Sādhanā (2022)47:170
https://doi.org/10.1007/s12046-022-01932-0

Nurse allocation in hospital: hybridization of linear regression, fuzzy set and game-theoretic approaches

REKHA SAHU[1], SUJIT DAS[2,*] and SATYA RANJAN DASH[3]

[1] School of Computer Engineering, KIIT Deemed to be University, Bhubaneswar, India
[2] Department of Computer Science and Engineering, NITW, Warangal, India
[3] School of Computer Applications, KIIT Deemed to be University, Bhubaneswar, India
e-mail: sahu_r@rediffmail.com; sujit.das@nitw.ac.in; sdashfca@kiit.ac.in

MS received 14 June 2021; revised 7 June 2022; accepted 10 June 2022

Abstract. Sufficient numbers of staff are required for any hospital for providing better services to patients with satisfactory treatment. In most hospitals, the patient's treatment and care depend on the availability of nurses. Less number of nurses are a reason for average care and treatment to patients, whereas more than the required number of nurses may cause wastage of expenditure and manpower. More importantly, an additional number of nurses may be assigned to some hospitals where a sufficient number of nurses are not available. Moreover, nurses are of different categories such as qualified nurses having experience in providing service to patients, qualified nurses without having experience, and nurses without qualification but have experience. The expenditure also depends on categories of nurses' appointments. To reach an equilibrium point in appointing nurses, we propose a hybridization model using linear regression, fuzzy set theory and game-theoretic approaches. Regression analysis is implemented for prediction based on different fuzzy membership values of independent variables. We implement two concepts of game theory, the Nash equilibrium and the perfect Nash equilibrium. Implementing the Nash equilibrium, different equilibria values are generated and by implementing perfect Nash equilibrium, a subgame is generated to reach one equilibrium value. We have illustrated the proposed approach using a case study in which the linear regression approach is implemented to predict the patients' arrival rate based on the monetary standard of the patient, communication facilities, patients curing chances, and patient choice towards the hospital, where these four features are quantified using fuzzy membership values. Nash equilibrium decides all possible ways of the nurses' allocations and perfect Nash equilibrium assist to reach an appropriate allocation. Thus, the hybridization of the regression approach, fuzzy membership and game theory approaches finalize the exact allocation of nurses. Finally, a comparative analysis is given to demonstrate the effectiveness of the proposed approach.

Keywords. Nurse allocation; regression analysis; fuzzy membership value; Nash equilibrium; perfect Nash equilibrium.

1. Introduction

The patients' arrival rate in a hospital varies from time to time and season to season. In response to these variations, the number of staff is generally varied over a day. Hence allocation of medical staff resources is considered to be an important problem for avoiding the problem of overloaded and underloaded staff and provinding better treatment to the patients. The major challenge is to appoint the staff in a hospital with time-varying demands. One of the vital staff resources in any health care system is nurses. In COVID-19 pandemic situations, nurses are supposed to provide high-quality care to the affected persons to mitigate the challenges of the healthcare system. The whole nation is continuously trying to improve the quality and safety of care for the patients efficiently by providing better nursing service [1]. The survey given in [1] regarding different hospitals has concluded that appointing nurses with their demanding salaries is a headache for hospital management. The nursing care survey [2] concluded that appointing nurses in the hospital with assigning time and tasks is complex work. Again, the qualitative, as well as quantitative aspect of the nurse, is an issue for some of the hospitals such as Merle West Medical Centre [2]. Nurse patient ratio, nurse staffing norms, nursing manpower requirement, nursing workforce requirement are the main issues in Indian hospitals and proper nurse staffing norms are emphasised in [3]. Computerised nurses' management is a low-cost approach to manage the nurses in a hospital [4]. In order to optimize the workload of the nurses, Giammona

*For correspondence

Published online: 20 August 2022

et al [5] developed Nursing Care Score system, which was approximately 75% effective with the flexibility of deciding nurse-to-patient ratio and staff allocation, but the authors did not discuss the quantitative aspect in appointing nurses in the hospital.

Nurses play a vital role in treating and taking care of patients. Since nurses frequently visit the patients and are considered to be the nearest person to a patient in a hospital, they should be well trained so that they can be the good personalized person for the patients. Registered nurses have a good impact on health care and provide better services to patients [6]. Also, the supports of nurses can deliver quality care to prevent and manage pressure injuries [7], whereas one nurse can manage the specific responsibility to reduce the patients' pain [8]. A survey of 42 nurses in [9] showed that the calibre and intention of nurses are major issues for patient treatment. The number of different backgrounds or quality nurses is equally important for a small and medium hospital [10]. Harley *et al* observed that the nurses' education was needed to enrich the service for the emergency patients [11]. In the case of childbirth [12], highly trained nurses can take care of the patient properly. The importance and efficiencies of the nurses for healthcare to provide better care and treatment were studied in [13]. Based on the experience and qualification of nurses, three categories of nurses are found: a) having good experience and qualified, b) qualified without experience and c) no qualification but have a good experience. These types of nurses serve patients in different ways like some can treat in absence of doctors, some give medicine and injection and some can make the accessories ready for treatment. The smaller number of nurses can create dissatisfaction to the patients whereas a greater number of nurses can result in a wastage of manpower and unnecessary expenditure of the system. Nurses' requirement in a hospital fully depends on the patients, and accordingly, the number of patients in a hospital influence the appointment of nurses. According to the statistics of USA based hospitals found in [14], the arrival of patients is dependent on the availability of nurses. UK emergency department simulated the availability of patients using different methods which are given in [15]. To keep stock of resources in the hospital, the number of patients' arrival is a major factor and the Markov Chain Model was used in [16] to predict the number of patients to be admitted. Hence, different categories of nurses and patients' arrival rates are necessary to consider when nurses are allocated to hospitals.

The number of nurses is finalized after knowing the arrival rate of patients. Different traditional machine learning and deep learning approaches are used for forecasting [17], whereas regression analysis is one of the machine learning approaches which is used mainly for prediction. Some significant contributions using linear regression are narrated below. A pair-wise interactions linear regression model was developed in [18] to study some features of patients and then predicted the risk. A LASSO regression model was studied in [19] which predicted mortality with severe sepsis. Cui *et al* [20] analysed that the prediction of multi-output regression improved the management of healthcare resources, and payment. Panay *et al* [21] studied that the regression model is necessary to predict the treatment cost to a patient. Thus, the regression analysis technique can predict the risk of patients, mortality rate, resources' requirements and price requirements. Besides these, regression analysis was used to predict different areas for resource allocation, such as a univariate nonlinear regression model with time as the independent variable. Corn price was used as the dependent variable to allocate the corn price and a multivariate regression model was proposed taking production-consumption, import and export volume as independent variables, and maize price as the dependent variable [22]. The regression model for the drought prediction was formulated based on geographical differences, seasonal influence, incorporation of climate indices and author bias [23]. A linear regression model was used to predict the building energy which is needed based on climate and thermo-physical parameters [24]. Park *et al* [25] developed a multiple regression model to predict renal function based on adjuvant treatment for gastric cancer, the impact of radiotherapy on estimated glomerular filtration rate after gastric surgery and the age group. A multiple regression model was formulated in [26] to predict the primary delay recovery of high-speed railways taking independent variables as dwell buffer time, running buffer time, the magnitude of primary delay, and individual sections' influence. Giretti *et al* [27] proposed a multiple regression predictor to predict blood urea in preterm infants based on nutrition and clinical data. To forecast the electrical load in the urban area, a linear regression model is proposed by Johannesen *et al* [28] by interpreting the independent variables as time, weather and random effects. Recently, Rath *et al* [29] predicted the coronavirus disease cases based on independent variables as number of positive, number of decreased and number of recovery cases using the multiple linear regression approach. To forecast road traffic death in Thailand, a multiple regression analysis was proposed in [30] with the independent variables such as death rate per 100,000 population, gross domestic product, the number of registered vehicles, and energy consumption of the transportation sector. The death rate is predicted in [31] using the multiple regression model considering independent variables as doctor availability, hospital availability, annual per capita income, and population density. Thus, regression analysis has shown the performance of forecasting in different areas based on different related factors. Al-Doghaither *et al* [32] studied the main factors for choosing a hospital for the patients. Pope *et al* [33] investigated that the number of emergency admissions to hospitals in England and Wales had risen sharply due to some nonclinical factors such as lack of time to support the seniors. The authors [33] observed that the requirement of patient's social support and community follow-up had significant influences on admission. Beckert *et al* [34] investigated the benefit of the public hospital in contrast to the private hospital and noticed that minority (who are relatively poor in terms of education and monetary standard) patients were getting less benefit due to geographical distribution. Kuklinski *et al* [35] observed that patients' choice of hospital depends upon the quality of treatment. In summary, we have observed that the number of patients in a hospital depends mainly on the following factors: total patients in the area, their monetary standard, communication facilities, probability of curing disease and patient personal choice towards a hospital. Moreover, a multiple regression model can forecast the patient arrival rate based on the above-mentioned factors.

The factors such as total patients in the area, their monetary standard, communication facilities, probability of curing disease and patient personal choice towards a hospital should be in measurable form for the required manipulation in regression analysis approach. Several antivirus therapies are used for the treatment of COVID-19 disease. To choose an appropriate treatment for a mild symptom of a COVID-19 patient, an additive ratio assessment approach with a hesitant fuzzy decision-making approach is fruitful. One case study with this approach showed that Remdesivir is the best medicine for mild symptoms of COVID-19 patients [36]. A fuzzy game theory approach can successfully allocate human resources and tasks in an organisation [37]. Fuzzy logic formulates the mathematical approach to assign membership values and to quantify and interpret the different variables in the required form [38]. Many extensions of fuzzy set have been continuously used to solve complex real-life problems [39, 40]. The variables (total patients in the area, their monetary standard, communication facilities, probability of curing disease and patient personal choice towards a hospital) influence patients' arrival rate and these variables can be quantified using fuzzy membership values. Considering these factors, a multiple regression equation can be generated to predict patients' arrival rates.

