NOTE ON HYPOTHESIS K OF HARDY AND LITTLEWOOD.

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1. LET $r_{s,k}(n)$ denote the number of *distinct** representations of n as a sum of s positive kth powers. Hardy and Littlewood have conjectured that

$$r_{k,k}(n) = O(n^{\epsilon})$$

for every positive ϵ. On the other hand it has not even been shown that for any[1] fixed k,

(1) *there exists an* n *such that* $r_{k,k}(n) \geqslant 2$.

I show that

Theorem 1. *For* $k=7$ *there are infinitely many* n *satisfying* (1).

Theorem 2. *For* $k=9$ *there are infinitely many* n *satisfying* (1).

2. We have

$$(2) \quad \sum_{a=2,16,21,25} \{ (x+a)^7 + (x-a)^7 \} = \sum_{b=5,14,23,24} \{ (x+b)^7 + (x-b)^7 \}$$

Integrating this twice *w.r.t.* x we get

$$\sum_{a=2,16,21,25} \{ (x+a)^9 + (x-a)^9 \} = \sum_{b=5,14,23,24} \{ (x+b)^9 + (x-b)^9 \} + cx + d.$$

Here $d=0$, but $c \neq 0$. Putting $x = \frac{y_1^9 - y_2^9}{c}$ we obtain

$$(3) \quad \sum_{a=2,16,21,25} \{ y_1^9 - y_2^9 + ac)^9 + (y_1^9 - y_2^9 - ac)^9 \} + (cy_2)^9$$

$$= \sum_{b=5,14,23,24} \{ (y_1^9 - y_2^9 + bc)^9 + (y_1^9 - y_2^9 - bc)^9 \} + (cy_1)^9.$$

By proper choice of the integers y_1 and y_2, each side of (3) is a sum of nine positive ninth powers. Hence Theorem 2.

3. If we start with

$$(4) \quad \sum_{a=7,14,21} \{ (x+a)^5 + (x-a)^5 \} = \sum_{b=1,18,19} \{ (x+b)^5 + (x-b)^5 \}$$

instead of (2), and proceed as in the last section, we obtain Theorem 1.

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* *i.e.*, permutation of the bases not allowed, *e.g.*, $r_{2,5}(33)=1$.

[1] That (1) is true for infinitely many n when $k=5,6,8$ is known. See papers by Rao and Sastry in *Journ. London Math. Soc.*, 1934, **9**, 170-71, 172-73, 242-46.