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Revisiting passive RC networks with over unity gain

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Abstract. In this letter, passive RC networks having over unity gain have been revisited. Several works are reviewed, analyzed, if necessary, and conditions of over unity gain are found out by analysis or simulation. A new network is also presented for the same purpose. The aim has been to bring the existing works to the notice of the concerned students and teachers, and also authors of text/reference books.

Keywords. Over unity gain; passive RC networks; simulation.

1. Introduction

More than seven decades ago, Epstein published a paper on the synthesis of passive RC networks having over unity gain [1]. Earlier than [1], there was only one (less circulated) journal paper [2] on the subject. It is surprising that [1] went unnoticed by authors of standard text and reference books; even in journals, only one [3] mentioned this work, while another [4] treated the topic, apparently unaware of [1–3]. Later, there appeared another paper [5].

The purpose of this letter is to bring these existing works on the topic to the attention of the concerned students, teachers, and also authors of text/reference books. A review of the existing works, analysis, if necessary, find the conditions of over unity gain, from the results, or from simulation have been presented. Also, an alternative configuration for the same purpose has been presented.

This communication is organized as follows. Section 2 deals with the derivation and analysis of some networks, other than Epstein's. This is followed by section 3, which gives the details of two of Epstein's networks, to be designated as N_a and N_b. Section 4 presents a network N_c, alternative to that of N_a, along with its analysis. The conclusions are given in the last section 5.

2. Derivation and analysis of some networks, other than Epstein's

2.1 *Networks derived from intuitive considerations*

From intuitive considerations, it appears that the two-section RC ladder shown in figure 1 may give over unity gain across the terminals A and B, because across B and ground, it will give a phase shift > 90°. Indeed, by assuming the input to be 1 volt, for simplicity, we get, by analysis,

$$|V_{AB}|^2 = \left(9x^2 + x^4\right)/\left(1 + 7x^2 + x^4\right) \qquad (1)$$

where $x = \omega CR$. This equation was simulated and the results are shown in figure 2, wherefrom we get cross over of the unity line at frequency $x_o = 0.707$ $(1/\sqrt{2})$, maximum at $x_{max} = 1.8$, and $|V_{AB}|^2_{max} \cong 1.18$, so that $|V_{AB}|_{max} \cong 1.09$. Clearly, it is a high pass filter.

2.2 *Mitros' network*

Mitros [5] considered the four section RC network shown in figure 3. He also gave the transfer function as

$$\begin{aligned} H(s) &= V_0/V_{in} \\ &= (1 + 8RCs)/\left(1 + 8RCs + 8R_2^2C_2^2s^2 + R_3^3C_3^3s^3\right). \end{aligned} \qquad (2)$$

For $R_2 = R_3 = R$, $C_1 = C_2 = C$, $s = j\omega$ and $x = \omega CR$, the magnitude squared function of Equation (2) becomes

$$|H(j\omega)|^2 = \left(1 + 64x^2\right)/\left[\left(1 - 8x^2\right)^2 + x^2\left(8 - x^2\right)^2\right]. \qquad (3)$$

Simulation results of Equation (3) are shown in figure 4, wherefrom we get the unity gain crossover frequencies as $x_1 = 0.000$, $x_2 = 0.575$, maximum at $x_0 = 0.236$, and $|H(j\omega)|^2_{max} = 1.194$. Thus the maximum gain is 1.093.

2.3 *Three section RC ladder*

The three section RC ladder network, shown in figure 5, is well known for its application in the so called phase-shift oscillator, either as it is, for which an amplifier of gain 29

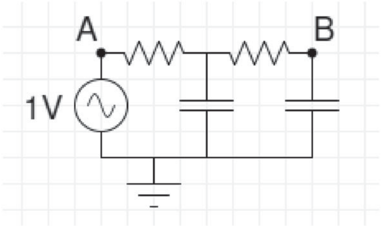

Figure 1. A simple two section RC ladder network.

