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Ordering and pricing decisions of regular products in a supply chain with the effects of product-specific gift cards

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Abstract. Fast development and increasing popularity of gift cards motivates many retailers to offer such cards to their customers. This paper aims to examine the effects of gift cards and inflation on optimal ordering policy for regular products and to analyze the advantages of providing product-specific gift cards. Two models are proposed. In the first model, benefit function of retailer and supplier are considered separately and in the second model both of them are considered as a chain, and benefit function is optimized for a limited planning horizon. In this inventory system, the current value of the whole profit for the limited planning horizon is first modeled. Next, an algorithm is proposed for each model to obtain the optimal order for retailer and supplier, the value of discount for gift cards and the number of replacement retailer and supplier during the planning horizon. Eventually, we show our model through numerical examples and also report the sensitivity analysis to find some managerial insights.

Keywords. Inventory; gift card; pricing; supply chain coordination.

1. Introduction

The present competitive market leads sellers to use different new techniques such as offering gift card in order to survive on the market and develop their selling market. Providing gift card may cost retailers and influence on profitability and amount of ordering. For example, in 2005 overall selling of gift cards was more than 60 billion dollars in the United States Offenberg, [1] and 82 percent of big retailer's stores presented gift card in Canada Bahta *et al* [2]. Gift cards have gained significant popularity in many countries. One example of gift card is Mooncake gift cards in China. Many companies, governmental parts and universities give gift cards to their employees before Mooncake festival to use for purchasing during different festivals. In fact, giving gift cards is a kind of discount to costumers in a certain time (the discount offered is a volume discount) and this discount causes to increase in sale.

Gift cards are two types with payment time. Free gift card which gives a percent of discount and encourages the customer to return faster and re-purchase. Pre-paid gift card in addition to incitement the customer for purchasing, which is used for presenting to others. Since customers pay the money of these gift cards before receiving their goods, seller can invest these monies in different fields. From another point of view, these cards can be divided into three categories: product-specific designed for products, retailer-specific designed for retailers and network-branded registered on networks.

Product-specific cards are those which are customized to purchase one or more products, and the name and number of specific products are written on these cards. The second type of gift cards is the retailer-specific cards. The customer can use these cards to purchase any goods offered in the retail stores. Some of the well-known retailers in the world such as Wal-Mart, Tesco or Carrefour, offer this type of retailer-specific cards on a daily basis. The third type, network-branded registered network cards could be traded on any of credit card basis such as American Express Gift Card Horne, [3] and Okcard issued by Bailian Group in China. The product-specific gift cards are usually used for seasonal products. In fact, the increased demand in a short period of time for some seasonal products leads to the use of this type of gift cards. In this paper, we focus on product-specific gift cards and will develop pre-paid gift cards.

The product-specific gift card has many advantages for retailers. First of all, since there is a certain time between offering gift cards and using them by customers, retailers gain significant amount of money before presenting their products. Hence when there is a shortage of money in company, retailers can utilize the motivational policy of gift card. In the other words, retailers receive money

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through selling gift card before selling the product to costumers. Secondly, customers can present gift cards to other people and this potentially increases the potential customers. In fact, gift card increases the demand for products through customers who do not buy without gift card. Finally, some people who have gift card may never buy with their gift cards because it can be lost expired. Therefore, in this case, we have received the price of the product without selling it. Given that gift card money is received before the sale of goods, so inflation and its impact must also be considered. When retailers use gift cards, the following questions arise for them. Does issuing gift card increase profit in a certain planning horizon? How much does it increase? How much discount should be given to gift cards? How much does gift card influence on the amount of ordering?

Khouja *et al* [4] have developed a free gift card on the sales newspaper model, which does not include the cost of the order. Zhang *et al* [5] a non-free gift card have been developed on the sales newspaper model, which also takes into account the financial impact of sold gift cards. Khouja and Zhou[6] developed a non-free gift card on the sales newspaper model, which includes purchase and ordering costs. The work done above is one level and only one level terms (for example retailer) are considered but then Khouja and Zhou [6], Khouja *et al* [7], Li *et al* [8] and Pan *et al* [9] considered the terms of income and cost of two levels (retailer and supplier) and developed the gift card. But one of the most important issues in inventory control is maintenance costs, which are not taken into account in the work done, and also the impact of inflation in the supply chain is not considered.

In this paper, a two-level supply chain model is developed where in addition to the retailer's costs, supplier's costs are also included. Accordingly, the optimal decisions for the supplier's order quantity, the retailer's order quantity, and the gift card price are obtained.

According to the above-mentioned research, it seems that there is still no research that has examined gift cards in regular products. Thus, in this research two optimal inventory control models with issuing product-specific gift card during the planning horizon are developed by considering inflation.

Using our model, we address several questions about the use of" product-specific and pre-paid" gift cards. What are the optimal replenishment policies for members of chain in presence of the strategy of gift card and inflation? How to propose a proper incentive mechanism to motivate supply chain (SC) members for accepting these optimal policies? How the percentage of gift card discount affect the model? How the inflation affects the model?

In order to give an appropriate response to the aforementioned questions, a two-echelon supply chain includes a supplier and a retailer is formed in which the gift card and inflation are also considered. In the first model, each of the members wants to optimize his own profits but in the second one they decide together.

Our study generates several important findings. When supply chain members use gift cards, more customers are attracted and the total number of customers in the chain increases. As a result, brand awareness also increases. Due to receiving the gift card money before selling the goods and not visiting the percentage of customers who bought the gift card, the chain profit will also increase.

Consider the conditions in which the retailer uses an inventory control model of goods ordered to the supplier in any H period for supplying his/her own goods. Besides, the retailer sells a number of gift cards to institutions, companies, and individuals at a price lower than the value of the goods before the period H and the sale of goods. Therefore, the retailer earns some money before selling goods from which he/she can make profits. Now, the revenue of retailer and supplier should be maximized in the inflationary conditions.

We configure the SC under two separate conditions:

1. A model in which each part of SC will decide only based on the reliable profit.
2. An open model in which each part will engage in order to maximize capital profit of the whole system.

The major contributions of this research are as follows.

- The effect of gift card on the ordering amounts of members of chain is examined.
- We develop a two-echelon supply chain which includes a supplier and a retailer by considering gift card.
- To show developed models and algorithms, numerical example is presented.
- Several regulations or rules about gift cards applied in actual life are considered.
- We find the optimal order quantity for the retailer and the supplier as well as the optimal price of the gift card for the retailer.

The rest of this paper is organized as follows. In section 2, the literature review is defined. In section 3, the problem and the notations are defined. In section 4, the problem is formulated. Numerical example is illustrated and a sensitivity analysis is performed on some parameters in section 5. Some indication of future research and the conclusions are drawn in section 6.

2. Literature review

Our article is related to three topics: motivational gift card policy, inventory control problems with motivational policies, and motivational policies supply chain management.

2.1 *Motivational gift card policy*

Aiming to get retailer's inventory level and optimum value of gift card, Khouja *et al* [4] used a model originally presented by Cachon and Swinney [10] and developed a model for analyzing the efficacy of gift card. They also compared the function of free gift card strategy with discount strategy and provided some conditions where the gift cards perform better than discount. In their model, each season is divided into two parts. In the first part, the price of the product is *p* and in second part, the remaining products are sold with discount. They also divide customers to three categories: 1- bargain- hunting 2- myopic 3- strategic customers. Zhang *et al* [11] proposed an optimized ordering model for news vendor problem by gift card, and compared it with classic news vendor model. They analyzed the improvement of the model profit by increasing in demand through gift card, receiving money at selling gift card time and non-redemption gift cards. In this paper, goods are supposed to be seasonal. The demand for these goods is denoted by *D* which is considered constant but a percentage of the sold goods, *S*, is added to the demand. In this article, the retail profit function is maximized due to the constant cost of ordering and the cost of buying and selling. Finally, once the amount of the discount on the gift card is considered as a parameter and the optimal amount of the order is obtained and once the discount amount on the gift card is considered as a variable and the optimal amount of the order amount and the discount price of the gift card is obtained. Khouja and Zhou [6] examined the effects of selling gift card on the optimum amount of inventory of Holiday product in pre-holiday period and pricing these products after holiday period form retailer's viewpoint. They supposed that a percent of gift cards will never be returned and people who buy with gift card are interested in Holiday product and some do not prefer to buy them. In fact, this model is similar to news vendor model where selling period is divided into two parts: Period before holiday and period after holiday when vendor must specify the price of the product at the initial period.