Allocation of nurse resources to the hospital is decided by the management considering the numbers and types of patients and it is a critical task for hospital management. Game theoretical approaches help to make a decision when there is a contrast between two groups in their requirement or choice for one thing but one has to decide at one equilibrium point [41]. A game theory approach is applied to decide on supply and demand criteria. Here friendliness to customers and higher profit of manufacturer are to be taken care and it is a non-cooperative form of game theory. A mathematical formulation and optimisation were performed to meet a decision in [42]. Hansen and Raskin [43] considered one or more equilibrium points called the Nash Equilibria [43] to meet a decision. Since the controversy can be interpreted between two groups of people, it can be said two-player games. When a game has more than one equilibrium, it is difficult to take one decision. So, all equilibria are manipulated as a cost function in terms of physical and behavioural cost and interpreted as a payoff matrix. Imposing constraints on the cost function values we get a sub-game from the payoff matrix and perfect Nash equilibrium is found out for two-player games [44]. In a two-player imitation game, the complexity is solved in finding approximate Nash equilibria [45]. A distributed Nash equilibrium finding algorithm for a quadratic game with nonlinear dynamics is proposed in [46]. A deep neural network-based algorithm is proposed to identify the Markovian Nash equilibrium of general large N-player stochastic differential games in [47]. A non-cooperative game is an interactive strategic decision situation. The players try to optimize according to their perception. The contradiction situation is raised since the requirement of both players are opposed to each other [48]. More investigations on Nash equilibrium under uncertainty for non-cooperative game are found in [49, 50]. The patients' requirement is to get treatment from a greater number of nurses and at the same time a huge number of unemployed nurses are ready to join the hospitals. But normally the goal of the hospital management is to appoint a relatively smaller number of nurses in order to have less expenditure. The interactive strategic decision is required where the patients' level of satisfaction and managements' expenditure are in contrast to each other. Hence, the nurses' appointment is considered to be the equivalent of a non-cooperative game. Following the Nash equilibrium approach and perfect Nash equilibrium approach, it can be possible to manipulate the required number of nurses for the hospital.

As per our findings, no research works are found regarding the number of nurses to be appointed in a hospital. The main contribution of this study is to determine the number of nurses to be allocated to a hospital according to the satisfaction of the patients and the criteria of management. Hence, we have proposed a hybridization of regression analysis with fuzzy membership values and the Nash equilibrium model to predict the number of nurses to be appointed. Since nurses' appointment depends on patients' quantity aspect, so we have implemented a multiple regression model as part of our proposed model. In the multiple regression approach, we have considered total patients in the surrounding, their monetary standard, communication facilities, probability of curing diseases, and patient's personal choice towards as independent variables, and the number of patients may admit in the hospital as dependent variable. To express the value of the above independent variables, we have used the fuzzy membership values. Based on the different categories of nurses, different types of patients and management decisions, we can follow the Nash equilibrium approach for finding different ways of appointing nurses. Finally, all hospitals do not provide the same treatment as some hospitals provide excellent, some good, some manageable and some average treatments. So, by following the perfect Nash equilibrium approach and selecting a treatment approach from excellent, good, manageable and average, we can conclude the exact number of nurses allocate to the hospital. In summary, the hospital data are converted into the appropriate form using fuzzy theory. From the filtered data, the regression equation is generated and then patient arrival rates are predicted. From patients' arrival rates, different options are manipulated in appointing nurses following the Nash equilibrium approach. From different options, the exact number of nurses is finalized following the perfect Nash equilibrium approach. The overall workflow diagram to represent the proposed work is depicted in figure 1.

The rest of the paper is organised as follows. In section 2, we have presented the basic ideas relevant to the proposed approach. The proposed approach is illustrated in section 3 followed by the case study related to nursing allocation in hospitals in section 4. Result discussion and comparative study is given in section 5. Finally, conclusions are provided in section 6.

2. Preliminaries

In this section, we discuss some basic concepts involved in the proposed hybridization model.

2.1 *Multiple Linear Regression Analysis [51]*

Multiple regression (MR) analysis is a method to define the relationship of a collection of independent variables to a single dependent variable. When we have quantitative information about the variables like $x_{i1}, x_{i2}, \ldots, x_{in-1}, x_{in}$ which combinedly influence the value of another variable like y_i, then we can quantify y_i based on the sum of the weights of variables $x_{i1}, x_{i2}, \ldots, x_{in-1}, x_{in}$. Thus, we can define mathematically the multiple linear regression equation with one dependent variable y_i and n number of independent variables $x_{i1}, x_{i2}, \ldots, x_{in-1}, x_{in}$ as given below in (1).

$$y_i = a_1 + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3} + \ldots\ldots + b_{n-1} x_{in-1} + b_n x_{in} \quad (1)$$

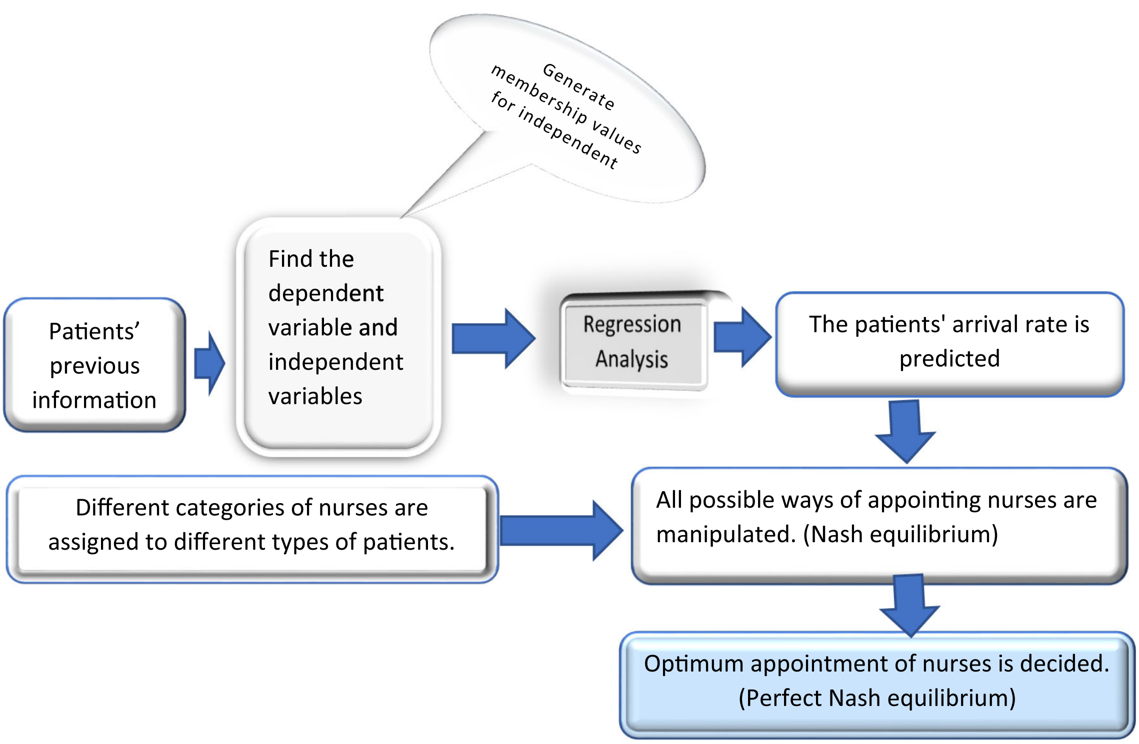

Figure 1. Workflow diagram of the proposed approach.

Here $x_{i1}, x_{i2}, x_{i3}, \ldots\ldots, x_{in-1}, x_{in}$ are the scores of cases i, y_i is the predictor and b_1, b_2, … b_n are partial regression coefficients or regression weights, and a_1 be the regression intercept.

To get equation 1 for predicting the value of dependent variable y_i, we have to find the value of a_1, and b_j, j = 1, 2, …, n. We generate a system of n equations from N numbers of instances as defined in (2) from instances $(y_i, x_{i1}, x_{i2}, x_{i3}, \ldots\ldots, x_{in-1}, x_{in})$ for i = 1 to N.

$$\left.\begin{array}{c} \sum y_i = Na_1 + b_1 \sum x_{i1} + b_2 \sum x_{i2} \\ + \ldots + b_n \sum x_{in} \\ \sum x_{i1} y_i = a_1 \sum x_{i1} + b_1 \sum x_{i1}^2 + b_2 \\ \sum x_{i1} x_{i2} + \ldots + b_n \sum x_{i1} x_{in} \\ \sum x_{i2} y_i = a_1 \sum x_{i2} + b_1 \sum x_{i1} x_{i2} \\ + b_2 \sum x_{i2}^2 + \ldots + b_n \sum x_{i2} x_{in} \\ \vdots \quad \vdots \qquad \vdots \\ \sum x_{in} y_i = a_1 \sum x_{in} + b_1 \sum x_{i1} x_{in} \\ + b_2 \sum x_{i2} x_{in} + \ldots + b_n \sum x_{in}^2 \end{array}\right\} \tag{2}$$

By solving the system of equations 2, we can find the values of a_1, b_1, b_2, b_3,…, b_n. The values of a_1, b_1, b_2, b_3,…, b_n can be optimized for the cost function as given in (3).

$$\text{Minimize} \sum (y - \hat{y})^2 \tag{3}$$

Here $\hat{y}$ is the predicted value and y is the real value.

2.2 *Fuzzy set [52, 53]*

Vagueness can be interpreted rigorously and preciously using the mathematical representation of the fuzzy theory. According to the application area, the vague concepts are measured in fuzzy theory. The fuzzy set is defined mathematically in the following way. For the set X, where X is a collection of our application objects, the fuzzy set $\overline{A}$ in X (FS(X)) is defined as ordered pairs as follows.

$$\tilde{A} = \left\{ (x, \mu_{\tilde{A}}(x)) | x \in X \right\}$$

Here $\mu_{\tilde{A}}(x)$ represents the membership value of x in A. The values of $\mu_{\tilde{A}}(x)$ lie between 0 and 1.

The membership function $\mu_{\tilde{A}}(x)$ is defined below in (4).

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, x \le l_1 \\ \dfrac{1}{1 + e^{-2(x-4)}}, l_1 < x \le l_2 \\ 1, x > l_2 \end{cases} \tag{4}$$

The membership value can be generated for the independent variables defined in section 2.1 in the multiple regression equation.

2.3 *Function as greatest integer*

In many applications, integer numbers are accepted. But our proposed approach consider integer as well as decimal values. In such cases, we have to convert the decimal value in terms of integer numbers using the greatest integer function. The integer linear function $g : R \to Z^+$ as for $x \in R$ is defined below in (5).

$$g(x) = \begin{cases} 0, x \le 0 \\ [x], x > 0 \end{cases} \tag{5}$$

2.4 *Nash Equilibrium [43]*

We can predict the number of objects required for performing our task using multiple regression analysis or any appropriate machine learning techniques by considering some related specific features or factors. In certain cases, we find some factors which have no relation with the objects but have a strong influence on the objects. Due to the influences of such different factors or resources and contrast in taking the decision, we face difficulties to decide on the number of objects that should be involved to perform the work. To reach saturation or optimize decisions, we would apply Nash equilibrium. The concept behind the Nash equilibrium is as follows.

The two players in game theory, are in the intention to reach a decision when they have to make decisions at the same time and the decision of one player takes into account the decision of another player. Nash equilibrium determines the optimal solution in a non-cooperative game. Many Nash equilibria may be shown in a game. Nash equilibrium requires several conditions to hold to apply:

1. All players are interested only to optimize their expected payoff and will act accordingly.
2. All players execute their strategies perfectly.
3. All players are infinitely intelligent, or at least intelligent enough to determine the solution.
4. Every player knows (or can deduce) the planned equilibrium strategy of all other players, and know that changing their strategy will not result in other players changing their strategy.
5. Every player knows every other player satisfies the above four conditions.

