VINOGRAWDOW'S SOLUTION OF WARING'S PROBLEM (II).

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Received December 12, 1935.
(Communicated by Dr. S. Chowla.)

In the first part of this paper it was shown by the method of Vinogradow, as modified by Pillai, that

**Theorem I.** If Hypothesis $P^*$ is true, then for $n > n_0$, there exists a number $w$ depending only on $n$ such that every large number $N \equiv 1 \pmod{w}$ can be expressed as a sum of $(3n + 2)'nth powers' \geq 0$.

In the rest of this paper we shall suppose that $n > n_0$. The method of my first paper enables us to formulate the more general

**Theorem II.** It is possible to find a number $w$ depending only on $n$ with the following property:

Let the sequence of positive integers $\xi$ have the property that

$$\sum_{\xi \leq x} \frac{1}{\xi^{1-\epsilon}} \ll x$$

for every positive $\epsilon$. Then every large $N \equiv (2f + 1) \pmod{w}$ is expressible in the form

$$(x_1^n + x_2^n + \cdots + x_{n+2}^n) + (\xi_1 + \xi_2)$$

where $\xi_1$ and $\xi_2$ are numbers of the "$\xi$ sequence" and the $x$'s are integers $\geq 0$.

The following special case is of interest:

Let the $\xi$ sequence consist of the primes $\equiv 1 \pmod{w}$ not exceeding $x$. Then the property $\sum_{\xi \leq x} \frac{1}{\xi^{1-\epsilon}} \ll x$ is true and hence (here $f = 1$):

**Theorem III.** There exists a number $w$ depending only on $n$ such that every large $N \equiv 3 \pmod{w}$ is expressible in the form

$$(x_1^n + x_2^n + \cdots + x_{n+2}^n) + (p_1 + p_2)$$

where the $x$'s are integers $\geq 0$ and $p_1, p_2$ are primes.

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* Hypothesis $P$ is that $H_{n,n}(x) \gg x^{1-\epsilon}$ for any $\epsilon > 0$. Here $H_{n,n}(x)$ denotes the number of numbers $\leq x$ which can be expressed as a sum of $n$ "$n$th powers" $\geq 0$. See the first part of this paper: Proc. Ind. Acad. Sci., (A), 1935, 2, 562-573.