ON SUMS OF POWERS.

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1. WE write $(m)^k = (n)^k$ when $\sum_{s=1}^{m} x_s^k = \sum_{t=1}^{n} y_t^k$ has a solution in positive integers $x_1, \cdots, x_m, y_1, \cdots, y_n$ with $(x_1, \cdots, x_m, y_1, \cdots, y_n) = 1$, and if (in case $m=n$) the y's are not merely a permutation of the x's. Rao[1] has recently given examples of

$$(3)^6 = (3)^6 \text{ and } (7)^8 = (7)^8.$$

It seems very likely that $(m)^k = (m)^k$ is always possible[2] for some $m \leqslant k$. We prove this here for $k=10$, $m=9$.

In contradistinction to Rao's work, ours involves only the simplest calculations.

2. Let

(1) $\quad a_1, a_2, \cdots \overset{k}{=} b_1, b_2, \cdots$

signify

(2) $\quad a_1^m + a_2^m + \cdots = b_1^m + b_2^m + \cdots (1 \leqslant m \leqslant k)$

Tarry[3] has observed that (1) implies

(3) $\quad a_1, a_2, \cdots, b_1 + x, b_2 + x, \cdots \overset{k+1}{=} b_1, b_2, \cdots, a_1 + x, a_2 + x, \cdots$

Applying Tarry's observation to his result[4]

(4) $\quad 1, 5, 10, 24, 28, 42, 47, 51 \overset{7}{=} 2, 3, 12, 21, 31, 40, 49, 50$

with[5] $x=9$, $x=19$, $x=17$, $x=6$ in succession (the order is relevant), we obtain finally

(5) $\quad 1, 5, 8, 9, 20, 21, 24, 36, 51, 52, 67, 79, 82, 83, 94, 95, 98, 102$
$\overset{11}{=} 2, 3, 7, 14, 17, 18, 27, 39, 43, 60, 64, 76, 85, 86, 89, 96, 100, 101.$

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[1] *Journ. London Math. Soc.*, 1934, **9**, 172-173.
Math. Ztschr., 1934, **39**, 240-243.
We remark that $(m)^k = (m)^k$ is trivial for $m \geqslant k+1$.

[2] If not, then Hypothesis K (in Hardy and Littlewood's *Researches on Waring's Problem*) is true. See *Math. Ztschr.*, 1925, **23**, 1-37 (4).

[3] See Dickson, *History of the Theory of Numbers*, II, page 710.

[4] Dickson, *loc. cit.*, page 710.

[5] The first step with $x=9$ is worked out on p. 710 of Dickson, *loc. cit.*

It is known that (1) implies

(6) $z+a_1,\ z+a_2, \cdots \stackrel{k}{=} z+b_1, z+b_2, \cdots$

Applying (6) to (5) with $z=-\frac{103}{2}$ we obtain

(7) $$-\frac{101}{2}, +\frac{101}{2}, -\frac{93}{2}, +\frac{93}{2}, -\frac{87}{2}, +\frac{87}{2}, -\frac{85}{2}, +\frac{85}{2}, -\frac{63}{2}, +\frac{63}{2}, -\frac{61}{2}, +\frac{61}{2}, -\frac{55}{2}, +\frac{55}{2}, -\frac{31}{2}, +\frac{31}{2}, -\frac{1}{2}, +\frac{1}{2}, \stackrel{11}{=} -\frac{99}{2}, +\frac{99}{2}, -\frac{97}{2}, +\frac{97}{2}, -\frac{89}{2}, +\frac{89}{2}, -\frac{75}{2}, +\frac{75}{2}, -\frac{69}{2}, +\frac{69}{2}, -\frac{67}{2}, +\frac{67}{2}, -\frac{49}{2}, +\frac{49}{2}, -\frac{25}{2}, +\frac{25}{2}, -\frac{17}{2}, +\frac{17}{2}.$$

Hence the desired result:

(8) $$1^{10}+31^{10}+55^{10}+61^{10}+63^{10}+85^{10}+87^{10}+93^{10}+101^{10}=17^{10}+25^{10}+49^{10}+67^{10}+69^{10}+75^{10}+89^{10}+97^{10}+99^{10}.$$