

# DECOMPOSITIONS INTO SQUARES OF PRIMES.

BY HANSRAJ GUPTA,  
*Government College, Hoshiarpur.*

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## 1. Introduction.

I. CHOWLA<sup>1</sup> has conjectured that every positive integer can be represented as the sum of the squares of at most eight primes  $\geq 1$ . He has verified that this is true for all positive integers  $\leq 30,000$ . I give here a table showing that the conjecture is true for all positive integers  $\leq 100,000$ .

I. Chowla has shown that the smallest number requiring as many as five squares is 24. The smallest numbers requiring six, seven and eight squares are 73; 266, and 795 respectively.

Supposing that the conjecture holds for all numbers  $< N$ , we find by trial a number  $M \leq N$ , such that

$$N \leq M + R, \text{ where } R = 794; 265; 72; \text{ or } 23;$$

according as  $M$  can be expressed in one or the other of the forms:

$$p^2; p^2 + q^2; p^2 + q^2 + r^2; p^2 + q^2 + r^2 + s^2;$$

$p, q, r$  and  $s$  being any primes  $\geq 1$ .

Then it follows that the conjecture is true for all numbers upto  $M + R$ .

For the sake of brevity, I write

$$a(p) \text{ for } p^2 + 794,$$

$$b(p, q) \text{ for } p^2 + q^2 + 265,$$

$$c(p, q, r) \text{ for } p^2 + q^2 + r^2 + 72,$$

$$\text{and } d(p, q, r, s) \text{ for } p^2 + q^2 + r^2 + s^2 + 23.$$

Within the limits of my table need for  $d$ 's does not arise.

If  $k$  and  $m$  be any two consecutive entries in the table, then  $m - k \leq 794; 265; \text{ or } 72$ ; according as  $m$  belongs to the class  $a, b$ , or  $c$ . The table itself requires no further explanation. I find<sup>2</sup> that

$$m - k \equiv 0 \pmod{24}.$$

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<sup>1</sup> I. Chowla, *Proc. Ind. Acad. Sci.*, 1935, 1, 451-453.

<sup>2</sup> This is easily proved, for if  $p$  be any prime  $\geq 5$ , then  $p^2 \equiv 1 \pmod{24}$ .

