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Continuous review inventory system for intuitionistic fuzzy random demand under service level constraint

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Abstract. An intuitionistic fuzzy random variable (IFRV) handles ambiguous, incomplete and ill-known data or information along with statistical variability, and deals with fuzzy number, grade of membership and non-membership functions and probability distribution function. So, taking such advantages of IFRV, we extend the classical continuous review inventory system in intuitionistic fuzzy random (IFR) environment by considering demand rate as IFRV. Due to uncertain variability, demand may suddenly increase, consequently, item shortage (stock out) may occur in the inventory system. To minimize the shortage quantity, fuzzy service level constraint and reorder level pertaining to safety stock are considered here. Service level constraint ensures that a certain fraction of demand to be fulfilled by on-hand inventory, whereas safety stock is kept against anticipation of shortage. Furthermore, we develop a methodology to compute the fuzzy expected shortage quantity for IFR inventory system, and determine the order quantity and reorder level. The proposed model is illustrated with numerical example and sensitivity analysis by changing the values of key parameters, which also helps in delineating the managerial insights.

Keywords. Inventory; continuous review; service level constraint; intuitionistic fuzzy random demand; score function; fuzzy expectation.

1. Introduction

Decision making about order releasing time and quantity and reorder level, is the most important issue in an inventory management problem, especially, when demand is uncertain, imprecise or both. Generally, demand estimation process depends on previous data records and many other uncertain factors such as price, availability, weather, etc. Sometimes past data records may not be available or if available, it may not carry full information. This type of ill-known data and lack of information result an imprecise boundary of the demand. Furthermore, this imprecise demand may vary due to statistical variability. Let us consider an example, n experts are assigned to estimate the demand of forthcoming year. The ith expert suggests that demand is about $\widetilde{d}_i$ unit with probability p_i; $i = 1, 2, \ldots, n$, here about $\widetilde{d}_i$ is interpreted as triangular intuitionistic fuzzy number (TIFN) $\widetilde{d}_i = (\underline{d_i}, d_i, \overline{d_i})$. This indicates that demand is imprecise, varying from minimum value $\underline{d_i}$ to maximum value $\overline{d_i}$. A TIFN is represented with membership, non-membership and score functions which will be discussed later in section 2. However, the final estimation of demand, after compiling all experts' opinion, may be $\left(\widetilde{d}_i, p_i\right)$; $i = 1, 2, \ldots, n$, which is a discrete fuzzy random variable (FRV), wherein fuzziness as well as randomness both appeared simultaneously [1, 2]. The final estimation of demand may also be viewed as a continuous FRV, as is considered in this paper. However, such type of demand may create sudden shortage of product in the system. Unsatisfied and impatience customer may go to another shop, that is not good for a long-term business. Continuous review policy, wherein inventory level is continuously supervised, is an appropriate technique to handle such type of situation by placing the order in advance. This paper is intended to extend the classical continuous review inventory system in Triangular intuitionistic fuzzy random (TIFR) environment by considering TIFR demand and fuzzy service level constraint.

In recent few decades, fuzzy set theory and its variant such as random fuzzy variable, fuzzy random variable, uncertain set, etc. were widely used in inventory modeling problems. A comprehensive literature survey on fuzzy inventory models is done by [3]. We here extensively review the literature of inventory models focusing on fuzzy (random) demand, and also find out the research gap. The same is also presented in table 1. The classical continuous

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review inventory model presented in [4] has been extended in many directions. [5] extended the same model in fuzzy environment by considering fuzzy cost parameters. Chang *et al* [6] and Lin [7] extended the continuous review and periodic review inventory models in fuzzy random environment by considering lead time (elapsed time between order place and its actual arrival [4]) demand as FRV and annual demand as fuzzy variable. Lead time reduction is also considered in both of the models. [1] also extended the continuous review inventory model by assuming fuzzy random lead time demand and annual demand as a fuzzy number. [8, 9] extended the periodic review and continuous review inventory systems by considering fuzzy random lead time demand and fuzzy annual demand. [10] further extended the fuzzy random periodic review system by considering lead time reduction and variable crashing cost. [11] extended the single-period probabilistic inventory model by considering discrete fuzzy random demand. [12] developed a production-inventory by considering fuzzy random demand and product reliability, and employed fuzzy differential equation to optimize the model. [13] modeled fuzzy demand in a two-level supply chain, wherein buyer adopted continuous review policy to place an order. [14] formulated an integrated inventory model in fuzzy random environment by considering trade-credit policy. The authors formulated a fuzzy random programming for said scenario and used genetic algorithm to optimize the same. Soni *et al* [15] noticed that fuzziness in demand can be reduced over the time due to the learning. Hence, they considered learning effect in fuzzy random demand. Kumar and Goswami [2] extended the continuous review inventory system by assuming fuzzy random demand, and employed fuzzy random renewal reward process and fuzzy expectation to optimize the model. [16] developed an integrated production-inventory for a single-vendor and a single-buyer by considering fuzzy random demand, and developed a methodology to find the fuzzy expected shortage quantity. [17] developed a fuzzy continuous review inventory system with service level constraint by considering fuzziness and randomness in demand and cost parameters, and employed fuzzy stochastic optimization approach. In recent years, [18–23] addressed inventory models in different framework considering FRV demand.

The notion of intuitionistic fuzzy set (IFS) and Atanassov intuitionistic fuzzy set (AIFS) were proposed by [24, 25]. IFS emphasizes more on grade of membership of the fuzzy set by adding an extra degree to model the hesitation and uncertainty about the degree of membership [26]. After adding this advantage, IFS is being started to apply in inventory management problems in recent days. [27] developed a fuzzy EPQ (economic production quantity) model by considering fuzzy demand and fuzzy cost parameters, and estimated an interval then used fuzzy intuitionistic programming technique to optimize the mathematical formulation. [28] developed an intuitionistic fuzzy inventory model by considering price and promotional effort dependent demand. The model estimated a crisp interval from the intuitionistic fuzzy model by using membership, non-membership and score functions, and then multi-objective approach has been applied to optimize it. [29] investigated a hill type EPQ model by considering fuzzy stochastic demand and other parameters as IFSs, and used intuitionistic fuzzy aggregation with Bonferroni mean for defuzzification.

The service level constraint ensures that a fraction of demand must be fulfilled from on-hand inventory, and remaining may be fulfilled in next shipment or may be lost of sales. As for example, if the administration decides that the minimum 80% of demand to be filled by the on-hand inventory that means shortage (or loss of sales) quantity cannot exceed 20% of the total demand. Service level constraint is more significant when shortage is not considered good for a business. [30] considered fill rate service constraint in a continuous review inventory system, wherein lead time can be reduced, and negative exponential investment function has been taken for reduction. [31] developed a single-vendor and single-buyer integrated model by considering fuzzy random demand during lead time and service level constraint. [32] addressed an inventory model considering fuzzy random demand, in which a service level constraint was imposed to restrict the stock-out level in each cycle.