2.2 *Inventory control problems with another motivational policies*

In the literature, in addition to gift card incentive policy, other incentive policies have been used for EOQ models and newspaper models for sales incentive policies. In trade credit/ delay payment policy, supplier consider a period for retailer to pay the purchasing cost after receiving the products. Indeed if the remained purchasing cost become paid before considered time, so no additional cost should be paid by purchaser. Goyal, dating back to 1985, was the first to introduce the EOQ (Economic Order Quantity) business credit financing model on inventory issues. After him, many research studies have been done in this direction. Among the notable works in this case are the following. Aggarwal and Jaggi [12], Taleizadeh *et al* [13–18], Lashgari *et al* [19], Diabat *et al* [20], Gholamian and Ebrahimzadeh-Afruzi [21], Giri and Sharma [3], Taleizadeh *et al* [22], Zhang *et al* [23] and Thangam and Uthayakumar [24] are among the authors who have developed the delay payment incentive policy, in fact they have addressed the impact of delay payment on retailer profitability, increasing retailer demand, and retailer costs including maintenance, ordering and payment interest.

Sometimes suppliers offer special selling prices to stimulate sales or reduce inventory. The buyer can make two decisions against this opportunity. (1) Continuing with a specific order or (2) with a normal order quantity. If a particular order is to be placed, it is difficult to determine its optimal size. As discussed in Silver *et al* [25], research on determining whether to place a special order and, if so, the quantity dates back at least as far as an approach proposed by Naddor [26] for the structurally similar problem of determining the amount to purchase immediately before a price increase. Naddor's approach, which he acknowledged would not necessarily yield an optimum solution, is to determine, over the time horizon determined by the size of the special order, whether placing a special order at the lower price versus placing a normal-sized order at the lower. Hadley and Whitin [27], Tersine and Toelle [22], Munson and Hu [28] , Taleizadeh *et al* [29] , Taleizadeh *et al* [30]; Taleizadeh *et al* [5], Bera *et al* [31], Pasandideh *et al* [32] and Archetti *et al* [33] are among the authors who have developed the incremental discounts incentive policy.

2.3 *Supply chain management with motivational policies*

Supply chain coordination makes the whole chain a single, centralized decision maker. Decisions that are locally optimal for each member are globally inefficient Whang [34]. Coordination mechanisms encourage SC members to make decisions that are optimal for the entire chain Giannoccaro and Pontrandolfo [35]. At best, the performance of a decentralized chain can be equivalent to that of a centralized chain. When a SC has a decentralized structure and each SC member has an independent business and their action plan should be implemented by an incentive scheme for a fair share of the benefits obtained Li and Wang [36]. The participation of all members in the coordination model can be guaranteed using an incentive scheme. In general, a coordination model in a decentralized chain has two main objectives: (1) to increase the profits of chain members and (2) to share the benefits of the coordination model between SC members to encourage all members of the chain Giannoccaro and Pontrandolfo [35]. Khouja and Zhou [6] proposed a two-echelon supply chain where in first echelon a server offers some services to its customers and also sell gift cards to them with the same

Table 1. Comparison between the gift card policy and the other motivational mechanism of sales.

Reference	Gift card	Delay payment	All-units discounts	Incremental discounts	Inflation	EOQ	Newspaper	Supply chain
Goyal, [42]		*				*		
Taleizadeh *et al* [13]		*				*		
Lashgari *et al* [37]		*						*
Lashgari *et al* [19]		*				*		
Diabat *et al* [20]		*				*		
Lashgari *et al* [38]		*						*
Hadley and Whitin, [28]				*		*		
Tersine and Toelle, [22]				*		*		
Munson and Hu, [28]			*			*		
Taleizadeh *et al* [29]				*			*	
Taleizadeh *et al* [5, 30]				*		*		
Bera *et al* [31]			*			*		
Archetti *et al* [33]				*			*	
Khouja *et al* [4]	*						*	
Taleizadeh and Nematollahi [14]		*			*	*		
Zhang *et al* [11]	*						*	
Khouja and Zhou [6]	*						*	
This Paper	*				*	*		*

price. However, in second echelon, there are retailers who sell gift cards with price less than real price. In this paper, it is supposed that first echelon is a leader in Stakelberg game. Note that gift cards are sold while products and services are sold by their price. When customer receives gift card from retailer, a holding cost which includes capital cost and risk of the non-redemption gift card is imposed on him or her. At last, profit function of first echelon of supply chain is obtained with respect to gift card price, maintenance cost, and transfer cost; and the profit function of second echelon (retailer) is obtained according to the price that the supplier sells to the retailer. Khouja *et al* [7] develops a single-manufacturer single-retailer supply chain with gift card in which the gift cards' redemption rate is constant or increasing in gift card value. Finally, to solve it using a Stackelberg game where the manufacturer is the leader. Li *et al* [8] focuses on both retailer-sponsored gift cards and manufacturer-sponsored gift card problems in a two-product supply chain. Then authors developed a cost-sharing mechanism that the retailer can use to improve the motivation to sponsor gift cards. Pan *et al* [9] analyzes the optimal pricing and ordering strategies of selling gift cards in supply chain with Stackelberg model. This article is presented in two modes: the case where gift cards cannot be given to others and the case that can be given as a gift. Other incentive policies have also been developed on supply chain models. Lashgari *et al* [37] have developed a two-tier delay payment model on a three-tier supply chain in which retailers serve their customers. N time unit and supplier from retailer M unit time unit receives percentage of purchase. Lashgari *et al* [38] have also developed a supply chain model in which goods are perishable, taking into account trade credit policy. Ebrahimi *et al* [39] and Tsao *et al* [40] and Ding *et al* [41] have also recently developed incentive policies on the supply chain model.

Table 1 may be helpful to better differentiate the works done by summarizing the literature of some of the articles on other motivational policies. In table 1, the comparison between the motivational policy of the gift card and the other motivational mechanism of sales is provided. According to table 1 and considering the literature review, it can be concluded that the gift card is a newly emerging research subject in the field of motivational policy of sales which has received great attention from the researchers.

In addition, as discussed above, this discount mechanism has many advantages over the others. All this together motivated us to develop a model for a supply chain by including discount mechanism of gift card. To justify the contribution of this paper, this article is compared with other works done in the field of gift cards from different directions in table 2,

As far as the research literature shows, there is still no study that examines product gift cards in the context of EOQ models for regular products. Previous models on the use of gift cards are few and are focused on the context of newspaper models. As shown in table 2, inventory holding cost which is one of the major costs in inventory control problems and accounts for a high proportion of the total cost has not been taken into account in all the performed work. Ignoring such an important cost, greatly influences on the models to be applied to real-world cases. Indeed, the presented models in the literature may be viewed as a developed case for the newspaper sales problem. This paper also considers the impact of inflation, which plays a critical role in reality.

Indeed, so far, the impact of inflation on the variables in the supply chain has not been considered in the literature. In this regard, only one research paper has investigated this matter in a simple newsvendor problem without involving the costs of ordering and inventory holding. By reviewing the literature, we found that only four research papers have studied the impact of gift cards on the supply chain where only the purchasing cost has been considered. They all have ignored the ordering and inventory holding costs.

3. Problem definition

Suppose a retailer who sells a kind of product to his costumers. Due to different reasons, such as the New Year, during part of year, people become more eager to buy things. The retailer issues a gift card to meet more demand in such situations, before this interval (L units before) he issues some gift card. We suppose that at zero moment retailer sells all gift cards to buyers, companies, universities and other organizations at a price below the real price of the product ($P_r(1-\beta)$). It should also be noted that retailer receives money when sells gift card. Therefore, seller receives the money before selling products and gets the benefit of this money.

Costumers request gift card for two objects: 1- using for themselves. 2- Giving gifts to others. After selling gift cards, buyers who have purchased a gift card for themselves will have to wait for the gift card to be sold ([0, M]). But if they buy the gift card to give as a gift to others, they must gift them in [0, H). People who have gift cards must buy their products in [max (M, at the moment of reception), H], otherwise the value of gift card is wasted. Hence retailer sells his product only in cash in [0, M], but in [M, H] people who have gift card buy with their gift cards and the rest of the costumers will buy in cash. The schematic diagram of problem is depicted in figure 1. Figure 2 shows the issues discussed above in chronological order.

It should be mentioned that the number of added demands is the function of the percentage of gift card discount i.e. the more the discount of gift cards, the more the demands for them. Though if we do not have any discount for gift card, we will have a constant demand for buying gift card (Those people who buy to gift). In this problem, the buyers of gift cards are divided into 3 categories: 1- costumers who have gift cards but if they did not have, they would buy (Old and loyal customers). 2- Customers who only buy with a gift card, whether they have received the gift card or bought it themselves. 3- People who have bought gift card (for themselves or others) but they do not use it. In other words, if the retailer does not use the gift card policy, the total demand is D, but if he uses the gift card policy, he will sell the S number gift cards and the number of customers will increase.

The demand from card users who purchase products even without a gift card is shown as $\alpha_1 S$, demand from card users who purchase products only with gift cards is shown as $\alpha_2 S$, and the demand of card users shown as $\alpha_3 S$ and never visit to purchase products with gift cards. It is straightforward that $\alpha_1 + \alpha_2 + \alpha_3 = 1$, and $0 \leq \alpha i \leq 1$, $i = 1, 2, 3$. Figure 2 shows the composition of the retailer's demand.