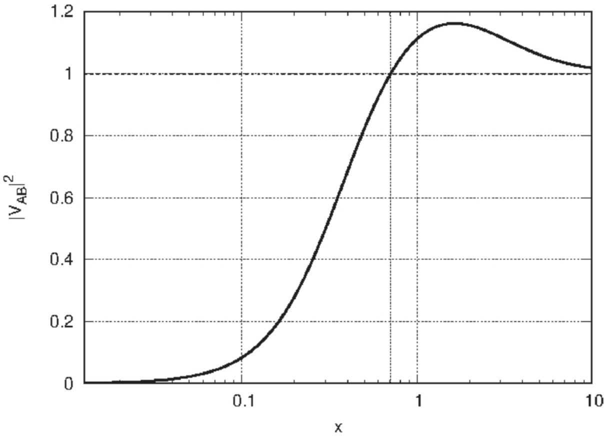

Figure 2. Simulation of Equation (1).

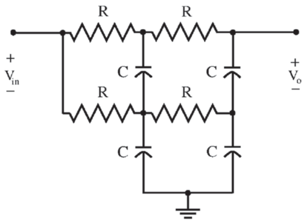

Figure 3. Mitros' network which can give over unity gain.

with a phase = 180° is required to generate sinusoidal oscillations at the frequency $\omega_0 RC = 1/(\sqrt{6})$ [6], or its reoriented form, which gives a gain of 1 + (1/29) with phase = 0°, for which an emitter follower is required for sinusoidal oscillations at the same frequency [3, 4].

Since this network is so well known, we shall not pursue it further.

3. Epstein's two networks

Epstein [1] investigated several networks, from which we select here only two, mainly because of their simplicity.

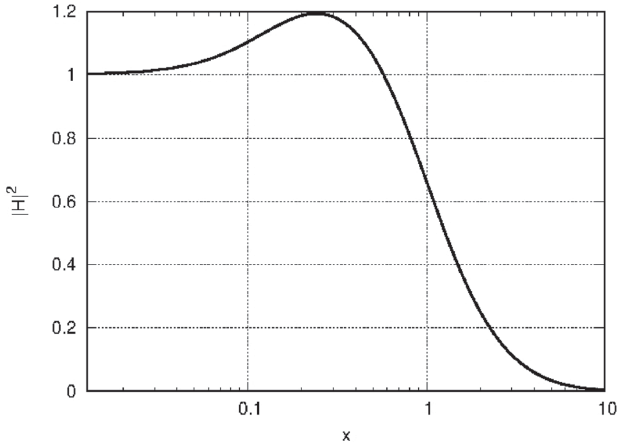

Figure 4. Simulation results for Equation (3).

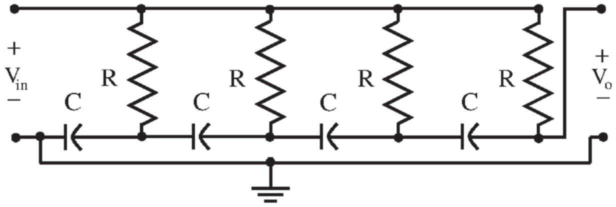

Figure 5. Three section RC ladder.

3.1 *Epstein's first network N_a*

Epstein set his aim to synthesize the transfer function (TF)

$$G_a(s) = V_{0,a}/V_{i,a} = (1 + sT_1)/[(1 + sT_2)(1 + sT_3)], \quad (4)$$

where $T_2 + T_3 > T_1$. Following the synthesis procedure of Bower and Ordung [7], he synthesized Equation (4) by the network N_a, shown in figure 6 (figure 4 in [1]), where,

$$\begin{aligned} &T_3 = R_0C_0,\ T_a = R_1R_2C_1, \\ &T_2 = R_1R_2C_1/(R_1 + R),\ \text{and} \\ &T_a = T_2T_3/(T_2 + T_3 - T_1). \end{aligned} \quad (5)$$

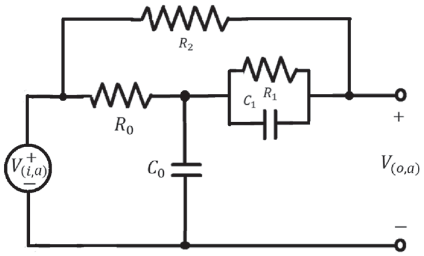

Figure 6. Epstein's network N_a.

