ON SUMS OF POWERS.

BY S. CHOWLA,
Andhra University, Waltair
AND
S. SASTRY,
Benares.

Received January 25, 1935.

1. We write \((m)^k = (n)^k\) when \(\sum_{s=1}^{m} x_s^k = \sum_{t=1}^{n} y_t^k\) has a solution in positive integers \(x_1, \ldots, x_m, y_1, \ldots, y_n\) with \((x_1, \ldots, x_m, y_1, \ldots, y_n) = 1\), and if (in case \(m=n\)) the \(y\)'s are not merely a permutation of the \(x\)'s. Rao\(^1\) has recently given examples of
\[(3)^6 = (3)^6\] and \[(7)^8 = (7)^8\].

It seems very likely that \((m)^k = (m)^k\) is always possible\(^2\) for some \(n \leq k\). We prove this here for \(k=10, m=9\).

In contradistinction to Rao's work, ours involves only the simplest calculations.

2. Let
\[
(1) \quad a_1, a_2, \ldots, b_1, b_2, \ldots
\]

signify
\[
(2) \quad a_1^m + a_2^m + \cdots = b_1^m + b_2^m + \cdots (1 \leq m \leq k)
\]

Tarry\(^3\) has observed that (1) implies
\[
(3) \quad a_1, a_2, \ldots, b_1 + x, b_2 + x, \ldots \leq k \leq b_1, b_2, \ldots, a_1 + x, a_2 + x, \ldots
\]

Applying Tarry's observation to his result\(^4\)
\[
(4) \quad 1, 5, 10, 24, 28, 42, 47, 51 \leq 2, 3, 12, 21, 31, 40, 49, 50
\]

with\(^5\) \(x=9, x=19, x=17, x=6\) in succession (the order is relevant), we obtain finally
\[
(5) \quad 1, 5, 8, 9, 20, 21, 24, 36, 51, 52, 67, 79, 82, 83, 94, 95, 98, 102
\]
\[
\leq 2, 3, 7, 14, 17, 18, 27, 39, 43, 60, 64, 76, 85, 86, 89, 96, 100, 101.
\]

---


We remark that \((m)^k = (n)^k\) is trivial for \(m \geq k+1\).

2 If not, then Hypothesis K (in Hardy and Littlewood's Researches on Waring's Problem) is true. See Math. Ztschr., 1925, 23, 1-37 (4).

3 See Dickson, History of the Theory of Numbers, II, page 710.

4 Dickson, loc. cit., page 710.

5 The first step with \(x=0\) is worked out on p. 710 of Dickson, loc. cit.
It is known that (1) implies

\[ z + a_1, z + a_2, \ldots \overset{k}{\rightarrow} z + b_1, z + b_2, \ldots \]

Applying (6) to (5) with \( z = -\frac{103}{2} \) we obtain

\[ -\frac{101}{2}, + \frac{101}{2}, - \frac{93}{2}, + \frac{93}{2}, - \frac{87}{2}, + \frac{87}{2}, - \frac{85}{2}, + \frac{85}{2}, - \frac{63}{2}, \]
\[ + \frac{63}{2}, - \frac{61}{2}, + \frac{61}{2}, - \frac{55}{2}, + \frac{55}{2}, - \frac{31}{2}, + \frac{31}{2}, - \frac{1}{2}, + \frac{1}{2}, = \]
\[ - \frac{99}{2}, + \frac{99}{2}, - \frac{97}{2}, + \frac{97}{2}, - \frac{89}{2}, + \frac{89}{2}, - \frac{75}{2}, + \frac{75}{2}, - \frac{69}{2}, \]
\[ + \frac{69}{2}, - \frac{67}{2}, + \frac{67}{2}, - \frac{49}{2}, + \frac{49}{2}, - \frac{25}{2}, + \frac{25}{2}, - \frac{17}{2}, + \frac{17}{2}. \]

Hence the desired result:

\[ 1^{10} + 31^{10} + 55^{10} + 61^{10} + 63^{10} + 85^{10} + 87^{10} + 93^{10} + 101^{10} = 17^{10} + 25^{10} \]
\[ + 49^{10} + 67^{10} + 69^{10} + 75^{10} + 89^{10} + 97^{10} + 99^{10}. \]