PILLAI'S EXACT FORMULÆ FOR THE NUMBER $g(n)$ IN WARING'S PROBLEM.

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1. Let n be a positive integer $\geqslant 7$; let $g(n)$ denote the least s such that every positive integer can be expressed as a sum of at most s non-negative nth powers. Denote by $[x]$ the greatest integer contained in x, by $\{x\}$ the fractional part of x. Write

$$l = [(\tfrac{3}{2})^n], \qquad j = [(\tfrac{4}{3})^n]$$

Pillai has recently proved the following remarkable results (1, 2):

Theorem 1: *If* $8 \leqslant n \leqslant 30$, *then* $g(n) = 2^n + l - 2$.

Theorem 2: *If* $n \geqslant 30$ *and* $\{(\tfrac{3}{2})^n\} \leqslant 1 - \dfrac{l+3}{2^n}$, *then* $g(n) = 2^n + (l-2)$.

When $\{(\tfrac{3}{2})^n\} > 1 - \dfrac{l+3}{2^n}$ (it seems not unlikely that this case never arises for $n \geqslant 8$) his previous results for $g(n)$ were not exact. Pillai has now found that

Theorem 3: *When* $\{(\tfrac{3}{2})^n\} \geqslant 1 - \dfrac{l-1}{2^n}$, *then*

(1) $g(n) = 2^n + l + j - 2$ if $\{(\tfrac{4}{3})^n\} \geqslant 1 - \dfrac{r}{3^n}$

(2) $g(n) = 2^n + l + j - 3$ if $\{(\tfrac{4}{3})^n\} < 1 - \dfrac{r}{3^n}$

where r *is defined by* $3^n = l \cdot 2^n + r$.

The only cases in which an exact formula for $g(n)$ has not been obtained are, therefore,

(α) $3^n = l \cdot 2^n + 2^n - l - 2$.
(β) $3^n = l \cdot 2^n + 2^n - l - 1$.
(γ) $3^n = l \cdot 2^n + 2^n - l$.

When n is even, (α), (β) and (γ) are impossible. *Hence Pillai's theorems* 1, 2 *and* 3, *above determine* g(n) *exactly when* n *is even.*

Pillai also finds that

Theorem 4: $g(7) = 143$

i.e., $g(n) = 2^n + (l - 2)$ *is also true for* $n = 7$.

REFERENCE.

Pillai 1, "On Waring's Problem," *The Journal of the Annamalai University*, March 1936, **5**, No. 2, 145—166.

Pillai 2, "On Waring's Problem," *Journal of the Indian Math. Society*, New Series, **2**, No. 1, 16—44.