DERIVATION OF ÅNGSTROM'S FORMULA FOR ATMOSPHERIC RADIATION AND SOME GENERAL CONSIDERATIONS REGARDING NOCTURNAL COOLING OF AIR-LAYERS NEAR THE GROUND.

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THE radiation S_T^w coming from an isothermal atmosphere[1] at temperature T with water-vapour content w is given by

$$S_T^w = 2\pi \int_0^\infty J_{\lambda T}\, H_3(0) d\lambda - 2\pi \int_0^\infty J_{\lambda T}\, H_3(k_\lambda w)\, d\lambda \qquad .. \qquad .. \quad (1)$$

where $J_\lambda d\lambda$ is the energy of radiation contained between the wave-length limits λ and $\lambda + d\lambda$ and emitted per unit solid angle by unit area of a black-body at temperature T in unit time, k_λ* is the (Naperian) coefficient of absorption at wave-length λ and

$$H_3(kw) = \int_1^\infty \exp\,(-kw\xi)\, \frac{d\xi}{\xi^3}.$$

The first term on the right represents the full black-body radiation at temperature T and the second term the radiation received by unit area from an infinitely extended plane black surface at temperature T (parallel to the unit area) after passing through an atmospheric layer containing w gms. of precipitable water.

To a first approximation[2]

$$H_3(kw) = \tfrac{1}{2} \exp\,(-\tfrac{3}{2}\, kw). \qquad .. \qquad .. \qquad .. \qquad .. \quad (2)$$

Since $H_3(0) = \frac{1}{2}$

$$S_T^w \doteq \pi \int_0^\infty J_{\lambda T}\, d\lambda - \pi \int_0^\infty J_{\lambda T}\, d\lambda\, \exp\,(-\tfrac{3}{2}\, k_\lambda w).$$

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* Throughout the paper, k is used to indicate the absorption coefficient when the transmission is expressed as a power of ϵ, and a the coefficient when transmission is expressed as a power of 10. $a = 0{\cdot}434\, k$.

Now, we can divide the total radiation from a black-body into n groups, $S_1, S_2, S_3, \ldots, S_n$ depending on the value of the absorption coefficient of the water-vapour in the order of increasing k. The radiation from an isothermal atmosphere will then be given by

$$S_T^w = S_1 - S_1 \exp\left(-\tfrac{3}{2} k_1 w\right) + S_2 - S_2 \exp\left(-\tfrac{3}{2} k_2 w\right) + \ldots + S_n - S_n \exp\left(-\tfrac{3}{2} k_n w\right).$$

If k_1 is very small, $\exp\left(-\tfrac{3}{2} k_1 w\right)$ is very nearly equal to unity and if k_n is very large, $\exp\left(-\tfrac{3}{2} k_n w\right)$ is negligibly small.

To a first approximation, we can divide[3] the wave-lengths of the atmospheric radiation into three groups.

(*a*) those for which k for water-vapour is very small, (*b*) those for which k is a definite single value, and (*c*) those for which k is large. If the black-body radiation comprised within these three groups be S_1, S_2 and S_3,

$$S_T^w = S_2 + S_3 - S_2 \exp\left(-\tfrac{3}{2} k_2 w\right) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

Equation (3) is similar in form to Ångstrom's empirical formula for atmospheric radiation, *viz.*, $\dfrac{S}{\sigma T^4} = A - B \cdot 10^{-\gamma e}$, where A, B and γ are constants, T is the temperature and e the aqueous vapour pressure at the place of observation. A and B have the values $0 \cdot 77$ and $0 \cdot 28$ and $\gamma = 0 \cdot 074$ when e is expressed in mm.

From the absorptive properties of water-vapour, it is possible to evaluate the values of the constants in the above equation if we make reasonable subdivision of the spectrum into regions with different mean values of absorption coefficients. A suitable division is obtained if we group together (1) regions in which the decimal absorption coefficient α lies below 1, (2) regoins where α lies between 1 and 10, and (3) regions for which α is greater than 10. Using the absorption coefficients calculated from Hettner's experimental results[4] and the spectral curve of black-body radiation, it is found that at 290°-300° A, the fraction of the energy comprised within those wave-lengths for which the decimal absorption coefficient is less than 1 ($8 \cdot 5 - 10 \cdot 5\mu$ and $3 \cdot 5 - 4 \cdot 3\mu$) is about $0 \cdot 14$; the fraction with decimal absorption coefficient greater than 10 ($5 \cdot 1 - 7 \cdot 9\mu$ and $> 15 \cdot 3\mu$) is $0 \cdot 54$ and the remainder is $0 \cdot 32$. Expressed in terms of the full black-body radiation at temperature T, S_1, S_2 and S_3 are respectively equal to $0 \cdot 14$, $0 \cdot 32$ and $0 \cdot 54$ when the temperature is 295° A.