The aforesaid literature survey and Table 1 indicate that no inventory model has been developed in IFRV environment, and very few models have been developed in intuitionistic fuzzy environment. IFRV captures probabilistic uncertainty as well as fuzziness, wherein fuzziness is delineated with membership as well as non-membership functions and probabilistic uncertainty is represented with distribution function. In real life business scenario, such type of probabilistic uncertainty and fuzziness are often intrinsic in the demand (one of the most important parameters) of inventory problem. In this study, we develop a continuous review inventory system by considering IFR demand. Such a demand may suddenly increase or decrease, consequently, situation of item shortage may occur. So, we impose a service level constraint in fuzzy sense to tackle such type of situation. As evidence of the above literature review and the best of our knowledge, no such inventory model has been developed till now.

The rest of the paper is organized as follows. Section 2 discusses the basic of fuzzy sets, intuitionistic fuzzy sets, FRV and IFRV. Besides this, the section also briefly reviews the classical continuous review inventory system ([4]). Section 3 lists the notations and assumptions which are to be used throughout the paper. In Section 4, we derive mathematical formulation in intuitionistic fuzzy random environment and also provide the solution procedure. Section 5 is a numerical example section, wherein the proposed model and solution methodology to be illustrated with numerical example, graphical illustration and

Table 1. Literature review.

			Uncertainty in demand representation			
				Fuzziness		
Author(s)	Reorder level	Service level constraint	Randomness	MF	NMF	SF
Dutta *et al* [1]	✔		✔	✔		
Kumar and Goswami [2]	✔		✔	✔		
Tutuncu *et al* [5]	✔		✔			
Change and Ouyang [6]	✔		✔	✔		
Lin [7]	✔		✔	✔		
Dey and Chakraborty [8–10]	✔		✔	✔		
Dutta *et al* [11]			✔	✔		
Bag *et al* [12]			✔	✔		
Pirayesh *et al* [13]	✔			✔		
Xu [14]			✔	✔		
Soni *et al* [15]	✔			✔		
Kumar *et al* [16]	✔		✔	✔		
Chakraborty and Bhuiya [17]	✔	✔	✔	✔		
Dey [19]	✔		✔	✔		
Mahapatra *et al* [20]	✔		✔	✔		
Zhang *et al* [21]			✔	✔		
Bhuiya and Chakraboty [22]	✔		✔	✔		
Wang and Yang [23]			✔	✔		
Chakrabortty *et al* [27]				✔	✔	✔
De and Sana [28]				✔	✔	✔
De and Sana [29]	✔		✔			
Moon *et al* [30]	✔	✔	✔			
Soni and Patel [31]	✔	✔	✔	✔		
Rong and Maiti [32]	✔	✔	✔	✔		
This study	✔	✔	✔	✔	✔	✔

MF = Membership function, NMF = Non-membership function, SF = Score function

sensitivity analysis. The discussion will be ended with concluding remarks and underlining the future research direction in Section 6.

2. Preliminaries

Before presenting the proposed inventory model, we here discuss some basic definitions and propositions of fuzzy sets, IFS and IFRV, and also briefly review the continuous review inventory system.

2.1 *Fuzzy set theory*

One can go through [33–36] for detail discussion on fuzzy set, FRV, IFS and IFRV. Here we only discuss the related facts of fuzzy theory which will be used in this study.

2.1a *Triangular fuzzy number*: A triangular fuzzy number (TFN) is denoted as a triplet $\tilde{A} = (a-\varepsilon, a, a+\delta); a, \varepsilon(\geq 0), \delta(\geq 0) \in \mathbb{R}$ (real line), and is defined as a fuzzy set on $\mathbb{R}$, whose membership function $\mu_{\tilde{A}}(x)$ satisfies the following properties:

(i) $\tilde{A}$ is normal, i.e., there exist $x \in \tilde{A}$ such that $\mu_{\tilde{A}}(x) = 1$.

(ii) $\mu_{\tilde{A}}(x)$ is a continuous mapping from $\mathbb{R}$ to $[0, 1]$ which is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a+\varepsilon}{\varepsilon}, & a-\varepsilon \leq x \leq a; \\ R(x) = \dfrac{a+\delta-x}{\delta}, & a \leq x \leq a+\delta; \\ 0, & \text{otherwise,} \end{cases} \quad (1)$$

where $L(x)$ is called left spread function and is strictly increasing in $[a-\varepsilon, a]$, and $R(x)$ is called right spread function and is strictly decreasing in $[a, a+\delta]$.

(iii) $\tilde{A}$ is a convex fuzzy subset of $\mathbb{R}$.

(iv) Support of $\tilde{A}$, $supp\ \tilde{A} = cl\{x \in \mathbb{R} : \mu_{\tilde{A}}(x) > 0\}$ is compact [2], where *cl* represents the closure of the set.

The α-cut (for $0 \le \alpha \le 1$) of a TFN $\widetilde{A}$ is defined as $A_\alpha = [A_\alpha^-, A_\alpha^+] = [a-(1-\alpha)\varepsilon, a+(1-\alpha)\delta]$.

2.1b *Fuzzy random variable (FRV)*: According to Kwakernaak [34], fuzziness may associate with an event in two ways: either in description of the event or in occurrence of the event. He also introduced the notion of FRV and fuzzy expectation. Later, FRV is further modified and simplified by [33] and [35]. Here we consider Gil *et al* [33] approach. Let us assume that (Ω, B, P) be a probability space, wherein B is the $\sigma-$ algebra of the subsets of sample space Ω and P is the probability measure. A mapping $\widetilde{X}: \Omega \to F_c(R)$ (the family of fuzzy numbers on real line) is called an FRV, if for all $\alpha \in [0, 1], X_\alpha^-: \Omega \to \mathbb{R}$ and $X_\alpha^+: \Omega \to \mathbb{R}$ are real-valued random variables. For every $\omega \in \Omega, \widetilde{X}_\alpha(\omega) = (\widetilde{X}(\omega))_\alpha = [X_\alpha^-(\omega), X_\alpha^+(\omega)]$, where $X_\alpha^-(\omega) = \inf(\widetilde{X}(\omega))_\alpha$ and $X_\alpha^+(\omega) = \sup(\widetilde{X}(\omega))_\alpha$. The expectation of FRV $\widetilde{X}$ is defined as $E[\widetilde{X}] = \{[\int_\Omega X_\alpha^- dP, \int_\Omega X_\alpha^+ dP] : 0 \le \alpha \le 1\}$. The $\alpha-$cut (for $0 \le \alpha \le 1$) of fuzzy expectation $E[\widetilde{X}]$ is $E[X_\alpha] = [E[X_\alpha^-], E[X_\alpha^+]]$.

2.1c *Intuitionistic fuzzy set (IFS)*: An intuitionistic fuzzy set (IFS) $\widetilde{A}$ on an universal set X is denoted by $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x), \upsilon_{\widetilde{A}}(x)) : x \in X\}$, where $\mu_{\widetilde{A}}(x)$: $X \to [0, 1]$ and $\upsilon_{\widetilde{A}}(x)$: $X \to [0, 1]$ are defined as the degree of membership and degree of non-membership, respectively ([37]). For each $x \in \widetilde{A}, 0 \le \mu_{\widetilde{A}}(x) \le 1, 0 \le \upsilon_{\widetilde{A}}(x) \le 1$ and $0 \le \mu_{\widetilde{A}}(x) + \upsilon_{\widetilde{A}}(x) \le 1$.