We intend to get the amount of discount in gift cards and also the amount of order for retailer and supplier in case of using gift card motivational policy in both states of cooperation and non-cooperation of members of the supply chain. Dependent variable values are the amount of retailer's and supplier's order in each period and selling price of gift card to customer which are obtained by obtaining the decision variables. It should be mentioned that our aim from obtaining these variables is to maximize profitability of retailer and supplier separately in first model and maximize profitability of the whole chain in second model.

The parameters and decision variables are introduced hereunder:

Parameters

- P_r Selling price of each item by retailer
- D_r Rate of demand for retailer
- H Duration of the planning horizon
- θ Rate of inflation
- h' The cost of providing each gift card
- I_e The interest rate determining the retailer's income for each dollar in a period of time.
- A_r Ordering cost of retailer
- I_{hr} Holding cost of each product per time unit for retailer
- I_h Holding cost of each product per unit time for supplier
- P_s Selling price of each item by supplier
- S The whole gift card sold at zero instant which is equal to sH. s is rate of the purchase with a gift card which is equal to $a + b\beta$. a and b are parameters and constant.
- C_s Purchasing price of each item by supplier
- F The cost of each delivery of goods from the supplier to the retailer
- A_s Ordering cost of supplier
- L The length of time between card selling and the start of selling products.

Decision Variables

- n_s Shipment quantity from the supplier to the retailer in each time period.
- n_r The number of replenishment during the planning horizon.
- β Discount percentage on each gift card

Table 2. The comparison between this article and other works done in the field of gift cards.

Reference	Cost elements					Solution		Variables			Optimality		Objective function		Inflation
	Ordering	holding	Purchasing	Newspaper	Supply chain	Closed form	Non-closed form	Retailer order quantity	Supplier order quantity	Price of gift card	Global	Local	Convex	Concave	
Khouja et al [4]			*	*		*		*		*	*			*	
Khouja et al [7]			*		*	*				*	*			*	
Pan et al [9]			*		*	*		*		*	*			*	
Zhang et al [11]	*		*	*				*		*				*	*
Li et al [8]			*		*	*				*	*			*	
Khouja and Zhou, [6]	*		*		*	*				*	*			*	
This paper	*	*	*		*	*		*	*	*	*			*	*

Dependent Variables

IN_s Income of supplier in the entire course of H at zero time

SE_s Ordering cost of supplier in the entire course of H at zero time

HO_s Holding cost of supplier in the entire course of H at zero time

PU_s Purchasing cost of supplier in the entire course of H at zero time

TE_s Transportation cost of supplier in the entire course of H at zero time

IN_r Income of retailer in the entire course of H at zero time

SE_r Ordering cost of retailer in the entire course of H at zero time

HO_r Holding cost of retailer in the entire course of H at zero time

PU_r Purchasing cost of retailer in the entire course of H at zero time

PC_r Profit of retailer in the entire course of H at zero time

PC_{rsi} Profit of supply chain in the entire course of H at zero time for i, $i = 1, 2, 3$

4. Model development

It should be mentioned that in this section, costs of retailer and supplier are calculated in the first part, then two models are developed in two cases of cooperation and non-cooperation of members.

4.1 *Costs and incomes of each echelon*

4.1.1 *Retailer's costs and incomes* ***Retailer's Income***
As shown in appendix A, retailer's incomes in the range of [0,H] include the following three parts:

$$\underbrace{P_rD_r\frac{H}{n_r}U(n_r)-P_r\alpha_1 s\frac{H}{n_r}U(n_r)}_{I}+\underbrace{S\left[P_r(1-\beta)-h^{'}\right]e^{\theta L}}_{II}$$
$$+\underbrace{\begin{array}{l}\left(I_e\left[P(1-\beta)-h^{'}\right]sHL\right)+\sum_{i=1}^{n_r}I_e\left[P(1-\beta)-h^{'}\right](\alpha_1+\alpha_2)s\frac{H^2}{n_r^2}\left((n_r-i)+\frac{1}{2}\right)e^{-i\theta\frac{H}{n_r}}\\+I_e\left[P(1-\beta)-h^{'}\right]\alpha_3 s\frac{H^2}{n_r^2}\sum_{i=1}^{n_r}e^{-i\theta\frac{H}{n_r}}\end{array}}_{III} \quad (1)$$

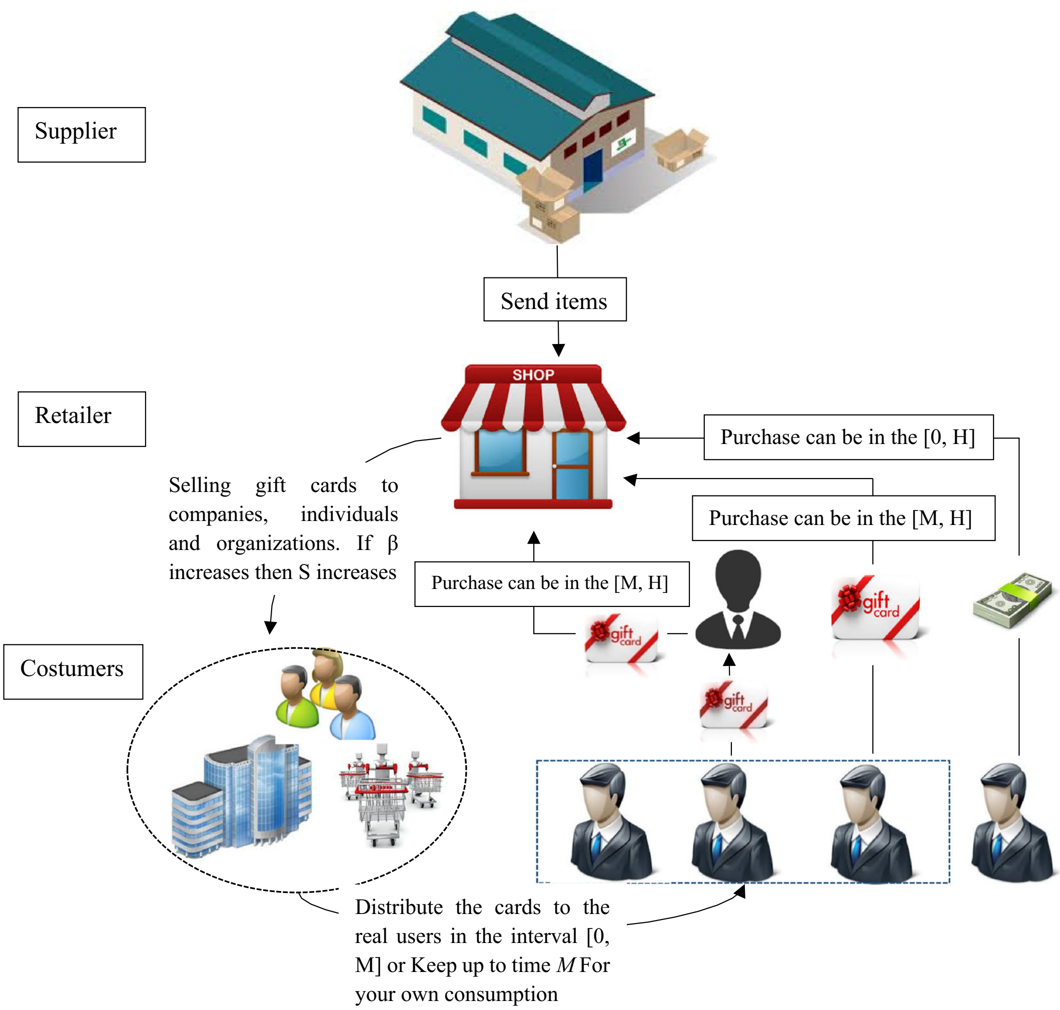

Figure 1. Schematic representation of the model presented.