Note that in Equations (5), T_a does not have the dimension of time. Instead of trying to find out where the error crept in, we carried out our own analysis by using the dual input technique [8], and a single application of Thevenin's theorem to the circuit at the left of the parallel R_1C_1 combination. Carrying out the algebra, our result for $H_a(s) = V_{0,a}/V_{i,a}$ becomes

$$H_a(s) = (\{sR_1[C_1(R_0+R_2)+C_0R_0]/((R_0+R_1+R_2)\} +1)/\left(As^2+Bs+1\right), \tag{6}$$

with

$$A = C_0C_1R_0R_1R_2/(R_0+R_1+R_2) \text{ and } B = [C_0R_0(R_1+R_2)+C_1R_1(R_0+R_2)]/(R_o+R_1+R_2). \tag{7}$$

Comparing Equations (6) and (7) with (4), we identify

$$T_1 = R_1[C_1(R_0+R_2)+C_0R_0]/((R_0+R_1+R_2),\ T_2T_3 = A \text{ and } T_2+T_3 = B. \tag{8}$$

Eliminating first T_3 and then T_2 from Equation (8), we get

$$T_{2,3} = \left[(B/2)^2+A\right]^{1/2} \pm (B/2). \tag{9}$$

These solutions do not have any constraints on the three time constants.

3.2 *Epstein's second network N_b*

The second network selected, N_b, is shown in figure 7. Rather than analyzing it as it is, we work on its reoriented version shown in figure 8, find its transfer function G_a', and then use $G_a = 1-G_a'$.

Figure 8 network can be analyzed by any of the techniques given in standard textbooks, e.g. [9]. We find it convenient to do this by repeated application of Thevenin's theorem. The result, after application of $G_a = 1-G_a'$ is, with $x = \omega RC$:

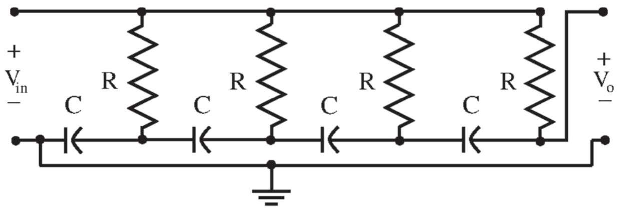

Figure 7. Epstein's second network N_b.

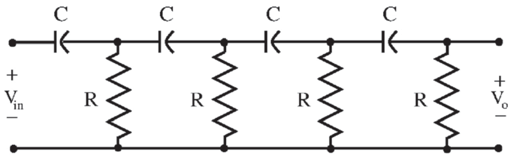

Figure 8. Reoriented form of chosen Epstein's second network N_b.

$$G_a = \left[\left(1-15x^2\right)+jx\left(7-10x^2\right)\right]/\left[\left(x^4-15x^2+1\right)+\ jx\left(7-10x^2\right)\right] \tag{10}$$

so that

$$|G_a|^2 = \left[\left(1-15x^2\right)^2+x^2\left(7-10x^2\right)^2\right]/\left[\left(x^4-15x^2+1\right)^2+x^2\left(7-10x^2\right)^2\right] \tag{11}$$

Clearly, this is a low-pass filter with $|G_a|^2 = 1$ at $x = 0$ and $|G_a|^2 = 0$ at x=infinity, with two cross over frequencies and a maximum in between. As usual, we find these critical frequencies and the maximum value by simulation, whose results are shown in figure 9; from this, we get $x_1 = 0.8$, $x_2 = 8.0$, $x_{max} = 2.3$, and $|G_a|^2_{max} = 1.28$. Thus $|G_a|_{max} = 1.13$.

4. The alternatively synthesized network N_c

It is known [9] that the transmission zeros of a network are obtained by series impedance poles and shunt impedance zeros. N_a was obtained by the pole created by the parallel R_1C_1 combination. An alternative network (N_c) can be obtained by a shunt impedance zero, as shown in figure 10.