Suppose, we have two players A and B. A want to maximize the values of variables u, v, and w for u, v and w $\in Z^+$ and B wants to minimize the values of variables p, q, and r for p, q, and $r \in Z^+$. Again, we have a function $f : (u, v, w, p, q, r) \to r \, for \, r \in Z^+$ which has to be optimized. The function is defined as,

Table 1. The generated values for u, v, and w corresponding to values of p, q, and r.

p1	p2	p3	p4	p5	p6	p7	p8	p9	p10	p11	p12
u1	u2	u3	u4	u5	u6	u7	u8	u9	u10	u11	u12
q1	q2	q3	q4	q5	q6	q7	q8	q9	q10	q11	q12
v1	v2	v3	v4	v5	v6	v7	v8	v9	v10	v11	v12
r1	r2	r3	r4	r5	r6	r7	r8	r9	r10	r11	r12
w1	w2	w3	w4	w5	w6	w7	w8	w9	w10	w11	w12

$$f(u,v,w,p,q,r) = u \times p + v \times q + w \times r \qquad (6)$$

There must be specific values for function f for considering as it the optimum values.

Thus, player A wants to maximize the values of u, v and w and the functional value f whereas in controversy B want to minimize the values of p, q and r and the functional value f. According to different values of the variables u, v, w, p, q, & r, we generate the payoff matrix. The values contained in the payoff matrix is interpreted as Nash equilibria. Hence for certain values are generated by player B for variables p, q and r, correspondingly player A generate the values of the variables u, v, and w so that the cost function f is optimized. For player A, different values of variable p are p_i for i = 1, 2, …12, different values of variable q are q_i for i = 1, 2, …12 and different values of variable r are r_i for i = 1, 2, …12. Similarly, for player B, different values of variable u are u_i for i = 1, 2, …12, different values of variable v are v_i for i = 1, 2, …12 and different values of variable w are w_i for i = 1, 2, …12. Considering the values given in table 1, the function f becomes optimum (say). Then we have the Nash equilibrium payoff matrix as in table 2.

Nash equilibria are generated with function f defined in (6). To find an equilibrium an appropriate subgame is considered and perfect Nash equilibrium is concluded for reaching the decision.

2.5 *Perfect Nash Equilibrium [44]*

For one decision, more than one option is generated by Nash equilibria. Perfect Nash equilibrium is implemented when there are Nash equilibria instead of one Nash equilibrium. For finding Perfect Nash equilibrium, we have to find a sub-game using the constraints. In section 2.3, we have the game with two players A and B. We have a function $f:(u,v,w,p,q,r) \to r \; for \; r \in Z^+$ which has to be optimized. The function is defined as, $f:(u,v,w,p,q,r) = u \times p + v \times q + w \times r$. There must be specific values for function f for considering as for it as optimum value. We have twelve options as defined in table 2. We have to choose one option and for the reason other criteria to be considered according to category or value of some variable t. So, we define a function

$$F:(u,v,w,p,q,r,t) = f(u,v,w,p,q,r),$$

for some specific value $t = t_0$, the value of u, v, w, p, q *and* r are considered.

Finally, generate a payoff matrix as table 3.

Table 3. Payoff matrix for perfect Nash equilibrium.

u	v	w
P	q	r

3. Proposed method

This section presents the proposed method below in a stepwise manner.

Step 1. The dependent variable and independent variables are extracted from the given information, where the required independent variables are expressed using fuzzy membership values.

Step 2. Multiple regression equations based on the dependent and independent variables are generated as given below.

$$y_i = a_1 + b_1 x_{i1} + b_2 x_{i2} + b_3 x_{i3} + \ldots\ldots + b_{n-1} x_{in-1} + b_n x_{in}$$

Here $x_{i1}, x_{i2}, \ldots, x_{in-1}, x_{in}$ are independent and y_i be the dependent variables, where values of the independent variables are considered as fuzzy membership values. Partial regression coefficients or regression weights are denoted by b_1, b_2, . . . b_n, and a_1 be the regression intercept.

Step 3. To obtain the weights of $a_1, b_1, b_2, b_3, \ldots\ldots, b_{n-1}, b_n$, the system of equations mentioned in (2) are solved.

Step 4. The value of the dependent variable (y_i) is converted into a positive integer using the greatest integer function using (5) as g(y_i) to transform the decimal value into a positive integer so that it can be countable.

Step 5. By assigning u to p, v to q, and w to r, the cost function $f(u,v,w,p,q,r) = u \times p + v \times q + w \times r$ mentioned in (6) is computed, where u = g(y_1), v = g(y_2), and w = g(y_3), and p = K_1, q = K_2, and r = K_3 are the constants which may be fixed according to the application area.

Table 2. The payoff matrix with Nash equilibria.

(p1, q1, r1, u1, v1, w1)	(p2, q2, r2, u2, v2, w2)	(p3, q3, r3, u3, v3, w3)	(p4, q4, r4, u4, v4, w4)
(p5, q5, r5, u5, v5, w5)	(p6, q6, r6, u6, v6, w6)	(p7, q7, r7, u7, v7, w7)	(p8, q8, r8, u8, v8, w8)
(p9, q9, r9, u9, v9, w9)	(p10, q10, r10, u10, v10, w10)	(p11, q11, r11, u11, v11, w11)	(p12, q12, r12, u12, v12, w12)

Table 4. Payoff matrix.

F_1	F_2	F_3	F_4
F_5	F_6	F_7	F_8
F_9	F_{10}	F_{11}	F_{12}

Step 6. Pay off matrix for Nash equilibria as defined in table 2 is computed according to the different values of u, v and w and their corresponding values of p, q, and r. Those are computed using (6) and given in table 2.

From (6), we have

$$f(u,v,w,p,q,r) = u \times p + v \times q + w \times r.$$

Now we have the payoff matrix as presented in table 4 where,

$$\begin{aligned} F_1 &= f(u1,v1,w1,p1,q1,r1),\\ F_2 &= f(u2,v2,w2,p2,q2,r2),\\ F_3 &= f(u3,v3,w3,p3,q3,r3),\\ F_4 &= f(u4,v4,w4,p4,q4,r4),\\ F_5 &= f(u5,v5,w5,p5,q5,r5),\\ F_6 &= f(u6,v6,w6,p6,q6,r6),\\ F_7 &= f(u7,v7,w7,p7,q7,r7),\\ F_8 &= f(u8,v8,w2,p8,q8,r8),\\ F_9 &= f(u9,v9,w9,p9,q9,r9),\\ F_{10} &= f(u10,v10,w10,p10,q10,r10),\\ F_{11} &= f(u11,v11,w11,p11,q11,r11),\\ F_{12} &= (u12,v12,w12,p12,q12,r12). \end{aligned}$$

Thus, we have twelve solutions.

Step 7. From Nash equilibria, we generate perfect Nash equilibrium as defined in table 3 by imposing criteria and generating a subgame. In step 6, we have twelve solutions and from them, we have to choose one solution following the perfect Nash equilibrium principle. A constraint or criteria is added as the value of the variable $t = t_0$. Then we evaluate the function $F(u,v,w,p,q,r,t) = f(u,v,w,p,q,r)$. According to the value of t, function F is evaluated and the optimum values of u, v, w, p, q and r are obtained.

4. Case study

This case study illustrates the proposed model for appointing the number of nurses in a hospital for good treatment, where the nurse resources expenditure is less than 1500. Doctors consider how many nurses should be assigned to a patient according to the disease and condition of the patient. Moreover, how many different categories of nurses are needed to be assigned to a patient is considered by doctors and management. For the case study, we have taken 60 very serious patients, 80 serious patients and 120 suffering but not serious patients. We have also recorded the monetary standard of the patient, communication facilities, patients curing chances, and patient choice towards the hospital. To quantify the above attributes, we have used fuzzy membership values. Using (4), we can generate the membership values which belongs to [0, 1] for the features like the monetary standard of the patient, communication facilities, patients curing chances, and patient choice towards the hospital. Nurses in a hospital are categorized as qualified nurses having experience in providing service to patients, qualified nurse without experience and experience nurse without qualification, where a highly experienced and qualified nurse can serve a patient in the absence of a doctor, qualified nurse treats a patient as instructed by the doctor and finally, experienced nurse may not have the qualification but can serve patients according to requirement.

We have symbolized C_1, C_2, and C_3 as qualified nurses having experience in providing service to patients, qualified nurses without experience and experienced nurses without qualification, respectively.

The patients are categorised into three categories; such as very serious, serious, and suffering but not serious. We have symbolized V as very serious, S as serious and N as suffering but not serious patients. The number of patients admitted in a hospital depends on the number of people who fall into disease in the nearby area, the monetary standard of patients, communication facilities, probability of curing disease, personal choice of patients regarding the hospital, etc. Assume that the number of very serious patients arrived in a hospital is X that depends on the factors or variables as x_1 stands for the number of patients who fall in disease in the nearby area, x_2 stands for the monetary standard of patients, x_3 stands for communication facilities surrounding the hospital, x_4 stands for the probability of disease curability, and x_5 stands in personal choice about the hospital. We have taken X as the dependent variable and the independent variables are x_1, x_2, x_3, x_4 and x_5.

Similar assuming Y as the number of serious patients depends on the factors as y_1 stands for the number of patients who fall in disease, y_2 stands for monetary standard, y_3 stands for communication facilities, y_4 stands for the probability of cure disease, and y_5 stands in personal choice. Similar assuming z as the number of suffering but not serious patients which depends on the factors as z_1 stands for the number of patients fall in disease, z_2 stands for monetary standard, z_3 stands for communication facilities, z_4 stands for the probability of cure disease, and z_5 stands in personal choice. Here $x_1, y_1, z_1 \in Z^+$ be the set of positive integers as it denotes the number of patients affected, $x_2, y_2, z_2 \in [0, 1]$ are membership values for the monetary standard, $x_3, y_3, z_3 \in [0, 1]$ are membership values for the communication facilities, $x_4, y_4, z_4 \in [0, 1]$ are membership values for the probability of curing disease and $x_5, y_5, z_5 \in [0, 1]$ are membership values for the personal choice value as the level of preference to admit the particular hospital and finally, X, Y, $Z \in Z^{+}$ be the set of positive integers that to be predicted i.e., different categories patients arrive at the hospital. All the information regarding the patients is noted in table 5.

The hospitals treat the patients differently like excellent, good, manageable, and average, which are respectively represented as E, G, M, and P. These differences are shown due to the different issues like the calibre of a doctor, financial status of the hospital, available treatment machinery, medicine availability, etc. In summary, regarding hospitals related to nurses, we have the following information.

i) Each department of the hospital contains three categories of nurses such as qualified nurses having experience in providing service to patients, qualified nurses without having experience and nurses without qualification but have experience. These three categories of nurses are named as C_1, C_2 and C_3 respectively.
ii) The patients are categorised as V, S and N for each department of a hospital. V stands for very serious, S stands for serious and N stands for suffering but not serious.
iii) Treatment of each department of a hospital can be categorized as E, G, M and P, where E stands for excellent, G stands for good, M stands for manageable, P stands for average.