(α, β)-cut is a way to decompose an IFS into a family of crisp intervals for given values of $\alpha, \beta \in [0, 1]$ and $(\alpha+\beta) \in [0, 1]$. This family of crisp intervals make easy to accomplish the arithmetical operations on IFSs and also useful in defuzzification of an IFS. The (α, β)-cut of an IFS $\widetilde{A}$ is denoted by $A_{(\alpha,\beta)}$ and is defined as

$$A_{(\alpha,\beta)} = \left\{ \begin{array}{l} (x, \mu_{\widetilde{A}}(x), \upsilon_{\widetilde{A}}(x)) : x \in X \\ \mu_{\widetilde{A}}(x) \ge \alpha, \upsilon_{\widetilde{A}}(x) \le \beta, \alpha, \beta \in [0, 1] \end{array} \right\}.$$

2.1d *Triangular intuitionistic fuzzy number* [37]: A triangular intuitionistic fuzzy number (TIFN), $\widetilde{A} = \left\langle (a-\varepsilon, a, a+\delta); \mu_{\widetilde{A}}(x), \upsilon_{\widetilde{A}}(x) \right\rangle; a, \varepsilon \ge 0, \delta \ge 0$ is an IFS on $\mathbb{R}$, which is defined by membership function $\mu_{\widetilde{A}}(x)$ and non-membership function $v_{\widetilde{A}}(x)$ as follows:

$$\mu_{\widetilde{A}}(x) = \begin{cases} \dfrac{x-a+\varepsilon}{\varepsilon}, & a-\varepsilon \le x \le a; \\ \dfrac{a+\delta-x}{\delta}, & a \le x \le a+\delta; \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

and

$$\upsilon_{\widetilde{A}}(x) = \begin{cases} \dfrac{a-x}{\varepsilon}, & a-\varepsilon \le x \le a; \\ \dfrac{x-a}{\delta}, & a \le x \le a+\delta; \\ 1, & \text{otherwise.} \end{cases} \tag{3}$$

The score function or net membership of $\widetilde{A}$ (see, [37]) is defined as follows:

$$S_{\widetilde{A}}(x) = \begin{cases} L(x) = \dfrac{x-a+\varepsilon}{\varepsilon} - \dfrac{a-x}{\varepsilon}, & a-\varepsilon \le x \le a; \\ R(x) = \dfrac{a+\delta-x}{\delta} - \dfrac{x-a}{\delta}, & a \le x \le a+\delta; \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

2.1e *Signed distance of TIFN*: The α-cut (see, [28]) of the score function $S_{\widetilde{A}}(x)$ of TIFN $\widetilde{A}$ is obtained as

$$\begin{aligned} [A_\alpha^-, A_\alpha^+] &= [L^{-1}(\alpha), R^{-1}(\alpha)] \\ &= \left[a - \frac{(1-\alpha)\varepsilon}{2}, a + \frac{(1-\alpha)\delta}{2}\right] \end{aligned} \tag{5}$$

The signed distance (see, [36]) of TIFN $\widetilde{A}$ can be easily obtained as

$$IA = \frac{1}{2}\int_0^1 (A_\alpha^- + A_\alpha^+) = a + \frac{\delta - \varepsilon}{8} \tag{6}$$

2.1f *Intuitionistic fuzzy random variable* [38, 39]: If (Ω, B, P) be a probability space (as defined in Section 2.1b), and $F_{ifn}(R)$ be the family of IFNs on $\mathbb{R}$, then an intuitionistic fuzzy random variable (IFRV) is a mapping $\widetilde{X}: \Omega \to F_{ifn}(R)$. A triangular IFRV (TIFRV) $\widetilde{X}$ is defined for each $\omega \in \Omega$, as $\widetilde{X}(\omega) = \langle [X(\omega)-\varepsilon, X(\omega), X(\omega)+\delta]; \mu_{\widetilde{X}}(\omega), \upsilon_{\widetilde{X}}(\omega) \rangle$ is a TIFN. For each $\omega \in \Omega$, membership and non-membership functions of $\widetilde{X}(\omega)$ can be defined similar as of Eqs. (2) and (3), respectively.

2.1g *Expectation of TIFRV*: Let $\widetilde{X}$ be a TIFRV, which is defined for each $\omega \in \Omega$, as a TIFN $\widetilde{X}(\omega) = \langle [X(\omega)-\varepsilon, X(\omega), X(\omega)+\delta]; \mu_{\widetilde{X}}(\omega), \upsilon_{\widetilde{X}}(\omega) \rangle$. The fuzzy expectation $E[\widetilde{X}]$ of $\widetilde{X}$ can be obtained using sections 2.1b and 2.1e as $E[\widetilde{X}] = \{[\int_\Omega (X-\varepsilon+\alpha\varepsilon)dP, \int_\Omega (X+\delta-\alpha\delta)dP] : 0 \le \alpha \le 1\}$, which is a TIFN. The α-cut of fuzzy expectation $E[\widetilde{X}]$ is $E[X_\alpha] = [E[X]-\varepsilon+\alpha\varepsilon, E[X]+\delta-\delta\alpha]$. After using signed distance method as discussed in section 2.1e, the defuzzified value of expected value of TIRRV $\widetilde{X}$ is $E[X] + (\delta-\varepsilon)/8$.

2.2 *Continuous review inventory system [4]*

In this section, we briefly review the classical continuous review inventory system for real-valued random demand, based on (*Q*, *r*) policy. In (*Q*, *r*) policy, due to random (uncertain) demand, inventory level is continuously reviewed, and an order of size *Q* is released when inventory level reaches at a certain level *r*, called reorder level. The placed order is replenished to the system after a fixed time *L*, called lead time. After that cycle repeats itself. Due to the random demand, shortage may or may not be occurred. However, if it occurs, is backlogged. The expected total cost comprises with ordering cost, holding cost and backlogging cost, which is given as

$$EC(Q,r) = (D/Q)A + h(Q/2 + r - LD) + (D/Q)bE[X - r]^{+}, \tag{7}$$

where *D* is the annual expected demand, *A* is the cost incurs for placing an order, a cost *h* is incurred for maintenance of per item per unit time, *b* is the per item backlogging cost. *X* is a random variable denotes demand during lead time *L*. In next section, we will extend the same into IFR environment by assuming demand rate as an TIFRV and also impose the service level constraint.

3. Continuous review inventory model in IFR environment

The following assumptions are made while developing the proposed model, mathematically.