Now it is a simple ladder and can be formally synthesized by an RC transfer function synthesis procedure [9, 10]. Instead of that, the network being so simple and known, we analyze it to get T_1, T_2 and T_3 in terms of circuit

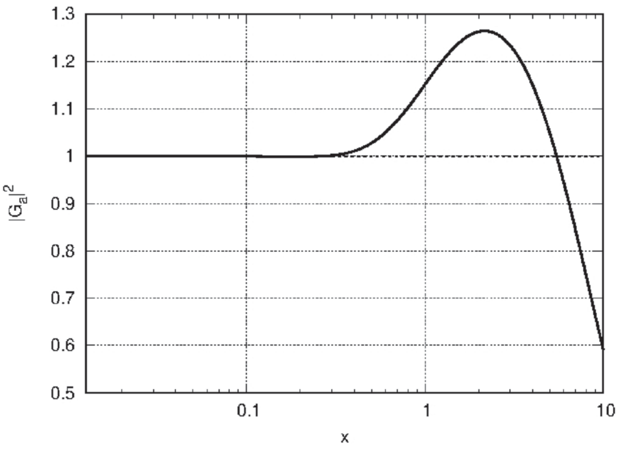

Figure 9. Simulation results of Equation (14).

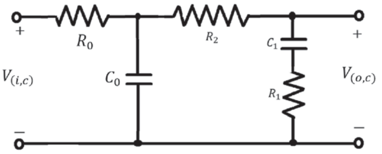

Figure 10. The alternatively synthesized network, N_c.

components. Analysis is simpler than that of N_a, and yields, after a little algebra, and rearrangement,

$$G_c(s) = (sC_1R_1 + 1)/\{s^2[C_0C_1R_0(R_1 + R_2)] + s[C_1(R_0 + R_1 + R_2) + C_0R_0] + 1\}. \tag{13}$$

Comparing Equations (13) with (4), we identify

$$T_1 = C_1R_1, T_2T_3 = C_0C_1R_0(R_1 + R_2) \text{ and } T_2 + T_3 = C_1(R_0 + R_1 + R_2) + C_0R_0. \tag{14}$$

As in the earlier Section, T_2 and T_3 can be solved for, and the results are

$$T_{2,3} = \Big([C_1(R_0 + R_1 + R_2) + C_0R_0] \pm \Big\{[C_1(R_0 + R_1 + R_2) + C_0R_0]^2 - 4C_0C_1R_0(R_1 + R_2)\Big\}^{1/2}\Big)/2. \tag{15}$$

It may be easily verified that the product T_2T_3 is given by Equation (10).

5. Conclusions

This Letter has revisited all the existing works on RC networks, including two of Epstein's, which have over unity gain. Some of them have low-pass, and some have high-pass characteristics, all with cross over of the unity line, and a maximum at some frequency. These critical frequencies, and the maximum values have been found out by simulation. A new configuration, alternative to Epstein's first chosen network, has also been discussed.

In all the networks investigated, the maximum gain is 2. This has been intuitively concluded, but a formal, rigorous proof is yet to be obtained.

Major application of such networks are in oscillators with emitter followers or unity gain connected op-amps as the active device. Of course, the well-known phase shift oscillators use an amplifier with gain more than unity. Other applications of over unity gain may be in situations where a transformer is to be replaced for boosting the voltage by a small amount.

Prior to the advent of integrated circuits (IC's), major application of RC networks, in general, were in low frequency circuits, where a high Q inductor is impossible to obtain. That is the reason why active RC circuits flooded the literature, see [11, 12] for examples, and some major efforts were directed to simulate a high Q inductor [13–15].

With the advent of IC's, efforts were also directed to use the metallic interconnections between components, which basically behaved as distributed RC networks, and were considered as a parasitic, before Kaufman's paper [16] appeared. Since then, many workers directed their efforts towards applying distributed RC networks for various useful purposes, including null networks [17, 18], and in oscillators with an op-amp connected in the unity gain connection [19].

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Declarations

Conflict of interest The author declares that he has no conflict of interest.

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