2. *The Table.*

|       |                        |       |                        |
|-------|------------------------|-------|------------------------|
| 795   |                        | 16923 | <i>a</i> (127)         |
| 1323  | <i>a</i> (23)          | 17187 | <i>b</i> (109, 71)     |
| 1755  | <i>a</i> (31)          | 17955 | <i>a</i> (131)         |
| 2475  | <i>a</i> (41)          | 18075 | <i>b</i> (113, 71)     |
| 3003  | <i>a</i> (47)          | 18267 | <i>b</i> (131, 29)     |
| 3603  | <i>a</i> (53)          | 18387 | <i>b</i> (131, 31)     |
| 4275  | <i>a</i> (59)          | 18603 | <i>b</i> (127, 47)     |
| 4515  | <i>a</i> (61)          | 18795 | <i>b</i> (131, 37)     |
| 5283  | <i>a</i> (67)          | 19563 | <i>a</i> (137)         |
| 5835  | <i>a</i> (71)          | 20115 | <i>a</i> (139)         |
| 6123  | <i>a</i> (73)          | 20283 | <i>b</i> (103, 97)     |
| 6267  | <i>b</i> (71, 31)      | 20547 | <i>b</i> (139, 31)     |
| 7035  | <i>a</i> (79)          | 20715 | <i>b</i> (137, 41)     |
| 7683  | <i>a</i> (83)          | 20955 | <i>b</i> (139, 37)     |
| 7875  | <i>b</i> (79, 37)      | 21147 | <i>b</i> (131, 61)     |
| 8115  | <i>b</i> (83, 31)      | 21267 | <i>b</i> (139, 41)     |
| 8715  | <i>a</i> (89)          | 21435 | <i>b</i> (127, 71)     |
| 8835  | <i>b</i> (83, 41)      | 21555 | <i>b</i> (109, 97)     |
| 9027  | <i>b</i> (89, 29)      | 21795 | <i>b</i> (139, 47)     |
| 9147  | <i>b</i> (89, 31)      | 21915 | <i>b</i> (107, 101)    |
| 9315  | <i>b</i> (73, 61)      | 21963 | <i>c</i> (139, 47, 19) |
| 9555  | <i>b</i> (89, 37)      | 22011 | <i>c</i> (137, 53, 19) |
| 10203 | <i>a</i> (97)          | 22083 | <i>c</i> (139, 43, 29) |
| 10995 | <i>a</i> (101)         | 22323 | <i>b</i> (107, 103)    |
| 11403 | <i>a</i> (103)         | 22995 | <i>a</i> (149)         |
| 11523 | <i>b</i> (97, 43)      | 23595 | <i>a</i> (151)         |
| 12243 | <i>a</i> (107)         | 23835 | <i>b</i> (149, 37)     |
| 12675 | <i>a</i> (109)         | 24027 | <i>b</i> (151, 31)     |
| 12723 | <i>b</i> (103, 43)     | 24147 | <i>b</i> (149, 41)     |
| 12987 | <i>b</i> (109, 29)     | 24363 | <i>b</i> (137, 73)     |
| 13563 | <i>a</i> (113)         | 24627 | <i>b</i> (139, 71)     |
| 13827 | <i>b</i> (109, 41)     | 24747 | <i>b</i> (151, 41)     |
| 13995 | <i>b</i> (113, 31)     | 25443 | <i>a</i> (157)         |
| 14187 | <i>b</i> (101, 61)     | 25515 | <i>c</i> (131, 89, 19) |
| 14355 | <i>b</i> (109, 47)     | 25755 | <i>b</i> (157, 29)     |
| 14523 | <i>b</i> (107, 53)     | 25923 | <i>b</i> (137, 83)     |
| 14715 | <i>b</i> (113, 41)     | 26187 | <i>b</i> (149, 61)     |
| 14883 | <i>b</i> (113, 43)     | 26259 | <i>c</i> (137, 83, 23) |
| 15075 | <i>b</i> (89, 83)      | 26475 | <i>b</i> (139, 83)     |
| 15243 | <i>b</i> (113, 47)     | 26595 | <i>b</i> (127, 101)    |
| 15435 | <i>b</i> (107, 61)     | 27363 | <i>a</i> (163)         |
| 15627 | <i>b</i> (109, 59)     | 27555 | <i>b</i> (151, 67)     |
| 15867 | <i>b</i> (109, 61)     | 27795 | <i>b</i> (163, 31)     |
| 16107 | <i>b</i> (89, 89)      | 28035 | <i>b</i> (131, 103)    |
| 16179 | <i>c</i> (113, 53, 23) | 28683 | <i>a</i> (167)         |

