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Flood routing using the Muskingum-Cunge method and application of different routing parameters

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Abstract. Considering the importance of flood studies and methods for reducing flood damage in river engineering projects, flood flow behavior in rivers should be investigated. The Muskingum-Cunge method is a flood routing method, whose parameters are calculated based on the physical characteristics of the river. Selection of an appropriate reference discharge and the kinematic wave velocity is of utmost importance for calculating the routing parameters (X, K). In the present study, the accuracy of the Muskingum-Cunge method is studied by using the method in the river reach between the Mollasani hydrometric station in the upstream and Ahwaz hydrometric station in the downstream of the Karun River, Iran. The results show that if instead of using constant values for the parameters of the Muskingum-Cunge method, different parameters are used based on changes in the flow depth, the accuracy of the Muskingum-Cunge method is increased, especially in the estimation of the hydrograph peak. In addition, taking into account the geometric characteristics of the river channel and flow conditions in the study river reach and the use of the Monoclinal wave relations to calculate the kinematic wave velocity increased the accuracy of the method. Accordingly, the Mean Relative Error (MRE) of total and flood peak section using the method presented in this study and the Monoclinal wave relations is equal to 2.78 and 3.37 percent, respectively. While by using constant values for the mentioned parameters and application of the conventional Muskingum-Cunge method, the error values are 6.18 and 13.70 percent, respectively.

Keywords. Different routing parameters; flood routing; muskingum-cunge method; monoclinal wave.

1. Introduction

Flood can cause many economic and fatal damages and therefore, acquisition of accurate flood information is important in order to design hydraulic structures facing the flood. Flood wave propagation along a river and the determination of flood discharge at specified sections and times is very useful in reducing flood damage, designing hydraulic structures and water resource planning.

Generally, flood routing methods can be divided into two groups of hydraulic routing and hydrological routing. The Muskingum method is one of the hydrological methods which, due to its simplicity, is widely used in the flood routing, and was introduced by McCarthy in 1938 [1]. Investigations done by Cunge (1969) showed that the Muskingum method equation is very similar to the advection-diffusion equation, and its results are close to the results of the kinematic wave method. By discretizing the kinematic wave equation as well as by matching the numerical diffusion with the physical diffusion, he modified the Muskingum method. In this way, the parameters of the Muskingum-Cunge method are calculated based on the physical characteristics of the river [2].

Ponce et al. [3] studied the Muskingum-Cunge method and given the large correlation between the computational results and observed hydrographs, they concluded that this method is a highly efficient one in comparison to other flood routing methods. [4–6] studied the Muskingum-Cunge method with variable two, three, and four-point parameters. [7] compared the Muskingum-Cunge method with the dynamic wave method and their results indicated the acceptable accuracy of the Muskingum-Cunge method. In the studies performed by [4, 5, 8–11] the Muskingum-Cunge method was investigated in the artificial channels and the results showed the acceptable accuracy of this method in the flood routing. Studies showed that, if different and non-constant values are used for parameters of the linear Muskingum method (X, K, and Δt), the accuracy of this method is increased, especially in estimation of the peak section of the flood hydrograph [12].

Flood routing, and especially the calculation of flood peak section, is widely used in the design of hydraulic structures and flood control. Considering the geometric characteristics of the river section as well as flow conditions

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including wave velocity and wave type is of great importance in the analysis of unsteady flow. In addition, accurate calculation of routing parameters of the Muskingum-Cunge method (X, K) is the critical factor in increasing the accuracy of flood routing using the mentioned method. In the study river reach of the present study, flow width is much greater than flow depth and the resulting wave is of Monoclinal type. In addition, due to the irregularity of the river section and changes in flow width and, consequently, changes in kinematic wave velocity at different depths, the X and K parameters change with depth. For this reason, in the present study, instead of using a constant value for the X and K parameters, the river section of the river was divided into 10 cm reaches considering the flow depth and using Monoclinal wave equations, different values for the kinematic wave velocity and the X and K parameters were calculated for each river reach. The solution presented in the present study increased the accuracy of the Muskingum-Cunge method in estimating the outflow hydrograph.