$$S_T^w = \sigma T^4 (0 \cdot 54 + 0 \cdot 32) - 0 \cdot 32 \sigma T^4 \exp\left(-\tfrac{3}{2} k_2 w\right).$$

If we have to give a single mean value to the absorption coefficient between $10 \cdot 5$ and $15 \cdot 3\mu$, the decimal absorption coefficient α will be, on the basis of Hettner's values, about 4.

$$S_T^w / \sigma T^4 = 0 \cdot 86 - 0 \cdot 32 \times 10^{-6w}$$

The above equation holds for an isothermal atmosphere. Actually, however, the temperature decreases with height and the constants would therefore require some modifications. As S_3 refers to wave-lengths for which the absorption coefficients are large, the effective radiating part of the atmosphere for these wave-lengths will be very near the place of observation (except at great heights where w will be very small or when the atmosphere is very dry) and will have nearly the same temperature as the air at the place of observation. Expressed in figures, a layer of the atmosphere containing 1 mm. of precipitable water will absorb 9/10 of the radiation coming from a black-body in the region where $\alpha = 10$. When the aqueous vapour pressure is e millimetres, this amount of precipitable water can be contained in a layer $960/e$ metres thick. With the ordinary vapour pressures and lapse-rates prevailing in the atmosphere, the mean temperature of this layer will differ little from the temperature at the place of observation. S_3 will therefore be only slightly less than $0 \cdot 54$.

Let us now consider the variable part S_2. If the mean absorption coefficient for this region be taken as 4, the exponential in the equation for radiation is, as we have seen, $-6w$ where w is the amount of precipitable water in centimetres. According to Hann, the amount of precipitation water in the atmosphere is on the average given by the equation $w = 0 \cdot 21e$ where e is the aqueous vapour pressure at the surface in mm. of mercury and the value of $3\,kw/2$ becomes $1 \cdot 26e$.

This value is far too high compared with the empirical constant ($\cdot 07$) determined from observations of atmospheric radiation. The reason for this discrepancy seems to be that Hettner's values of absorption coefficient are too high in the region $10 \cdot 5 - 15\mu$. Both Fowle's measurements[5] as well as the recent ones made by L. R. Weber and H. M. Randall[6] show this. The latter authors used long columns of water-vapour and larger resolution than Hettner. The values of α calculated from their results are compared to those obtained from Hettner's measurements in Figs. 1 and 2. Even in the region of large absorptivities, there seem to be gaps of smaller absorption as for example between $21 \cdot 5$ and $22 \cdot 5\mu$. The mean absorptivities from Fowle's measurements are: $\cdot 04$ for $11 - 12\mu$, $\cdot 17$ for $12 - 13\mu$, $\cdot 42$ for $13 - 14\mu$ and $3 \cdot 0$ for $14 - 15\mu$. If we use the average value of the absorption coefficient between $10 \cdot 5$ and 15μ obtained from Weber and Randall's observations, which is near $0 \cdot 3$, the multiplier in the exponent in Ångstrom's equation will be very nearly $\cdot 09$, which nearly agrees with the value obtained from observations of sky radiation. Decisive measurements of the absorption coefficient of water in different regions of the infra-red spectrum are very much to be desired.