Retailer's costs

<u>Ordering cost</u>

Let $SE_r(t)$ denote the ordering cost at time t. Thus, the total cost of ordering in $[0, H]$ by considering inflation in each period is equal to:

$$\begin{aligned} SE_r &= SE_r(0) + SE_r(T_r) + \ldots + SE_r((n_s-1)T_r) \\ &= \sum_{j=0}^{n_r-1} A_{(jT_r)} = \sum_{j=0}^{n_r-1} A_r e^{-(j\theta T_r)} = A_r \left[\frac{e^{-n_r\theta T_r}-1}{e^{-\theta T_r}-1}\right] \\ &= A_r U(n_r) \end{aligned} \tag{2}$$

Holding cost

Given the fact that in each period, inventory is $\frac{(D_r+\alpha_2 s)T_r^2}{2}$, Let $HO_r(t)$ denote the holding cost at time t. Total holding cost at zero instance by considering inflation is:

$$\begin{aligned} HO_r &= HO_r(0) + HO_r(T_r) + \ldots + HO_r((n_r-1)T_r) \\ &= \sum_{j=0}^{n_r-1} I_{hr} P_{s(jT_r)} \frac{(D_r+\alpha_2 s){T_r}^2}{2} e^{-(j\theta T_r)} \\ &= I_{hr} P_s \frac{(D_r+\alpha_2 s){T_r}^2}{2} \left[\frac{e^{-n_r\theta T_r}-1}{e^{-\theta T_r}-1}\right] \\ &= I_{hr} P_s \frac{(D_r+\alpha_2 s){T_r}^2}{2} U(n_r) \end{aligned} \tag{3}$$

Purchasing cost

The purchasing cost for retailer during planning horizon is calculated as follows:

$$\begin{aligned} PU_r &= PU_r(0) + PU_r(T_r) + \cdots + PU_r((\frac{n_r}{n_s}-1)T_r) \\ &= \sum_{j=0}^{\frac{n_r}{n_s}-1} P_{s(j)}(D_r+\alpha_2 s)T_r = \sum_{j=0}^{\frac{n_r}{n_s}-1} P_s e^{-(j\theta T_s)}(D_r+\alpha_2 s)T_r \\ &= P_s(D_r+\alpha_2 s)T_r \left[\frac{e^{-n_r\theta T_r}-1}{e^{-\theta T_r}-1}\right] \\ &= P_s(D_r+\alpha_2 s)\frac{H}{n_r}\left[\frac{e^{-\theta H}-1}{e^{-\theta\frac{n_s H}{n_r}}-1}\right] \overset{O(n_r,n_s)=\frac{H}{n_r}\left[\frac{e^{-\theta H}-1}{e^{-\theta\frac{n_s H}{n_r}}-1}\right]}{=} \\ &= P_s(D_r+\alpha_2 s)O(n_r,n_s) \end{aligned} \tag{4}$$

4.1.2 *Supplier's costs and Income* The total cost of supplier in the whole of H period contain holding cost, set up, transportation, and purchasing in which these costs are obtained as follows:

Holding cost

In order to obtain the supplier holding cost, first the average inventory in each period is calculated and is multiplied by holding cost, but the average inventory of final period is varied based on whether n_r can be divided by n_s or not. This calculation was performed in Appendix B. Now, according to the results presented in Appendix B and considering the inflation and the number of orders, the total holding cost for cases 1, 1-2 and 2-2 is presented as follows.

Mode 1

$$HO_{s1} = I_h C_s \frac{(D_r+\alpha_2 s)H}{2n_r}\left(\frac{n_r}{n_s}-1\right)V(n_s) \tag{5}$$

Mode 1-2

$$\begin{aligned} &HO_{s2} = I_h C_s \frac{(D_r+\alpha_2 s)H}{2n_r} \\ &\left(\left(\left[\frac{n_r}{n_s}\right]-1\right)\left[\frac{e^{-(n_s-1)\theta\left[\frac{n_r}{n_s}\right]\frac{H}{n_r}}-1}{e^{-\theta\left[\frac{n_r}{n_s}\right]\frac{H}{n_r}}-1}\right] + \left(n_r - \left[\frac{n_r}{n_s}\right](n_s-1)\right) \right. \\ &\left. e^{-\left(\theta(n_s-1)\left[\frac{n_r}{n_s}\right]\frac{H}{n_r}\right)}\right) \end{aligned} \tag{6}$$

Mode 2-2

$$\begin{aligned} &HO_{s2} = I_h C_s \frac{(D_r+\alpha_2 s)H}{2n_r} \\ &\left(\left[\frac{n_r}{n_s}\right]\left[\frac{e^{-(n_s-1)\theta\left(\left[\frac{n_r}{n_s}\right]+1\right)\frac{H}{n_r}}-1}{e^{-\theta\left(\left[\frac{n_r}{n_s}\right]+1\right)\frac{H}{n_r}}-1}\right]\right. \\ &\left.+\left(n_r - \left(\left[\frac{n_r}{n_s}\right]+1\right)(n_s-1)\right)e^{-\left(\theta(n_s-1)\left(\left[\frac{n_r}{n_s}\right]+1\right)\frac{H}{n_r}\right)}\right) \end{aligned} \tag{7}$$

Setup cost

The number of times ordering in the first and second modes is equal and is lower than the third mode by 1. Noting that during H period, supplier orders N_s times and also by considering inflation and the results presented in Appendix C, the whole ordering cost at zero instance is:

$$SE_s = \begin{cases} SE_{s1} & if\ \frac{n_r}{n_s} = \left[\frac{n_r}{n_s}\right] \\ SE_{s2} & if\ \frac{n_r}{n_s} \neq \left[\frac{n_r}{n_s}\right] \&\ Round\ down(\left[\frac{n_r}{n_s}\right]) \\ SE_{s3} & if\ \frac{n_r}{n_s} \neq \left[\frac{n_r}{n_s}\right] \&\ Round\ up(\left[\frac{n_r}{n_s}\right]) \end{cases} \tag{8}$$

Transport cost

With respect to N_r times order from retailer to supplier, the whole cost of transportation in H period is:

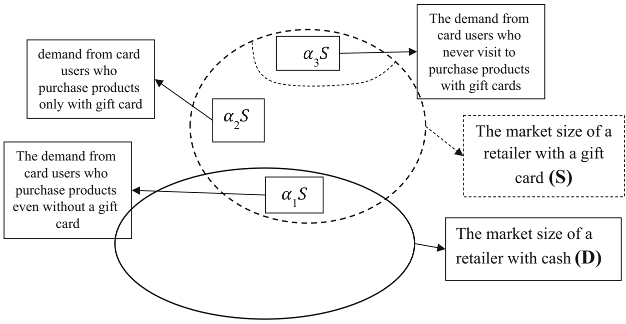

Figure 2. The final demand of the retailer with gift card.

$$TE_s = TE_s(0) + TE_s(T) + \ldots + TE_s((n_r-1)T) = \sum_{j=0}^{n_r-1} F_{(jT_r)} = \sum_{j=0}^{n_r-1} F e^{-(j\theta T_r)} = F\left[\frac{e^{-n_r\theta T_r}-1}{e^{-\theta T_r}-1}\right] = FU(n_r) \quad (9)$$

Purchasing cost

Let $PU_s(t)$ denote the purchasing cost at time t. according to the results presented in appendix D, The purchasing cost during planning horizon is:

$$PU_s = \begin{cases} PU_{s1} & if\ \frac{n_r}{n_s} = \left[\frac{n_r}{n_s}\right] \\ PU_{s2} & if\ \frac{n_r}{n_s} \neq \left[\frac{n_r}{n_s}\right] \&\ Round\ down(\left[\frac{n_r}{n_s}\right]) \\ PU_{s3} & if\ \frac{n_r}{n_s} \neq \left[\frac{n_r}{n_s}\right] \&\ Round\ up(\left[\frac{n_r}{n_s}\right]) \end{cases} \quad (10)$$

Supplier's Income

Since only one retailer receives product from this supplier, so the supplier's demand is equal to retailer's demand. Retailer's demand rate by cash is $(D_r - \alpha_1 s)$ and by gift card is $(\alpha_1 + \alpha_2)s$. Hence the total demand rate is $(D_r + \alpha_2 s)$. Therefore, Let $IN_s(t)$ denote the income of supplier at time t and considering inflation in the whole H period at zero instance, the income of supplier by is:

$$IN_s = IN_s(0) + IN_s(T_r) + \ldots\ldots + IN_s((n_r-1)T_r) = \sum_{j=0}^{n_r-1} P_{s(j)} D_s T_r = \sum_{j=0}^{n_r-1} P_s e^{-(j\theta T_r)} D_s T_r = P_s D_s T_r \left[\frac{e^{-n_r\theta T_r}-1}{e^{-\theta T_r}-1}\right] = P_s(D_r + \alpha_2 s)\frac{H}{n_r}U(n_r) \quad (11)$$

4.2 *Model 1: Non-integrated supplier–retailer model*

In this model, the benefit function of each member of chain (supplier and retailer) is maximized separately. In fact, each member of chain aims to maximize his benefit and does not consider other members' interests. In the following, first benefit function of retailer is written, then the value of decision variables is optimized with respect to the algorithm 1. Next benefit function of supplier is written separately and the value of its decision variable is optimized with respect to the algorithm 2.