2. Materials and methods

2.1 *Study area*

In the present study, the data recorded by the Water Resources Management Company of Iran, coming from the Mollasani (station no: 21-308, 48°53’E, 31°35’N) and Ahvaz (station No: 21-309, 48°40’E, 31°20’N) hydrometric stations, which were located in the upstream and downstream of the study river reach of the Karun River, Iran, was used (figure 1). Data recorded on 02.02.2012 to 05.02.2012 in the desired river reach was used to investigate the flood routing using the Muskingum-Cunge method. It is worth noting that the two hydrometric stations were located at a distance of 60.5 km from each other. In addition, the average slope of the river reach (between the Mollasani and the Ahwaz hydrometric stations) was equal to S0 = 0.00011.

2.2 *The Muskingum-Cunge method*

Cunge proved that the Muskingum equation is similar to the Advection-Diffusion and obtained the approximate Eq. 1. The obtained equation is similar to Muskingum equation [13].

$$Q_{i+1}^{n+1} = C_1 Q_i^{n+1} + C_2 Q_i^n + C_3 Q_{i+1}^n \tag{1}$$

Where the coefficients C_1, C_2 and C_3 are expressed as Eqs. 2 to 4.

$$C_1 = \frac{0.5\Delta t - KX}{K - KX + 0.5\Delta t} \tag{2}$$

$$C_2 = \frac{0.5\Delta t + KX}{K - KX + 0.5\Delta t} \tag{3}$$

$$C_3 = \frac{K - KX - 0.5\Delta t}{K - KX + 0.5\Delta t} \tag{4}$$

Where Δt is the time step, X and K are the routing parameters that are calculated using Eqs. 5 and 6, respectively.

$$X = \frac{1}{2}\left[1 - \frac{Q_r}{BS_0 C_k \Delta x}\right] \tag{5}$$

$$K = \frac{\Delta x}{C_k} \tag{6}$$

Where Q_r is the reference discharge, C_k is the kinematic wave velocity and Δx is the spatial step.

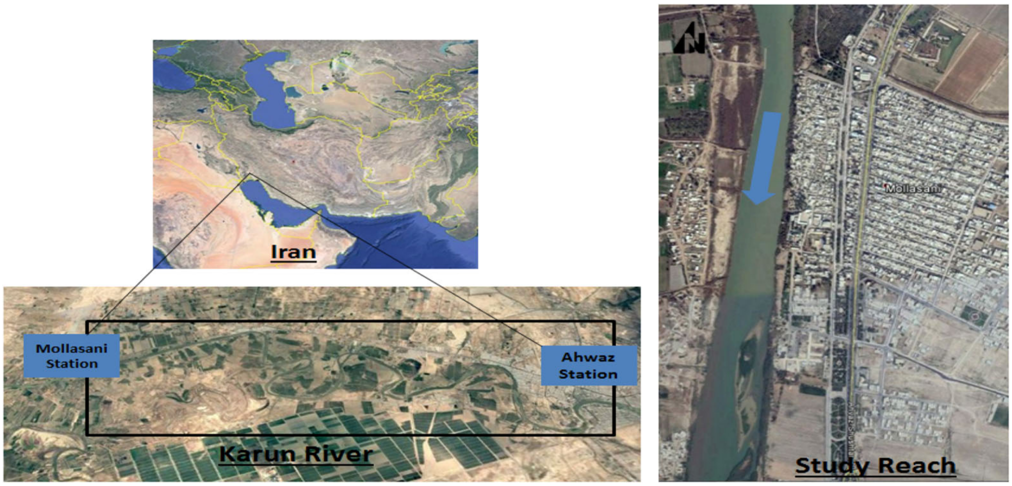

Figure 1. Schematic view of the study area.

