NOTE ON DIRICHLET'S L-FUNCTIONS.

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Let

\[ L_s(x) = \sum_{n=1}^{\infty} \chi(n)n^{-s} \quad (s > 0) \]

where \( \chi(n) \) is a real non-principal character mod \( k \):

\[ S_1(x) = \sum_{n \leq x} \chi(n), \quad S_m(x) = \sum_{n \leq x} S_{m-1}(n) \quad (m \geq 2) \]

Let \( m = m(x) \) be the least positive integer (if any) such that

\[ S_m(x) \geq 0 \quad (x \geq 1) \]

If \( m \) exists, then\(^1\)

\[ L_s(x) > 0 \quad (s > 0) \]

For

\[ S_m(1) = 1, \quad S_m(n) \geq 0 \quad (n = 2, 3, \ldots) \]

whence \( (s > 0) \)

\[ L_s(x) = \sum_{n=1}^{\infty} \chi(n)n^{-s} = \sum_{n=1}^{\infty} S(n)(n^{-s} - (n+1)^{-s}) \]

\[ = \sum_{n=1}^{\infty} S_2(n)(n^{-s} - 2(n+1)^{-s} + (n+2)^{-s}) = \cdots \]

\[ = \sum_{n=1}^{\infty} S_m(n) \sum_{t=0}^{m} (-1)^t \frac{m!}{t! (m-t)!} (n+t)^{-s} \]

\[ = s(s+1) \cdots (s+m-1) \sum_{n=1}^{\infty} S_m(n) \int_0^1 \int_0^1 \cdots \int_0^1 (n+u_1+\cdots \cdots + u_m)^{-s} \cdot du_1 \cdots du_m > 0. \]

K. Subbarao calculated \( m \) for the primitive real characters corresponding to

\[ k = 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 47, 53, 59, 61, 71, 73, 79, 83, 89, 97. \]

He showed that \( m = 3 \), for \( k = 53 \) and in the other cases \( m \leq 2. \)

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\(^1\) This theorem and the proof given here are due to S. Chowla in a paper "Note on Dirichlet's L-functions," *Acta arithmetica*, 1935, 1, 113-114.
I. Chowla proved that $m$ is finite for the real primitive characters corresponding to

$$k = 15, 21, 33, 35, 51, 55, 57, 77, 87, 91, 95, 101, 103, 105, 107, 127, 131, 191, 203, 421$$

$m$ being $= 3$ for $k = 91 = 7$ for $k = 77$ and $\leq 2$ otherwise.

I have calculated $m$ for the real primitive characters corresponding to

$$k = 149, 151, 157, 167, 179, 181 \text{ and } 193$$

$m$ being $= 2$ for $k = 179, 181 \text{ and } 193$.

$m = 3$ for $k = 149$ and $m = 1$ for $k = 151 \text{ and } 167$.

Hence $L(s, \chi) > 0$ for $0 < s < 1$ when $\chi$ is a real primitive character corresponding to the above values of $k$.

\[2 \text{ m = 1 also for } k = 7. \text{ It would be interesting to know whether there are infinitely many } k \text{ with } m = 1.\]