The nurses are assigned to patients based on the decision of expert doctors and management. Allocation of three categories of nurses (C1, C2 and C3), i.e., how many nurses are allocated to the three categories of patients (V, S and N) and the treatment outcome is shown in table 6.

The assumed information about the very serious patients (X), serious patients (Y), and suffering but not serious patients (Z) in a particular hospital is summarised in tables 7, 8, and 9, respectively, which are collected from the previous record of a hospital.

The regression model is implemented with the information given in tables 7, 8 and 9. Three regression equations are generated to forecast the number of very serious (X), serious (Y) and suffering but not serious (Z) patients in the hospital.

The regression model for predicting the number of very serious patients who may visit the hospital is derived from (1) as follows.

$$X = a_{11} + b_{11}x_1 + b_{21}x_2 + b_{31}x_3 + b_{41}x_4 + b_{51}x_5$$

The above forecasting equation is solved by finding the value of a_{11}, b_{11}, b_{21}, b_{31}, b_{41} and b_{51}. With the system of equations given in (2) and table 7, we have generated the six systems of equations.

Table 7. The number of very serious patients and their information.

X	x_1	x_2	x_3	x_4	x_5
10	50	0.6	0.9	1	0.4
20	25	0.7	1	0.5	0.7
30	60	0.8	1	0.9	0.8
10	60	0.4	0.6	0.8	1
10	20	0.9	0.8	0.7	0.9
20	30	0.7	1	0.9	0.9
30	40	0.7	0.7	0.7	0.8
30	50	0.3	0.5	0.6	0.5
60	90	0.5	0.4	0.5	0.4
50	80	0.2	0.6	0.3	0.6

Table 5. Patient information.

Categories of patients	No of patients surrounding the hospital (x_1)	Monetary standard of patients (x_2)	Communication facilities for the hospital (x_3)	Chances of getting the cure for the patients at the hospital (x_4)	Patients' choice towards the hospital for treatment (x_5)
Very serious (V)	60	0.8	1	0.7	0.9
Serious (S)	80	0.7	0.8	0.8	0.6
Suffering but not serious (N)	120	0.4	0.9	0.9	0.4

Table 6. Nurse allocation to patients and treatment outcome.

Patients/treatment	Excellent(E) (C_1, C_2, C_3)	Good(G) (C_1, C_2, C_3)	Manageable(M) (C_1, C_2, C_3)	Average(P) (C_1, C_2, C_3)
Very serious (V)	(3, 4, 5)	(2, 4, 5)	(2, 3, 4)	(1, 2, 0)
Serious (S)	(1, 3, 4)	(1, 2, 3)	(1, 1, 3)	(0, 1, 1)
Not serious (N)	(1, 2, 3)	(1, 2, 2)	(0, 1, 2)	(0, 1, 1)

Table 8. The number of serious patients and their information.

Y	y_1	y_2	y_3	y_4	y_5
40	50	0.6	0.9	1	0.4
20	25	0.7	1	0.5	0.7
60	60	0.8	1	0.9	0.8
10	40	0.4	0.6	0.8	1
70	90	0.9	0.8	0.7	0.9
60	80	0.7	1	0.9	0.9
10	40	0.7	0.7	0.7	0.8
30	50	0.8	0.3	1	0.7
70	80	0.4	0.7	0.8	0.4
60	70	0.5	0.8	0.4	0.8

$$\sum X = Na_{11} + b_{11}\sum x_1 + b_{21}\sum x_2 + b_{31}\sum x_3 + b_{41}\sum x_4 + b_{51}\sum x_5$$
$$\rightarrow \quad 10a_{11} + 505\ b_{11} + 5.8\ b_{21} + 7.5\ b_{31} + 6.9\ b_{41} + 7\ b_{51} = 270$$

$$\sum x_1X = a_{11}\sum x_1 + b_{11}\sum x_1^2 + b_{21}\sum x_1x_2 + b_{31}\sum x_1x_3 + b_{41}\sum x_1x_4 + b_{51}\sum x_1x_5$$
$$\rightarrow \quad 505a_{11} + 30225\ b_{11} + 262.5\ b_{21} + 349.0\ b_{31} + 332.5\ b_{41} + 331.5\ b_{51} = 16300$$

$$\sum x_2X = a_{11}\sum x_2 + b_{11}\sum x_1x_2 + b_{21}\sum x_2^2 + b_{31}\sum x_2x_3 + b_{41}\sum x_2x_4 + b_{51}\sum x_2x_5$$
$$\rightarrow \quad 5.8a_{11} + 262.5b_{11} + 3.82\ b_{21} + 4.66\ b_{31} + 4.23\ b_{41} + 4.24\ b_{51} = 141$$

$$\sum x_3X = a_{11}\sum x_3 + b_{11}\sum x_1x_3 + b_{21}\sum x_2x_3 + b_{31}\sum x_3^2 + b_{41}\sum x_3x_4 + b_{51}\sum x_3x_5$$
$$\rightarrow \quad 7.5a_{11} + 349\ b_{11} + 4.66\ b_{21} + 6.07\ b_{31} + 5.41\ b_{41} + 5.41\ b_{51} = 183$$

$$\sum x_4X = a_{11}\sum x_4 + b_{11}\sum x_1x_4 + b_{21}\sum x_2x_4 + b_{31}\sum x_3x_4 + b_{41}\sum x_4^2 + b_{51}\sum x_4x_5$$
$$\rightarrow \quad 6.9a_{11} + 332.5\ b_{11} + 4.23b_{21} + 5.41\ b_{31} + 5.19\ b_{41} + 4.95\ b_{51} = 164$$

$$\sum x_5X = a_{11}\sum x_5 + b_{11}\sum x_1x_5 + b_{21}\sum x_2x_5 + b_{31}\sum x_3x_5 + b_{41}\sum x_4x_5 + b_{51}\sum x_5^2$$
$$\rightarrow \quad 7a_{11} + 331.5\ b_{11} + 4.24\ b_{21} + 5.41\ b_{31} + 4.95\ b_{41} + 5.32\ b_{51} = 172$$

Thus, we have a system of equations as follows.

Table 9. The number of suffering but not serious patients and their information.

Z	z_1	z_2	z_3	z_4	z_5
10	20	0.6	0.9	1	0.4
20	25	0.7	1	0.5	0.7
60	60	0.8	1	0.9	0.8
30	40	0.4	0.6	0.8	1
40	50	0.9	0.8	0.7	0.9
10	30	0.7	1	0.9	0.9
30	40	0.7	0.7	0.7	0.8
20	20	0.6	0.4	0.3	1
30	30	0.2	0.1	0.8	0.5
60	70	0.4	0.6	0.5	0.8

$$10a_{11} + 505\ b_{11} + 5.8\ b_{21} + 7.5\ b_{31} + 6.9\ b_{41} + 7\ b_{51} = 270,$$
$$505a_{11} + 30225\ b_{11} + 262,5\ b_{21} + 349.0\ b_{31} + 332.5\ b_{41} + 331.5\ b_{51} = 16300,$$
$$5.8a_{11} + 262.5b_{11} + 3.82\ b_{21} + 4.66\ b_{31} + 2.23\ b_{41} + 4.24\ b_{51} = 141,$$
$$7.5a_{11} + 349\ b_{11} + 4.66\ b_{21} + 6.07\ b_{31} + 5.41\ b_{41} + 5.41\ b_{51} = 183,$$
$$6.9a_{11} + 332.5\ b_{11} + 4.23b_{21} + 5.41\ b_{31} + 5.19\ b_{41} + 4.95\ b_{51} = 164,$$
$$7a_{11} + 331.5\ b_{11} + 4.24\ b_{21} + 5.41\ b_{31} + 4.95\ b_{41} + 5.32\ b_{51} = 172$$

Solving the above system of equations using Guess elimination method, the values of a_{11}, b_{11}, b_{21}, b_{31}, b_{41}, b_{51} are respectively obtained as 20.93, 0.55, 28.68, 0.03, −43.63, and −11.63, which are used to formulate the multiple linear regression expression given below in (7)

$$X = 20.93 + 0.55x_1 + 28.68x_2 + 0.03x_3 - 43.63x_4 - 11.63x_5 \tag{7}$$

The given information noted in table 5 are $x_1 = 60$, $x_2 = 0.8$, $x_3 = 1$, $x_4 = 0.7$ and $x_5 = 0.9$, then the number of very serious patients may arrive to the hospital can be predicted as

$$X = 20.93 + 0.55 \times 60 + 28.68 \times 0.8 + 0.03 \times 1 - 43.63 \times 0.7 - 11.63 \times 0.9 \approx 35.9$$

Using (5), we obtain that the number of very serious patients (N_1) who may come to the hospital is $g(35.9) = 36$.

The regression model for predicting the number of serious patients who may visit the hospital is derived from (1) as follows.

$$Y = a_{12} + b_{12}y_1 + b_{22}y_2 + b_{32}y_3 + b_{42}y_4 + b_{52}y_5$$

The above forecasting equation is solved by finding the value of a_{12}, b_{12}, b_{22}, b_{32}, b_{42} and b_{52}. With the system of equations given in (2) and table 8, we have generated the six systems of equations.

$$\sum Y = Na_{12} + b_{12}\sum y_1 + b_{22}\sum y_2 + b_{32}\sum y_3 + b_{42}\sum y_4 + b_{52}\sum y_5 \rightarrow 10a_{12} + 585b_{12} + 6.5\ b_2 + 7.8\ b_3 + 7.7\ b_4 + 7.4\ b_5 = 430$$

$$\sum y_1 Y = a_1\sum y_1 + b_1\sum y_1^2 + b_2\sum y_1y_2 + b_3\sum y_1y_3 + b_4\sum y_1y_4 + b_5\sum y_1y_5 \rightarrow 585a_{12} + 38225b_{12} + 383.5b_2 + 461\ b_3 + 453.5\ b_4 + 433.5b_5 = 29300$$

$$\sum y_2 Y = a_1\sum y_2 + b_1\sum y_1y_2 + b_2\sum y_2^2 + b_3\sum y_2y_3 + b_4\sum y_2y_4 + b_5\sum y_2y_5 \rightarrow 6.5a_{12} + 383.5b_{12} + 4.49\ b_2 + 5.11\ b_3 + 5.06\ b_4 + 4.89\ b_5 = 284$$

$$\sum y_3 Y = a_1\sum y_3 + b_1\sum y_1y_3 + b_2\sum y_2y_3 + b_3\sum y_3^2 + b_4\sum y_3y_4 + b_5\sum y_3y_5 \rightarrow 7.8a_{12} + 461\ b_{12} + 5.11\ b_2 + 6.52\ b_3 + 5.91\ b_4 + 5.77\ b_5 = 351$$

$$\sum y_4 Y = a_1\sum y_4 + b_1\sum y_1y_4 + b_2\sum y_2y_4 + b_3\sum y_3y_4 + b_4\sum y_4^2 + b_5\sum y_4y_5 \rightarrow 7.7a_{12} + 453.5b_{12} + 5.06\ b_2 + 5.91\ b_3 + 6.29\ b_4 + 5.61\ b_5 = 332$$

$$\sum y_5 Y = a_1\sum y_5 + b_1\sum y_1y_5 + b_2\sum y_2y_5 + b_3\sum y_3y_5 + b_4\sum y_4y_5 + b_5\sum y_5^2 \rightarrow 7.4a_{12} + 433.5b_{12} + 4.89\ b_2 + 5.77\ b_3 + 5.61\ b_4 + 5.84\ b_5 = 310$$