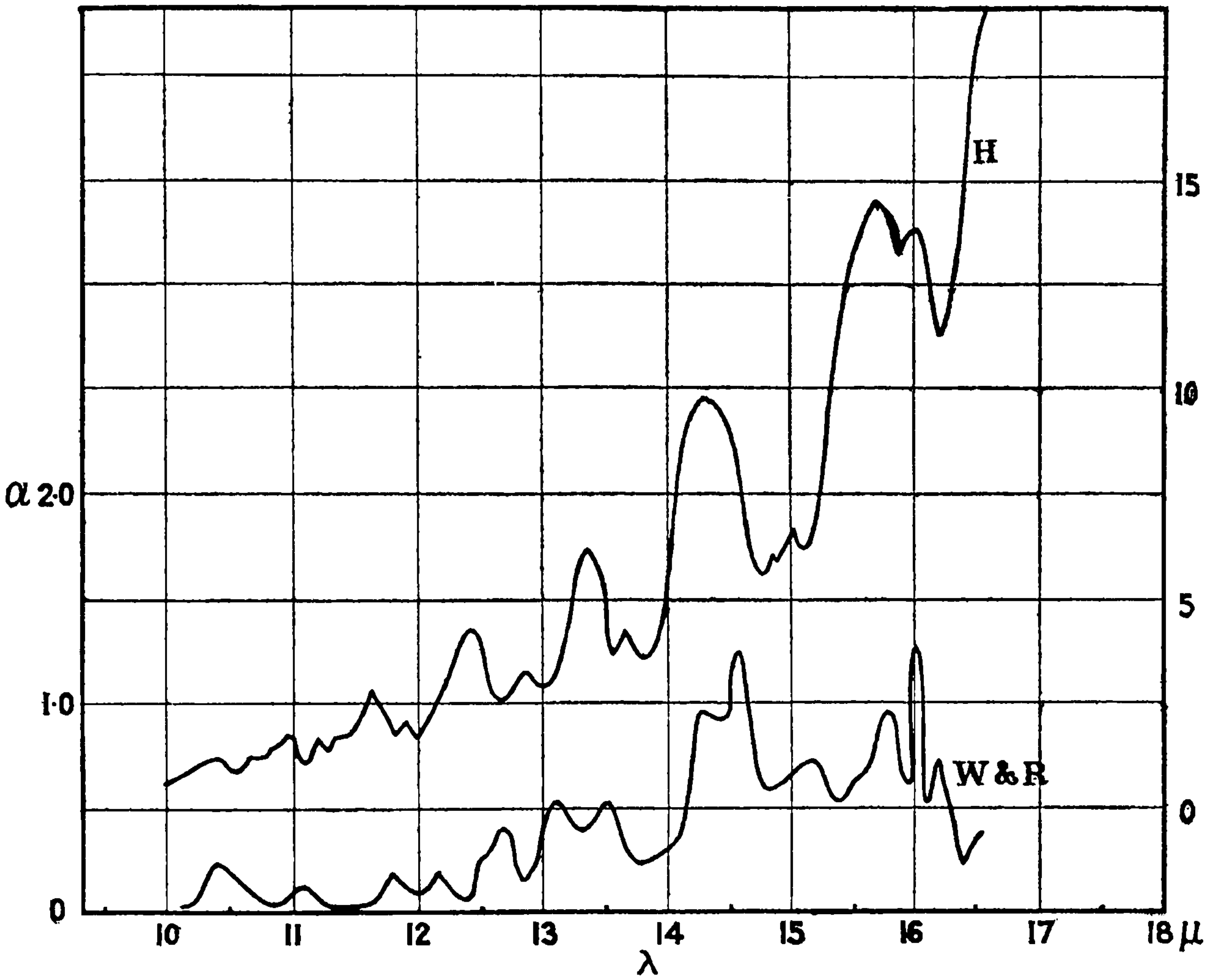

FIG. 1. H = Hettner; W & R = Weber and Randall.
The scale for Hettner's curve is given on the right hand side and that for Weber and Randall's on the left.

If the average coefficient of absorption in the regions of moderate absorption be as low as 0·3, S_2 will come from more distant and cooler layers. The moisture required to produce a depletion of energy of 9/10 will now be 3·3 cm. of precipitable water and to contain this a thickness of atmosphere of $33 \times 960/e$ metres of air will be required. If the average value of e be 2 millimetres, the thickness will be 15840 metres. It is thus not permissible to regard the temperature of the atmospheric layer responsible for S_2 as that near the place of observation. The effective temperature will be lower, the greater the dryness of the atmosphere. The "constant" B in Ångstrom's formula corresponding to S_2 radiation may be expected to be markedly smaller than 0·32; A also will be smaller than 0·86.

