NOTE ON HYPOTHESIS K OF HARDY AND LITTLEWOOD.

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1. Let \( r_{s,k}(n) \) denote the number of distinct* representations of \( n \) as a sum of \( s \) positive \( k \)th powers. Hardy and Littlewood have conjectured that

\[
r_{s,k}(n) = O(n^e)
\]

for every positive \( e \). On the other hand it has not even been shown that for any\(^1\) fixed \( k \),

(1) there exists an \( n \) such that \( r_{s,k}(n) \geq 2 \).

I show that

Theorem 1. For \( k=7 \) there are infinitely many \( n \) satisfying (1).

Theorem 2. For \( k=9 \) there are infinitely many \( n \) satisfying (1).

2. We have

\[
\sum_{a=2,16,21,25} \{ (x+a)^7 + (x-a)^7 \} = \sum_{b=5,14,23,24} \{ (x+b)^7 + (x-b)^7 \}
\]

Integrating this twice w.r.t. \( x \) we get

\[
\sum_{a=2,16,21,25} \{ (x+a)^9 + (x-a)^9 \} = \sum_{b=5,14,23,24} \{ (x+b)^9 + (x-b)^9 \} + c x + d.
\]

Here \( d = 0 \), but \( c \neq 0 \). Putting \( x = \frac{y_1^6 - y_2^6}{c} \) we obtain

\[
\sum_{a=2,16,21,25} \{ y_1^9 - y_2^9 + ac \}^9 + (y_1^9 - y_2^9 - ac)^9 \} + (cy_2)^9
\]

\[
= \sum_{b=5,14,23,24} \{ (y_1^9 - y_2^9 + bc)^9 + (y_1^9 - y_2^9 - bc)^9 \} + (cy_1)^9.
\]

By proper choice of the integers \( y_1 \) and \( y_2 \), each side of (3) is a sum of nine positive ninth powers. Hence Theorem 2.

3. If we start with

\[
\sum_{a=7,14,21} \{ (x+a)^5 + (x-a)^5 \} = \sum_{b=1,18,19} \{ (x+b)^5 + (x-b)^5 \}
\]

instead of (2), and proceed as in the last section, we obtain Theorem 1.

* i.e., permutation of the bases not allowed, e.g., \( r_{2,5} (33) = 1 \).

\(^1\) That (1) is true for infinitely many \( n \) when \( k=5,6,8 \) is known. See papers by Rao and Sastry in *Journ. London Math. Soc.*, 1934, 9, 170-71, 172-73, 242-46.

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