Thus, we have the system of equations as follows.

$$10a_{12} + 585b_{12} + 6.5\ b_2 + 7.8\ b_3 + 7.7\ b_4 + 7.4\ b_5 = 430$$

$$585a_{12} + 38225b_{12} + 383.5b_2 + 461\ b_3 + 453.5\ b_4 + 433.5b_5 = 29300$$

$$6.5a_{12} + 383.5b_{12} + 4.49\ b_2 + 5.11\ b_3 + 5.06\ b_4 + 4.89\ b_5 = 284$$

$$7.8a_{12} + 461\ b_{12} + 5.11\ b_2 + 6.52\ b_3 + 5.91\ b_4 + 5.77\ b_5 = 351$$

$$7.7a_{12} + 453.5b_{12} + 5.06\ b_2 + 5.91\ b_3 + 6.29\ b_4 + 5.61\ b_5 = 332$$

$$7.4a_{12} + 433.5b_{12} + 4.89\ b_2 + 5.77\ b_3 + 5.61\ b_4 + 5.84\ b_5 = 310$$

Solving the above system of equations using Guess elimination method, we have

$$a_{12} = -12.36, b_{12} = 1.01, b_{22} = 11.94, b_{32} = 21.62, b_{42} = -9.17, b_{52} = -28.92,$$

which are used to formulate the multiple linear regression expression given below in (8)

$$Y = -12.36 + 1.01y_1 + 11.94y_2 + 1.62y_3 - 9.17y_4 - 28.92y_5 \tag{8}$$

We have used the information regarding serious patients of an area is as defined in table 5 as $y1 = 80$, $y2 = 0.7$, $y3 = 0.8$, $y4 = 0.8$ and $y5 = 0.6$. Hence, the number of serious patients may arrive at the hospital can be predicted as

$$Y = -12.36 + 1.01 \times 80 + 11.94 \times 0.7 + 21.62 \times 0.8 - 9.17 \times 0.8 - 28.92 \times 0.6 = 69.4$$

Using (5), we obtain that the number of serious patients (N_2) who may come to the hospital is $g(69.4) = 70$.

The regression model for predicting the number of suffering but not serious patients who may visit the hospital is derived from (1) as follows.

$$z = a_{13} + b_{13}z_1 + b_{23}z_2 + b_{33}z_3 + b_{43}z_4 + b_{53}z_5$$

The above forecasting equation is solved by finding the value of a_{13}, b_{13}, b_{23}, b_{33}, b_{43} and b_{53}. With the system of equations given in (2) and table 9, we have generated the six systems of equations.

$$\sum Z = \mathrm{N}a_{13} + b_{13}\sum z_1 + b_{23}\sum z_2 + b_{33}\sum z_3 + b_{43}\sum z_4 + b_{53}\sum z_5$$
$$\rightarrow \quad 10a_{13} + 385b_{13} + 6b_{23} + 7.1b_{33} + 7.1b_{43} + 7.8\, b_{53} = 310$$

$$\sum z_1 Z = a_{13}\sum z_1 + b_{13}\sum z_1^2 + b_{23}\sum z_1 z_2 + b_{33}\sum z_1 z_3 + b_{43}\sum z_1 z_4 + b_{53}\sum z_1 z_5$$
$$\rightarrow \quad 385a_{13} + 17425b_{13} + 233.5b_{23} + 278b_{33} + 273.5b_{43} + 308.5\, b_{53} = 14500$$

$$\sum z_2 Z = a_1\sum z_2 + b_1\sum z_1 z_2 + b_2\sum z_2^2 + b_3\sum z_2 z_3 + b_4\sum z_2 z_4 + b_5\sum z_2 z_5$$
$$\rightarrow \quad 6a_{13} + 233.5b_{13} + 4b_{23} + 4.69b_{33} + 4.28b_{43} + 4.79\, b_{53} = 186$$

$$\sum z_3 Z = a_1\sum z_3 + b_1\sum z_1 z_3 + b_2\sum z_2 z_3 + b_3\sum z_3^2 + b_4\sum z_3 z_4 + b_5\sum z_3 z_5$$
$$\rightarrow \quad 7.1a_{13} + 278b_{13} + 4.69b_{23} + 5.83b_{33} + 5.23b_{43} + 5.57\, b_{53} = 217$$

$$\sum z_4 Z = a_1\sum z_4 + b_1\sum z_1 z_4 + b_2\sum z_2 z_4 + b_3\sum z_3 z_4 + b_4\sum z_4^2 + b_5\sum z_4 z_5$$
$$\rightarrow \quad 7.1a_{13} + 273.5b_{13} + 4.28b_{23} + 5.23b_{33} + 5.47b_{43} + 5.37\, b_{53} = 216$$

$$\sum z_5 Z = a_1\sum z_5 + b_1\sum z_1 z_5 + b_2\sum z_2 z_5 + b_3\sum z_3 z_5 + b_4\sum z_4 z_5 + b_5\sum z_5^2$$
$$\rightarrow \quad 78a_{13} + 308.5b_{13} + 4.79b_{23} + 5.57b_{33} + 5.37b_{43} + 6.44\, b_{53} = 248$$

Thus, we have the system of equations as follows.

$$\begin{aligned}
&10a_{13} + 385b_{13} + 6b_{23} + 7.1b_{33} + 7.1b_{43} + 7.8\, b_{53} = 310\\
&385a_{13} + 17425b_{13} + 233.5b_{23} + 278b_{33} + 273.5b_{43} + 308.5\, b_{53} = 14500\\
&6a_{13} + 233.5b_{13} + 4b_{23} + 4.69b_{33} + 4.28b_{43} + 4.79\, b_{53} = 186\\
&7.1a_{13} + 278b_{13} + 4.69b_{23} + 5.83b_{33} + 5.23b_{43} + 5.57\, b_{53} = 217\\
&7.1a_{13} + 273.5b_{13} + 4.28b_{23} + 5.23b_{33} + 5.47b_{43} + 5.37\, b_{53} = 216\\
&78a_{13} + 308.5b_{13} + 4.79b_{23} + 5.57b_{33} + 5.37b_{43} + 6.44\, b_{53} = 248
\end{aligned}$$

Solving the above system of equations using Guess elimination method, we have $a_{13} = -0.05$, $b_{13} = 1.00$, $b_{23} = 8.10$, $b_{33} = -13.20$, $b_{43} = -4.40$, $b_{53} = 0.18$, which are used to formulate the multiple regression equation given below in (9).

$$Z = -0.05 + z_1 + 8.10z_2 - 13.20z_3 - 4.40z_4 + 0.18z_5 \tag{9}$$

We have used the information regarding patients as given in table 5 as $z_1 = 120$, $z_2 = 0.4$, $z_3 = 0.9$, $z_4 = 0.9$ and $z_5 = 0.4$. Hence the number of suffering but not serious patients may arrive to the hospital can be predicted as

$$\begin{aligned}
Z &= -0.05 + 120 + 8.10 \times 0.4 - 13.20 \times 0.9\\
&\quad - 4.40 \times 0.9 + 0.18 \times 0.4\\
&= 107.4
\end{aligned}$$

Using (5), we obtain the number of suffering but not serious patients (N_3) come to the hospital is $g(107.4) = 108$.

Now we compute the nurse requirements for very serious patients ($N_1 = 36$), serious patients ($N_2 = 70$), and suffering but not serious patients ($N_3 = 108$) using table 6 and section 2.4.

The number of C_1 category nurses (NC_1) requirement according to different types of treatment is computed as

$$NC_1 = \begin{cases}
N_1n_1 + N_2n_{13} + N_3n_{25} = 36 \times 3 + 70 \times 1 + 108 \times 1 = 126, & \text{if excellent treatment is provided.}\\
N_1n_4 + N_2n_{16} + N_3n_{28} = 36 \times 2 + 70 \times 1 + 108 \times 1 = 250, & \text{if good treatment is provided}\\
N_1n_7 + N_2n_{19} + N_3n_{31} = 36 \times 2 + 70 \times 2 + 108 \times 0 = 142, & \text{if managing treatment is provided.}\\
N_1n_{10} + N_2n_{22} + N_3n_{34} = 36 \times 1 + 70 \times 0 + 108 \times 0 = 36, & \text{if average treatment is provided.}
\end{cases}$$

The number of C_2 category nurses (NC_2) requirements according to different types of treatment is computed as

Table 10. Payoff matrix information.

	Excellent treatment			Good treatment			Manageable treatment			Average treatment		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
V	N_1n_1	N_1n_2	N_1n_3	N_1n_4	N_1n_5	N_1n_6	N_1n_7	N_1n_8	N_1n_9	N_1n_{10}	N_1n_{11}	N_1n_{12}
S	N_2n_{13}	N_2n_{14}	N_2n_{15}	N_2n_{16}	N_2n_{17}	N_2n_{18}	N_2n_{19}	N_2n_{20}	N_2n_{21}	N_2n_{22}	N_2n_{23}	N_2n_{24}
N	N_3n_{25}	N_3n_{26}	N_3n_{27}	N_3n_{28}	N_3n_{29}	N_3n_{30}	N_3n_{31}	N_3n_{32}	N_3n_{33}	N_3n_{24}	N_3n_{25}	N_3n_{36}

$$NC_2 = \begin{cases} N_1n_2 + N_2n_{14} + N_3N_{26} = 36 \times 4 + 70 \times 3 \\ +108 \times 2 = 570, \text{ if excellent treatment} \\ \text{is provided.} \\ N_1n_5 + N_2n_{17} + N_3n_{29} = 36 \times 4 + 70 \times 2 \\ +108 \times 2 = 500, \text{ if good treatment} \\ \text{is provided} \\ N_1n_8 + N_2N_{20} + N_3n_{32} = 36 \times 3 + 70 \times 1 \\ +108 \times 1 = 286, \text{ if managing treatment} \\ \text{is provided.} \\ N_1n_{11} + N_2n_{23} + N_3n_{35} = 36 \times 2 + 70 \times 1 \\ +108 \times 1 = 250, \text{ if average treatment} \\ \text{is provided.} \end{cases}$$

The number of C_3 category nurses (NC_3) requirements according to different types of treatment is computed as

$$NC_3 = \begin{cases} N_1n_3 + N_2n_{15} + N_3n_{27} = 36 \times 5 + 70 \times 4 \\ +108 \times 3 = 784, \text{ if excellent treatment} \\ \text{is provided.} \\ N_1n_6 + N_2n_{18} + N_3n_{30} = 36 \times 5 + 70 \times 3 \\ +108 \times 2 = 606, \text{ if good treatment} \\ \text{is provided} \\ N_1n_9 + N_2n_{21} + N_3n_{33} = 36 \times 4 + 70 \times 3 \\ +108 \times 2 = 570, \text{ if managing treatment} \\ \text{is provided.} \\ N_1n_{12} + N_2n_{24} + N_3n_{36} = 36 \times 0 + 70 \times 1 \\ +108 \times 1 = 178, \text{ if average treatment} \\ \text{is provided.} \end{cases}$$

Here n_i ($i = 1, 2, \ldots, 36$) refers to the number of nurses {where number of type C1 nurses are n_i (i = 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31, and 34), type C2 nurses are n_i (i = 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, and 35) and type C3 nurses are n_i (i = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, and 36) assigned for various category of patients (very serious, serious and suffering but not serious patients).