Table 1. Characteristics of the recorded flow at the Mollasani hydrometric station (upstream) and the calculated parameters.

					$C_k = \left(\frac{5}{3}\right)V$			$C_k = \frac{1}{B}\frac{dQ}{dh}$		
Time (h)	Inflow (m^3/s)	Input Depth (m)	B (m)	Q_r (m^3/s)	C_k	X	K (h)	C_k	X	K (h)
0	287	1.63								
1	287	1.63								
2	292	1.65								
3	292	1.65								
4	294	1.66	172.3	293.5	1.71	0.43	9.82	1.26	0.40	13.36
5	296	1.67								
6	298	1.68								
7	300	1.69								
8	305	1.71								
9	307	1.72								
10	311	1.74	172.8	312.5	1.73	0.42	9.74	1.24	0.39	13.55
11	316	1.76								
12	320	1.78								
13	325	1.8								
14	329	1.82								
15	334	1.84	173.7	332.5	1.74	0.42	9.67	1.23	0.38	13.62
16	340	1.87								
17	349	1.91								
18	356	1.94	180.3	358	1.70	0.41	9.90	1.25	0.38	13.47
19	367	1.99								
20	380	2.05								
21	392	2.1	186	393.5	1.67	0.40	10.05	1.21	0.37	13.89
22	407	2.17								
23	421	2.23	187	427.5	1.69	0.40	9.97	1.16	0.35	14.50
24	434	2.29								
25	450	2.35								
26	462	2.4								
27	471	2.44	188	466	1.71	0.39	9.83	1.22	0.35	13.82
28	478	2.47								
29	482	2.49								
30	487	2.51								
31	489	2.52								
32	491	2.53								
33	494	2.54								
34	494	2.54								
35	491	2.53	189.5	489.5	1.71	0.39	9.84	1.19	0.34	14.15
36	491	2.53								
37	489	2.52								
38	489	2.52								
39	487	2.51								
40	485	2.5								
41	482	2.49								
42	478	2.47								
43	475	2.46	188	473	1.71	0.39	9.82	1.20	0.34	14.04
44	473	2.45								
45	469	2.43								
46	464	2.41								
47	459	2.39								
48	455	2.37								
49	450	2.35	187.5	450	1.70	0.39	9.87	1.20	0.35	14.00
50	446	2.33								
51	441	2.31								
52	434	2.28								
53	430	2.26	187	426.5	1.69	0.40	9.93	1.15	0.35	14.67
54	423	2.23								

Table 1 continued

					$C_k = \left(\frac{5}{3}\right)V$			$C_k = \frac{1}{B}\frac{dQ}{dh}$		
Time (h)	Inflow (m^3/s)	Input Depth (m)	B (m)	Q_r (m^3/s)	C_k	X	K (h)	C_k	X	K (h)
55	419	2.21								
56	412	2.18								
57	405	2.15	186	402	1.69	0.40	9.97	1.19	0.36	14.07
58	398	2.12								
59	392	2.09								
60	383	2.05	180.8	379.5	1.72	0.41	9.78	1.29	0.38	13.02
61	376	2.02								
62	369	1.99								
63	367	1.98								
64	356	1.93	180.3	359	1.71	0.41	9.84	1.23	0.38	13.64
65	349	1.9								
66	342	1.87								
67	338	1.85	173.7	334.5	1.75	0.42	9.61	1.23	0.38	13.62
68	331	1.82								
69	327	1.8								
70	320	1.77								
71	316	1.75	172.8	313.5	1.74	0.42	9.67	1.25	0.39	13.40
72	311	1.73								
73	307	1.71								
74	303	1.69								
75	298	1.67								
76	292	1.64	172.3	294	1.72	0.43	9.76	1.31	0.40	12.87
77	289	1.63								
78	285	1.61								
79	279	1.58								
80	274	1.56								
81	270	1.54	171.6	271	1.70	0.43	9.88	1.33	0.41	12.62
82	265	1.52								
83	263	1.51								
84	259	1.49								
85	255	1.47								
86	250	1.45	160.1	249	1.79	0.43	9.37	1.39	0.42	12.11
87	246	1.43								
88	242	1.41								
89	239	1.4								
90	235	1.38								
91	233	1.37								
92	231	1.36								
93	229	1.35	128.8	228.5	2.19	0.44	7.67	1.68	0.42	9.99
94	226	1.34								
95	224	1.33								
96	222	1.32								

