NOTE ON WARING'S PROBLEM.

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Received July 3, 1936.

1. Let $f(m)$ denote the number of representations of m as a sum of k non-negative kth powers. On the assumption that $f(m) = O(m^{\epsilon})$ for every positive ϵ, Hardy and Littlewood proved that $G(k) \leq 4k$. Mahler has recently made the surprising observation (based on a simple algebraic identity) that this assumption is false for $k = 3$. It is therefore of interest to know whether the results of Hardy and Littlewood based on "Hypothesis K" can be proved under a less drastic assumption. Dr. Walfisz remarked to me in a recent letter that we can obtain these results on the less stringent hypothesis that

$$(1) \qquad \sum_{1}^{x} f^2(m) = O(x^{1+\epsilon})$$

for every positive ϵ. It is the sole purpose of this note to embody a slight generalisation of this remark in the form of the

Theorem. If there exists a number $n = n(k)$ *such that*

$$(2) \qquad \sum_{1}^{x} f^2(m)\ (x - m)^n = O(x^{n+1+\epsilon})$$

for every positive ϵ, then

$$(3) \qquad G(k) \leq \text{Max}\ [2k + 1, \Gamma(k)]$$

Proof.—A reference to P. N. VI shows that

(3) is true if

$$(4) \qquad \sum_{1}^{\infty} f^2(m) x^{2m} = O\ \{(1 - x)^{-1-\epsilon}\}$$

for every $\epsilon > 0$ $(0 < x < 1)$. But (4) is certainly true if (2) is true. Hence our result.