Next, the payoff matrixes are explored for various types of treatment (excellent, good, manageable, average) based on the information given in table 10 and the corresponding values are computed in table 11.

The number of different categories of nurses' requirements according to different types of treatment is summarized in table 12.

We have four Nash equilibria as defined in table 11 and so perfect Nash equilibrium has been extracted.

According to the service provided by the hospitals, the hospitals can be categorized as excellent treatment, good treatment, manageable treatment and average treatment. When more than one Nash equilibrium or the payoff matrices are generated, the subgame is considered for getting perfect Nash equilibrium as follows.

$F(v, t_i) = N_i$, where $N_i \in Z^+$, v is value according to values in table 11 and t_1 = excellent treatment, t_2 = good treatment, t_3 = manageable treatment, and t_4 = average treatment.

Since the case study is based on good treatment and the nurse resource expenditure is less than 1500, so we have the subgame according to t_2 = good treatment for getting perfect Nash equilibrium. Thus, we have a perfect Nash equilibrium, a single payoff matrix as in table 13 extracted from table 12.

Finally, from tables 12 and 13, we have to appoint the total number of nurses in the hospital for good treatment are 250 category C_1 nurses (qualified and experienced), 500 category C_2 nurses (qualified), 606 category C_3 nurses (not qualified but good experienced).

The simulation result of the case study is summarised in figure 2.

5. Discussion and Comparative study

The proposed approach is based on hybridization of regression analysis, fuzzy membership values and game theoretic approach which can predict the number of nurses require in a particular period in a hospital. This approach calculates the number of nurses based on parameters: the number of patients fall in disease in an area surrounding the hospital, the monetary standard of patients, communication facilities of the hospital, the probability of curing disease, and personal likeness towards the hospital, doctors' and management's opinions regarding the nurse requirement, quality of treatment required and condition of the patients. This study has considered three different types of nurses

Table 11. Payoff matrix for various types of treatment.

	Excellent treatment			Good treatment			Manageable treatment			Average treatment		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
V	108	144	180	72	144	180	72	108	144	36	72	0
S	70	210	280	70	140	210	70	70	210	0	70	70
N	108	216	324	108	216	216	0	108	216	0	108	108

Table 12. Total nurses' allotments according to categories of treatment.

	C_1	C_2	C_3
Excellent	286	570	784
Good	250	500	606
Managing	142	286	570
Average	36	250	178

Table 13. Perfect Nash equilibrium values.

Good treatment			
	C_1	C_2	C_3
V	72	144	180
S	70	140	210
N	108	216	216

according to their calibres such as qualified nurses having experience in providing service to patients, qualified nurses without experience and experience nurses without qualification. All the independent variables influencing patients' arrival rate are quantified using fuzzy membership values and the patients' arrival rate is forecasted using regression analysis. Nash equilibria and perfect Nash equilibrium have allocated the number of different categories of nurses to the hospital.

The proposed approach determines the appropriate number of nurses required to be allocated to a hospital and it can be justified from the following three statements which are presented mathematically using Lemma 1, Lemma 2 and Lemma 3 respectively.

- Prediction of the arrival rate of patients is approximately equal to the actual arrival rate.
- Assigning satisfiable numbers of nurses to patients is evaluated by Nash equilibria values.
- Perfect Nash equilibrium find out the particular number of nurses for the patient from the Nash equilibria.

Our approach satisfies the three statements which has been justified through the following mathematical analysis (three lemmas).

Lemma 1 *Prediction of the arrival rate of the patient is approximately equal to the actual arrival rate which can be found using multiple regression equations and the mathematical proof of it is given below.*

Proof Using $n = 5$, the multiple regression equation mentioned in eq. (1) can be written as

$$Y = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5.$$

Here, a, b_1, b_2, b_3, b_4 and b_5 are weight values for the variables X_1, X_2, X_3, X_4 and X_5.

X_1 is the number of patients who fall in disease in the nearby area,

X_2 is the monetary standard of patients,

X_3 is communication facilities surrounding the hospital,

X_4 is the probability of disease curability,

X_5 is the personal choice about the hospital,

Y is the number of patients.

The system of equations is generated using (2) for finding the values of a, b_1, b_2, b_3, b_4 and b_5 which is given below

$$\sum Y = \mathrm{N}a + b_1\sum X_1 + b_2\sum X_2 + b_3\sum X_3 + b_4\sum X_4 + b_5\sum X_5$$

$$\sum X_1Y = a\sum X_1 + b_1\sum X_1^2 + b_2\sum X_1X_2 + b_3\sum X_1X_3 + b_4\sum X_1X_4 + b_5\sum X_1X_5$$

$$\sum X_2Y = a\sum X_2 + b_1\sum X_1X_2 + b_2\sum X_2^2 + b_3\sum X_2X_3 + b_4\sum X_2X_4 + b_5\sum X_2X_5$$

$$\sum X_3Y = a\sum X_3 + b_1\sum X_1X_3 + b_2\sum X_2X_3 + b_3\sum X_3^2 + b_4\sum X_3X_4 + b_5\sum X_3X_5$$

$$\sum X_4Y = a\sum X_4 + b_1\sum X_1X_4 + b_2\sum X_2X_4 + b_3\sum X_3X_4 + b_4\sum X_4^2 + b_5\sum X_4X_5$$

$$\sum X_5Y = a\sum X_5 + b_1\sum X_1X_5 + b_2\sum X_2X_5 + b_3\sum X_3X_5 + b_4\sum X_4X_5 + b_5\sum X_5^2$$

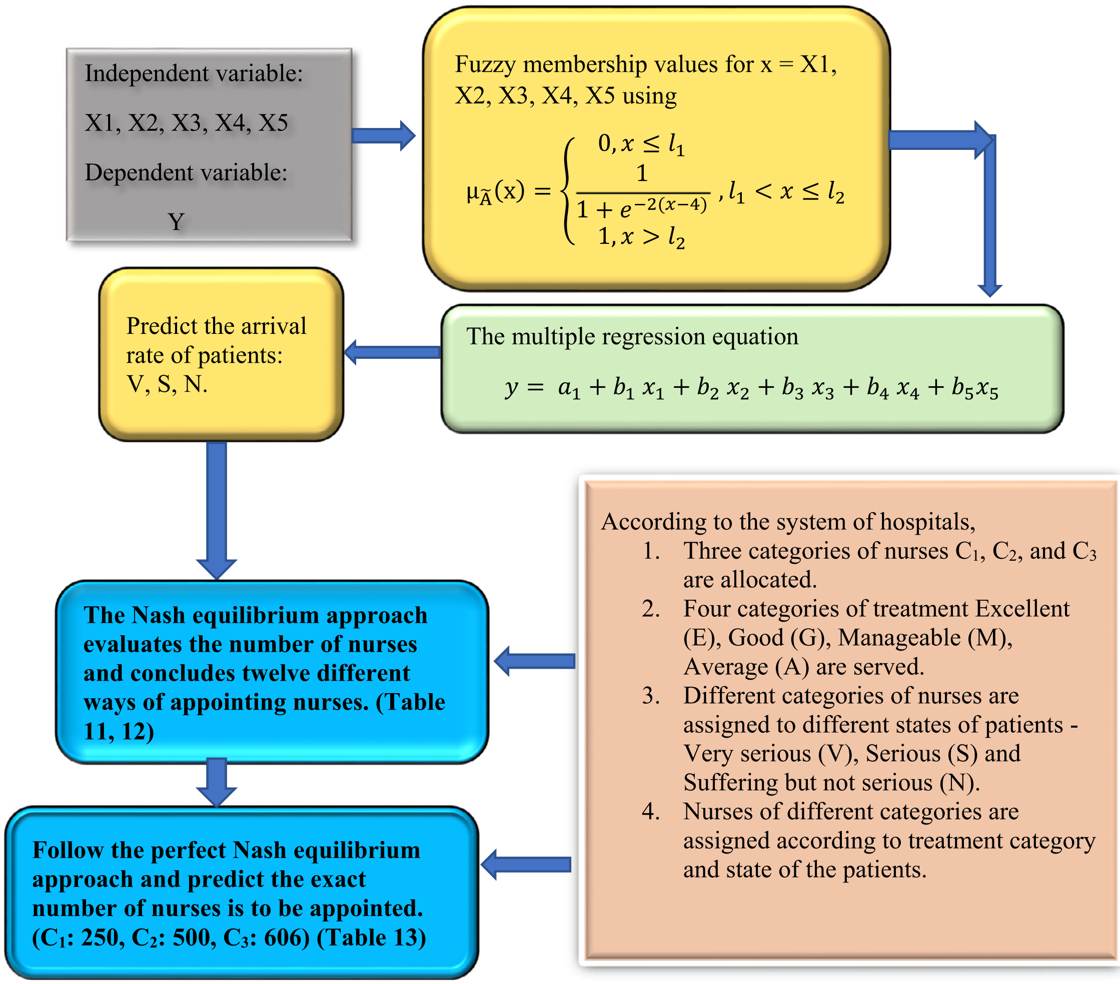

Figure 2. Sketch of simulation result of the case study.

Solving the above system of equations, we can find the values of a, b_1, b_2, b_3, b_4 and b_5. These values are optimized minimizing the cost function $\sum(y-\hat{y})^2$ mentioned in eq. (3), where $\hat{y}$ is the predicted value and y is the real value. Finally, the generated equation $Y = a + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5$ will predict the approximate arrival rate.

Lemma 2 *Assigning satisfiable numbers of nurses to patients is evaluated by Nash equilibria values.*

Proof Consider C_1, C_2, and C_3 are three categories of nurses, where C_1 stands for qualified nurses having experience in providing service to patients, C_2 is qualified nurses without experience and C_3 is experienced nurses without qualification. Assume c1, c2 and c3 are the costs for C_1, C_2, C_3 categories of nurses respectively and P be the set of patients, x ∈ P. Say n1, n2, and n3 numbers of C_1, C_2, and C_3 categories of respective nurses are demanded by patient x and for x ∈ P, m1, m2, and m3 numbers of C_1, C_2, and C_3 categories of nurses assigned by the management respectively.