If the Manning equation is used, the kinematic wave velocity for the wide rectangular channels is considered as Eq. 7 [14].

$$C_k = \left(\frac{5}{3}\right)V \tag{7}$$

The reference discharge is one that can be used to correctly estimate the routing parameters. Several equations have been proposed for calculating the reference discharge [15–17]. The investigations carried out in the present study showed that if Eq. 8 is used to calculate the reference discharge, the accuracy of the Muskingum-Cunge method is increased in the estimation of the outflow hygrograph compared to the cases where Eqs. 9 and 10 are used.

$$Q_r = \sum_{n=1}^{M} \frac{Q_{in}}{M} \quad (8)$$

$$Q_r = Q_b + 0.5\left(Q_{pi} - Q_b\right) \quad (9)$$

$$Q_r = 0.5\left(Q_{pi} - Q_b\right) \quad (10)$$

Where Q_b and Q_{pi} are the minimum and maximum inflow discharge, respectively, Q_{in} is the inflow discharge at the n^{th} time step and M is the number of inflow discharges.

2.3 *Monoclinal wave*

In a wide rectangular channel with a small slope, a uniform flow with a depth of h_1, a velocity of V_1, and a discharge of Q_1 flows. Due to the arrival of another wave, the flow discharge suddenly increases to Q_2 and a uniform flow with a depth of h_2 and a velocity of V_2 is formed. Upon the discharge increase step, a transitional wave with a stable geometric shape, known as the Monoclinal wave, is formed and released to the downstream after its formation. After time Δt, the wave passes a distance of $\Delta x = C_k \Delta t$. By applying the principle of the volume conservation to the control volume moving with the kinematic wave velocity Eq. 11 is obtained [18, 19].

$$C_k A_2 - Q_2 = C_k A_1 - Q_1 \quad (11)$$

According to Eq. 15, the kinematic wave (C_k) is obtained using Eq. 12.

$$C_k = \frac{Q_2 - Q_1}{A_2 - A_1} = \frac{dQ}{dA} = \frac{1}{B}\frac{dQ}{dy} \quad (12)$$

According to Eq. 12, the kinematic wave velocity is proportional to changes in the discharge (Q) and the flow depth (y). Therefore, the kinematic wave is resulted from the continuity equation, which in free surface flows represents the principle of mass conservation and therefore, it is referred to as the kinematic wave and is only a function of the flow depth.

3. Results and discussion

According to the equations (5 and 6), the value of X coefficient depends on the reference discharge (Q_r), flow upper width (B), river bed slope (S_0) and kinematic wave velocity (C_k). The coefficient K also depends on C_k. It is worth noting that Δx is the distance between the upstream and downstream and is not considered as a problem variable.

According to the above equations, the following can be considered:

1) The reference discharge depends on the inflow hydrograph and three equations (Eqs. 8 to 10 in the main manuscript text) have been proposed by previous researchers. The results of the present study indicated the high accuracy of Eq. 8. In other words, in Eq. 8, the effect of all inflow discharges on the reference discharge is considered, which increases the accuracy of the calculations.
2) According to the characteristics of the cross sections of natural rivers with irregular topographic up and downs, changes in the flow depth cause changes in the flow upper width (B), and consequently, the use of a constant value of B will cause error in calculation of X coefficient as well as outflow hydrograph. Usually, due to the greater width of rivers to their depth, the river cross section at different depths can be considered as a wide rectangle with different widths of the upper flow.
3) The kinematic wave velocity (Ck) influences the calculation of both the X and K coefficients. Due to the mild