Retailer model

With respect to incomes and calculated costs, retailer's profit function is:

$$\begin{aligned} PC_r(\beta, n_r) = & \underbrace{P_r D_r \frac{H}{n_r} U(n_r) - P_r \alpha_1 s \frac{H}{n_r} U(n_r)}_{\text{Income from saling rigular demand}} + \underbrace{S\left[P_r(1-\beta) - h'\right] e^{\theta L}}_{\text{Income from saling gift card}} \\ & + \underbrace{\begin{matrix} \left(I_e\left[P_r(1-\beta) - h'\right] sHL\right) + I_e\left[P_r(1-\beta)-h'\right] s \frac{H^2}{n_r^2} \sum_{i=1}^{n_r} (\alpha_1+\alpha_2)\left((n_r - i) + \frac{1}{2}\right) e^{-i\theta\frac{H}{n_r}} \\ + I_e\left[P_r(1-\beta) - h'\right] s \frac{H^2}{n_r^2} \sum_{i=1}^{n_r} \alpha_3 e^{-i\theta\frac{H}{n_r}} \end{matrix}}_{\text{Intreset rate from gift card}} \\ & - \left[\underbrace{A_r U(n_r)}_{\text{Fixed ordering cost}} + \underbrace{I_{hr} P_s \frac{(D_r+\alpha_2 s)T_r^2}{2} U(n_r)}_{\text{Holding cost}} + \underbrace{P_s(D_r + \alpha_2 s)\frac{H}{n_r} U(n_r)}_{\text{Purchasing cost}}\right] \end{aligned} \quad (12)$$

As calculated in Appendix E, Equation 12 is concave and the value of $\beta(n_r)$ is equal to

$$\beta(n_r) = \frac{\begin{bmatrix} P_r bH\left(e^{\theta L} + I_e L + I_e \frac{H}{n_r^2} XX\right) - I_e HL\left(Pa + bh'\right) - I_e Pa \frac{H^2}{n_r^2} XX - h' I_e b \frac{H^2}{n_r^2} XX \\ -P_s \alpha_2 b \frac{H}{n_r}\left[I_{hr} \frac{H}{2n_r} U(n_r) + O(n_r, n_s)\right] - P_r \alpha_1 b \frac{H}{n_r} U(n_r) - He^{\theta L}\left[bh' + P_r a\right] \end{bmatrix}}{2P_r bH\left[e^{\theta L} + I_e L + I_e \frac{H}{n_r^2} \sum_{i=1}^{n_r} (\alpha_1 + \alpha_2)\left((n_r - i) + \frac{1}{2}\right) e^{-i\theta \frac{H}{n_r}}\right]} \tag{13}$$

Now with respect to the following algorithm, the optimal value of $\beta(n_r)$ and n_r is obtained.

Algorithm 1 (solving retailer model)

Step 1) set $n_r = 1$ and obtain $\beta(n_r)$ from equation 13.
Step 2) with respect to obtained values for n_r and $\beta(n_r)$, obtain the value of objective function $PC(n_r, \beta(n_r))$ from equation 12.
Step 3) set $n_r = n_r + 1$, then obtain $\beta(n_r)$ from equation 13.
Step 4) obtain the objective function $PC(n_r, \beta(n_r))$ with respect to the obtained values in previous step from equation 12.
Step 5) if $PC_r(n_r, \beta(n_r)) < PC_r(n_r - 1, \beta(n_r - 1))$, so $n_r^* = n_r - 1$, $\beta^* = \beta(n_r - 1)$ and go to step 6. Otherwise return to step 3. Continue to meet stop criterion.
Step 6) with respect to the obtained results from previous steps (obtained values for β^* and n_r^*), obtain the values of dependent variables.

1. Sale's price of each gift card from $P_r(1 - \beta^*)$
2. Length of time between orders for retailer form H/n_r^*
3. Order quantity in each cycle for retailer form $[(D_r + (a + \beta^* b))H]/n_r^*$

Supplier model

As shown in Appendix F, With respect to income and calculated costs, If n_r is divisible by n_s, supplier's profit function is:

$$PC_s(n_s) = \begin{cases} PC_{s1}(n_s) & \text{if } \frac{n_r}{n_s} = \left[\frac{n_r}{n_s}\right] \\ PC_{s2}(n_s) & \text{if } \frac{n_r}{n_s} = \left[\frac{n_r}{n_s}\right] \& \text{ Round Down } \left[\frac{n_r}{n_s}\right] \\ PC_{s3}(n_s) & \text{if } \frac{n_r}{n_s} = \left[\frac{n_r}{n_s}\right] \& \text{ Round Up } \left[\frac{n_r}{n_s}\right] \end{cases} \tag{14}$$

Since n_s is an integer, we use from following algorithm to obtain the optimal value of it.

Algorithm 2 (solving supplier model)

Step 1) set $n_s = 1$ and obtain $PC_s(n_s)$ from equation 14.
Step 2) set $n_s = n_s + 1$, then obtain $PC_s(n_s)$ from equation 14.
Step 3) if $PC_s(n_s) < PC_s((n_s - 1))$, so $n_s^* = n_s - 1$ and go to step 4. Otherwise return to step 2. Continue to meet stop criterion.
Step 4) with respect to the obtained results from previous steps (obtained values for n_s^*), obtain the values of dependent variables.

1. Length of time between orders for supplier form $\frac{H}{n_s^*}$
2. Order quantity in each cycle for supplier form $\frac{(D_r + (a + \beta^* b))H}{n_s^*}$

4.3 *Model 2: Combined model with a gift card and a negotiation procedure*

Generally, sharing data is one of the methods of reducing cost in uniform case compared with the non-uniform case. It must be considered that some of retailer's costs may increase in uniform case compared with separate case but the overall income of whole chain increases. For this reason, to encourage retailer, supplier gives a percentage of benefit to retailer which obtain from uniform model. Therefore, in this model a benefit function regarding to obtained costs and incomes of all members of chain (supplier and retailer) is written and regarding to algorithm 3, the value of its decision variables are optimized. It should be mentioned that in algorithm 3 after obtaining decision variables, regarding to compromise between retailer and supplier for sharing benefit, the optimal value of benefit function is also obtained.

Given that the supplier holding cost has three modes, thus, the total chain profit has three modes, the first of which is as follows. The second and third modes are also provided in appendix G.

$$PC_{rs1}(n_s,n_r,\beta(n_s,n_r)) = \underbrace{P_rD_r\frac{H}{n_r}U(n_r) - P_r\alpha_1 s\frac{H}{n_r}U(n_r)}_{\text{Income from saling rigular demand}} + \underbrace{S\left[P_r(1-\beta(n_s,n_r)) - h'\right]e^{\theta L}}_{\text{Income from saling gift card}}$$
$$\underbrace{\begin{aligned} &+\left(I_e\left[P(1-\beta(n_s,n_r)) - h'\right]sHL\right) \\ &+I_e\left[P(1-\beta(n_s,n_r)) - h'\right]s\frac{H^2}{n_r^2}\sum_{i=1}^{n_r}(\alpha_1+\alpha_2)\left((n_r-i)+\frac{1}{2}\right)e^{-i\theta\frac{H}{n_r}} \\ &+I_e\left[P(1-\beta(n_s,n_r)) - h'\right]s\frac{H^2}{n_r^2}\sum_{i=1}^{n_r}\alpha_3 e^{-i\theta\frac{H}{n_r}}\end{aligned}}_{\text{Intreset rate from gift card}} \tag{15}$$
$$-\left[\underbrace{A_rU(n_r)}_{\text{Fixed ordering cost}} + \underbrace{I_{hr}P_s\frac{(D_r+\alpha_2 s)T_r^2}{2}U(n_r)}_{\text{Holding cost}} + \underbrace{P_s(D_r+\alpha_2 s)\frac{H}{n_r}O(n_r,n_s)}_{\text{Purchasing cost}}\right]$$
$$+P_s(D_r+\alpha_2 s)\frac{H}{n_r}V(n_s) - A_sV(n_s) - FU(n_r) - C_s(D_r+\alpha_2 s)\frac{H}{n_s}V(n_s)$$
$$-I_hC_s\frac{(D_r+\alpha_2 s)H}{2n_r}\left(\frac{n_r}{n_s}-1\right)V(n_s)$$

As calculated in appendix H, equation 15 is concave and the value of $\beta_1(n_s,n_r)$ is equal to

$$\beta_1(n_s,n_r) = \frac{\left[\begin{aligned} &P_rbH\left(e^{\theta L}+I_eL+I_e\frac{H}{n_r^2}XX\right) + P_s\alpha_2 b\frac{H}{n_r}V(n_s) - I_eHL\left(Pa+bh'\right) - I_ePa\frac{H^2}{n_r^2}XX - h'I_eb\frac{H^2}{n_r^2}XX \\ &-P_s\alpha_2 b\frac{H}{n_r}\left[I_{hr}\frac{H}{2n_r}U(n_r)+O(n_r,n_s)\right] - P_r\alpha_1 b\frac{H}{n_r}U(n_r) - He^{\theta L}\left[bh'+P_ra\right] \\ &-C_s\alpha_2 b\frac{H}{n_s}V(n_s) - I_hC_s\frac{\alpha_2 bH}{2n_r}\left(\frac{n_r}{n_s}-1\right)V(n_s)\end{aligned}\right]}{2P_rbH\left[e^{\theta L}+I_eL+I_e\frac{H}{n_r^2}\sum_{i=1}^{n_r}(\alpha_1+\alpha_2)\left((n_r-i)+\frac{1}{2}\right)e^{-i\theta\frac{H}{n_r}}\right]} \tag{16}$$

Also, as shown in "Appendix I" the objective function also for second and third modes is concave and has a global minimum, and Eqs. (I4) and (I5) will be the optimum values of β^* for second and third modes, respectively.

