HEAT RADIATION FROM THE CLEAR ATMOSPHERE AT NIGHT.

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Introduction.

SINCE the application by Simpson of the detailed absorption spectrum of water-vapour to the problem of atmospheric radiation, interest in the subject has revived and many papers have appeared dealing with different aspects of it. In a paper published recently,[1] D. Brunt has discussed the question of the net loss of heat by radiation from the ground and the air layers in its neighbourhood, and, from the analogy of radiative diffusion to eddy diffusion, has put forward a simple empirical formula for the downward radiation S incident on one square centimetre from the atmosphere,

$$S = \sigma T^4 (a + b \sqrt{e})$$

where T is the temperature in degrees absolute and e the aqueous vapour pressure of the air at the place of observation, σT^4 is the black-body radiation at temperature T, and a and b are constants. Brunt observes that the values of a and b and b/a vary from one set of observations to another. The formula in common use for S was one suggested by Å. Angstrom,[2]

$$S/\sigma T^4 = A - B \cdot 10^{-\gamma e}$$

where A, B and γ are constants. The purpose of the present paper is to examine the values of the constants with the help of all available data and to point out the connection between the two formulæ.

Brunt's Formula.

Brunt has given the values of a and b calculated from observations at a number of places and also the correlation coefficients between $S/\sigma T^4$ and $\sqrt{e}$. They are reproduced in Table I; e is expressed in millibars.

There appears to be an arithmetical error in Brunt's calculation of the constants from Poona values. The values as calculated by the author are $a = 0 \cdot 47$, $b = 0 \cdot 061$ and $r = 0 \cdot 92$.

The correlation coefficients are high at most of the places but the following points are to be remembered in this connection. The radiation values

TABLE I.

Author	Place	Height above sea-level in metres	a	b	b/a	r	Remarks
Dines	Benson	..	·52	·065	..	·97	12 monthly mean values. S was measured at Benson while e refers to Kew.
Asklöf ..	Upsala	24	·43	·082	·19	·83	28 individual observations.
Ångstrom ..	Bassour	1160	·48	·058	·12	·73	38 " "
Boutaric ..	Montpellier and Pic du Midi	2859	·60	·042	·07	..	6 mean values.
Robitzsch ..	Lindenberg	40	·34	·110	·32	1·0	11 grouped means from registration curves on 30 clear nights.
Ramanathan and Desai	Poona	564	·26	·12	·46	·93	12 monthly means.

for Benson were 12 monthly means measured with Dines's thermopile radiometer, while the vapour pressures used in the calculation were from observations at Kew. The values for Lindenberg[3] are based on 11 grouped means of 1350 individual observations obtained by Robitzsch from registration curves on 30 clear nights. As the correlation coefficient in this case is as high as 1·0 it is necessary to look into this somewhat closely. Robitzsch used a self-registering instrument for recording the night radiation. His apparatus consisted of two similar thin circular metal plates one blackened and the other polished, placed horizontally inside a Dewar's flask and exposed to the zenith sky subtending an angle of 60°. The difference of temperature between the strips was recorded galvanometrically with thermojunctions. Robitzsch's method of calculation and of extrapolation to obtain the radiation from the whole sky are open to objection as he apparently assumes that the sides of the Dewar's flask contribute nothing to the net radiation received by the element. But actually the sides of the flask would be at different temperature from the receiver, and glass being a good absorber for infra-red radiation, the sides of the flask would contribute an appreciable amount to the net radiation. The values of S given by Robitzsch cannot therefore be used for testing the formula. The exceptionally low value of "a" calculated from the observations of Robitzsch is without doubt due to this cause.

It was considered that it would be desirable to calculate the values of the constants in Brunt's formula from longer series of individual observations. Besides the series of 144 night observations taken at Poona by Drs. Ramanathan and Desai in 1930-31, a second series of 114 observations on clear nights taken by me in 1933-34 at the Agricultural Meteorological Observatory was available. A search in the literature showed that the following series of observations could also be made use of for the same purpose.