3.1 *Assumptions*

1. Demand rate is assumed as a TIFRV. As we discussed in Section 2.1g, expectation of TIFRV is a TIFN. Hence, the annual expected (average) demand is considered here an TIFN as $\widetilde{D} = \left\langle (D - \Delta_3,\, D,\, D + \Delta_4);\, \mu_{\widetilde{D}}, \upsilon_{\widetilde{D}} \right\rangle$, where Δ_3 and Δ_4 are spreads of fuzziness which can be fixed by decision makers. However, it must be restricted as $0 \le \Delta_3 \le D$ and $\Delta_4 \ge 0$ in order to ensure that demand is not negative. The variance in demand is a crisp quantity and is σ^2.
2. The inventory level is continuously reviewed, and an order is placed when it reaches to the reorder level *r*. The order quantity is delivered after a lead time *L*.
3. The IFR demand $\widetilde{X}_i$, of the *i*th unit of time is independent to the demands of the previous as well as the forthcoming epochs. So, $\widetilde{X}_i;\ i: 1, 2, \ldots$ are independent fuzzy random variables with identical mean $\widetilde{D}$ and variance σ^2. Thus, demand during *L*, $\widetilde{X}$ is convolution of the demand rate $\widetilde{X}_i$ and lead time *L* (see, [2]. We now assumed that demand during lead time is TIFRV as $\widetilde{X}(\omega) = \left\langle (X(\omega) - \Delta_1,\, X(\omega),\, X(\omega) + \Delta_2);\, \mu_{\widetilde{X}}, \upsilon_{\widetilde{X}} \right\rangle$, where $\Delta_1 \ge 0, \Delta_2 \ge 0$ *and* $X \sim N(D_L, \sigma_L^2)$ is normally distributed with mean D_L=*DL* and variance is $\sigma_L^2 = L\sigma^2$.
4. In order to restrict the shortage in a scheduling period, service level constraint is imposed to ensure that a fraction of demand must be satisfied by on-hand inventory.
5. Due to IFR demand, inventory level may suddenly decrease and shortage may occur in the system, which may drastically affect the business. To mitigate such an effect, service level constraint is imposed here. If the fraction $1 - \gamma\,(0 \le \gamma \le)$ of lot size *Q* must be fulfilled from on-hand inventory, then shortage quantity should not exceed γQ. Then service level constraint can be incorporated as:

$$E[\widetilde{X} - r]^{+} \le \gamma Q, \quad 0 \le \gamma \le 1, \tag{8}$$

where $E[\widetilde{X} - r]^{+}$ is the expected value of IFR shortage quantity $[\widetilde{X} - r]^{+}$.

3.2 *Mathematical model*

The continuous review inventory model discussed in Section 2.2 is now extended in IFR environment by assuming (as mention in assumption 3) expected annual demand as a TIFN $\widetilde{D} = \left\langle (D - \Delta_3, D, D + \Delta_4); \mu_{\widetilde{D}}, \upsilon_{\widetilde{D}} \right\rangle$ and lead time demand as a TIFRV $\widetilde{X} = \left\langle (X - \Delta_1, X, D + \Delta_2); \mu_{\widetilde{X}}, \upsilon_{\widetilde{X}} \right\rangle$, where $\mu_{\widetilde{D}}, \upsilon_{\widetilde{D}}, \mu_{\widetilde{X}}$ and $\upsilon_{\widetilde{X}}$ are membership and non-membership functions as defined in Eqs. (2) and (3). The cost function given in Eq. (7) can be extended in IFR sense as follow:

$$\widetilde{EC}(Q,r) = (\widetilde{D}/Q)A + h(Q/2 + r - L\widetilde{D}) + (\widetilde{D}/Q)bE[\widetilde{X} - r]^{+} \tag{9}$$

Now, in order to accomplish an easier arithmetical operation on IFNs, we rearrange the Eq. (9) as follows:

$$\widetilde{EC}(Q,r) = \left(\frac{A}{Q} + \frac{bE[\widetilde{X} - r]^{+}}{Q} - hL\right)\widetilde{D} + h\left(\frac{Q}{2} + r\right) \tag{10}$$

Subject to the service level constraint given in Eq. (8).

3.3 *Defuzzified cost function*

In this section, we use fuzzy expectation (Section 2.1g) and signed distance (Section 2.1e) methods to convert the IFR cost function into deterministic one. We first approximate an interval of IFR cost function by using membership, non-membership and score functions, and then employ the signed distance method [36].

By using simple arithmetic operations on interval ([16]), α-cut of $\widetilde{EC}(Q,r)$ is obtained here as follows:

$$\begin{aligned} EC_\alpha(Q,r) &= \left[EC_\alpha^-(Q,r), EC_\alpha^-(Q,r)\right] \\ &= \left[\left(\frac{A}{Q} + \frac{b\left(E[\widetilde{X}-r]^+\right)_\alpha^-}{Q} - hL\right)D_\alpha^- \right. \\ &\quad + h\left(\frac{Q}{2}+r\right), \left(\frac{A}{Q} + \frac{b\left(E[\widetilde{X}-r]^+\right)_\alpha^+}{Q} - hL\right)\widetilde{D}_\alpha^+ \\ &\quad \left. + h\left(\frac{Q}{2}+r\right)\right] \end{aligned} \tag{11}$$

We now proceed to obtain $D_\alpha = [D_\alpha^-, D_\alpha^+]$, $\left(E[\widetilde{X}-r]^+\right)_\alpha^-$ and $\left(E[\widetilde{X}-r]^+\right)_\alpha^+$ in the following sections.

3.2a *Evaluation of* $D_\alpha = [D_\alpha^-, D_\alpha^+]$: The annual expected demand $\widetilde{D} = \left\langle (D-\Delta_3,\, D,\, D+\Delta_4); \mu_{\widetilde{D}}, \upsilon_{\widetilde{D}} \right\rangle$ is a TIFN, which is defined with membership and non-membership functions as follows:

$$\mu_{\widetilde{D}}(y) = \begin{cases} \dfrac{y-(D-\Delta_3)}{\Delta_3}, & D-\Delta_3 \le y \le D; \\ \dfrac{(D+\Delta_4)-y}{\Delta_4}, & D \le y \le D+\Delta_4; \\ 0, & \text{otherwise.} \end{cases} \tag{12}$$

and

$$v_{\widetilde{D}}(y) = \begin{cases} \dfrac{D-y}{\Delta_3}, & D-\Delta_3 \le y \le D; \\ \dfrac{y-D}{\Delta_4}, & D \le y \le D+\Delta_4; \\ 1, & \text{otherwise.} \end{cases} \tag{13}$$

The score function or net membership, as discussed in Eq. (3), of $\widetilde{D}$ is obtained as follows:

$$S_{\widetilde{D}}(y) = \begin{cases} \dfrac{2(y-D)+\Delta_3}{\Delta_3}, & D-\Delta_3 \le y \le D; \\ \dfrac{2(D-y)+\Delta_4}{\Delta_4}, & D \le y \le D+\Delta_4; \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

The α-cut (for $0<\alpha\le 1$) of score function $S_{\widetilde{D}}(y)$ (as said in Eq. (5)) is

$$[D_\alpha^-(\alpha),\, D_\alpha^+(\alpha)] = \left[D - \frac{(1-\alpha)\Delta_3}{2}, D + \frac{(1-\alpha)\Delta_4}{2}\right] \tag{15}$$

3.2b *Evaluation of* $\left[\left(E[\widetilde{X}-r]^+\right)_\alpha^-,\ \left(E[\widetilde{X}-r]^+\right)_\alpha^+\right]$: Due to TIFRV demand, shortage occurrence phenomenon and shortage quantity both are IFRVs, and depend on reorder level and the spreads of TIFRV demand during lead time. Shortage will occur when $\widetilde{X} > r$, and shortage quantity is $[\widetilde{X}-r]^+$. For simplicity we denote $\widetilde{Y} = [\widetilde{X}-r]^+$, where $\widetilde{X} - r = \left\langle (x-r-\Delta_1,\, x-r,\, x-r+\Delta_2); \mu_{\widetilde{X}}, \upsilon_{\widetilde{X}} \right\rangle$. Since, x is a variable, so, for fixed values of r, Δ_1 and Δ_2, there are four possible cases (i) $x \in [r+\Delta_1, \infty)$, (ii) $x \in [r,\, \Delta_1 + r]$, (iii) $x \in [r-\Delta_2,\, r]$ and (iv) $x \in (-\infty,\, r-\Delta_2]$ as shown in figures 1 to 4. Now, we find the IFR shortage quantity for each case and then finally merge all to find the intuitionistic fuzzy expected value.