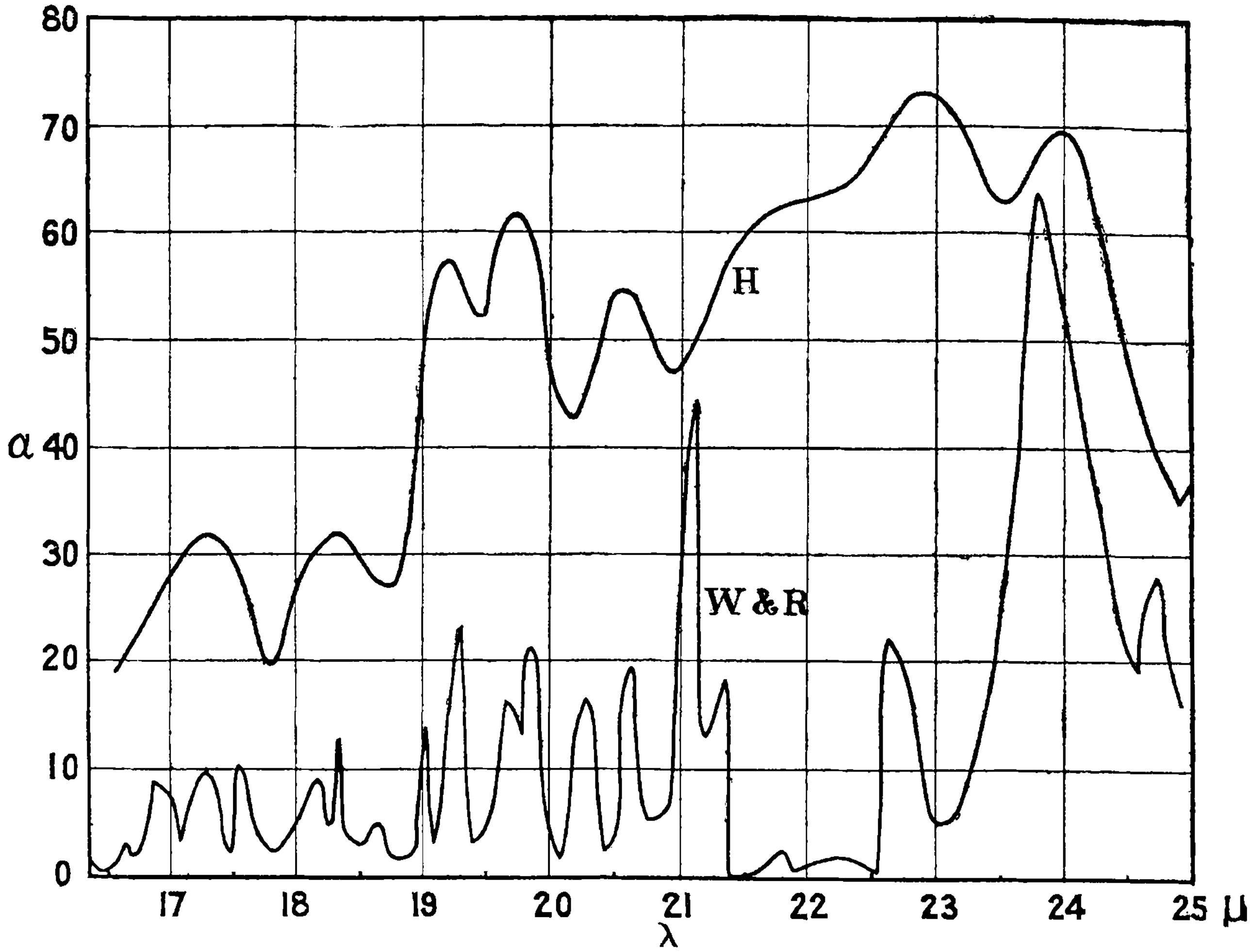

FIG. 2. H = Hettner ; W & R = Weber and Randall.

Nocturnal Cooling of Air Layers near the Ground.

From an altogether different line of evidence, the conclusion had been reached by the authors that air itself should be considered as exchanging by radiation appreciable quantities of energy with distant layers of the atmosphere. On calm, clear nights in winter, the lowest temperature is observed to occur, not in immediate contact with the ground but at a distance of half to one foot above ground.*[7]

The following table gives the temperatures at different heights above ground observed at the Agricultural Meteorological Observatory, Poona, on a calm, clear night in January 1933 (5th to 6th).

* It has been verified by observations at distant places and also by simultaneous observations in the neighbourhood of the Agricultural Meteorological Observatory at Poona that this effect is not due to advection and is also not peculiar to any particular locality.

