WE write \( \beta (k) \) for the least value of \( n \) such that
\[
\sum_{s=1}^{m} x_s^k = \sum_{t=1}^{n} y_t^k
\]
is possible in positive integers \( x_s (s \leq m) \), \( y_t (t \leq n) \) with \( m < n \). When \( (x_1, \ldots, x_m) = (y_1, \ldots, y_n) = 1 \) we shall express the possibility of (1)
by the notation
\[
(m)^k = (n)^k.
\]
In a series of recent papers Rao has shown that
\[
\beta (k) \leq k - 1
\]
for \( 1 \leq k \leq 8 \); he has also obtained numerous relations of the type
\[
(5)^7 = (5)^7, \quad (2)^9 = (12)^9, \quad \text{etc.}
\]
All these results were obtained by the method of direct calculation. In this paper we use Tarry's process (see Wright 1, 2) which involves only the simplest calculations, to obtain some new results of the same type, namely,

**Theorem 1.**
\[
(6)^9 = (8)^9
\]
i.e., \( \beta (9) \leq 8 \)

Thus (3) is true for \( k = 9 \).

**Theorem 2.**
\[
(7)^9 = (7)^9
\]
It is likely that \( \beta (k) \leq k - 1 \) for all \( k \), but from Vinogradov's method we can only deduce that \( \beta (k) = 0 \) \((k \log k)\).

**Notation.** Let
\[
a_1, \ldots, a_n \quad \text{signify that}
\]
\[
\sum_{r=1}^{n} a_r^m = \sum_{r=1}^{n} b_r^m \quad (1 \leq m \leq k)
\]
Thus it is obvious that

Lemma 1. \[ \sum_{r=1}^{n} (x + a_r)^r = \sum_{r=1}^{n} (x + b_r)^r. \quad [1 \leq s \leq k] \]

Lemma 2 (Tarry). If \[ a_1, \ldots, a_n \leq b_1, \ldots, b_n \]

then

\[ a_1, \ldots, a_n, b_1 + x, \ldots, b_n + x^{k+1} b_1, \ldots, b_n, a_1 + x, \ldots, a_n + x. \]

Lemma 3 (Tarry).

\[ 1, 5, 10, 24, 28, 42, 47, 51 \}

Lemmas 1 and 2 are immediately obvious on comparing coefficients of powers of \( x \). For lemmas 1 to 3 see also Dickson 1, Wright 1, 2 and I. Chowla 1, 2.

Proofs of Theorems 1 and 2.

Applying lemma 2 to lemma 3 with \( k = 7 \) and \( x = 19 \), we get

\[ (7) \quad 1, 5, 10, 24, 28, 42, 47, 51 \]

Applying lemma 2 to (7) with \( k = 8 \) and \( x = 17 \), we get

\[ (8) \quad 1, 5, 10, 24, 28, 42, 47, 51, 59, 63, 69 \]

Applying lemma 2 to (8) with \( k = 9 \) and \( x = 9 \), we get

\[ (9) \quad 1, 5, 11, 21, 36, 42, 48, 52, 54, 58, 79, 83, 94, 95 \]

Applying lemma 1 to (9) with \( x = -47 \), we get

\[ (10) \quad -46^r - 42^r - 26^r + 1^r + 7^r + 32^r + 47^r + 49^r \]

or

\[ (11) \quad 1^r + 4^r + 7^r + 32^r + 3^r + 44^r + 47^r + 49^r \]

The special case \( s = 9 \) of (11) implies

\[ (12) \quad (6)^9 = (8)^9 \]

i.e.,

\[ (13) \quad \beta (9) \leq 8. \]
Applying lemma 1 to (9) with \( x = -\frac{97}{2} \), we get

\[(14) \quad 7s + 11s + 19s + 61s + 69s + 91s + 93s = 1s + 13s + 25s + 55s + 75s + 87s + 95s. \quad [s = 1, 3, 5, 7, 9]\]

The case \( s = 9 \) of (14) implies

\[(15) \quad (7)^9 = (7)^9\]

Thus theorems 1 and 2 are completely proved.

**BIBLIOGRAPHY.**