Therefore, the values of $\beta(n_s,n_r)$ and $PC_{rs}(n_s,n_r,\beta(n_s,n_r))$ are equal to

$$\beta(n_s,n_r)=\begin{cases}\beta_1(n_s,n_r) & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\\ \beta_2(n_s,n_r) & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\&A_s>I_h\left(n_r-\left[\frac{n_r}{n_s}\right]n_s\right)D_sT_r\times\left[\frac{n_r}{n_s}\right]T_r\\ \beta_3(n_s,n_r) & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\&A_s<I_h\left(n_r-\left[\frac{n_r}{n_s}\right]n_s\right)D_sT_r\times\left[\frac{n_r}{n_s}\right]T_r\end{cases}\tag{17}$$

$$PC_{rs}(n_s,n_r,\beta(n_s,n_r))=\begin{cases}PC_{rs1} & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\\ PC_{rs2} & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\&A_s>I_h\left(n_r-\left[\frac{n_r}{n_s}\right]n_s\right)D_sT_r\times\left[\frac{n_r}{n_s}\right]T_r\\ PC_{rs3} & if\ \frac{n_r}{n_s}=\left[\frac{n_r}{n_s}\right]\&A_s<I_h\left(n_r-\left[\frac{n_r}{n_s}\right]n_s\right)D_sT_r\times\left[\frac{n_r}{n_s}\right]T_r\end{cases}\tag{18}$$

Since n_s and n_r are an integer, we use following algorithm to obtain the optimal value of decision variables.

Algorithm 3 (solving integrated model)

Step 1) set $n_s=1$, $n_r=1$ and obtain $\beta(n_s,n_r)$ from equation (17) and then obtain the objective function $PC(n_s,n_r,\beta(n_s,n_r))$ from equation (18) with respect to the obtained values of n_s, n_r and $\beta(n_s,n_r)$.
Step 2) set $n_r=n_r+1$, then obtain $\beta(n_s,n_r)$ from equation (17) and then obtain the objective function $PC(n_s,n_r,\beta(n_s,n_r))$ from equation (18) with respect to the obtained values of $n_s,n_r$and $\beta(n_s,n_r)$.
Step 3) if $PC_r(n_s,n_r,\beta(n_s,n_r))<PC_r(n_s,n_r-1,\beta(n_s,n_r-1))$, so $n_r^*=n_r-1$, $\beta^*=\beta(n_s,n_r-1)$ and go to step 4. Otherwise return to step 2 Continue to meet stop criterion.
Step 4) set $n_s=n_s+1$ and go to next step.
Step 5) set $n_r=n_s$, then obtain $\beta(n_s,n_r)$ from equation (17) and then obtain the objective function $PC(n_s,n_r,\beta(n_s,n_r))$ from equation (18) with respect to the obtained values of $n_s,n_r$and $\beta(n_s,n_r)$
Step 6) if $PC_r(n_s,n_r,\beta(n_s,n_r))<PC_r(n_s,n_r-1,\beta(n_s,n_r-1))$, so $n_r^*=n_r-1$, $\beta^*=\beta(n_s,n_r-1)$ and go to step 8
Step 7) if $PC_{sc}(n_s,n_r,\beta(n_s,n_r))<PC_{sc}(n_s-1,n_r-1,\beta(n_s,n_r-1))$, so $n_s^*=n_s-1$ and go to step 8. Otherwise return to step4. Continue to meet stop criterion.
Step 8) According to the results in previous step (The value obtained for n_r^*, β^* and n_s^*), obtain the value of decision variables.

1. Obtain the sales price of each gift card from $P_r\left(1-\beta^*\left(n_r^*,n_s^*\right)\right)$
2. Compute the length of time between orders for retailer form $\frac{H}{n_r^*}$
3. Calculated the order quantity in each cycle for retailer form $\frac{\left(D_r+\left(a+\beta^*\left(n_r^*,n_s^*\right)b\right)\right)H}{n_r^*}$
4. Determine the length of time between orders for supplier form $\frac{H}{n_s^*}$

Table 3. The results of solved model for case study.

Amount	Model 0 Retailer	Model 0 Supplier	Model 1 Retailer	Model 1 Supplier	Model 2 Retailer	Model 2 Supplier
Income	195,990,000	111,990,000	251,250,000	128,730,000	253,130,000	130,460,000
Transfer cost	–	4,199,700	–	4,478,300	–	2,528,200
Setup Cost	4,199,700	1,285,200	4,478,300	1,703,000	2,528,200	1,285,200
Holding cost Cost	4,479,700	411,270	4,827,400	440,600	8,697,600	397,930
Purchasing cost	111,990,000	68,546,000	128,730,000	78,328,000	130,460,000	79,586,000
β	–	–	0.31	–	0.34	–
Ordering quantity	400	2,000	431	1725	774	2,322
The number of replenishment	15	3	16	4	9	3
Demand rate	200	–	230	–	232	–
Profit the whole chain	75,314,000	37,550,000	113,210,000	43,781,000	113,772,500	44,343,500

Table 4. The sensitivity analysis for the first example.

Parameter	% Changes	Optimal values				% Changes in
		β^*	n_r^*	n_s^*	$PC_{rs}(\beta^*, n_r^*, n_s^*)$	$PC_{rs}(\beta^*, n_r^*, n_s^*)$
D_r	+50	0.28	12	4	232,760,000	35.70%
	+25	0.28	12	4	202,213,000	17.89%
	-25	0.27	9	3	141,460,000	− 17.53%
	− 50	0.27	8	2	111,255,000	− 35.14%
α_3	+50	0.28	9	3	172,689,000	0.68%
	+25	0.28	9	3	172,103,000	0.33%
	− 25	0.27	9	3	170,955,000	− 0.33%
	− 50	0.27	9	3	170,393,000	− 0.66%
I_{hr}	+50	0.27	12	4	168,360,000	− 1.85%
	+25	0.28	12	4	170,018,000	− 0.88%
	− 25	0.28	8	4	173,894,000	1.38%
	− 50	0.28	6	3	176,475,000	2.88%
H	+50	0.28	15	5	258,562,000	50.74%
	+25	0.28	12	4	214,995,000	25.34%
	− 25	0.27	8	2	128,390,000	− 25.15%
	− 50	0.27	4	2	85,373,000	− 50.23%
I_e	+50	0.29	9	3	187,668,000	9.41%
	+25	0.28	9	3	179,589,000	4.70%
	− 25	0.26	9	3	163,498,000	− 4.68%
	− 50	0.25	9	3	155,524,000	− 9.33%
a	+50	0.21	9	3	186,094,000	8.49%
	+25	0.24	9	3	178,601,000	4.12%
	− 25	0.31	9	3	164,856,000	− 3.89%
	− 50	0.34	9	3	158,594,000	− 7.54%
A_r	+50	0.27	8	4	170,277,000	− 0.73%
	+25	0.27	9	3	170,889,000	− 0.37%
	− 25	0.27	10	2	171,683,000	0.09%
	− 50	0.27	12	4	173,356,000	1.07%
A_s	+50	0.27	9	3	170,886,000	− 0.37%
	+25	0.27	9	3	171,207,000	− 0.19%
	− 25	0.27	9	3	171,850,000	0.19%
	− 50	0.27	9	3	172,171,000	0.37%
θ	+50	0.28	10	5	171,613,000	0.05%
	+25	0.28	10	5	171,618,000	0.05%
	− 25	0.27	9	3	171,921,000	0.23%
	− 50	0.27	10	2	172,480,000	0.55%
P_r	+50	0.30	9	3	307,429,000	79.23%
	+25	0.29	9	3	239,439,000	39.59%
	− 25	0.25	9	3	103,784,000	− 39.49%
	− 50	NAN	NAN	NAN	NAN	NAN
P_s	+50	0.27	13	1	164,930,000	− 3.85%
	+25	0.28	12	4	170,015,000	− 0.88%
	− 25	0.28	8	4	173,899,000	1.38%
	− 50	0.28	6	3	176,478,000	2.89%
h'	+50	0.27	9	3	169,959,000	− 0.92%
	+25	0.27	9	3	170,744,000	− 0.46%
	− 25	0.28	9	3	172,313,000	0.46%
	− 50	0.28	9	3	173,108,000	0.92%
b	+50	0.32	9	3	187,126,000	9.09%
	+25	0.30	9	3	179,213,000	4.48%
	− 25	0.23	9	3	164,275,000	− 4.23%
	− 50	0.15	9	3	158,111,000	− 7.82%

Table 4 continued

Parameter	% Changes	Optimal values				% Changes in $PC_{rs}(\beta^*, n_r^*, n_s^*)$
		β^*	n_r^*	n_s^*	$PC_{rs}(\beta^*, n_r^*, n_s^*)$	
L	+50	0.29	9	3	186,553,000	8.76%
	+25	0.28	9	3	178,995,000	4.35%
	− 25	0.26	9	3	164,148,000	− 4.30%
	− 50	0.25	9	3	156,866,000	− 8.55%

5. Determine the order quantity in each cycle for supplier form $\frac{(D_r+(a+\beta^*(n_r^*,n_s^*)b))H}{n_s^*}$

Step 9) Regarding the compromise among members of chain for sharing benefit of share data, the value of objective function of each member of chain obtains as follow:

5. Numerical example, sensitivity analysis and managerial insight

In this section, to illustrate the proposed solution, some numerical examples are presented. The first one is associated with a real-world case study and the rest of them are presented to carry out sensitivity analysis and explain managerial insight.

