = \rho . e^2/mc^2$ ($e^2/mc^2 = 2,80 \cdot 10^{-13}$ cm.), where ρ is a numerical factor, which will not differ considerably from unity, we obtain

$$|V| \sim m\, c^2\, \frac{\alpha^{-2(m+n)+3}}{\rho^{2(m+n)+5}} \cdot \tfrac{1}{4}\, (2\, m + 2\, n + 2)\,! \tag{13}$$

It is apparent that V will be of the required order of magnitude, mc^2, for $2m + 2n \sim 3$, thus $m + n = 1$ or 2. With Fermi's value, (12) contains an additional factor α^3; therefore $2m + 2n \sim 6$ is required, and $m + n = 3$.

A similar result concerning $m + n$ is obtained on introducing the new value for g in the expression, which Heisenberg[4] obtained for the ratio of the energy of the electrons produced in showers and the energy of the proton,

$$\frac{E}{M\, c^2} \sim \frac{\hbar}{M\, c} \left[\frac{g}{\hbar\, c} \left(\frac{\hbar}{m\, c} \right)^{m+n} \right]^{-\frac{1}{2+m+n}} \tag{14}$$

Using Fermi's value g_F he calculated the numerical values for this ratio, which are compared with those obtained with g (10′), in the following table :

$m+n$	0	1	2	3
E/Mc^2 with g_F	580	5,7	0,31	0,14
E/Mc^2 with g	15	0,45	0,087	0,033

Heisenberg concluded that the result is reasonable only for $m + n = 2$ or 3. We see that with g (10′) the ratio E/Mc^2 becomes of the same order of magnitude already for $m + n = 1$. Thus both problems considered seem to point to the same conclusion. It is remarkable that also Konopinski and Uhlenbeck[5] obtained the best agreement between the theoretical curves for the intensity distribution of the β-ray spectra, derived with the help of a generalisation of Fermi's interaction formula, and those observed experimentally, for $m + n = 1$. The discrepancy between this result and Heisenberg's and Iwanenko's estimate, $m + n = 3$ (v.L.c.), would not arise under the assumptions made in this note.

Summary.

Planck's derivation of the formula for the number of oscillators per unit volume, together with the relation between the frequency and the natural line width, suggest that a factor of the order of magnitude α^{-3} may have to be introduced in Fermi's expression for the transition probability in the theory of β-decay. The new value for Fermi's constant g is such that we obtain satisfactory aggreement between Konopinski–Uhlenbeck's conclusions concerning the intensity distribution of β-ray spectra and the results of Heisenberg's and Iwanenko's application of Fermi's theory to the production of electron showers by cosmic rays and the interaction of heavy particles respectively.

REFERENCES.

1. Fermi, *Zeits. f. Phys.*, 1933, **88**, 161.
2. Iwanenko, *Zeits. f. Phys.*, 1936, **102**, 119.
3. Heitler, *Theory of Radiation*, 1936, p. 107, form. (15).
4. Heisenberg, *Zeits. f. Phys.*, 1936, **101**, 533.
5. Konopinski-Uhlenbeck, *Phys. Rev.*, 1935, **48**, 7, 107.