(1) Two series of 121 observations by Kimball[4] at Mount Weather, Washington and other places;

(2) 50 individual observations on 12 clear nights at Mount Whitney by Ångstrom[2];

(3) 28 observations at Kew made with the Benson radiometer[5]; and

(4) 14 grouped means of observations near Vienna by O. Eckel.[6]

In Table II are given the values of the constants a and b and the correlation coefficients between $S/\sigma T^4$ and $\sqrt{e}$ calculated from all these series of observations.

TABLE II.

Author	Place	Height above sea-level in metres	a	b	b/a	r	Remarks
Kimball	Washington	137	·44	·027	·061	·29	22 clear nights.
	Mt. Weather	540	·52	·066	·127	·84	50 do.
	Different places in U.S.A.	..	·53	·062	·117	·88	121 do.
Ångstrom ..	Mt. Whitney	4420	·50	·032	·064	·30	50 do.
O. Eckel	Kanzelhöhe (Austria)	1500	·47	·063	·134	·89	14 grouped means.
Observatory Staff ..	Kew	5	·62	·056	·090	·64	28 observations with Dines's radiometer.
Ramanathan and Desai, 1930 ..	Poona	564	·55	·038	·070	·63	144 clear nights.
Raman	Poona	564	·62	·029	·047	·68	114 do.

The correlation coefficients calculated from the individual daily values are not so good as those from the grouped mean and the monthly mean values and there are two instances of very small coefficients, one at Washington and the other at Mount Whitney. The mean value of the constant "a" is 0·53 and its variations are not so high as cannot be explained by the difference in the nature of the instruments and the methods of their use. The average value of b is ·047. The variations are, however, large.

Ångstrom's Formula.

We shall next consider Ångstrom's formula. This is based mainly on experimental observations, but Ångstrom has given reasons[2] for choosing an exponential form for the vapour pressure in his equation. In one of his recent papers[7] Ångstrom has given the equation in the form:

$$S/\sigma T^4 = 0 \cdot 75 - 0 \cdot 32 \times 10^{-\cdot 069e}$$

where e is the vapour pressure in *millimetres* of mercury. The values of the constants calculated from a number of published series of observations and also from the two series of observations taken at Poona are given in Table III. All the observations were plotted on graph paper, the mean curve drawn and the equation of the curve obtained assuming it to be exponential in form.

TABLE III.

Place of observation	Author	A	B	γ	No. of observations	Remarks
—	Ångstrom	·75	·32	·069	..	As given by Ångstrom[7]
Upsala	Asklöf	·79	·299	·069	..	As given by Asklöf[7]
—	Ångstrom	·79	·262	·071	..	As given by Mosby[8]
Mt. Weather	Kimball	·80	·325	·093	121	As calculated by the author
Kanzelhöhe	Eckel	·71	·24	·098	13 grouped means	do.
Poona	Ramanathan and Desai	·78	·27	·053	144	do.
Poona	Raman	·79	·273	·068	114	do.

The mean values of A, B and γ are 0·77, 0·28 and 0·074 respectively. When e is expressed in millibars, the factor in the exponent will be changed to ·055.

It may be mentioned that although in a general way, the formula expresses the nature of the variation of atmospheric radiation with temperature and vapour pressure, the fit in any individual series is by no means very good.

Relation between Brunt's and Ångstrom's Formulæ.

It is easy to show that Brunt's form of the equation is only a variation of Ångstrom's, holding approximately good within the range of usual variation of vapour pressure and of the accuracy of the measurement. $A - B \cdot 10^{-\gamma e}$ can be written in the form $(A - B) + B(1 - 10^{-\gamma e})$, the first term of which is nearly equal to Brunt's "*a*" and the second term has the value 0 when $e = 0$ and B when $e = \infty$. But we are generally concerned only with values of e up to about 30 millibars and within this range, the course of the curve can be roughly reproduced by the expression $b\sqrt{e}$ if b is suitably chosen. The following table gives the values of $0 \cdot 28\ (1 - 10^{-\cdot 055e})$ and of $\cdot 05\sqrt{e}$ and $\cdot 06\ \sqrt{e}$ for values of e ranging from 2 to 30 millibars.