5.1 *Case study*

As explained earlier, gift card is one of the stimulating ways to sell further and many companies use this policy to stimulate their customers.

Ofogh Kurosh company is one of the biggest chain store companies in Iran that cooperates with more than one-hundred-fifty suppliers. This company has negotiated with some suppliers to issue gift card for some occasions like anniversary of the company establishment, different celebrations and so on. Then this company sells the gift cards to different organizations during L time units before that occasion and sells its products to gift card owners in a certain period of time. We have implemented the proposed models of our paper in this company.

Table 5. Effects of changing in transportation cost on three models.

%Changes in parameter F	%Changes in total profit		
	Model 0	Model 1	Model 2
− %50	1.86%	1.31%	1.07%
− %25	0.93%	0.66%	0.09%
+%25	− 0.93%	− 0.66%	− 0.37%
+%50	− 1.86%	− 1.31%	− 0.73%

Chosen product is a detergent package that is sold in the final three weeks of each year. This company starts issuing gift cards since late November and sells this product through gift card. Maintenance costs are calculated based on the rent of space per square meter and some of the costs that the company has identified as maintenance costs. The cost of ordering is also calculated according to the expenses of the central headquarters and its employees.

The daily average demand for this product has been 200 customers. Since the average customer of the branch was 2000, its demand has been 10% of the total customers of this store. So, as the number of its customers has increased by 15% in the past months, its demand prediction is 2000 × 15% × 10%. The interest rates paid and received are determined by the interest rate on the loans and the interest rate received by the country banks, respectively. With respect to our experience in selling gift cards and using them by customers, α_3 is considered to be 0.2 and it is consonant regardless of supplier. Since company identified its customers and categorized them with respect to the number and amount of buying in previous periods, we consider $\alpha_1 = 0.1$ since it reached 0.1 of loyal customers. Since $\alpha_1 + \alpha_2 + \alpha_3 = 1$, the value of α_2 is 0.7. Other parameters are as follows:

$P_r = 350$ thousand Rials, $D_r = 200$ person/day, $H = 30$ day, $\theta = 0.005$, $h' = 10000$ Rials, $I_e = 0.02$, $A_r = 3000000$ Rials, $I_{hr} = 0.04$, $P_s = 200$ thousand Rials, $a = 25$, $b = 100$, $I_h = 0.015$, $C_s = 120$ thousand Rials, $F = 3000$ thousand Rials, $A_s = 4500$ thousand Rials, $L = 30$ day.

The results of presented model by using the parameters of the problem are showed in table 3.

It can be inferred from the table that the benefit of model zero, where the supplier and the retailer individually try to optimize their incomes and costs and don't use gift card policy, is less than first and second model. The whole benefit of the first model is also less than the second (table 4).

In the zero model, the company do no use gift card policy and this leads to no demand for gift card in period H and products are only sold in cash. This leads to a sharp decrease in demand compared to the second model. In fact,

Table 6. Effects of changes in interest rate and time between card selling and the start of selling products on profit of supply chain

			%Changes in parameter L	%Changes in parameter I_e
	− %50	− %25	+%25	+%50
− %50	− 12.44%	− 9.91%	− 4.79%	− 2.21%
− %25	− 10.26%	− 7.00%	− 0.40%	2.93%
+%25	− 5.75%	− 1.02%	8.55%	13.36%
+%50	− 3.44%	2.04%	13.10%	18.65%

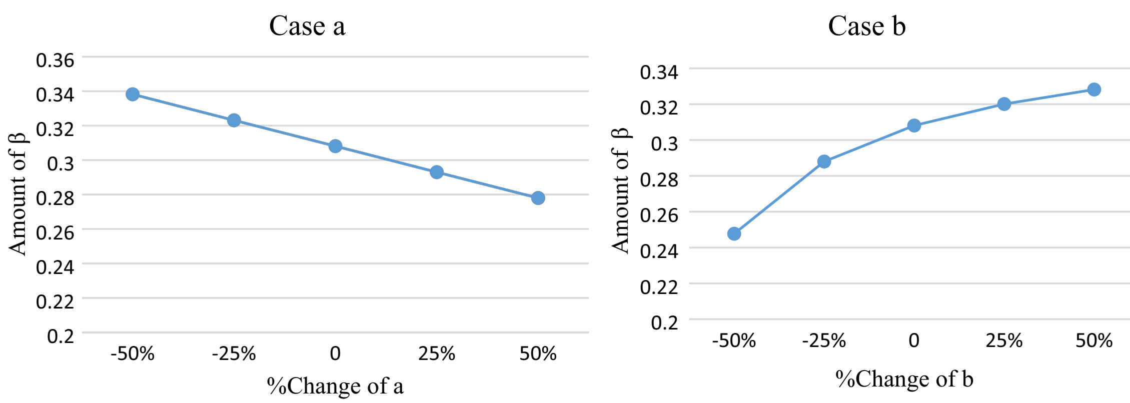

Figure 3. Effects of change in a and b on percentage discount on each gift card.

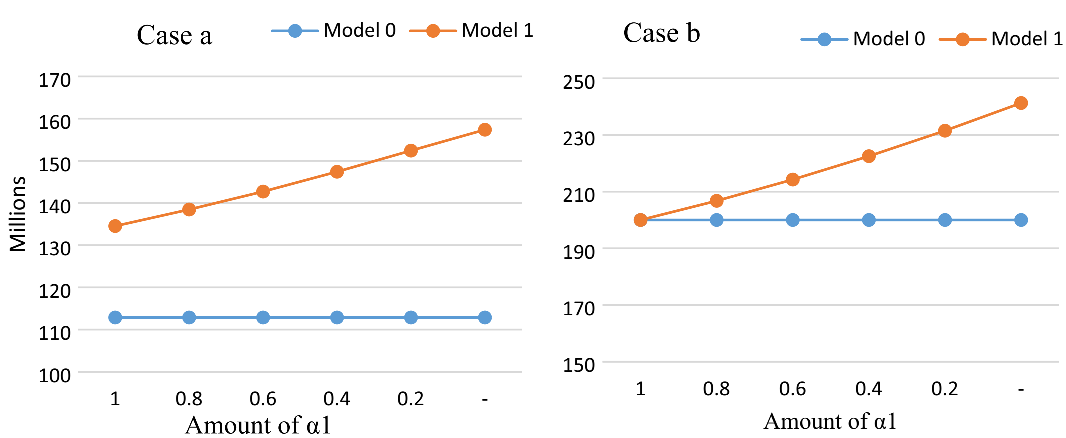

Figure 4. The effects of change in α_1 on total profit and order quantity of retailer.

company only has its regular demand and calculate its incomes and costs and optimize the values of T and Q according to that. In addition, supplier maximizes the benefit function with respect to the retailer's decisions.

In the first model, retailer uses gift card stimulating policy but he only notes to his costs and incomes and do not consider supplier's costs and incomes and values $\beta = 0.24$ and $n_r = 16$ are considered with respect to them. However, The decision variables to maximize the total profit of the chain do not give the highest profit of the chain because the model is optimized in two steps and supplier's income and cost is not considered for the optimization of T^* and β^*. However, in the second model that there are some information exchanges and distributer's incomes as well as suppliers are included simultaneously, the final total income increases 0.6% compared with the first model.