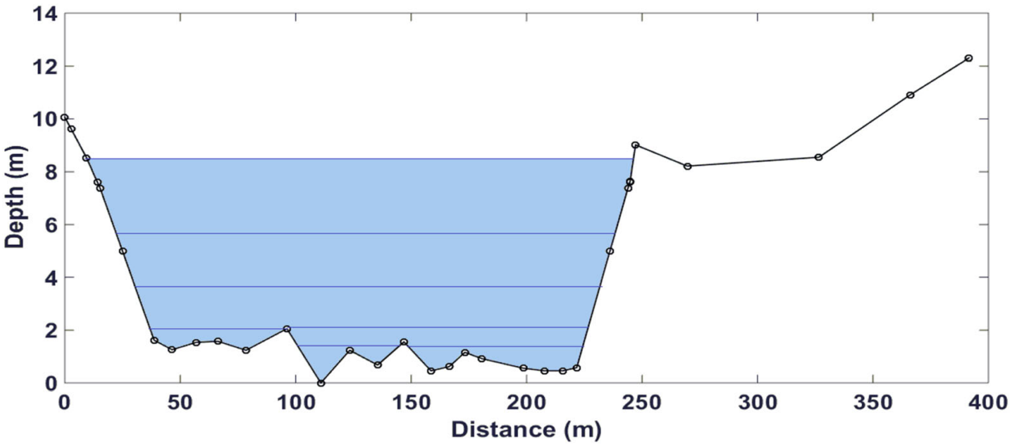

Figure 2. Geometric characteristics of the section of the Mollasani hydrometric station.

slope of most rivers and according to the descriptions in section 2, the Monoclinal wave is developed in large rivers.

According to the above explanations, all the effective parameters in flood routing were investigated in the present study using the Muskingum-Cunge method. Theoretically, and considering the equations of the mentioned method and the explanations provided, increase in the accuracy of flood routing using the solution presented in the present study was expected and is justified. To indicate the accuracy and efficiency of the proposed method, the actual flood data recorded at the Mollasani and Ahwaz hydrometric stations of the Karun river were also used.

According to the investigations of the present study and based on the explanations presented about the Monoclinal wave and the observed flood characteristics presented in table 1, the formed wave was a Monoclinal wave, and therefore, by using Eq. 12 to calculate the kinematic wave velocity (C_k), the accuracy of the Muskingum-Cunge method increased. In other words, since the upper width of the flow (B) was much larger than the flow depth (y) (table 1), and the average slope of the Karun river in the studied river reach was very small ($S_0 = 0.00011$), a Monoclinal wave was formed.

According to figure 2, which shows the irregular sections of the Karun river at the Mollasani hydrometric station and has been recorded by the Water Resources Research Institute of Iran, if the depth changes occur only in one of the intervals indicated in figure 2, the kinematic wave (C_k), reference discharge (Q_r) and, consequently, the routing parameters of the Muskingum-Cunge method (X, K) are constant. In other words, in previous studies, the average flow width was used to calculate C_k. In addition, the reference discharge in the Muskingum-Cunge method with constant parameters, was equal to the average total inflow discharges. However, if depth changes due to changes in discharge at different times vary at the intervals indicated in figure 2, the use of different values for the routing parameters of the Muskingum-Cunge (X, K) method would increase the accuracy of this method in the estimation of the outflow hydrograph.

In other word, in the present study, the mentioned section (figure 2) was divided into 10 cm intervals based on the depth changes and each interval was considered as wide rectangular sections, and then C_k, Q_r and the flood routing parameters (X, K) were calculated individually for each interval. According to the characteristics of the desired section and river reach (Mollasani hydrometric station to Ahwaz hydrometric station), the formed wave was a Monoclinal one and the results indicated that the use of the method presented in this study and the Monoclinal wave equations increased the accuracy of outflow hydrograph calculations, especially the peak section that is of great importance. It is worth noting that the values of the flow width (B), kinematic wave velocity (C_k), reference discharge (Q_r) and routing parameters (X, K) for each interval were calculated separately.