Case (*i*) When $x \in [r+\Delta_1,\, \infty)$, then membership and non-membership functions of $\widetilde{Y}$ are shown in figure 1. The α-cut and β-cut of $\widetilde{Y}$ are: $Y_\alpha = (x-r-\Delta_1+\alpha\Delta_1,\, x-r+\Delta_2-\alpha\Delta_2)$, $0\le\alpha\le 1$ and $Y_\beta = [x-r-\beta\Delta_1,\, x-r+\beta\Delta_2]$, $0\le\beta\le 1$. The said two functions are:

$$\mu_{\widetilde{Y}}(y) = \begin{cases} \dfrac{y-(x-r-\Delta_1)}{\Delta_1}, & x-\Delta_1-r \le y \le x-r; \\ \dfrac{(x-r+\Delta_2)-y}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

and

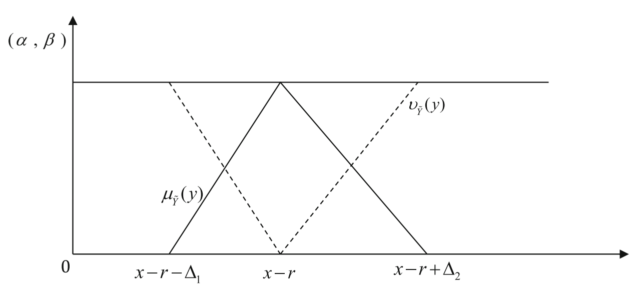

Figure 1. Intuitionistic fuzzy shortage quantity when $x \in [r+\Delta_1,\, \infty)$.

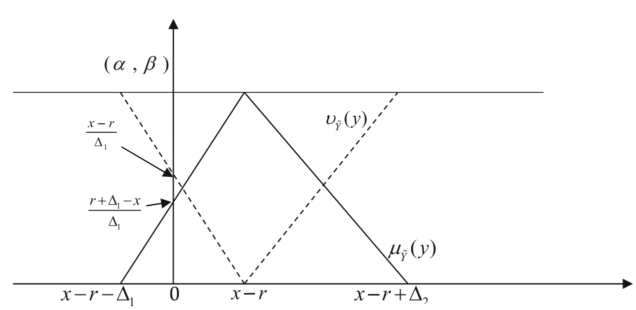

Figure 2. Intuitionistic fuzzy shortage quantity when $x \in [r,\, \Delta_1 + r]$.

$$\upsilon_{\tilde{Y}}(y) = \begin{cases} \dfrac{(x-r)-y}{\Delta_1}, & x-\Delta_1-r \le y \le x-r; \\ \dfrac{y-(x-r)}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 1, & \text{otherwise.} \end{cases} \quad (17)$$

The score function of $\widetilde{Y}$ is:

$$S_{\tilde{Y}}(y) = \begin{cases} \dfrac{2\{y-(x-r)\}+\Delta_1}{\Delta_1}, & x-\Delta_1-r \le y \le x-r; \\ \dfrac{2\{(x-r)-y\}+\Delta_2}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

The α-cut of the score function is:

$$\left[Y_\alpha^-(\alpha), Y_\alpha^+(\alpha)\right] = \left[(x-r)-\frac{(1-\alpha)\Delta_1}{2}, (x-r)+\frac{(1-\alpha)\Delta_2}{2}\right], \quad 0 \le \alpha \le 1 \quad (19)$$

Case (*ii*) When $x \in [r, \Delta_1 + r]$, the membership and non-membership functions are shown in figure 2. The α-cut and β-cut of $\widetilde{Y}$ for this case are:

$$Y_\alpha = \begin{cases} [0,\ x-r+\Delta_2-\alpha\Delta_2],\ 0 \le \alpha \le \dfrac{r-x+\Delta_1}{\Delta_1}; \\ [x-r-\Delta_1+\alpha\Delta_1,\ x-r+\Delta_2 \\ -\alpha\Delta_2],\ \dfrac{r-x+\Delta_1}{\Delta_2} \le \alpha \le 1. \end{cases} \quad (20)$$

and

$$Y_\beta = \begin{cases} [x-r-\beta\Delta_1,\ x-r+\beta\Delta_2],\ 0 \le \beta \le \dfrac{x-r}{\Delta_1}; \\ [0,\ x-r+\beta\Delta_2],\ \dfrac{x-r}{\Delta_1} \le \beta \le 1. \end{cases} \quad (21)$$

The membership function of $\widetilde{Y}$ is:

$$\mu_{\tilde{Y}}(y) = \begin{cases} \dfrac{y-(x-r-\Delta_1)}{\Delta_1}, & 0 \le y \le x-r; \\ \dfrac{(x+\Delta_2-r)-y}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \quad (22)$$

The non-membership function of $\widetilde{Y}$ is:

$$\upsilon_{\tilde{Y}}(y) = \begin{cases} \dfrac{(x-r)-y}{\Delta_1}, & 0 \le y \le x-r; \\ \dfrac{y-(x-r)}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 1, & \text{otherwise.} \end{cases} \quad (23)$$

Hence, the score function of $\widetilde{Y}$ is:

$$S_{\tilde{Y}}(y) = \begin{cases} \dfrac{2\{y-(x-r)\}+\Delta_1}{\Delta_1}, & 0 \le y \le x-r; \\ \dfrac{2\{(x-r)-y\}+\Delta_2}{\Delta_2}, & x-r \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \quad (24)$$

and the α-cut of score function for this case is:

$$\left[Y_\alpha^-(\alpha), Y_\alpha^+(\alpha)\right] = \left[(x-r)-\frac{(1-\alpha)\Delta_1}{2}, (x-r)+\frac{(1-\alpha)\Delta_2}{2}\right]. \quad (25)$$