|       |                         |       |                         |
|-------|-------------------------|-------|-------------------------|
| 28875 | <i>b</i> (131, 107)     | 40995 | <i>c</i> (199, 31, 19)  |
| 29115 | <i>b</i> (167, 31)      | 41235 | <i>b</i> (199, 37)      |
| 29355 | <i>b</i> (149, 83)      | 41283 | <i>b</i> (197, 47)      |
| 29523 | <i>b</i> (167, 37)      | 41547 | <i>b</i> (199, 41)      |
| 29787 | <i>b</i> (139, 101)     | 41715 | <i>b</i> (199, 43)      |
| 30003 | <i>b</i> (167, 43)      | 41883 | <i>b</i> (197, 53)      |
| 30723 | <i>a</i> (173)          | 42075 | <i>b</i> (199, 47)      |
| 30963 | <i>b</i> (167, 53)      | 42099 | <i>c</i> (193, 67, 17)  |
| 31155 | <i>b</i> (173, 31)      | 42171 | <i>c</i> (193, 67, 19)  |
| 31323 | <i>b</i> (163, 67)      | 42435 | <i>b</i> (181, 97)      |
| 31563 | <i>b</i> (173, 37)      | 42675 | <i>b</i> (199, 53)      |
| 31635 | <i>b</i> (167, 59)      | 42843 | <i>b</i> (193, 73)      |
| 31875 | <i>b</i> (167, 61)      | 42987 | <i>b</i> (191, 79)      |
| 32043 | <i>b</i> (173, 43)      | 43227 | <i>b</i> (181, 101)     |
| 32835 | <i>a</i> (179)          | 43347 | <i>b</i> (199, 59)      |
| 33555 | <i>a</i> (181)          | 43563 | <i>b</i> (197, 67)      |
| 33675 | <i>b</i> (179, 37)      | 43755 | <i>b</i> (193, 79)      |
| 33867 | <i>b</i> (181, 29)      | 43995 | <i>b</i> (163, 131)     |
| 33987 | <i>b</i> (181, 31)      | 44187 | <i>b</i> (179, 109)     |
| 34155 | <i>b</i> (179, 43)      | 44403 | <i>b</i> (197, 73)      |
| 34395 | <i>b</i> (181, 37)      | 44667 | <i>b</i> (191, 89)      |
| 34515 | <i>b</i> (179, 47)      | 45315 | <i>a</i> (211)          |
| 34707 | <i>b</i> (181, 41)      | 45435 | <i>b</i> (193, 89)      |
| 34875 | <i>b</i> (181, 43)      | 45627 | <i>b</i> (211, 29)      |
| 35115 | <i>b</i> (179, 53)      | 45795 | <i>b</i> (181, 113)     |
| 35235 | <i>b</i> (181, 47)      | 45963 | <i>b</i> (197, 83)      |
| 35283 | <i>c</i> (157, 101, 19) | 46155 | <i>b</i> (191, 97)      |
| 35523 | <i>b</i> (157, 103)     | 46203 | <i>c</i> (199, 79, 17)  |
| 35787 | <i>b</i> (179, 59)      | 46467 | <i>b</i> (211, 41)      |
| 36027 | <i>b</i> (179, 61)      | 46635 | <i>b</i> (211, 43)      |
| 36243 | <i>b</i> (163, 97)      | 46755 | <i>b</i> (199, 83)      |
| 36435 | <i>b</i> (173, 79)      | 46947 | <i>b</i> (191, 101)     |
| 36507 | <i>b</i> (181, 59)      | 47115 | <i>b</i> (157, 149)     |
| 37275 | <i>a</i> (191)          | 47355 | <i>b</i> (191, 103)     |
| 38043 | <i>a</i> (193)          | 47595 | <i>b</i> (211, 53)      |
| 38115 | <i>b</i> (191, 37)      | 47787 | <i>b</i> (199, 89)      |
| 38355 | <i>b</i> (193, 29)      | 47811 | <i>c</i> (167, 139, 23) |
| 38595 | <i>b</i> (191, 43)      | 47883 | <i>c</i> (151, 151, 47) |
| 38643 | <i>c</i> (193, 31, 19)  | 48123 | <i>b</i> (193, 103)     |
| 38883 | <i>b</i> (193, 37)      | 48267 | <i>b</i> (211, 59)      |
| 39603 | <i>a</i> (197)          | 48507 | <i>b</i> (211, 61)      |
| 40395 | <i>a</i> (199)          | 48627 | <i>b</i> (191, 109)     |
| 40443 | <i>b</i> (197, 37)      | 48675 | <i>c</i> (211, 61, 19)  |
| 40707 | <i>b</i> (199, 29)      | 48723 | <i>c</i> (191, 109, 17) |
| 40923 | <i>b</i> (197, 43)      | 48963 | <i>b</i> (193, 107)     |