In other words, according to Eqs. 7 and 12, the kinematic wave velocity is a function of the inflow discharge and flow width, and according to Eq. 8, the reference discharge is also a function of the inflow discharge and for each interval in which the flow depth and width has changed, different values for these parameters must be calculated.

In general, the present study consisted of the following steps:

1) Division of the river section at Mollasani hydrometric station to 10 cm depths, due to the changes in the flow

Table 2. Flow characteristics and routing parameters in the case of using a constant value.

		$C_k = \left(\frac{5}{3}\right)V$			$C_k = \frac{1}{B}\frac{dQ}{dh}$		
B (m)	Q_r (m^3/s)	C_k (m/s)	X	K (h)	C_k (m/s)	X	K (h)
159.15	358	1.89	0.41	8.85	1.40	0.38	11.99

Table 3. Comparison of the calculated and observed outflow hydrographs.

	$C_k = \left(\frac{5}{3}\right)V$		$C_k = \frac{1}{B}\frac{dQ}{dh}$	
Errors value	Constant parameters	Different parameters	Constant parameters	Different parameters
Mean Relative Error (MRE) %	6.18	5.26	3.47	2.78
MRE (Peak section, T=31 to T=38 (h))	13.70	11.48	7.46	3.37
DPO %	7.64	7.09	6.31	4.77

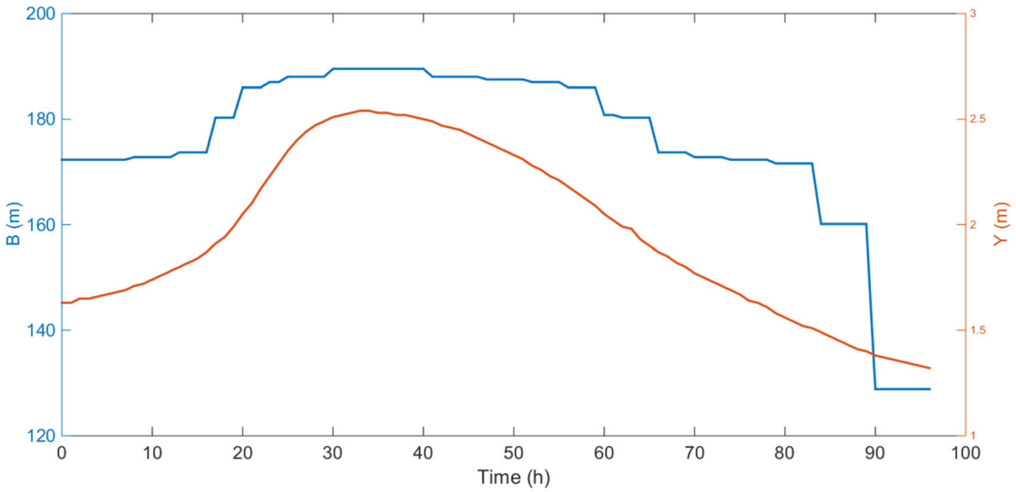

Figure 3. Width and depth of flow of the Mollasani Hydrometric Station (upstream) of the Karun River at different times.