Case (*iii*) When $x \in [r-\Delta_2, r]$, the membership and non-membership functions are delineated in figure 3. The α-cut and β-cut of $\widetilde{Y}$ for this case are:

$$Y_\alpha = \begin{cases} [0,\ 0], & \dfrac{x-r+\Delta_2}{\Delta_2} \le \alpha \le 1; \\ [0,\ x+\Delta_2-r-\alpha\Delta_2], & 0 \le \alpha \le \dfrac{x-r+\Delta_2}{\Delta_2}. \end{cases} \quad (26)$$

$$Y_\beta = \begin{cases} [0,\ 0], & 0 \le \beta \le \dfrac{r-x}{\Delta_2}; \\ [0,\ x-r+\beta\Delta_2], & \dfrac{r-x}{\Delta_2} \le \beta \le 1. \end{cases} \quad (27)$$

The membership and non-membership functions of $\widetilde{Y}$ for this case are:

$$\mu_{\tilde{Y}}(y) = \begin{cases} \dfrac{(x-r+\Delta_2)-y}{\Delta_2}, & 0 \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \quad (28)$$

$$\upsilon_{\tilde{Y}}(y) = \begin{cases} \dfrac{y-(x-r)}{\Delta_2}, & 0 \le y \le x-r+\Delta_2; \\ 1, & \text{otherwise.} \end{cases} \quad (29)$$

The score function is:

$$S_{\tilde{Y}}(y) = \begin{cases} \dfrac{2\{(x-r)-y\}+\Delta_2}{\Delta_2}, & 0 \le y \le x-r+\Delta_2; \\ 0, & \text{otherwise.} \end{cases} \tag{30}$$

Similarly, the right end point of expected value of score function is:

$$\begin{aligned} E(Y_\alpha^+) &= \int_{-\infty}^{\infty} Y_\alpha^+(\alpha) f(x)dx \\ &= \int_{r-\Delta_2}^{r} \frac{2(x-r)+\Delta_2(1-\alpha)}{2} f(x)dx + \int_{r}^{r+\Delta_1} \frac{2(x-r)+\Delta_2(1-\alpha)}{2} f(x)dx \\ &\quad + \int_{r+\Delta_1}^{\infty} \frac{2(x-r)+\Delta_2(1-\alpha)}{2} f(x)dx \\ &= \frac{(1-\alpha)\Delta_2}{2}\left[1-\Phi(\frac{r-\Delta_2-D_L}{\sigma_L})\right] + \sigma_L^2 \phi\left(\frac{r-\Delta_2-D_L}{\sigma}\right) + (D_L-r)\left[1-\Phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right] \end{aligned} \tag{34}$$

Hence, α-cut of score function is:

$$\left[Y_\alpha^-(\alpha), Y_\alpha^+(\alpha)\right] = \left[0, (x-r) + \frac{\Delta_2(1-\alpha)}{2}\right] \tag{31}$$

Case *(iv)* When $x \in (-\infty, r-\Delta_2]$, then there will be no shortage as shown in figure 4. The membership and non-membership functions are 0 and 1 for all $y \in \widetilde{Y}$. The α-cut of $\widetilde{Y}$ is [0, 0] and β-cut of $\widetilde{Y}$ is [0, 0]. $\mu_{\tilde{Y}}(y) = 0$ and $\upsilon_{\tilde{Y}}(y) = 1$. Consequently, $S_{\widetilde{Y}}(y) = 0$. Hence, α-cut of score function is:

$$\left[Y_\alpha^-(\alpha), Y_\alpha^+(\alpha)\right] = [0, 0] \tag{32}$$

Now, we combine all the above discussed cases in order to estimate the α-cut of expected shortage quantity. For this, we merge the Eqs. (19), (25), (31) and (32). The left end point of expected value of score function is:

$$\begin{aligned} E(Y_\alpha^-) &= \int_{-\infty}^{\infty} Y_\alpha^-(\alpha) f(x)dx \\ &= \int_{r}^{r+\Delta_1} \frac{(\alpha-1)\Delta_1 + 2(x-r)}{2} f(x)dx \\ &\quad + \int_{r+\Delta_1}^{\infty} \frac{(\alpha-1)\Delta_1 + 2(x-r)}{2} f(x)dx \\ &= \frac{(\alpha-1)\Delta_1}{2}\left[1-\Phi\left(\frac{r-D_L}{\sigma_L}\right)\right] \\ &\quad + (D_L-r)\left[1-\Phi\left(\frac{r-D_L}{\sigma_L}\right)\right] + \sigma_L^2\phi\left(\frac{r-D_L}{\sigma_L}\right) \end{aligned} \tag{33}$$

where $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{\frac{-1}{2}z^2}$ and $\Phi(z) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{z} e^{\frac{-1}{2}t^2} dt$ are standard normal distribution and cumulative normal distribution functions, respectively.

After substituting the Eqs. (15), (33) and (34) in Eq. (11), we can easily find the α-cut of fuzzy cost function $\widetilde{EC}(Q,r)$. The signed distance (see, [36]) of $\widetilde{EC}(Q,r)$ is denoted here by $IEC(Q,r)$ and is obtained as follows:

$$\begin{aligned} IEC(Q,r) &= \frac{1}{2}\int\limits_0^1 (EC_\alpha^- + EC_\alpha^+)d\alpha \\ &= \left(D + \frac{1}{8}(\Delta_4 - \Delta_3)\right) \\ &\quad \left\{\frac{A}{Q} - hL + \frac{b}{Q}(D_L - r)\left[1 - \frac{1}{2}\Phi\left(\frac{r-D_L}{\sigma_L}\right)\right.\right. \\ &\quad \left. - \frac{1}{2}\Phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right]. \\ &\quad + \frac{\sigma_L^2}{2}\frac{b}{Q}\left[\phi\left(\frac{r-D_L}{\sigma_L}\right)\right. \\ &\quad \left. + \phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right. \\ &\quad + \frac{1}{4}\frac{b}{Q}\left[\frac{\Delta_2}{2}\left(1 - \Phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right)\right. \\ &\quad \left.\left.\left. - \frac{\Delta_1}{2}\left(1 - \Phi\left(\frac{r-D_L}{\sigma_L}\right)\right)\right]\right]\right\} \\ &\quad + h\left(\frac{Q}{2} + r\right) \end{aligned} \tag{35}$$

The IFR service level constraint is mathematically given in Eq. (8). That soft constraint is here converted into equivalent deterministic one by using the methods of fuzzy expectation and signed distance. The equivalent deterministic constraint is

$$(D_L - r)\left[1 - \frac{1}{2}\Phi\left(\frac{r-D_L}{\sigma_L}\right) - \frac{1}{2}\Phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right] + \frac{\sigma_L^2}{2}\left[\phi\left(\frac{r-D_L}{\sigma_L}\right) + \phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right] + \frac{1}{4}\left[\frac{\Delta_2}{2}\left\{1 - \Phi\left(\frac{r-\Delta_2-D_L}{\sigma_L}\right)\right\} - \frac{\Delta_1}{2}\left\{1 - \Phi\left(\frac{r-D_L}{\sigma_L}\right)\right\}\right] \le \gamma Q \quad (36)$$

4. Solution methodology

Eq. (35) represents the equivalent deterministic cost function of the proposed IFR inventory model. Our target is to minimize it subject to the service level constraint (36). For this we first show that for a given value of Q, the cost function $IEC(Q, r)$ is convex with respect to r. Foremost we differentiate Eq. (35) with respect to Q.