|       |                         |       |                         |
|-------|-------------------------|-------|-------------------------|
| 49155 | <i>b</i> (181, 127)     | 60027 | <i>b</i> (241, 41)      |
| 49275 | <i>b</i> (211, 67)      | 60195 | <i>b</i> (241, 43)      |
| 49515 | <i>b</i> (191, 113)     | 60315 | <i>b</i> (193, 151)     |
| 49563 | <i>c</i> (193, 109, 19) | 60555 | <i>b</i> (241, 47)      |
| 49827 | <i>b</i> (211, 71)      | 60795 | <i>b</i> (233, 79)      |
| 50523 | <i>a</i> (223)          | 60915 | <i>b</i> (211, 127)     |
| 50595 | <i>c</i> (211, 71, 31)  | 61107 | <i>b</i> (239, 61)      |
| 50835 | <i>b</i> (223, 29)      | 61275 | <i>b</i> (197, 149)     |
| 51075 | <i>b</i> (179, 137)     | 61443 | <i>b</i> (233, 83)      |
| 51123 | <i>c</i> (211, 79, 17)  | 61491 | <i>c</i> (241, 53, 23)  |
| 51363 | <i>b</i> (223, 37)      | 61539 | <i>c</i> (233, 83, 17)  |
| 51411 | <i>c</i> (199, 107, 17) | 61611 | <i>c</i> (233, 83, 19)  |
| 51675 | <i>b</i> (211, 83)      | 61875 | <i>b</i> (239, 67)      |
| 52323 | <i>a</i> (227)          | 62115 | <i>b</i> (229, 97)      |
| 52395 | <i>c</i> (199, 109, 29) | 62163 | <i>c</i> (241, 61, 17)  |
| 52635 | <i>b</i> (199, 113)     | 62427 | <i>b</i> (239, 71)      |
| 53235 | <i>a</i> (229)          | 62667 | <i>b</i> (199, 151)     |
| 53475 | <i>b</i> (227, 41)      | 62907 | <i>b</i> (229, 101)     |
| 53715 | <i>b</i> (223, 61)      | 62955 | <i>b</i> (181, 173)     |
| 53907 | <i>b</i> (191, 131)     | 63003 | <i>c</i> (229, 101, 17) |
| 54075 | <i>b</i> (229, 37)      | 63795 | <i>a</i> (251)          |
| 54195 | <i>b</i> (211, 97)      | 63963 | <i>b</i> (233, 97)      |
| 54387 | <i>b</i> (229, 41)      | 64227 | <i>b</i> (251, 31)      |
| 55083 | <i>a</i> (233)          | 64347 | <i>b</i> (179, 179)     |
| 55323 | <i>b</i> (223, 73)      | 64515 | <i>b</i> (199, 157)     |
| 55515 | <i>b</i> (229, 53)      | 64755 | <i>b</i> (233, 101)     |
| 55563 | <i>c</i> (181, 149, 23) | 64947 | <i>b</i> (251, 41)      |
| 55611 | <i>c</i> (229, 53, 17)  | 65163 | <i>b</i> (233, 103)     |
| 55683 | <i>c</i> (229, 53, 19)  | 65403 | <i>b</i> (193, 167)     |
| 55923 | <i>b</i> (233, 37)      | 65643 | <i>b</i> (197, 163)     |
| 56187 | <i>b</i> (229, 59)      | 65787 | <i>b</i> (181, 181)     |
| 56427 | <i>b</i> (229, 61)      | 66003 | <i>b</i> (233, 107)     |
| 56667 | <i>b</i> (211, 109)     | 66267 | <i>b</i> (241, 89)      |
| 56883 | <i>b</i> (223, 83)      | 66843 | <i>a</i> (257)          |
| 57123 | <i>b</i> (227, 73)      | 66987 | <i>b</i> (251, 61)      |
| 57915 | <i>a</i> (239)          | 67155 | <i>b</i> (223, 131)     |
| 58035 | <i>b</i> (233, 59)      | 67323 | <i>b</i> (233, 113)     |
| 58275 | <i>b</i> (233, 61)      | 67587 | <i>b</i> (239, 101)     |
| 58875 | <i>a</i> (241)          | 67755 | <i>b</i> (199, 167)     |
| 59067 | <i>b</i> (239, 41)      | 67995 | <i>b</i> (239, 103)     |
| 59235 | <i>b</i> (239, 43)      | 68163 | <i>b</i> (257, 43)      |
| 59403 | <i>b</i> (223, 97)      | 68307 | <i>b</i> (251, 71)      |
| 59595 | <i>b</i> (233, 71)      | 68523 | <i>b</i> (257, 47)      |
| 59715 | <i>b</i> (241, 37)      | 68787 | <i>b</i> (191, 179)     |
| 59883 | <i>b</i> (233, 73)      | 69003 | <i>b</i> (197, 173)     |