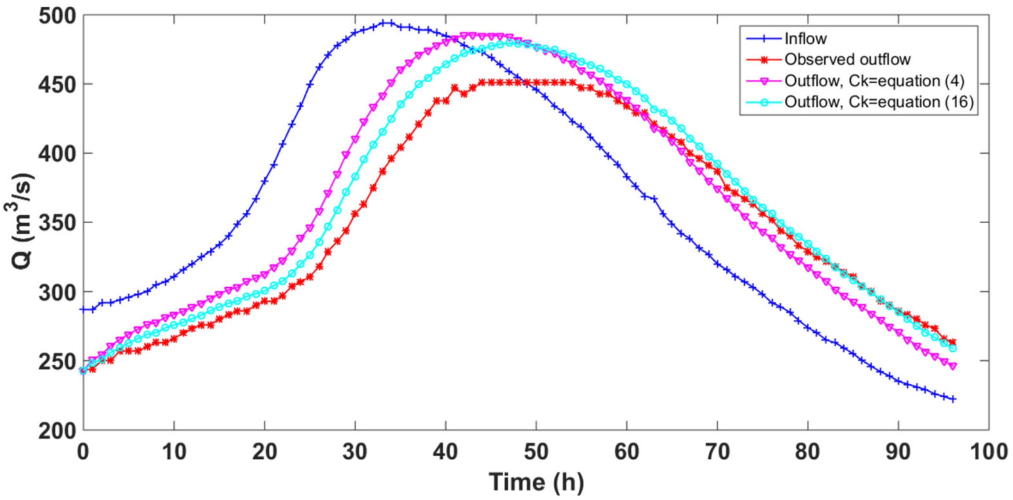

Figure 4. Observed and computed outflow hydrographs in case of using constant parameters.

width (B) in this interval and recording the flow width in each interval;

2) Calculation of the kinematic wave velocity using Eqs. 7 and 12 for each of the intervals;
3) Calculation of the reference discharge for the intervals according to step 1, i.e. for each 10 cm interval, a different value for the reference discharge (Q_r) was calculated using the inflow discharge to each interval;
4) Calculation of the Muskingum-Cunge routing parameters (X, K) for each of the intervals;
5) The use of these equations to calculate the outflow hydrograph using the method presented in this study and the previous Muskingum-Cunge method with constant parameters

It can be said that in the present study, for calculating the outflow hydrograph, a constant flow width (average of the maximum and minimum flow width) and a constant reference discharge was first used. Then, as mentioned above, the section of the Mollasani hydrometric station was divided into 10 cm intervals based on the depth changes and for each interval, a different flow width and reference discharge was calculated (table 1). It is worth noting that, in order to calculate the cinematic wave velocity (C_k), Eq. 7 and Eq. 12 were both used and the results showed that Eq. 12 is more accurate.

Also, the calculated values of the flow width (B), reference discharge (Q_r), kinematic wave velocity (C_k), and the routing parameters of the Muskingum-Cunge method (X, K) using a constant value are listed in table 2.

Results showed that if different values are calculated for the flow width (B), reference discharge (Q_r), kinematic wave velocity (C_k), and routing parameters of the

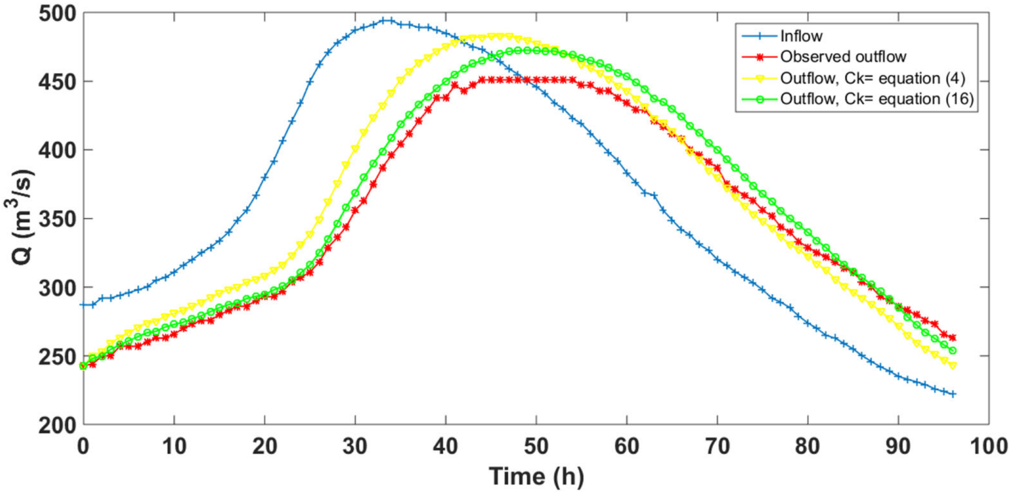

Figure 5. Observed and computed outflow hydrographs in case of using different parameters.