TABLE I. *Temperature* (°C.)

Time hrs.	Ground (surface)	Air just above ground	1″	3″	6″	1′	2′	4′	6′	10′
1700	33·9	29·0	28·5	28·5	28·5	28·3	28·3	28·0	28·0	27·5
1800	29·7	27·0	27·0	26·7	26·5	26·5	26·5	26·3	26·3	26·3
1900	22·2	20·0	19·5	19·0	19·0	19·0	19·5	20·2	21·0	21·0
2000	19·7	17·5	17·0	16·5	16·5	17·0	17·0	17·5	18·5	19·5
2200	17·2	15·2	15·0	14·5	14·5	14·5	15·0	16·0	16·5	15·5
2400	15·0	13·0	12·7	12·5	12·2	12·0	12·2	13·0	13·5	14·7
0600	11·7	10·0	9·5	9·2	9·0	9·0	10·0	11·0	11·5	12·0

Throughout the night, there is a shallow layer of unstable lapse-rate close to the ground and it is only above this that the well-known night inversion exists.

The usual explanation for the formation and development of night inversions near the ground is that when the sun's radiation is withdrawn, the ground cools as a black-body exposed to the dark radiation of the atmosphere and although the fall of temperature due to this is partly compensated by the flow of heat from inside the earth to the surface, the net effect is a fall of temperature. The fall of temperature of the air above the ground is primarily due to the spreading upward of the ground cooling by the processes of eddy diffusion and radiation exchange. If these were however the only causes operative, we should expect that the lowest temperature should occur at the surface of the ground, the distribution of temperature near the ground after the inversion sets in being somewhat as shown by A B C in Fig. 3.

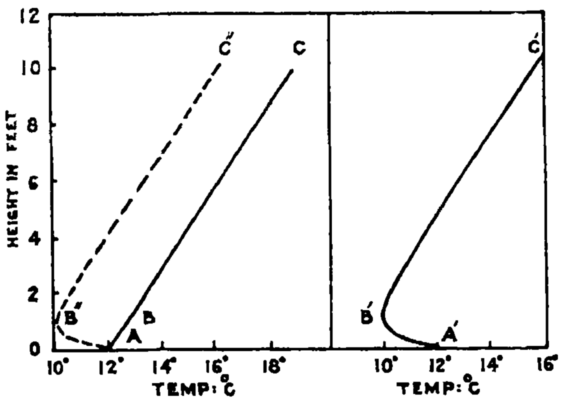

FIG. 3.

The actual distribution of temperature is however like A′ B′ C′.

The difference in the character of the curves can be explained if, besides the cooling which spreads upward from the ground, there is an additional general cooling of the air layers. The effect of such an addition will be to change the curve A B C to A B″ C″.

A mechanism for such general cooling is provided by the exchange of radiation of the air layers with the upper atmosphere. The portions of the heat spectrum for which water vapour has either very low or very high absorption will be ineffective for this purpose—the former because there will be no radiation from the air layer in that region and the latter because the radiation loss will be balanced by radiation income from neighbouring layers. Only the region of the spectrum which we have designated as S_2 will be effective.

Let the temperature of the air near the ground be 300° A. and the moisture-content 5×10^{-6} gm./c.c. corresponding to a vapour pressure of 5 mm. The decimal absorption coefficient in the region of S_2 absorption ($10{\cdot}5 - 15{\cdot}3\mu$) is $0{\cdot}3$ and the fraction of the energy of black-body radiation contained in this region about $0{\cdot}32$. Assuming that the effective temperature of the distant layer contributing to radiation in the region $11 - 16\mu$ is 270°A., the rate of fall of temperature due to this cause comes out to be 0·11°C./hour. If the effective temperature of the radiating layer be taken as 250°A., the rate of cooling will be increased to 0·16°C./hour. The minimum temperature which is observed at 0·5 to 1·5 ft. above ground is presumably therefore due to the fact that the air cools not only by eddy diffusion and radiation exchange with the ground but also by radiation exchange with the upper atmosphere. The coolest air does not settle down to the ground because the effects of viscosity and heat conduction keep the thin ground layer in stable equilibrium in spite of higher density above.[8]

The processes taking place in the neighbourhood of the ground on radiation nights are, as we have seen, complicated by a number of factors. For their full elucidation, the changes taking place both in the soil and at higher levels in the atmosphere have to be studied. Such a study has been commenced.

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