|       |                         |       |                         |
|-------|-------------------------|-------|-------------------------|
| 69267 | <i>b</i> (239, 109)     | 80187 | <i>b</i> (281, 31)      |
| 69963 | <i>a</i> (263)          | 80883 | <i>a</i> (283)          |
| 70155 | <i>b</i> (251, 83)      | 81075 | <i>b</i> (281, 43)      |
| 70395 | <i>b</i> (263, 31)      | 81315 | <i>b</i> (283, 31)      |
| 70563 | <i>b</i> (227, 137)     | 81483 | <i>b</i> (277, 67)      |
| 70803 | <i>b</i> (257, 67)      | 81723 | <i>b</i> (227, 173)     |
| 70875 | <i>c</i> (241, 109, 29) | 81795 | <i>c</i> (271, 89, 19)  |
| 71115 | <i>b</i> (197, 179)     | 82035 | <i>b</i> (283, 41)      |
| 71355 | <i>b</i> (257, 71)      | 82203 | <i>b</i> (263, 113)     |
| 71475 | <i>b</i> (229, 137)     | 82443 | <i>b</i> (257, 127)     |
| 71643 | <i>b</i> (263, 47)      | 82707 | <i>b</i> (281, 59)      |
| 71907 | <i>b</i> (199, 179)     | 82827 | <i>b</i> (269, 101)     |
| 72027 | <i>b</i> (229, 139)     | 82995 | <i>b</i> (241, 157)     |
| 72243 | <i>b</i> (263, 53)      | 83235 | <i>b</i> (277, 79)      |
| 72315 | <i>c</i> (241, 89, 79)  | 83475 | <i>b</i> (257, 131)     |
| 72555 | <i>b</i> (257, 79)      | 83715 | <i>b</i> (281, 67)      |
| 73155 | <i>a</i> (269)          | 83907 | <i>b</i> (271, 101)     |
| 73323 | <i>b</i> (233, 137)     | 84075 | <i>b</i> (283, 61)      |
| 73587 | <i>b</i> (269, 31)      | 84315 | <i>b</i> (271, 103)     |
| 74235 | <i>a</i> (271)          | 84507 | <i>b</i> (269, 109)     |
| 74475 | <i>b</i> (269, 43)      | 84747 | <i>b</i> (229, 179)     |
| 74667 | <i>b</i> (271, 31)      | 84915 | <i>b</i> (241, 163)     |
| 74835 | <i>b</i> (269, 47)      | 85155 | <i>b</i> (271, 107)     |
| 75075 | <i>b</i> (271, 37)      | 85395 | <i>b</i> (283, 71)      |
| 75123 | <i>c</i> (269, 43, 29)  | 85635 | <i>b</i> (257, 139)     |
| 75387 | <i>b</i> (271, 41)      | 85683 | <i>c</i> (271, 109, 17) |
| 75555 | <i>b</i> (271, 43)      | 85755 | <i>c</i> (271, 109, 19) |
| 75723 | <i>b</i> (257, 97)      | 85803 | <i>c</i> (257, 139, 19) |
| 75915 | <i>b</i> (271, 47)      | 86067 | <i>b</i> (251, 151)     |
| 76107 | <i>b</i> (269, 59)      | 86643 | <i>a</i> (293)          |
| 76347 | <i>b</i> (199, 191)     | 86691 | <i>c</i> (263, 131, 17) |
| 76563 | <i>b</i> (223, 163)     | 86955 | <i>b</i> (293, 29)      |
| 76755 | <i>b</i> (233, 149)     | 87075 | <i>b</i> (293, 31)      |
| 77523 | <i>a</i> (277)          | 87315 | <i>b</i> (233, 181)     |
| 77667 | <i>b</i> (241, 139)     | 87363 | <i>c</i> (277, 101, 19) |
| 77883 | <i>b</i> (197, 197)     | 87603 | <i>b</i> (277, 103)     |
| 77955 | <i>c</i> (271, 59, 31)  | 87795 | <i>b</i> (293, 41)      |
| 78195 | <i>b</i> (271, 67)      | 87963 | <i>b</i> (293, 43)      |
| 78363 | <i>b</i> (277, 37)      | 88203 | <i>b</i> (263, 137)     |
| 78435 | <i>c</i> (251, 109, 59) | 88443 | <i>b</i> (277, 107)     |
| 78675 | <i>b</i> (199, 197)     | 88635 | <i>b</i> (281, 97)      |
| 78843 | <i>b</i> (277, 43)      | 88875 | <i>b</i> (277, 109)     |
| 79035 | <i>b</i> (271, 73)      | 89043 | <i>b</i> (227, 193)     |
| 79755 | <i>a</i> (281)          | 89307 | <i>b</i> (211, 211)     |
| 79947 | <i>b</i> (271, 79)      | 89427 | <i>b</i> (281, 101)     |