Muskingum-Cunge method (X, K) for each 10 cm interval based on the depth changes, then the accuracy of the method is increased in comparison to the case in which constant values are used. In addition, if Eq. 12 is used to calculate C_k, because the formed wave is a Monoclinal one due to the geometric characteristics of the studied river reach (Mollasani hydrometric station to Ahwaz hydrometric station) and the explanations presented, then the Muskingum-Cunge method is more accurate in estimation of the outflow hydrograph than Eq. 7. The calculation results in both cases are shown in figures 4 and 5.

Further studies showed that estimation of the flood hydrograph peak section in downstream of the studied interval using constant parameters and Eq. 7 for calculation of the kinematic wave velocity (C_k) yielded a Mean Relative Error (MRE) of 13.70%. However, when using Eq. 12 to calculate C_k, the error decreased to 7.46%. While in the case of using different values for the mentioned parameters and consequently, for the routing parameters of the Muskingum-Cunge method (X, K), the Mean Relative Error (MRE) of the peak section by using Eq. 7 and Eq. 12 was equal to 11.48 and 3.37%, respectively. The Mean Relative Error (MRE) of the total flood, the Mean Relative Error (MRE) of the flood peak section and the relative error of the peak discharge (DPO) is listed in table 3.

The results showed that if the flow width (B) and flow depth (y) are plotted versus time (T) in accordance with figure 3, and then instead of using a constant value for the flow width, the weighted average of the flow width (Eq. 13) is used for each interval (according to table 1) considering the depth changes and subsequently, the flow width changes, then the accuracy of the Muskingum-Cunge method is increased, especially in estimation of the peak section of the inflow hydrograph.

$$\overline{B} = \frac{\sum B * \Delta t}{\sum \Delta t} \qquad (13)$$

In addition, the calculation results in case of using constant parameters and variable parameters (10 cm intervals based on depth changes) are shown in figure 4 and figure 5, respectively.

4. Conclusions

In general, the following results were obtained in the present study:

1) The Mean Relative Error (MRE) of the total flood and the relative error of the peak discharge (DPO) in the case of using the conventional Muskingum-Cunge method with constant parameters were equal to 6.18 and 7.64%, respectively. While using the solution presented in the present study (using different routing parameters and Monoclinal wave equations) the corresponding values of 2.78 and 4.77%, were calculated respectively.
2) The MRE of the peak section of the inflow hydrograph (T = 31 to T = 38 (h)) using constant routing parameters and do not taking into account the Monoclinal of the generated wave was calculated as 13.70%. However, when using the Monoclinal wave equations to calculate the kinematic wave velocity (C_k) and constant values for the routing parameters, the Mean Relative Error (MRE) was obtained as 7.46%, and in the case of using the Monoclinal wave equations and different values for the routing parameters, the Mean Relative Error (MRE) of 3.37% was obtained. In other words, error values in this case improved by 45.54 and 75.40%, respectively.

In other words, the division of the inlet section (upstream) to 10 cm depths and the use of different values for the kinematic wave velocity (C_k), reference discharge (Q_r) and, consequently, for the routing parameters of the Muskingum-Cunge method (X, K) for each interval increased the accuracy of this method, especially in estimation of the peak section of the flood hydrograph, which is of great importance in the design of flood control structures. In addition, considering the geometric characteristics of the cross section and the flood flow in the study interval and the use of the Monoclinal wave equations for calculating the kinematic wave velocity (C_k), improved the accuracy of the Muskingum-Cunge method.

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Declaration

Conflict of interest The authors declare that they have no conflict of interest.

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