As per the patient view, a function is defined as f (x, n1, n2, n3) = c1n1 + c2n2 + c3n3, x ∈ P, where n1 ∈ [N1, N2], n2 ∈ [N3, N4], n3 ∈ [N5, N6] and N1, N2, N3, N4, N5, N6 are the natural numbers fixed by patient. Patient x is satisfied with the nurses' assignment for l1 ≤ f (x, n1, n2, n3) ≤ l2, where [l1, l2] is maximum values fixed by patient.

Again, according to management view, a function is defined as g (x, m1, m2, m3) = c1m1 + c2m2 + c3m3, x ∈ P, where m1 ∈ [M1, M2], m2 ∈ [M3, M4], m3 ∈ [M5, M6] and M1, M2, M3, M4, M5, M6 are the natural numbers fixed by management. Management is satisfied with the nurses' assignment to patient x for l3 ≤ g (x, n1, n2, n3) ≤ l4, where l3, l4 are minimum values fixed by the management.

The numbers of nurses assigned to patient is satisfiable by Nash equilibria values if f (x, k1, k2, k3) = g (x, k1, k2, k3) for k1, k2, k3 number of C_1, C_2, C_3 categories of nurses respectively. We have, f (x, k1, k2, k3) = c1k1 + c2k2 + c3k3 if k1 ∈ [N1, N2], k2 ∈ [N3, N4], k3 ∈ [N5, N6] and g (x, k1, k2, k3) = c1k1 + c2k2 + c3k3 if k1 ∈ [M1, M2], k2 ∈ [M3, M4], k3 ∈ [M5, M6]. We can find K1, K2, K3, K4, K5 and K6 such that N1 ≤ K1 ≤ K2 ≤ N2, N3 ≤ K3 ≤ K4 ≤ N4, N5 ≤ K5 ≤ K6 ≤ N6, and M1 ≤ K1 ≤ K2 ≤ M2, M3 ≤ K3 ≤ K4 ≤ M4, M5 ≤ K5 ≤ K6 ≤ M6. Then for k1 ∈ [K1, K2], k2 ∈ [K3, K4], k3 ∈ [K5, K6], we have f (x, k1, k2, k3) = g (x, k1, k2, k3).

Hence, Nash equilibria values (k1, k2, k3) are used to assign satisfiable numbers of nurses to patient.

Lemma 3 *Perfect Nash equilibrium find out the particular number of nurses for the patient from the Nash equilibria.*

Proof The lemma 2 conclude that for k1 ∈ [K1, K2], k2 ∈ [K3, K4], k3 ∈ [K5, K6], we have Nash equilibria (k1, k2, k3) where k1, k2, k3 number of C_1, C_2, C_3 categories of nurses are respectively assigned to x ∈ P and P be the set of patients.

We have different values of k1, k2, k3 as 0, 1, 2, 3 etc. according to the values of K1, K2, K3, K4, K5 and K6. Particular values of k1, k2, and k3 are to be found out and perfect Nash equilibrium is selected from Nash equilibria (k1, k2, k3).

As perfect Nash equilibrium is defined in section 2.5 and step 7 of section 3, we generate a constraint for getting perfect Nash equilibrium. Consider patient can afford excellent service (E), good service (G), manageable service (M), and average service (A), and T = {E, G, M, A}. As defined in lemma 2, l1 ≤ f (x, n1, n2, n3) ≤ l2, where [l1, l2] is maximum values fixed by patient. For t ∈ T, consider the affordable value as l, where l1 ≤ l ≤ l2. We can define a function h (t, x, k1, k2, k3) = f (x, k1, k2, k3) = c1k1 + c2k2 + c3k3 for k1 ∈ [K1, K2], k2 ∈ [K3, K4], k3 ∈ [K5, K6]. Then, we can find t1 ∈ [K1, K2], t2 ∈ [K3, K4], t3 ∈ [K5, K6] such that h (t, x, t1, t2, t3) = f (x, t1, t2, t3) = c1t1 + c2t2 + c3t3 = l. Thus, the particular number of nurses (t1, t2, t3) are assigned to a patient and perfect Nash equilibrium find out the particular number of nurses for a patent.

Lemmas 2 and 3 have shown the number of different categories of nurses are assigned to a patient. Lemma 1 has justified that the prediction arrival rate is equal to the approximate arrival rate. Hence, the total number of nurses allocated to a hospital is finalized based on the state of the patients, management provisions, and patients' affordable standards.

Comparative analysis of the proposed approach with some existing and relevant approaches are given below. Fagerström and Rainio [54] proposed a linear regression model to predict the optimal requirement of nursing care intensity based on patients' required intensity values in each department. The model [54] ignored the number of patients in the hospital, and doctors' and management's opinions on appointing nurses based on excellent, good, managing and average care. To optimize the nursing workload and staff allocation, an electronic health record system was developed in [5] and it was approximately 75% effective with the flexibility of deciding nurse-to-patient ratio and staff allocation, but the experiment did not explain clearly regarding the quantitative and qualitative aspects in appointing nurses. But in our proposed model, we have considered the nurses' appointments based on the patients' availability, doctors and management opinion, quality of treatment required, condition of the patients, etc. In the proposed study, the arrival of patients is considered using regression analysis with the factors like the number of patients who fall into disease, the monetary standard of patients, communication facilities, the probability of cure disease, and personal choice towards a hospital. So, in comparison to the proposed model in [5, 54], our approach can yield a more accurate prediction on appointing the number of nurses. Moreover, the approach discussed in [55] is based on the queueing theory to predict the number of patients available in the hospital using poison distribution and the number of nurses' requirements is calculated according to the demands of the patients such as triage patient, placing a patient in cubicle/bed, taking samples (blood, tissue, etc. as requested), arranging diagnosis (X-ray, ultrasound, CT scan, etc.), paying attention to patient needs and making referral arrangements, and the ratio of the number of occupied nurses which is not available to the resource pool and the total number staff available. This model [55] has not considered the different calibre of the nurses, factors influencing the availability of patients, doctors and managements' opinions, and patient's condition, etc., for appointing nurses in the hospital. The authors [55] only emphasize the number of patients available, the current availability of nurses and more nurses required. As per our knowledge, none of the studies considered the different calibre of the nurses, factors influencing the availability of patients, doctors and management's opinion, and patient's condition etc, which are emphasised in the proposed model. Very few research contributions are found that have an emphasis on the quantitative discussion on the nurses and their appointments in the hospital, rather, the research contributions are found on the qualitative approach for nurses' appointments. Since the hybridization of regression analysis, fuzzy membership values and game theory approach is used to calculate the number of different types of nurses' appointment based on factors like the number of patients falling in disease in an area, the monetary standard of patients, communication facilities, the probability of cure disease, and personal likeness towards the hospital by considering the doctors' and management's opinion, quality of treatment required and condition of the patients, therefore our proposed model is an improved model to predict the more accurate requirement of the different types of nurses in the hospital.

6. Conclusion

Since nurses play a vital role in the treatment and care of patients especially in COVID-19 pandemic situation, the proposed model might help a hospital to decide how many different categories of nurses (high experience and qualified nurse, qualified nurse without experience, and nurse without qualification but have experience) are required to provide necessary treatments to the patients. The model emphasizes the categories of nurses to be appointed to overcome the lack of the required number of doctors. It also reduces the unnecessary workloads of the expert nurses to provide better treatment on behalf of the hospital. In general, patients might be categorized as very serious, serious and suffering but not serious patients, and based on the categories of the patients, expert or general nurses are allotted. Generally, hospitals provide excellent, good, manageable, and average treatment as per the requirements and demands of the patients, and accordingly nurses are appointed. Nurses' appointment depends upon the patients' arrival and, in turn, the arrival of patients to the hospital depends on factors like the number of people falling in disease in the nearby area, the monetary standard of the patients, communication facilities, probability of cure disease, personal choice towards the hospital, etc. For various kinds of prediction using the available information, regression analysis has been proved to be effective. We have used fuzzy membership grades to quantify the variables between 0 and 1. The game theory approaches; Nash Equilibrium is an approach to decide on different controversial situations by deriving all possible equilibria, and perfect Nash equilibrium is an approach to generate the one equilibrium or one decision considering the subgame. The proposed approach is a hybridization of regression analysis, fuzzy membership values and game theory, which provides a conclusion on the number of nurses of different categories to be appointed. We have presented a case study to decide the number of different types of nurses to be appointed at the hospital. The proposed approach has derived the four equilibria or alternatives on appointing nurses and finally, the perfect Nash equilibrium has concluded the exact number of nurses to be required. The novelty of the proposed approach is that it can well determine the requirement of nurses in a hospital so that the hospital can run economically. Main advantage of this approach is that both of the patients and management can be satisfied in their treatment level. The study has analysed the appointment of nurses based on patients' availability and the hospital treatment process. The prediction of accurate result is difficult when the patients' arrival rate is wrongly calculated due to error in the generation of the regression equation. The predicted result is not applicable when there is a sudden variation of patients' arrival rate. Also, less availability of nurses is a major limitation to appoint the appropriate number of nurses in hospitals. In future, researchers may consider other factors for nurses' appointments. One may improve this proposed approach by considering more features and other machine learning approaches to optimize the weight values of the features. The regression equation is generated by giving weightage to different factors and the weightage may not be exact values for the factors. So, one has to optimize the error or may use other machine learning techniques like SVM, Naïve bay's ML, decision tree, deep learning, etc. for accurate forecasting of patients' arrival. The concept may be applied to decide on different resources of a healthcare organization, educational organisation, the banking system and office management.