|       |                         |        |                         |
|-------|-------------------------|--------|-------------------------|
| 89595 | <i>b</i> (223, 199)     | 93867  | <i>b</i> (239, 191)     |
| 89835 | <i>b</i> (251, 163)     | 94083  | <i>b</i> (263, 157)     |
| 89955 | <i>b</i> (229, 193)     | 94155  | <i>b</i> (277, 131)     |
| 90147 | <i>b</i> (239, 181)     | 94179  | <i>c</i> (263, 157, 17) |
| 90387 | <i>b</i> (241, 179)     | 94251  | <i>c</i> (263, 157, 19) |
| 90603 | <i>b</i> (227, 197)     | 95043  | <i>a</i> (307)          |
| 90867 | <i>b</i> (271, 131)     | 95307  | <i>b</i> (251, 179)     |
| 91107 | <i>b</i> (241, 181)     | 95475  | <i>b</i> (307, 31)      |
| 91155 | <i>b</i> (293, 71)      | 95523  | <i>c</i> (269, 151, 17) |
| 91395 | <i>b</i> (269, 137)     | 95763  | <i>b</i> (277, 137)     |
| 91635 | <i>b</i> (263, 149)     | 96027  | <i>b</i> (251, 181)     |
| 91803 | <i>b</i> (233, 193)     | 96195  | <i>b</i> (239, 197)     |
| 91995 | <i>b</i> (281, 113)     | 96363  | <i>b</i> (307, 43)      |
| 92235 | <i>b</i> (263, 151)     | 96483  | <i>b</i> (283, 127)     |
| 92475 | <i>b</i> (271, 137)     | 96723  | <i>b</i> (307, 47)      |
| 92523 | <i>c</i> (293, 79, 19)  | 97515  | <i>a</i> (311)          |
| 92571 | <i>c</i> (283, 109, 23) | 97563  | <i>b</i> (293, 107)     |
| 92643 | <i>c</i> (271, 137, 19) | 97827  | <i>b</i> (311, 29)      |
| 92883 | <i>b</i> (257, 163)     | 97995  | <i>b</i> (307, 59)      |
| 93123 | <i>b</i> (283, 113)     | 98763  | <i>a</i> (313)          |
| 93363 | <i>b</i> (233, 197)     | 99003  | <i>b</i> (307, 67)      |
| 93435 | <i>c</i> (269, 139, 41) | 99195  | <i>b</i> (313, 31)      |
| 93483 | <i>c</i> (229, 199, 37) | 99363  | <i>b</i> (263, 173)     |
| 93531 | <i>c</i> (257, 163, 29) | 99555  | <i>b</i> (307, 71)      |
| 93603 | <i>c</i> (211, 211, 67) | 99747  | <i>b</i> (251, 191)     |
|       |                         | 99915  | <i>b</i> (313, 41)      |
|       |                         | 100083 | <i>b</i> (313, 43)      |