As shown in table 3, the number of replenishment and order of the retailer when using the gift card are higher than

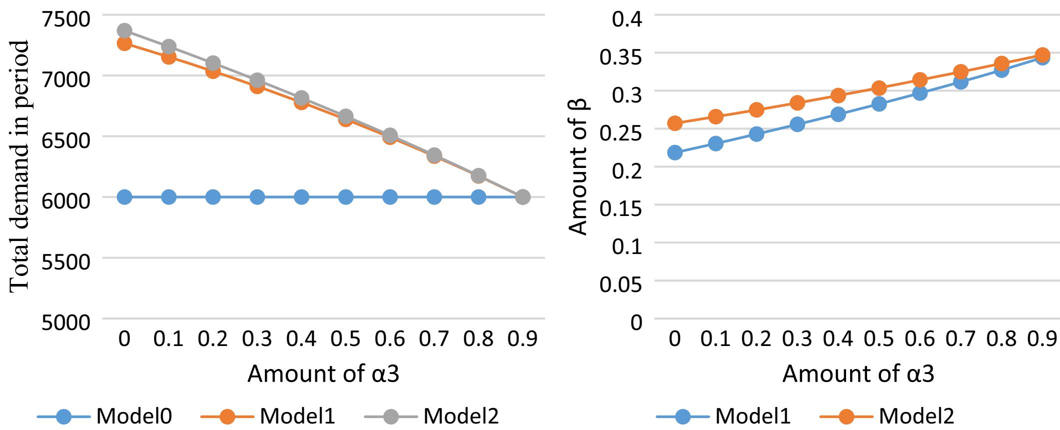

Figure 5. Effects of change in α_3 on total demand on period and amount of β.

those in Model 0, i.e. no use of the gift card. In the first model, a gift card is used and there are 7035 customers during the 30-day period, i.e. 1035 customers more than Model 0 (no use of the gift card). In other words, demand has increased from 200 to 235. In fact, using a gift card helps increase the number of customers and make the store more popular; this will increase the chain profit by 39%. Furthermore, in the third model, given that both members of the chain are integrated to maximize profits, they have paid 2% more interest to the gift cards; this has led to an increase in demand up to 237 customers and would also increase the chain profit by 0.6%.

5.2 *Sensitive analyze and managerial insight*

In this section, in order to illustrate the applicability of the solution procedure as well as to gain some managerial insights, several sensitivity analyses are performed on some key parameters of the model for the first example. For this purpose, the parameters D_r, α_3, I_{hr}, H, a, b, A_r, A_s, θ, P_r, P_s, h' and F are changed at four levels. The effects of the changes are shown in table (4) and also the succeeding conclusions are attained.

Impact to the retailer

The performed sensitivity analyses reveal that increasing the length of the prepayment period of gift card money, leads to the increase of the supply chain total profit and percentage discount on each gift card. Therefore, the retailer tends to sell their gift card as soon as possible before customers start buying goods. According to this fact, it is recommended that when gift cards are used, sales managers should give discounts on these cards to companies, institutions and customers, depending on when they are sold.

In other words, the amount of gift card discount depends on when they are sold. They can also inform customers before starting to sell the gift card and this will encourage them to buy gift cards at an earlier time. For example, the sales team can use discounted according to staircase method to sell time to persuade the customer to sell the gift card earlier and increase the profit of the store as well.

The higher the profit rate received by the retailer, the higher the chain income. One of the reasons is the increase in the retailer's received interest in gift card sales. Therefore, when using a gift card policy, managers can consult with banks and financial institutions as well as the chain profit will also increases by increasing the profit rate received.

As the cost of issuing a gift card (h') increases, the income of the chain members decreases. This indicates that it is better for the retailer to collaborate with gift producers who produce gift cards at less price than others and /or use gift card materials in order to reduce the cost of issuing a gift card and, in this way, increase its discounts and chain members' income.

Increasing product sales prices will increase the chain profit, and vice versa. In other words, as a product's margin increases, the retailer has more power to offer discounts on each gift card, and the increased discount increases demand which consequently leads to the increase in chain profits. Therefore, retailers need to negotiate with their suppliers so they can get more margin from the supplier so that they increase the profits of both members of the chain.

Moreover, as the number of gift card sales increases, (a and b), the chain profitability increases. This means that by increasing the total number of gift cards, increase the profits of the chain members. In fact, the second model has less impact on the profit of the chain than Models 1 and 0, which is due to the fair look to the members of the chain and maximizing the total profit of the chain.

The increasing of discount-dependent demand of gift card parameter and the length of the progressive credit period will push up the ordering quantity and replenishment cycle for getting more profit. The result implies two directions that the firms should pay much attention: first, increasing the sensitivity of the consumer on discount, and second, enhancing bargaining power to obtain a long progressive credit period.

Impact to the consumer

The higher the profit rate received by the retailer, the higher the gift card discounts. One of the reasons is the increase in the retailer's received interest in gift card sales. Therefore, when using a gift card policy, managers can the power to increase the discount rate on each gift card.

As the cost of issuing a gift card (h') increases, the ability to discount per gift card decrease.

Increasing product sales prices will increase the percentage of gift card discounts and vice versa. In other words, as a product's margin increases, the retailer has more power to offer discounts on each gift card. Therefore, retailers need to negotiate with their suppliers so they can get more margin from the supplier so that they can offer more discounts for the gift card.

Moreover, as the number of gift card sales increases, (a and b), the retailer's ability to increase gift card discounts. Therefore, gift card sales managers are advised to offer staircase discount method to customers when they want to sell them to organizations. This means that by increasing the total number of gift cards, the discounts can be increased to encourage them to buy more gift cards.

As it is specified in table 5 when the transportation cost increases, the benefit difference decreases between the third model and the first and the second models. One of the most important reasons is disregarding this cost as calculating β in the first two models in which this analysis shows the necessity of coordination between members of the chain. In fact, cost increases in the second model has less impact on the profit of the chain than the Model 1 and Model 0. This is because of integrating the members of the chain and maximizing the profits of the whole chain.

Two parameters, L and I_e, that influence the gift card and, ultimately, the profitability of the supply chain are already discussed. However, in table 6, the effect of both parameters is specified. As can be found in table 5, if the retailer can sell the gift card 50% earlier and also invest the money in projects that have 50% interest rate higher than normal, the profit would increase by 18.65.

Figure 3 shows the effect of gift card purchase parameters (a and b) on the gift card discount percentage. In the right-hand diagram, the gift card discount is reduced by increasing the parameter a. In fact, as the demand for gift card purchases increases, the amount of discounts on gift cards decreases. This clarifies that whenever the rate of giving and receiving gifts is high and we use gift card strategy, we can succeed even with less discounts. But the left-hand diagram shows that as parameter b increases, the discount rate also increases. In fact, if customers are sensitive to the discount, the model will apply a larger discount to attract the customer. Nevertheless, there is an important point to note that although the amount of b increases by the increase in discount, the slope of this increase is decreasing.

The left-hand diagram (Case a) of figure 4 shows the amount of supply chain profit changes for Models 0 and 1 relative to value α_1. In the diagram outline, it is assumed to be zero. As it is clear the more the customers who buy gift cards are among older and loyal retail customers (that is to say, α_1 increases), the more the chain profit in Model 1 and the closer to the chain profit in Model 0. As it can be seen, $\alpha_1 S<D$ and this is why even when Model 1 is $\alpha_1 = 1$, the profit of Model 1 is higher than Model 0. Now sales managers need to keep in mind that they sell more gift cards to customers who are not their customers. In this case, they can increase their store and own profits as well.

The right-hand diagram (case b) of figure 4 shows the number of changes of each retailer order for Model 0 and Model 1 relative to the value α_1. It is obvious that as α_1 increases, the retailer order quantity decreases and when $\alpha_1 = 1$ the order of both models is equal. In fact, if $S< D$, and in case $\alpha_1 = 1$, the order quantity of both models is equal, and if $S> D$, the amount of order value of Model 1 is always greater than Model 0.

In the left-hand graph of figure 5, the changes in the sales volume of the whole chain relative to the α_3 are shown. Since $\alpha_1 + \alpha_2 + \alpha_3 = 1$ and α_1 is a binary parameter and $\alpha_2 = 1 - \alpha_1 - \alpha_2$ so α_3 can summarized to be between zero to 0.9. Since the gift cards amount can't be 0, so the change in quantity does not affect the total number of embassies in the period and the amount is always constant, but in Model 1 and Model 2, as shown in figure increasing α_3 could lead to a decrease in volume. In other words, when the percentage of customers who buy a gift card but do not use it increases, the amount of the order decreases throughout the period.

6. Conclusion and future research

In this paper, two inventory models have been developed by considering gift card and inflation on the optimal ordering policy for regular product in an inventory control system for retailer and supplier in two cases cooperating and non-cooperating of members. This paper has contributed to the relatively limited academic knowledge on the examination of the impact of product-specific gift cards in optimal ordering and pricing decisions of regular products. In order to validate and verify the proposed model for optimal solution some numerical experiments were designed and performed. Findings show that using gift cards based on economic ordering models will result into the increase in retailer's demand while attracting more customers and making the brand more known. In fact, as it is shown in the first and second models, the profit of the supply chain is more in comparison to the situation that the gift card is not used. Each model has a unique approach and to solve this approach, convexity of all objective functions has been proved after being derived and a similar optimal solution has been provided for each model. For obtaining key factors of the model a sensitivity test has been performed.

For future studies partial backorders and multi product can be assumed and also deterioration rate, incorporating and other discount schemes pricing policies could be another background for future works.

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