References

[1] Thrall T H 2020 Optimal Allocation of Nursing Resources: An Interview with Robyn Begley, DNP, RN, NEA-BC; Pamela F. Cipriano, PhD, RN, NEA-BC, FAAN; and Todd Nelson, FHFMA, MBA. *JONA. The Journal of Nursing Administration*, 50(10): 502-504

[2] Gran-Moravec M B and Hughes C M 2005 Nursing time allocation and other considerations for staffing. *Nursing & Health Sciences* 7(2): 126–133

[3] Sharma S K and Rani R 2020 Nurse-to-patient ratio and nurse staffing norms for hospitals in India: A critical analysis of national benchmarks. *Journal of Family Medicine and Primary Care* 9(6): 2631

[4] Heslop L and Plummer V 2012 Nurse staff allocation by nurse patient ratio vs. a computerized nurse dependency management system: a comparative cost analysis of Australian and New Zealand hospitals. *Nursing Economics*, 30(6): 347

[5] Giammona S, Calò Arena G and M, Barone M A, Scelsa D, Lepre A and Schlenk E A, 2016 Nursing workload and staff allocation in an Italian hospital: a quality improvement initiative based on nursing care score. *Central European Journal of Nursing and Midwifery* 7(2): 420–427

[6] Ahlstedt C, Lindvall C E, Holmström I K and Athlin Å M 2019 What makes registered nurses remain in work? An ethnographic study. *International journal of nursing studies* 89: 32–38

[7] Barakat-Johnson M, Lai M, Wand T and White K 2019 A qualitative study of the thoughts and experiences of hospital nurses providing pressure injury prevention and management. *Collegian* 26(1): 95–102

[8] Peterson A, Berggården M, Schaller A S and Larsson B 2019 Nurses' Advocacy of Clinical Pain Management in Hospitals: A Qualitative Study. *Pain Management Nursing* 20(2): 133–139

[9] Toro-Flores R, Bravo-Agüi P, Catalán-Gómez M V, González-Hernando M, Guijarro-Cenisergue M J, Moreno-Vázquez M and Velasco-Sanz T R 2019 Opinions of nurses

regarding conscientious objection. *Nursing Ethics* 26(4): 1027–1038
[10] Park K O and Kim J K 2019 A Study of Desired Work Conditions of Nurses in Small-Medium Hospitals. *Journal of Korean Academy of Nursing Administration* 25(1): 1–13
[11] Harley A, Johnston A N B, Denny K J, Crilly Keijzers G and J and Massey D, 2019 Emergency nurses' knowledge and understanding of their role in recognising and responding to patients with sepsis: A qualitative study. *International Emergency Nursing* 43: 106–112
[12] Gombolay M, Golen T, Shah N and Shah J 2019 Queueing theoretic analysis of labour and delivery. *Health Care Management Science* 22(1): 16–33
[13] Eklöf N, Hupli M and Leino-Kilpi H 2015 Nurses' perceptions of working with immigrant patients and interpreters in Finland. *Public Health Nursing* 32(2): 143–150
[14] McKenna P, Heslin S M, Viccellio P, Hernandez Mallon W K and C and Morley E J, 2019 Emergency department and hospital crowding: causes, consequences, and cures. *Clinical and Experimental Emergency Medicine* 6(3): 189
[15] Bean D M, Taylor P and Dobson R J 2019 A patient flow simulator for healthcare management education. *BMJ Simulation and Technology Enhanced Learning* 5(1): 46–48
[16] He L, Madathil S C, Oberoi A, Servis G and Khasawneh M T 2019 A systematic review of research design and modeling techniques in inpatient bed management. *Computers & Industrial Engineering* 127: 451–466
[17] Sahu R, Dash S R, Cacha L A, Poznanski R R and Parida S 2020 Epileptic seizure detection: a comparative study between deep and traditional machine learning techniques. *Journal of Integrative Neuroscience* 19(1): 1–9
[18] Caruana R, Lou Y, Gehrke J, Koch P, Sturm M and Elhadad N 2015 Intelligible models for healthcare: Predicting pneumonia risk and hospital 30-day readmission. In: *Proceedings of the 21th ACM SIGKDD international conference on knowledge discovery and data mining* pp. 1721-1730
[19] Zhang Z and Hong Y 2017 Development of a novel score for the prediction of hospital mortality in patients with severe sepsis: the use of electronic healthcare records with LASSO regression. *Oncotarget* 8(30): 49637
[20] Cui L, Xie X, Shen Z, Lu R and Wang H 2018 Prediction of the healthcare resource utilization using multi-output regression models. In: *IISE Transactions on Healthcare Systems Engineering*, 8(4): 91-302
[21] Panay B, Baloian N, Pino J A, Peñafiel S, Sanson H and Bersano N 2020 Feature Selection for Health Care Costs Prediction Using Weighted Evidential Regression. *Sensors* 20(16): 4392
[22] Ge Y and Wu H 2019 Prediction of corn price fluctuation based on multiple linear regression analysis model under big data. *Neural Computing and Applications*, 1-13
[23] Anshuka A, van Ogtrop F F and Vervoort R W 2019 Drought forecasting through statistical models using standardised precipitation index: a systematic review and meta-regression analysis. *Natural Hazards* 97(2): 955–977
[24] Ciulla G and D'Amico A 2019 Building energy performance forecasting: A multiple linear regression approach. *Applied Energy* 253: 113500
[25] Park J S, Yu J I, Lim D H, Nam H, Kim Y I, Lee J and Bae J M 2021 Impact of Radiotherapy on Kidney Function among Patients Who Received Adjuvant Treatment for Gastric Cancer: Logistic and Linear Regression Analyses. *Cancers* 13(1): 59
[26] Jiang C, Huang P, Lessan J, Fu L and Wen C 2019 Forecasting primary delay recovery of high-speed railway using multiple linear regression, supporting vector machine, artificial neural network, and random forest regression. *Canadian Journal of Civil Engineering* 46(5): 353–363
[27] Giretti I, Correani A, Antognoli L, Monachesi C, Marchionni P, Biagetti C and Carnielli V P 2021 Blood urea in preterm infants on routine parenteral nutrition: A multiple linear regression analysis. *Clinical Nutrition* 40(1): 153–156
[28] Johannesen N J, Kolhe M and Goodwin M 2019 Relative evaluation of regression tools for urban area electrical energy demand forecasting. *Journal of Cleaner Production* 218: 555–564
[29] Rath S, Tripathy A and Tripathy A R 2020 Prediction of new active cases of coronavirus disease (COVID-19) pandemic using multiple linear regression model. *Diabetes & Metabolic Syndrome: Clinical Research & Reviews* 14(5): 1467–1474
[30] Jomnonkwao S, Uttra S and Ratanavaraha V 2020 Forecasting road traffic deaths in Thailand: Applications of time-series, curve estimation, multiple linear regression, and path analysis models. *Sustainability* 12(1): 395
[31] Dash S R and Sahu R 2019 Prediction of death rate using regression analysis. In: *Emerging Technologies in Data Mining and Information Security* Springer, Singapore, pp 735–745
[32] Al-Doghaither A H, Abdelrhman B M, Saeed A W and Magzoub M E M A 2003 Factors influencing patient choice of hospitals in Riyadh, Saudi Arabia. *The journal of the Royal Society for the Promotion of Health* 123(2): 105–109
[33] Pope I, Burn H, Ismail S A, Harris T and McCoy D 2017 A qualitative study exploring the factors influencing admission to hospital from the emergency department. *BMJ open* 7(8): e011543
[34] Beckert W and Kelly E 2021 Divided by choice? For-profit providers, patient choice and mechanisms of patient sorting in the English National Health Service. *Health Economics* 30(4): 820–839
[35] Kuklinski D, Vogel J and Geissler A 2021 The impact of quality on hospital choice. Which information affects patients' behavior for colorectal resection or knee replacement? *Health Care Management Science*, 1-18
[36] Mishra A R, Rani P, Krishankumar R, Ravichandran K S and Kar S 2021 An extended fuzzy decision-making framework using hesitant fuzzy sets for the drug selection to treat the mild symptoms of Coronavirus Disease 2019 (COVID-19). *Applied Soft Computing* 103: 107155
[37] Khanizad R and Montazer G 2018 Optimal allocation of human resources based on operational performance of organizational units using fuzzy game theory. *Cogent Engineering* 5(1): 1466382
[38] Das S, Malakar D, Kar S and Pal T 2018 A brief review and future outline on decision making using fuzzy soft set. *International Journal of Fuzzy System Applications* 7(2): 1–43
[39] Ali Z, Mahmood T, Ullah K and Khan Q 2021 Einstein geometric aggregation operators using a novel complex interval-valued pythagorean fuzzy setting with application in

green supplier chain management. *Reports in Mechanical Engineering* 2(1): 105–134

[40] Ashraf A, Ullah K, Hussain A and Bari M 2022 Interval-Valued Picture Fuzzy Maclaurin Symmetric Mean Operator with application in Multiple Attribute Decision-Making. *Reports in Mechanical Engineering* 3(1): 301–317

[41] Bhattacharjee P and Ray P K 2014 Patient flow modelling and performance analysis of healthcare delivery processes in hospitals: A review and reflections. *Computers&Industrial Engineering* 78: 299–312

[42] Ghosh P K, Manna A K, Dey J K and Kar S 2021 Supply chain coordination model for green product with different payment strategies: A game theoretic approach. *Journal of Cleaner Production* 290: 125734

[43] Hansen K A and Raskin M 2019 A Stay-in-a-Set Game without a Stationary Equilibrium. Tenth International Symposium on Games, Automata, Logics, and Formal Verification (GandALF'19). EPTCS 305, 2019, pp. 83–90

[44] Caruso F, Ceparano M C and Morgan J 2019 Subgame Perfect Nash Equilibrium: A Learning Approach via Costs to Move. *Dynamic Games and Applications* 9(2): 416–432

[45] Murhekar A and Mehta R 2020 Approximate nash equilibria of imitation games: Algorithms and complexity. In: *Proceedings of the 19th International Conference on Autonomous Agents and MultiAgent Systems* pp. 887-894

[46] Huang B, Zou Y and Meng Z 2020 Distributed-Observer-Based Nash Equilibrium Seeking Algorithm for Quadratic Games With Nonlinear Dynamics. *IEEE Transactions on Systems, Man, and Cybernetics: Systems* 51(11): 7260–7268

[47] Han J and Hu R 2020 Deep fictitious play for finding Markovian Nash equilibrium in multi-agent games. In: *Mathematical and Scientific Machine Learning* pp. 221-245

[48] A Brief Introduction to non-cooperative game theory https://web.archive.org/web/20100610071 152/http://www.ewp.rpi.edu/hartford/~stoddj/BE/IntroGameT.htm, 2010

[49] Romanuke V 2021 Refinement of acyclic-and-asymmetric payoff aggregates of pure strategy efficient Nash equilibria in finite noncooperative games by maximultimin and superoptimality. *Decision Making: Applications in Management and Engineering* 4(2): 178–199

[50] Romanuke V 2022 Pareto-efficient strategies in 2-person games in staircase-function continuous and finite spaces. *Decision Making: Applications in Management and Engineering* 5(1): 27–49

[51] Aiken L S, West S G, Pitts S C, Baraldi A N and Wurpts I C 2012 Multiple linear regression. *Handbook of Psychology, Second Edition*, *2*

[52] Sahu R, Dash S R and Das S 2021 Career selection of students using hybridized distance measure based on picture fuzzy set and rough set theory. *Decision Making: Applications in Management and Engineering* 4(1): 104–126

[53] De A, Kundu P, Das S and Kar S 2020 A ranking method based on interval type-2 fuzzy sets for multiple attribute group decision making. *Soft Computing* 24: 131–154

[54] Fagerström L and RAINIO A K, 1999 Professional assessment of optimal nursing care intensity level: a new method of assessing personnel resources for nursing care. *Journal of Clinical Nursing* 8(4): 369–379

[55] Thorwarth M, Rashwan W and Arisha A 2016 An analytical representation of flexible resource allocation in hospitals. *Flexible Services and Manufacturing Journal* 28(1–2): 148–165