TABLE IV.

e	$\cdot 28\ (1-10^{-\cdot 055e})$	$\cdot 05\ \sqrt{e}$	$\cdot 06\ \sqrt{e}$
2	·063	·070	·084
4	·111	·100	·120
6	·149	·123	·147
9	·191	·150	·180
12	·219	·173	·208
16	·243	·200	·240
20	·258	·223	·268
25	·268	·250	·300
30	·275	·273	·328

As the physical ideas underlying Ångstrom's formulæ are clearer,[9] this form of the relation is to be preferred to Brunt's.

Considering that the atmospheric radiation comes from the whole atmosphere above a place, it is not surprising that there is not better agreement with formulæ which take into account only the surface temperature and vapour pressure. The observed agreement is due to the fact that water vapour has large absorptivity over a considerable part of the infra-red spectrum and that the concentration of water vapour is usually a maximum near the ground.

The Minimum Value of Atmospheric Radiation.

The constant "*a*" in Brunt's formula or "A − B" in Ångstrom's can be interpreted as the fraction of black-body radiation at the temperature of the place of observation which is emitted by a small quantity of water-vapour. When the atmosphere is very dry, we should expect that $S/\sigma T^4$ may even go below 0·5. It is interesting to examine the minimum value which this quantity takes under very favourable conditions. In the following table are collected together a number of selected values of $S/\sigma T^4$ observed by different authors when the aqueous vapour pressure was exceptionally small.

TABLE V.

Author	Place	Height above sea-level	*e* mm.	Net radiation	S	σT^4	$S/\sigma T^4$
Eckel[6] ..	Kanzelhöhe	1500	1·5	0·179	0·206	0·385	0·53
Boutaric[10] ..	Mt. Blanc	4500	1·9	0·168	0·280	0·448	0·62
			2·4	0·186	0·275	0·461	0·60
Kimball[4] ..	Washington	137	1·7	0·194	0·268	0·462	0·58
Ångstrom[11]	Abisko	0	1·4	0·186	0·202	0·388	0·52
Do. ..	San Antonio	3000	1·6	0·225	0·275	0·500	0·55
Do. ..	San Gorganio	3500	1·1	0·221	0·215	0·436	0·49
			1·5	0·215	0·225	0·440	0·51
Do. ..	Mt. Whitney	4420	0·5	0·193	0·217	0·410	0·53
			1·4	0·183	0·234	0·417	0·56
Do. ..	Balloon ascent	3000	0·5	0·226	0·154	0·380	0·41
		to	0·7	0·216	0·163	0·379	0·43
		5100	1·6	0·248	0·157	0·405	0·39
			0·5	0·216	0·127	0·343	0·38
			0·6	0·224	0·140	0·364	0·38
			0·8	0·222	0·137	0·359	0·38
			1·0	0·226	0·156	0·382	0·41
			1·2	0·185	0·210	0·395	0·53
Mosby[8] ..	N. Polar			0·192	0·224	0·416	0·54
	Regions	0	1	0·144	0·166	0·310	0·53
Raman	Poona	564	1·5	0·242	0·410	0·652	0·63
			2·6	0·242	0·403	0·645	0·62
Do.	Sinhagad	1280	<0·2	0·258	0·351	0·609	0·57
			0·5	0·242	0·353	0·595	0·59

Measurements made at the surface of the earth whether at plain or mountain stations do not show a value lower than 0·49. A few observations made by the author at Sinhagad, a hill station 4200 ft. above sea-level and 15 miles to the south-west of Poona on a very dry night (9–3–1935, vapour pressure <0·2 mm.) showed a ratio as high as 0·57. Only Ångstrom's observations[12] taken from balloons at great heights show values as low as 0·38. More observations under exceptionally dry weather conditions or in high balloon ascents are to be desired.

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