$$\frac{\partial IEC(Q,r)}{\partial Q} = -\frac{I(\widetilde{D})[A + bIE[\widetilde{X} - r]^+]}{Q^2} + \frac{h}{2}$$

and

$$\frac{\partial^2 IEC(Q,r)}{\partial Q^2} = \frac{2I(\widetilde{D})[A + bIE[\widetilde{X} - r]^+]}{Q^3} > 0.$$

Thus, $IEC(Q, r)$ is convex function with respect to Q. Hence for a given value of r, the local optimal Q_{loc} can be obtained from the equation $\frac{\partial IEC(Q,r)}{\partial Q} = 0$. This implies $Q_{loc} = \sqrt{\frac{2I(\widetilde{D})(A+bIE[\widetilde{X}-r]^+)}{h}}$. Now, we substitute $Q = \sqrt{\frac{2I(\widetilde{D})(A+bIE[\widetilde{X}-r]^+)}{h}}$ in Eq. (35).

$$IEC(r) = \sqrt{2I(\widetilde{D})h(A + bIE[\widetilde{X} - r]^+)} + h(r - I(\widetilde{D})L) \quad (37)$$

Now,

$$\frac{dIEC(r)}{dr} = \sqrt{\frac{hI(\widetilde{D})}{2(A + bIE[\widetilde{X} - r]^+)}}\, b\frac{dIE[\widetilde{X} - r]^+}{dr} + h \quad (38)$$

Optimal r^* can be obtained from the equation $\frac{dIEC(r)}{dr} = 0$ that must satisfy $\left[\frac{d^2IEC(r)}{dr^2}\right]_{r=r^*} > 0$. If $Q_{loc} \ge \frac{IE[\widetilde{X}-r^*]^+}{\gamma}$ and $\left[\frac{d^2IEC(r)}{dr^2}\right]_{r=r^*} > 0$, then optimal order quantity is $Q_{opt} = Q_{loc}$, otherwise $Q_{opt} = \frac{IE[\widetilde{X}-r]^+}{\gamma}$, and accordingly the optimal total cost can be calculated from (35).

5. Illustrative example

The mathematical model and methodology developed in sections 3-4 are testified here with numerical experiment. For this we consider a hypothetical problem of a shoe retailer. The data set presented here is partially taken from [6] after adequate modification as per our IFR model's requirement. A small shoe retailer procures shoes from a nearby manufacturer on an ordering cost of ₹200 per order. The retailer's past data record and experience indicate that there is not a fixed trend in demand pattern. So, the final estimation of demand for forthcoming year is a IFRV with mean (550, 600, 650) and standard deviation 25 units. That means average annual demand varies between 550 units pair of shoes to 650 units. This demand pattern can be viewed as TIFRV that completely described with probability distribution, fuzzy membership and non-membership functions. Here, we assume that randomness follows

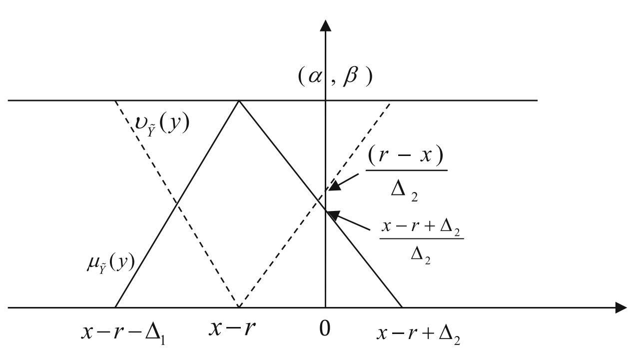

Figure 3. Intuitionistic fuzzy shortage quantity when $x \in [r - \Delta_2, r]$.

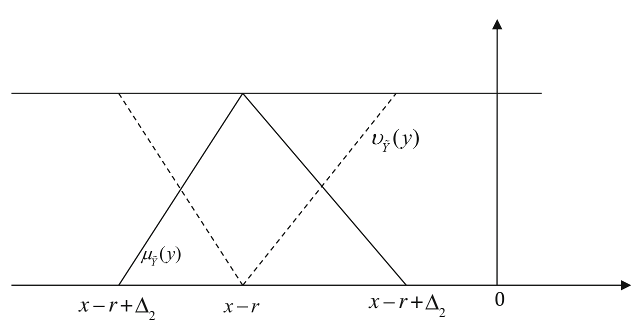

Figure 4. Intuitionistic fuzzy shortage quantity when $x \in (-\infty, r - \Delta_2]$.

Table 2. Sensitivity analysis.

Parameters	% Change	Value	r^*	Q^*	$IEC(Q^*, r^*)$
Δ_3	−37.5%	31.25	86.1372	515.036	10170.1
	−25%	37.5	86.1373	514.702	10165.8
	−12.5%	43.75	86.1373	514.367	10161.5
	0	50	86.1374	514.033	10154.3
	12.5%	56.25	86.1374	513.698	10153
	25%	62.5	86.1375	513.363	10148.7
	37.5%	68.75	86.1375	513.028	10144.4
Δ_4	−37.5%	31.25	86.1375	513.028	10144.4
	−25%	37.5	86.1375	513.363	10148.7
	−12.5%	43.75	86.1374	513.698	10153
	0	50	86.1374	514.033	10157.3
	12.5%	56.25	86.1373	514.367	10161.5
	25%	62.5	86.1373	514.702	10165.8
	37.5%	68.75	86.1372	515.036	10170.1
b	−37.5%	31.25	86.161	411.857	8114.21
	−25%	37.5	86.151	448.51	8847.06
	−12.5%	43.75	86.1435	482.385	9524.42
	0	50	86.1374	514.033	10157.3
	12.5%	56.25	86.1324	543.842	10753.3
	25%	62.5	86.1281	572.1	11318.4
	37.5%	68.75	86.1245	599.026	11856.9
h	−37.5%	12.5	86.1197	650.226	8050.47
	−25%	15	86.1261	593.566	8810.77
	−12.5%	17.5	86.1319	549.53	9508.7
	0	20	86.1374	514.033	10157.3
	12.5%	22.5	86.1425	484.63	10765.5
	25%	25	86.1473	459.756	11339.9
	37.5%	27.5	86.1519	438.355	11885.5
L	−37.5%	5	51.1372	370	7278.74
	−25%	6	62	422.832	8328.11
	−12.5%	7	74.47	470.639	9287
	0	8	86.1374	514.033	10157.3
	12.5%	9	97.7764	553.268	10944
	25%	10	109.397	588.702	11654.3
	37.5%	11	121	620.75	12296.5

normal distribution. The manufacturer takes 8 weeks in delivering the product after receiving the order from the retailer side. A maintenance (including holding) cost of ₹20 incurs for a pair of shoes at the retailer's warehouse. The retailer is penalized ₹50 per pair of shoes for not satisfying the customer's demand.

From the above information, we have: $\widetilde{D} = (550, 600, 650)$, A = ₹200 per order, h = ₹20, L = 8 weeks ⇒ L = 8/52 year, b = ₹50, Δ_3=50, Δ_4=50, γ=0.2, σ=25. The above information also indicates that demand during lead time is a TIFRV, $\widetilde{X} = (X - \Delta_1, X, X + \Delta_2)$, where $X \sim N(600, 25^2)$, $\Delta_1 = L\Delta_3$, $\Delta_2 = L\Delta_4$ and $\sigma_L = \sqrt{L}\sigma = 9.8$.

The optimal policy for the retailer is: $r^* = 86.1374$ units, $Q^* = 514.033$ units and the total cost is $IEC(r^*, Q^*)$ = ₹10157.3.

5.1 *Sensitivity analysis and managerial insight*

In order to delineate the managerial insight and to prove the robustness of the proposed model, sensitivity analysis by changing the value of key parameters is carried out, here. For this we change Δ_1, Δ_2, Δ_3, Δ_4, h, b and L from −37.5 to 37.5% with the increment of 12.5% in each parameter. The optimal policies for these changes are shown in Table 2 and also depicted in figure 5.

Sensitivity of lead time: When lead time is increased from 5 to 11 weeks, then the reorder level, order quantity and *IEC* increase, rapidly. It is because, for a larger lead time, the organization has to keep more quantity as a safety stock, consequently, order quantity and inventory holding cost increase, and hence total cost. Thus, we can conclude that lead time highly influences the optimal policy. Thus, management should try to reduce it.

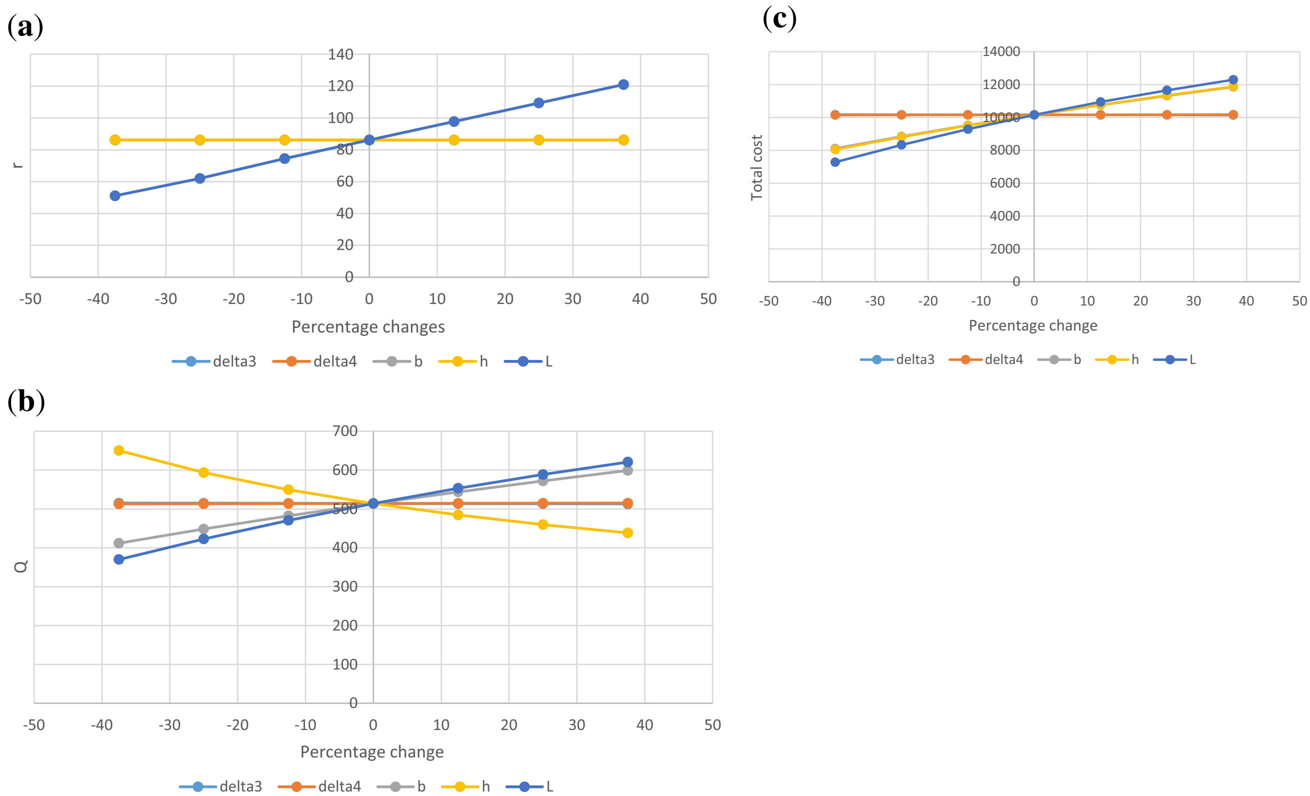

Figure 5. **a** Sensitivity in order level. **b** Sensitivity in order quantity. **c** Sensitivity in total cost.

Sensitivity of holding and backlogging costs: When holding cost is increased, reorder level (figure 5a) is almost fixed, order quantity (figure 5b) decreases while total cost (figure 5c) increases. Order quantity decreases because higher holding cost discourage in keeping of larger stock. Increasing holding cost also increases the total cost. When backordering cost is increased, reorder level is almost fixed and as expected, order quantity and total cost increase.

Sensitivity of spreads of TIFRV demand: Changes in Δ_3 and Δ_4 make change as Δ_1=$L\Delta_3$ and Δ_2=$L\Delta_4$. When Δ_4 is fixed and Δ_3 increases, then r is remains constant while Q and total cost slightly increase. When Δ_3 is fixed and Δ_4 is increased, then r remains constant while Q and total cost slightly increase as shown in figures 5a, 5b and 5c. These effects are noticed because changes in left and right spreads of TIFRV demand, significantly change the defuzzified demand. Thus, we conclude that spreads in intuitionistic fuzzy random demand influence the optimal policy.

6. Conclusion

In this paper, a continuous review inventory system has been developed in IFR environment by assuming demand rate as a TIFRV and average expected demand as a TIFN. Fuzzy service level constraint is imposed to minimize the stock out situation and to maximize the customer satisfaction level. Furthermore, fuzzy expectation and signed distance methods are also used in mathematical formulation and optimization of the proposed IFR inventory model. In this process, we developed a methodology to find the expected value of IFR shortage quantity. Finally, we illustrated the model and solution methodology with the help of numerical experiment. In order to prove the robustness and to delineate the managerial insights, the sensitivity analysis for changing the values of key parameters is also carried out. It is noticed that higher values of spreads of fuzziness of TIFRV demand and larger lead time significantly influence the optimal policy. Larger lead time significantly increases the overall cost of the system. This theoretical model can be applied in real life businesses where customers' demand is imprecise and random, especially, in a new business where demand is totally unknown and is just predicted on presumption of forthcoming scenarios.

The spreads of fuzziness can be reduced over the time due to the learning [15]. Furthermore, lead time can also be reduced through some investment. Leaning effect and lead time reduction may improve the customers' satisfaction level at the reduced cost. Thus, the proposed model can be further extended by assuming learning effect in FR demand, and lead time reduction possibility can be also considered as a potential research direction.

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